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Skeleton-based Generalized Cylinder Deformation under the Relative Curvature Condition

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Abstract

Deformation of a generalized cylinder with a parameterized shape change of its centerline is a non-trivial task when the surface is represented as a high-resolution triangle mesh, particularly when self-intersection and local distortion are to be avoided. We introduce a deformation approach that satisfies these properties based on the skeleton (densely sampled centerline and cross sections) of a generalized cylinder. Our approach uses the relative curvature condition to extract a reasonable centerline for a generalized cylinder whose orthogonal cross sections will not intersect. Given the desired centerline shape as a parametric curve, the displacements on the cross sections are determined while controlling for twisting effects, and under this constraint a vertex-wise displacement field is calculated by minimizing a quadratic surface bending energy. The method is tested on complicated generalized cylindrical objects. In particular, we discuss one application of the method for human colon (large intestine) visualization.

CCS Concepts

•***Computing methodologies*** → *Mesh models; Parametric curve and surface models; Shape analysis;*

1. Introduction

Deformation of generalized cylinders (GC) that are represented as triangle meshes is an important problem for synthesizing mutated models from an existing model or producing animation, as shown in Fig 1. In this paper, we focus on a task where the target shape of a long and complicated quasi-tube is given as a parametric center curve. This task, although can be conceptually intuitive (as in Fig 1), is practically difficult for traditional cage-based free-form deformations and direct manipulation methods like [SA07], because it is hard to impose precise control of the centerline curvature by manipulating several surface vertices or extraneous control points. Another category of large mesh deformations is based on the skeleton-driven skinning method such as [KCvO07]. Joints and edges are often used as skeletons to represent the structure of objects. Such skeletons are either manually given [KG07] or computed by methods like the Voronoi diagram [YBS07] or harmonic skeleton [AHLD07], which are also hard to parameterize.

In this paper, we use a centerline and discrete profile cross sections (CS) as the skeleton. We propose a skeletonization algorithm to find a centerline that prevents intersections between CSs. This is achieved by incorporating the relative curvature condition (RCC) [Dam08] to modify a initial/tentative centerline. We deform the centerline into a target curve and map all CSs using rotation minimizing frames [WJZL08]. Finally, the displacements of inter-

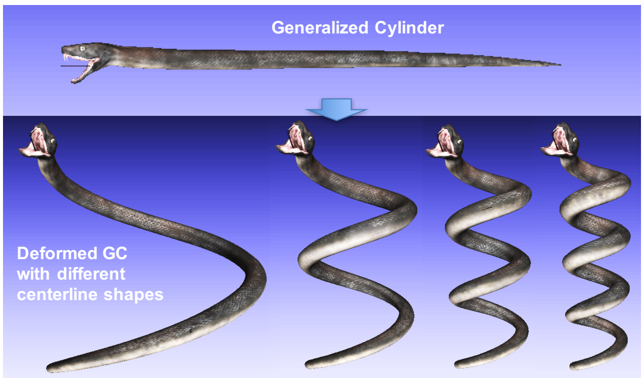

Figure 1: *Centerline-guided deformation: deform a generalized cylindrical surface into an target shape without unnecessary local distortion or cross-sectional intersections. Left: the original mesh of a generalized cylinder. Right: shape variations of a mesh whose centerlines are parameterized as helices of different rotation radius.*

cross-section regions are solved by minimizing a quadratic Thin Shell bending energy [BWH*06, ZPP*15]. As such, a GC is deformed without unnecessary distortion.

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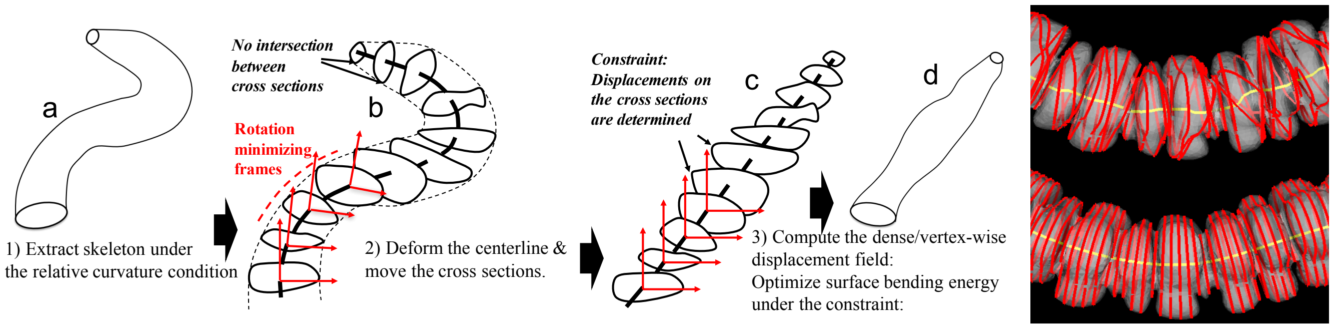

