Eurographics Symposium on Geometry Processing (2006)
Konrad Polthier, Alla Sheffer (Editors)

Size functions for 3D shape retrieval

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Abstract

This paper sketches a technique for 3D model retrieval built on size functions, a mathematical tool to compare shapes. Size functions are introduced for the first time to discriminate among 3D objects, through the proposal of an innovative method to construct size graphs independently of the underlying triangulation. We demonstrate the potential of our approach in a series of comparative experiments with respect to existing techniques.

Categories and Subject Descriptors (according to ACM CCS): I.3.6 [Computer Graphics]: Methodoloy and Techniques

1. Introduction

3D shape classification and retrieval is a very lively research topic. In this paper, we propose a framework to extend to the 3D domain the use of size functions, a shape descriptor which has been extensively applied to content-based image retrieval (see e.g. [VUFF93, CFG05]).

The theory of size functions has been developed since the beginning of the 90's in order to get a new geometric-topological approach to shape discrimination [VUFF93]. The idea is to analyze the growth of a topological space S, according to the increasing values of a real function φ defined on it. In particular, size functions code the topological evolution of S counting the number of connected components which remain disconnected passing from a lower level set of S to another. Since the growth of S is driven by the real function φ, size functions encode geometrical properties in the topological history. Hence they take into account both local and global properties of a shape. A similar approach has been introduced in [ELZ02].

The main contribution of this paper is to exploit and fruitfully enhance their potential for 3D shape comparison. The result is the definition of a technique for 3D shape description and retrieval, which is able to interpret the knowledge embedded in the shape, taking into account structure, topology and geometry.

2. Related work

The majority of the methods proposed in the literature for 3D shape retrieval mainly focuses on the low-level geometry of shapes, in the sense of considering its spatial distribution or extent in the 3D space [BKS*06, KFR03]. Nevertheless, there is a growing consensus towards high-level descriptors which merge a global topological analysis with local geometric attributes [CZCG04]. For example, the method presented in [HSKK01] addresses 3D shape similarity by using the Reeb graph in a multi-resolution fashion and performs retrieval by means of graph-matching techniques. Similarly, the importance of structural descriptions for shape matching has been recently pointed out in [BMM*03,ZS*05]. Exhaustive surveys on 3D shape searching techniques can be found for example in [TV04, IJL*05].

3. Approach

The aim of this paper is to provide a high-level technique based on size functions to fully reveal the topological information on the shape which is encoded in the representation model. The attractive feature of size functions is that they provide a high-level description which can be readily used to establish a similarity measure between shapes, formalizing qualitative aspects of shapes in a quantitative way.

3.1. Size functions

Given a *size pair* (S,φ), where S is a topological space and $\varphi : S \rightarrow \mathbf{R}$ is a continuous function called a *measuring function*, the size function $\ell_{(S,\varphi)} : \{(x,y) \in \mathbf{R}^2 : x < y\} \rightarrow \mathbf{N}$ is defined by setting $\ell_{(S,\varphi)}(x,y)$ equal to the number of connected components of the lower level set $S_y = \{P \in S : \varphi(P) \leq y\}$, containing at least one point of S_x [dFL05].

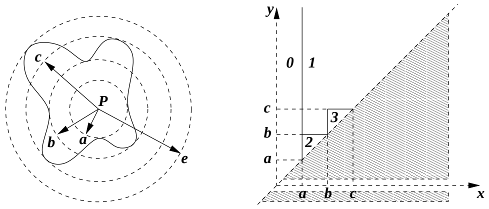


Figure 1: *A size pair and the corresponding size function.*

In the example in Figure 1 we consider the size pair (S,φ), where S is the curve represented by a continuous line in Figure 1(left), and φ is the function "distance from the point P". The size function associated with (S,φ) is shown in Figure 1(right). The value displayed in each region is the value taken by the size function in that region.

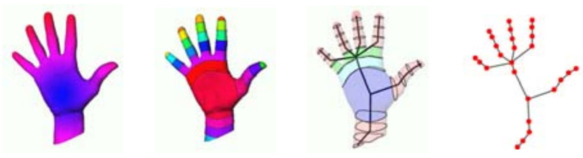
An important property of size functions is that they can always be seen as linear combinations of characteristic functions of triangles (possibly unbounded triangles with vertices at infinity). Hence, by taking the formal series of vertices associated with their right angles (called *cornerpoints* for the bounded triangles and *cornerlines* for the unbounded ones) we get a simple and compact representation [FL01].

