Topology Optimization for Computational Fabrication

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Part 4: Topology Optimization for Appearance and Structure Synthesis

Sylvain Lefebvre
Inria
Textures in Computer Graphics

- Rock Generic Granite
- Rock Generic Obsidian
- Rock Wall Smooth
- Rock Wall Wind Eroded
- Pavement Path
- Rock Pavement 001
- Rock Pavement 01
- Stone Tiles 03
Authoring textures

Forza Horizon 3, Microsoft Studios  https://www.forzamotorsport.net/en-us/games/fh3
Authoring textures

Too much content to be done entirely manually
Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g., erosion)

We will see both in the context of fabrication.
Texture Synthesis

• Three main directions
  – By-example synthesis
  – Procedural synthesis
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Texture synthesis: color formulation

Assumption (MRF):
Same neighborhoods at all scales $\Rightarrow$ Same visual content
Volume Texture Synthesis

Solid Texture Synthesis [Kopf07]

Lazy Solid Texture Synthesis [Dong08]
On-surface texture synthesis

[Lefebvre and Hoppe 2006]
On-surface texture synthesis, the easier way
On-surface texture synthesis, the easier way
On-surface texture synthesis, the easier way

Distortion!
On-surface texture synthesis, the easier way
On-surface texture synthesis, the easier way
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Random planes
Select Best
Random planes
On-surface texture synthesis, the easier way

Select Best
Random planes
On-surface texture synthesis, the easier way

Plane choices
On-surface texture synthesis, the easier way

Shifts + Rotations
Labelling Problem

- Surface neighborhood (2D)

Transition error
Distortion error
Multiresolution Synthesis

- Upsample, jitter, correction [Lefebvre and Hoppe 2005]
Results

- Time 28.6s
- thing:168602 (Steelyd)

- Time 14.7s
- thing:5506 (chylld)

- Time 18.7s
Texture as structure?

Model + appearance  + structure

Texture Synthesis?  ???
Texture synthesis: structure formulation

Exemplar

(density field)

Neighborhoods capture \textit{local geometry} across scales
Printability

1. Connected components
2. Minimum thickness
3. No weak part (rigidity)
Key ideas for structure synthesis

Pattern is stochastic

- Exhibits degrees of freedom
- Use pattern itself to locally reinforce structure
Key ideas for structure synthesis

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Exemplar specifies local geometry
  – Large scale arrangement can be optimized ‘orthogonally’
  – Combination with topology optimization?
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Pipeline

1. Input Mesh
2. Voxelize
3. Texture Synthesis
4. Structural Analysis
   - Shape Not OK
   - Local Update
   - Shape OK
5. Post-Processing Mesh Extraction
Pipeline

1. **Input Mesh**
   - Voxelize
2. **Texture Synthesis**
3. **Structural Analysis**
   - Local Update
   - Shape Not OK
   - Shape OK
4. **Post-Processing Mesh Extraction**
Pipeline

Input Mesh → Voxelize

Texture Synthesis → Structural Analysis

Structural Analysis → Post-Processing Mesh Extraction

Shape Not OK → Local Update

Shape OK
How to evaluate weak parts?

• Similar to SIMP method, we consider ‘weak’ and ‘strong’ material
• Issues:
  – Voxel grid is huge (~ 5M voxels)
  – Weak and strong ➔ hard to converge
  – We need 20-30 iterations synthesis/analysis

gọ Too expensive
➡ Approximate the pattern
Abstract Pattern Graph
Physical Simulation

- Basic idea: replace graph by finite elements

  In 2D: Quad & Triangle
  In 3D: Hex & Wedge

- Local planarity assumption
- Few elements: fast solution (1s)
Edge Selection Process

Solid
Empty
Selected
Simulation on the Final Mesh

Stress 99th%

153.9 KPa

30.5 KPa
Results – Structure + Color

$t_{\text{total}}$: 34.8s

$t_{\text{total}}$: 40.0s

$t_{\text{total}}$: 14.6s
From surface structure to final mesh
Results - Printouts