Figure 2: *Left: Algorithm pipeline. The input is a GC and an target centerline shape. The output is a deformed version of that GC. Right: illustration of step 1. Yellow: centerline. Red: CSs. A tentative centerline ($ctl_{initial}$) is extracted first (top). It is then modified ($ctl_{modified}$) based on the relative curvature condition (bottom).*

In general, our proposed algorithm leverages (a) an efficient geometrical deformation (RCC, rotation-minimizing frames) to produce accurate, large-scale, intersection-free CS mapping; and (b) a physical Thin Shell model to improve fine-scale deformations in local subdivided regions between CSs. Our contributions include 1) proposing a GC skeletonization method to extract a centerline and CSs that do not intersect based on RCC; 2) deforming the skeleton with twisting effect controlled by rotation minimizing frames; 3) computing a vertex-wise displacement field using the displacements on the CSs as constraints efficiently in linear time.

2. Method

As its input our method takes a triangle mesh forming the surface of a GC. It outputs a deformed mesh by a vertex-wise mapping. The pipeline is in Fig 2. The whole process is divided into the skeletonization step and the deformation step.

2.1. Skeletonization

The centerline of a GC, as defined in [AEIR03], is a curve inside the GC that maximizes the distance from the boundary. We adopt the method in [AEIR03] to extract a tentative centerline $ctl_{initial}$ based on the Voronoi diagram of the triangle mesh. We uniformly resample discrete points on the centerline and compute a CS at each point. At each point, we intersect the orthogonal plane (to the local tangent) with the triangle mesh and take the connected component closest to the local centerline point as a CS. There are a large number of intersections between CSs because the $ctl_{initial}$ may have large and even discontinuous curvatures, as shown in Fig 2 (top right). Therefore, they cannot be used directly as the skeleton to deform because CS intersections lead to surface folding. In [Dam08] J. Damon proposed the *relative curvature condition* (RCC) to detect CS intersections.

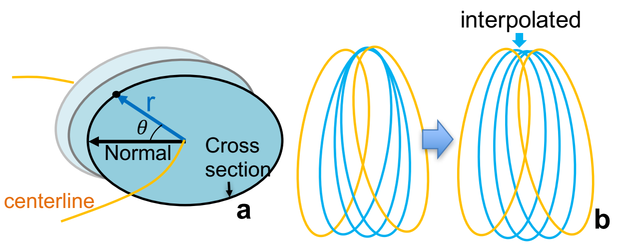

Figure 3: *a) Relative curvature condition; b) Replace intersecting CSs with interpolated ones.*

Fig 3(a) shows a CS at position s ($s \in [0,1]$) along a centerline. We specify angular location on the CS by the angle θ relative to the curve's Frenet normal at position s. The RCC is

$$r(\theta, s) < \frac{1}{k(s)cos(\theta)} \quad \text{when } cos(\theta) > 0. \tag{1}$$

In this inequality, $r(\theta,s)$ is the distance from the point on the tube surface to its corresponding centerline point. $k(s)$ is the local curvature of the centerline. The intuition is that regions with large centerline curvatures generally cannot tolerate large tube radius on the side of the normal of the centerline. As intuition suggests, there

Algorithm 1 Centerline Smoothing Algorithm

$it = 0, flag = False, maxIter = 20, ctl = ctl_{initial}, M$=num' of points on ctl
CS is M cross sections. T is the M tangent vectors of ctl
while $it < maxIter$ and not $flag$ **do**
  1-D Gaussian smoothing of ctl $(x_{n\times1}, y_{n\times1}, z_{n\times1})$ separately.
  δctl = M-tuple of zero vectors, $flag = True$
  for $i = 1 : M$ **do**
    Compute local ctl curvature k_i and the (Frenet) Normal direction N_i.
    Update $T[i]$ and $CS[i]$ using ctl
    if any point on $CS[i]$ violates RCC **then**
      $\delta ctl[i] = -k_i N_i$, $flag = False$
    end if
  end for
  1-D Gaussian smoothing on the 3 coord' of δctl seperately
  $ctl = ctl + \delta ctl$, $it = it + 1$
end while
output CS, $ctl_{modified} = ctl$