The discrete counterpart of a size pair is a *size graph* (G,φ), where $G=(V(G),E(G))$ is a finite graph, with $V(G)$ and $E(G)$ the set of vertices and edges respectively, and $\varphi : V(G) \rightarrow \mathbf{R}$ is a measuring function labelling the nodes of the graph [d'A00].

3.2. Building size graphs for 3D shapes

Our idea is to associate with a 3D object a size graph (G^f,φ), where G^f is a centerline skeleton representing S, f is a real continuous function driving the centerline extraction, and φ is a measuring function labelling each node of the graph with local geometrical properties of the original model. This model signature, which combines the structural information provided by the mapping function f with the different information provided by the measuring function φ, produces informative size functions. Beside the obvious improvement in computational efficiency, the skeletal structure reduces the dimensionality of the problem, meanwhile storing sufficient information about the original object. Further details about our method can be found in [BGSF06].

The construction of the centerline skeleton G^f relies on the discretization of the Reeb graph theory defined in [Bia04]. Given a shape represented by a regular triangle mesh M, we subdivide the co-domain $[f_{min}, f_{max}]$ of $f : M \rightarrow \mathbf{R}$ considering nv regular values of f, $f_i \in [f_{min}, f_{max}]$, $i = 1,\dots,nv$. The level sets of f that correspond to these values partition the mesh M into regions, see Figure 2(b). Hence all points belonging to a region or a contour are identified and represented as nodes and arcs of a traditional graph, see Figure 2(c,d).



(a) (b) (c) (d)

Figure 2: *(a) Evaluation of the distance from the barycenter on the hand model in [aim]. Red and blue colors respectively represent maximum and minimum values. (b) The mesh partition. (c-d) The centerline skeleton.*

Four different mapping functions f are considered in our framework, namely the distance from the barycenter, the distance from the center of the bounding sphere, the integral geodesic distance in [HSKK01] and the topological distance from curvature extrema in [MP02].

Once the centerline G^f has been extracted, the *size graph* (G^f,φ) is obtained by defining the measuring function $\varphi : V(G^f) \rightarrow \mathbf{R}$ on the nodes of G^f. For each node $v_R \in V(G^f)$ corresponding to a region R, the value of $\varphi(v_R)$ is defined as a property characterizing the region R or its boundary $B_M(R)$. In particular, we are proposing to use:

- the *area* of the region R;
- the *minimum, maximum and average distance* of the barycenter of R from the region vertices;
- the *length of* $B^+_M(R)$ (resp.$B^-_M(R)$), where $B^+_M(R)$ (resp. $B^-_M(R)$) is the set of connected components of $B_M(R)$ such that the outgoing directions for the mapping function f are ascending (resp. descending);
- the sum of the *pseudo-cone lateral areas* computed for each component of R in $B^+_M(R)$ (resp. $B^-_M(R)$);
- a set of distance functions from selected points on the minimal bounding box of the model.

3.3. 3D model comparison

Once that a size graph (G^f,φ) has been obtained, the definition of the size functions follows the classical one. Denoting by G^f_y the subgraph of G^f obtained by erasing all vertices of G^f at which φ takes a value strictly greater than y, and all edges that connect those vertices to other vertices, the size function of (G^f,φ) is defined by setting $\ell_{(G^f,\varphi)}(x,y)$ equal to the number of connected components of G^f_y, containing at least a vertex of G^f_x.

In order to compute size functions, we have followed the algorithm introduced in [d'A00]. To compare two models we use the matching distance between their size functions, whose suitability for shape comparison has been discussed from the theoretical point of view in [dFL05].

Figure 3: *Our testing models.*

4. Experimental results

We have constructed a database of regular triangle meshes consisting of 5 classes of 20 elements, plus 20 unclassified manufactured models, see Figure 3. The original models of our database were collected from several web repositories ([dre], [aim], [SMKF04], [cae], [mcg]). To validate our results, we have considered the spherical harmonic descriptor [KFR03], the view-based approach in [COTS03] and the Multiresolution Reeb graph described in [HSKK01].