$t_{\text{total}}$: 52.4s

$t_{\text{total}}$: 11.4s

$t_{\text{total}}$: 14.5s
Other recent references

• Designing Structurally-Sound Ornamental Curve Networks
  J. Zehnder, S. Coros, B. Thomaszewski, SIGGRAPH 2016

• Stenciling: Designing Structurally-Sound Surfaces with Decorative Patterns
  C. Schumacher, B. Thomaszewski, M. Gross, SGP 2016

• Synthesis of Filigrees for Digital Fabrication

All these works use a different point of view: discrete element distributions
Key ideas for structure synthesis

Pattern is stochastic
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Exemplar specifies local geometry
  - Large scale arrangement can be optimized ‘orthogonally’
  - Combination with topology optimization?
Our Goal

Exemplar

Synthesize shapes under structural and appearance objectives
Local geometry

minimise

\[
E(\Omega) = \int \min_{q \in \partial X} D(N_X(q), N_\Omega(p)) \quad \text{Local geometry}
\]

\(X\) Example shape

\(\Omega\) Synthesized shape

\(N_X(q)\)

\(N_\Omega(p)\)
Structural properties

\[ E(\Omega) = \int_{\partial \Omega} g \cdot u_g(\Omega) \, dx \]

minimise compliance

rigidity

\[ \minimise \]

\[ E(\Omega) = \int_{\partial \Omega} g \cdot u_g(\Omega) \, dx \]
Structural properties

minimise $E(\Omega) = \int_{\partial\Omega} g \cdot u_g(\Omega) dx$

compliance

rigidity

Gravity

Topology optimization

[Osher, Allaire, Sigmund]
Structural properties

\[ E(\Omega) = \int_{\partial \Omega} g u_g(\Omega) dx \]

minimise compliance

[Osher, Allaire, Sigmund]
Challenge

minimise

\[ E_0(\Omega) = \int_{\partial\Omega} \min_{q \in \partial\Omega} D(N_\Omega(p), N_\Omega(q)) \]

local geometry

minimise

\[ E_1(\Omega) = \int_{\partial\Omega} g \cdot u_g(\Omega) dx \]

rigidity
Challenge

\[ E_0(\Omega) = \int \min_{q \in \Omega} D(N_\Omega(p), N_X(q)) \] (local geometry)

\[ E_1(\Omega) = \int g \nabla g(\Omega) dx \] (rigidity)

Gravity
Weighted sum

Minimize $G(x) + \lambda C(x)$
minimise

\[ E_0(\Omega) = \int_{\partial \Omega} \min_{q \in \partial X} D(N_{\Omega}(p), N_X(q)) \]

such that

\[ E_1(\Omega) = \left( \int_{\partial \Omega} g \cdot u_g(\Omega) dx \right) < \alpha C_{\text{max}} \]

appearance

Gravity

rigidity
<table>
<thead>
<tr>
<th>Objective</th>
<th>Derivatives</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appearance</td>
<td>A(x)</td>
<td>Neighborhood matching [Barnes09, Busto10, Kaspar15], Not great due to combinatorial matching</td>
</tr>
<tr>
<td>Compliance constraint</td>
<td>C(x)</td>
<td>Linear elasticity (FEM), Derivatives C(x)</td>
</tr>
<tr>
<td>Volume constraint</td>
<td>sum(x)</td>
<td>Derivatives sum(x)</td>
</tr>
</tbody>
</table>

Gradient-based Optimization

GCMMA [Svanberg95]
Compliance Relaxation

\( \alpha = 1.2, V_{\text{max}} = 30\% \)

\( \alpha = 1.2, V_{\text{max}} = 35\% \)

\( \alpha = 1.2, V_{\text{max}} = 40\% \)

\( \alpha = 1.4, V_{\text{max}} = 30\% \)

\( \alpha = 1.6, V_{\text{max}} = 30\% \)
Multiresolution

Level 0

Level 1

Level 2

Compliance optimization

C_\text{opt} 0

C_\text{opt} 1

C_\text{opt} 2

Appearance and compliance optimization
Fabricated Objects

Contour extraction
Fabricated Objects: Shelves

Floor attachment
Fabricated Objects: Tables
Fabricated Objects: Phone Stands
Fabricated Objects: Chairs
Texture Synthesis

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Foams in nature

Coral reef

Metallic foam (chemical reaction)
Challenges: scale, fabricability, mechanical properties

- Data size
  - 4 GB (.ply)

- Fabrication

- Mechanical properties
Standard approach: periodic structures
Homogenisation

Representative Volume Element (RVE)

Homogenised elasticity tensor
[Andreassen and Andreasen 2014]
Drawbacks

[Pannetta et al. SIGGRAPH 2015]
Periodic grid

- Mapping?
  - Hard problem

- Graded properties:
  - Possible, but transitions?