is no restriction for the points on the other side ($cos(\theta) \leqslant 0$). This condition typically cannot hold for regions of $ctl_{initial}$ with large centerline curvatures. We thus compute $ctl_{modified}$ from $ctl_{initial}$ by Algorithm 1 and recompute the CSs as shown in Fig 2 (bottom right). In parts of the quasi-tube where r is large, some CSs still have points violating the RCC. We replace these CSs by ones that are interpolated from the planes containing two bounding CSs that are satisfactory (Fig 3(b)). The interpolation guarantees a uniform angle change between the planes [Dam18]. The slight non-orthogonality introduced is preserved during deformation.

2.2. Deformation

When we have a satisfactory skeleton of a GC, we deform the mesh by first reshaping the skeleton and second reconstructing the mesh.

As shown in Fig 2 (step 2), we construct a new centerline of the same length as $ctl_{modified}$. At each point on $ctl_{modified}$, a rotation minimizing frame [WJZL08] is calculated which consists of the local curve tangent and two orthogonal directions. For the frames on the new centerline, one direction is constrained to be the tangent direction of the curve. The other two directions are specified by users, e.g., all frames will be in the same direction for simple GC straightening without twisting. The CSs are mapped according to the corresponding frames. No unnecessary twisting is introduced thanks to rotation minimizing frames. Artificial twisting (Fig 5 (bottom)) can be introduced by assigning phase difference to the new frames.

The skeletonization only imposes the RCC to CSs, so we first fix the vertices (V^{fixed}) that are within a small distance (0.1 $\times$ centerline point spacing) of CSs and also satisfying the RCC. We then solve for the displacements of the rest ($V^{unknown}$) by minimizing the Thin Shell bending energy [ZPP*15, BWH*06], namely the squared Laplacian [ZPP*15, BWH*06] of the displacement function on the mesh ($(\Delta f)^2$). This Laplace-Beltrami operator characterizes the change of mean curvatures. The total bending energy is the integral of the bending energy across the mesh. Assume we have n vertices with unknown displacements and m fixed vertices. The following energy is quadratic and can be minimized efficiently:

$$E_{total} = \sum_{j \in \{x,y,z\}} \left(\sum_{i=1}^{n} (\Delta f^j(v_i^{unknown}))^2 + \sum_{i=1}^{m} (\Delta f^j(v_i^{fixed}))^2\right) \tag{2}$$

3. Results

3.1. Deformation Examples

Fig 4 shows the deformation results of some GC objects. Our method successfully deforms complicated meshes like the human colon without introducing self-intersections. The local curvature patterns are preserved without introducing unnecessary visual distortion. Fig 4(c) shows an example of the deformation of a colon at a close distance. It is shown that our method can deal with large-curvature regions and small surface handles quite well.

3.2. Parameterized Mesh Morphing

We show two examples of continuous shape variations and illustrate the deforming capability when using parameterized centerlines. Fig 5 shows the shape variations of a flexible tube whose centerline is bent by different angles and a cuboid twisted by different degrees. Fig 1 in the beginning of this paper also illustrates a similar procedure.

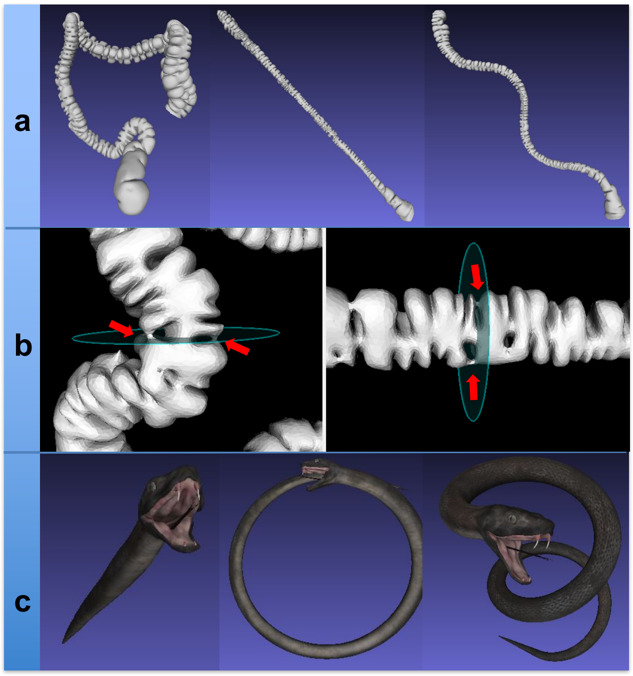

Figure 4: *3 examples of GC deformations. a) human colon. b) deformation detail of a colon: The red arrows point to small handles. Our method can deal with this topological noise (small handles) quite well. c) deformation of a snake*