As a first performance parameter, we have considered the *percentage recall*. The recall histogram in Figure 4(a) has been obtained computing, for the rank thresholds $N = 10, 20, \ldots, 120$, the percentage of models in the same class of the query retrieved within the first N items. Results are averaged over the whole database, and indicate that almost 80% of relevant items are retrieved within top 25% of the database (i. e. within the first 30 models; remember that each class contains 20 elements). Figure 4(b) compares the *average rank* for the whole database obtained using size functions with the values obtained by the other techniques. The value obtained with size functions is the lowest one; recall that for this indicator lower values indicate better performance. A further measure we are using to assess the retrieval performance is the *last place ranking* defined in [EBG98], whose values are reported in Figure 4(c). High values within the interval $[0, 1]$ indicate good results.

One of the attractive features of our approach is its flexibility. In fact, the core idea of our method is the analysis of properties of real functions describing the shape under study. The role of the real functions is to take into account only the shape properties of the object which are relevant to the problem at hand, as well as to impose the desired invariance properties. When changing the functions, the resulting configurations can give insights on the shape from different perspectives, see Figure 5. These results suggest that our approach could also be used as a finer tool, after a rough filter has been used, or as an instrument to refine queries. Using our technique would allow the user to readily indicate the shape idea he has in mind, through the selection of a set of a features (i.e. mapping and measuring functions) which have a clear and intuitive geometric (and perceptual) significance.

5. Concluding remarks

The proposed shape descriptor presents many desirable properties. Indeed it is:

1. quick to compute: the computation of 120 size functions for the 120 models in the database requires 1.53 second on a 1.73GHz laptop PC-M; the off-line step of computing the size graphs requires 1 minute and 12 seconds;
2. concise to store: on average it requires less than $1K$;
3. easy and quick to compare: evaluating 120×120 matching distances requires 8.55 seconds;
4. invariant under similarity transformations: imposing the desired invariance simply means requiring the invariance for the mapping and measuring functions, without any change in the mathematical model;
5. robust against noise and small extra features;
6. able to discriminate among shapes at many scales, conveying information about their global and local properties.

By summarizing, we have proposed an original framework to extend the use of size functions in the 3D context. We have derived a signature to be extracted from 3D models, which guarantees the topological coding and the geometrical description, and is computationally efficient. Such a representation has been used as a size graph for computing discrete size functions. The experimental results have shown that this approach is promising, and goes into the direction of developing tools to automatically annotate the shape semantic, and to encapsulate it in a digital shape representation.

Acknowledgements

The authors thank Prof. M. Ferri and the Vision Mathematics Group at the Univ. of Bologna for fruitful discussions. This work has been developed in the CNR research activity (ICT-P03) and partially supported by the EU NoE “AIM@SHAPE” (http://www.aimatshape.net).

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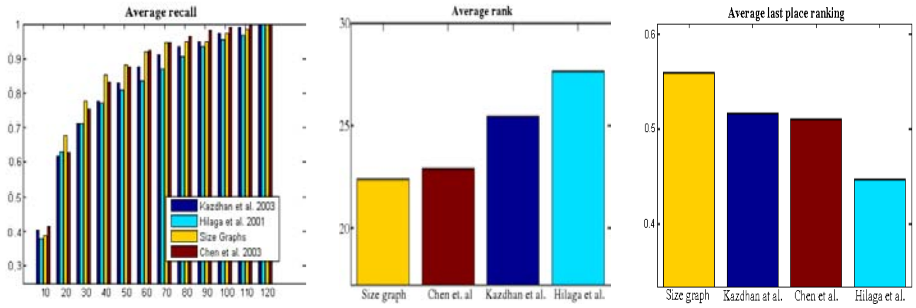

(a) (b) (c)

Figure 4: *Comparison with existing methods. (a) Recall, (b) average rank and (c) last place ranking histograms.*

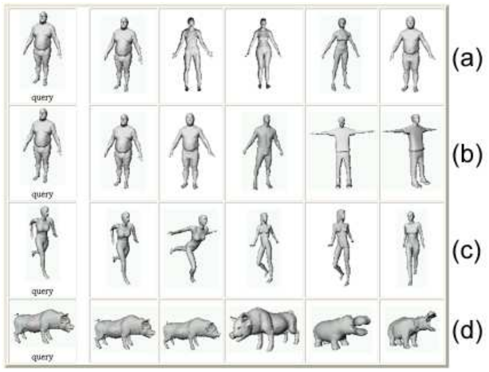

Figure 5: *Query results choosing different mapping and measuring functions. (a) Emphasis on spatial position, (b) robustness, (c) human poses; (d) fine results on animals.*

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