Hexahedral-dominant meshing
[Sokolov et al. 2015]
Procedural Voronoi Foams

- Aperiodic, stochastic, stationary
  Mimics nature.

- Trivially scales.
  O(1) time + memory.

- Fabricable.
  Few pockets, connected, thickness ok.

- Controllable elasticity
Procedural synthesis

F(x,y) called in every slice ‘pixel’
Procedural synthesis

F(x,y): is q=(x,y) inside?

Target density

Neighboring seeds

Bisectors

Voronoi edges

Local computations, O(1)

Trivially parallel (GPU)
Gradation (stackless)
Gradation (stackless)
Elasticity control

<table>
<thead>
<tr>
<th></th>
<th>Density</th>
<th></th>
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<tbody>
<tr>
<td>Family 1</td>
<td>0.0097</td>
<td>0.0168</td>
<td>0.0250</td>
<td>0.0332</td>
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<tr>
<td>Family 2</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Homogenisation

Young’s modulus

Density

Radius
Results

Crusty Knight
Results

Articulated Finger
Cute Octopus

- Results
Results

Anisotropy
## Performances

<table>
<thead>
<tr>
<th>Example</th>
<th>Extent (mm)</th>
<th># Voxels</th>
<th>Volume</th>
<th>% Filtered</th>
<th>Time per slice (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moomin</td>
<td>fig. 1</td>
<td>$26.7 \times 40.8 \times 51.9$</td>
<td>$534 \times 815 \times 1038$</td>
<td>6.44%</td>
<td>0.005%</td>
</tr>
<tr>
<td>Ellipsoid</td>
<td>fig. 13</td>
<td>$30.9 \times 30.9 \times 41.1$</td>
<td>$617 \times 617 \times 822$</td>
<td>6.30%</td>
<td>0.001%</td>
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<tr>
<td>Knight</td>
<td>fig. 14</td>
<td>$26.1 \times 30.0 \times 50.55$</td>
<td>$521 \times 600 \times 1011$</td>
<td>12.50%</td>
<td>0.023%</td>
</tr>
<tr>
<td>Finger</td>
<td>fig. 15</td>
<td>$25.0 \times 23.25 \times 70.5$</td>
<td>$500 \times 465 \times 1410$</td>
<td>23.35%</td>
<td>0.006%</td>
</tr>
<tr>
<td>SIGGRAPH logo</td>
<td>fig. 16</td>
<td>$20.0 \times 40.0 \times 80.0$</td>
<td>$400 \times 800 \times 1600$</td>
<td>5.73%</td>
<td>0.003%</td>
</tr>
<tr>
<td>Half-dome</td>
<td>fig. 17</td>
<td>$25.0 \times 50.0 \times 25.0$</td>
<td>$500 \times 1000 \times 500$</td>
<td>19.49%</td>
<td>0.025%</td>
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<tr>
<td>Octopus</td>
<td>fig. 18</td>
<td>$41.7 \times 41.1 \times 28.8$</td>
<td>$833 \times 822 \times 576$</td>
<td>17.27%</td>
<td>0.009%</td>
</tr>
<tr>
<td>Anisotropic cube</td>
<td>fig. 19</td>
<td>$40.0 \times 40.0 \times 40.0$</td>
<td>$800 \times 800 \times 800$</td>
<td>26.86%</td>
<td>0.005%</td>
</tr>
<tr>
<td>Forest dragon</td>
<td>fig. 20</td>
<td>$770.1 \times 990.7 \times 961.7$</td>
<td>$15402 \times 19814 \times 19234$</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Thank you for your attention!

Questions?

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