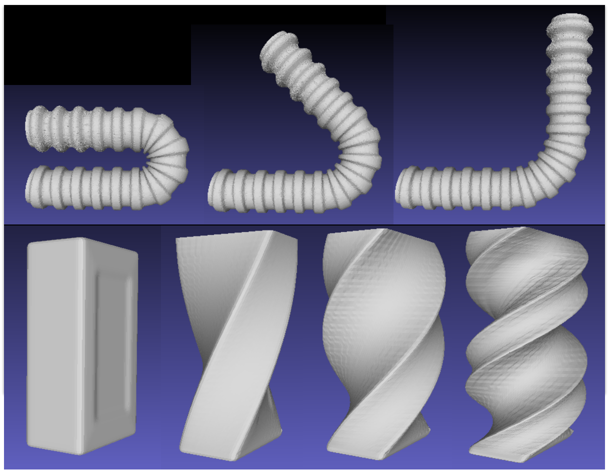

Figure 5: *Top: the mesh of a tube is bent by three different angles; Bottom: artificial twisting effect is applied to a cuboid. Although the points on the centerline stay static in this example, there is relative rotation between CSs.*

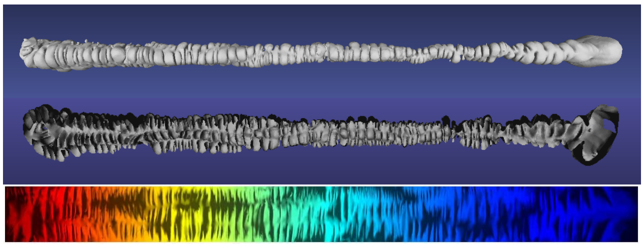

Figure 6: *a) straightened colon; b) slit-open colon; c) conformally flattened colon colored with Fiedler information [NMGK17]*

3.3. Colon Visualization

We introduce one particular application on colon visualization. Human colons reconstructed from medical images like CT are highly curved and difficult to view at a glance. Unfolding approaches like the conformal mappings [HGQ*06, ZMG*10, NMGK17] map tubular surfaces to a plane while keeping the curvature or volume rendering information as color intensities. However, these methods eliminate 3D geometry, which usually contains important diagnostic information [NGK17]. More importantly, although a flattened colon can be color-coded using geometry information, this prevents showing inherent texture simultaneously, e.g., mapping texture from endoscopic videos to reconstructed surfaces [ZPP*16].

A potentially more natural visualization is to straighten the colon while keeping the tubular structure and local curvature patterns using our method. We then open the tube along a longitudinal slitting line and map the cylinder-like mesh to a semi-cylinder-like mesh, so that we can view the interior directly. We compare our method with a conformal mapping method [NMGK17] in Fig 6. Notice our method semi-flattens the colon while keeping the geometry context. The area distortion is avoided and the surface can be colored with additional textural information.

4. Conclusion and Discussion

We have proposed a deformation approach of generalized cylinders based on their skeletons. We deal with long quasi-tubular generalized cylinders whose centerlines are densely sampled curves. By mathematically specifying the target centerline as a parametric curve and moving the cross sections along with it, we are able to deform a high-resolution quasi-tubular triangle mesh into a different shape. A vertex-wise displacement is calculated. The process avoids intersections between cross sections by incorporating the relative curvature condition. We use the moved cross-sectional curves to fix the displacements of a subgroup of the vertices that are close to the cross sections. Using cross-sectional displacements as the constraint, we solve for the displacements of all the other vertices and minimize the Thin Shell bending energy to preserve local curvature patterns. We have shown some experimental results that successfully deformed generalized cylinders without unnecessary visual distortion and the surfaces stayed smooth.

Limitation and Future Work: One major limitation of our method is that it depends on the chain-structured skeleton. This prevents us from applying it to more complex objects like tree-structured vessels. Another threat is that centerline modification becomes harder for regions with larger curvatures and radii, but we lack a proof of the existence of a centerline that guarantees no intersection between cross sections under extreme cases, although in practice our method is sufficient for quite complicated surfaces like human colons.

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