Compressive sensing: how to sample data from what you know!

Laurent Jacques (ICTEAM/ELEN, UCL)
First Part:

- Sparsity, low-rankness and relatives: “From information to structures”

- Compress while you sample: “From structure to scrambled sensing”

- and Reconstruct! “From scrambled sensing to information”
First Part:

- Sparsity, low-rankness and relatives: “From information to structures”

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Informative signals are composed of structures ...
2-D Example: Using Wavelets!

Representing this image ...
2-D Example: Using Wavelets!

...with those “wavelets”

different sizes, scales

different orientations

Representing this image ...
2-D Example: Using Wavelets!

Representing this image ...

...with those “wavelets”

different sizes, scales

different orientations
2-D Example: Using Wavelets!

Set of coefficients in $\Psi$
(gray=0)

$s.t. \ x = \sum_i \alpha_i \Psi_i$
2-D Example: Using Wavelets!

Set of coefficients in $\Psi$
(gray=0)

$\alpha$ s.t. $x = \sum_i \alpha_i \Psi_i$

Thresholding = Keep $K$ first values

Wavelet basis $\Psi$
2-D Example: Using Wavelets!

Wavelet basis $\Psi$

Set of coefficients in $\Psi$
(gray=0)

$\mathbf{x}$

$\alpha$ s.t. $\mathbf{x} = \sum_{i} \alpha_{i} \Psi_{i}$

$\mathbf{x}_K = \Psi \alpha_K$

$K = N/10$

Thresholding =
Keep $K$ first values

"Information" is still there!

Laurent Jacques - “Compressive sensing: how to sample data from what you know!”
Generalization: Sparsity principle

- Hypothesis: an image (or any signal) can be decomposed in a “sparsity basis” $\Psi$ with few non-zero elements $\alpha$:

$$x = \sum_{j=1}^{D} \alpha_j \Psi_j = \Psi \alpha, \quad \Psi = (\Psi_1, \cdots, \Psi_D) \in \mathbb{R}^{N \times D}$$
Generalization: Sparsity principle

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x = \sum_{j=1}^{D} \alpha_j \Psi_j = \Psi \alpha, \quad \Psi = (\Psi_1, \ldots, \Psi_D) \in \mathbb{R}^{N \times D}
\]

- $\Psi$ can be a ONB (e.g. Fourier, wavelets) or a dictionary (atoms)
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Non-linear approximation

$\#$ “atoms” $\Leftrightarrow$ improved quality
In summary: if “information” ...

... in a signal $x \in \mathbb{R}^N$ (e.g. $N =$ pixel number, voxels, graph nodes, ...)

there exists a “sparsity” basis (e.g. wavelets, Fourier, ...)

$$\Psi = (\Psi_1, \cdots, \Psi_D) \in \mathbb{R}^{N \times D}$$

where $x$ has a linear representation

$$x = \sum_{j=1}^{D} \alpha_j \Psi_j = \Psi \alpha$$

and

$$\|\alpha\|_0 := \# \{ i : \alpha_i \neq 0 \} \ll N \quad \| \alpha - \alpha_K \| \ll \| \alpha \|$$

Counterexample: Noise!
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Counterexample: Noise!

Small $\ell_1$-norm

$$\|\alpha\|_1 = \sum_i |\alpha_i|$$

Convex! (see after)
Other “informative” models
Other “informative” models

- Structured sparsity for high-dimensional data

\[ e_3 = t, \lambda \text{ or } z \]

Consider a data volume (3-D or more)
(e.g. video, hyperspectral data, medical data)

Possible models:
- 3-D sparsity basis (see before)
  (sometimes costly, sometimes \( \# \))
- or structured sparsity
  idea: consecutive “slices” vary “slowly”
Other “informative” models

- Structured sparsity for high-dimensional data

\[ e_3 = \lambda \]

Hyperspectral data cube

Hyperspectral slices

2-D \( \Psi \)

Hyperspectral sparsity coeff.

close coefficient supports!
Other “informative” models

- Structured sparsity for high-dimensional data

Hyperspectral sparsity coeff.

\[ e_3 = \lambda \]

Spatially Sparse

\[ \ell_1 \text{-norm} \]

\[ \ell_2 \text{-norm} \]

correlated & not sparse

Hyperspectral sparsity coeff.
Other “informative” models

- Structured sparsity for high-dimensional data

\[ e_3 = \lambda \]

Hyperspectral sparsity coeff.

Spatially Sparse

\[ \ell_1\text{-norm} \]

\[ \ell_2\text{-norm} \]

\[ \| \alpha \|_{2,1} = \sum_{i,j} (\sum_k |\alpha_{i,j,k}|^2)^{1/2} \]

Correlated & not sparse

\( \lambda \leftrightarrow k \)

\( e_1 \leftrightarrow i \)

\( e_2 \leftrightarrow j \)

\( e_1 = \lambda \)

sparse

not sparse
Other “informative” models

- Low-rank models in high dimensions

Example:
Video (PETS DB)

Moving foreground

Static background

\[ x = e_1 e_2 e_3 = t \]
Other “informative” models

- Low-rank models in high dimensions

Example:
Video (PETS DB)

- Example:
  Video (PETS DB)
  
  \[ x = e_1 + e_2 + \ldots + e_3 = t \]

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Other “informative” models

- Low-rank models in high dimensions

Example:
- Video (PETS DB)

\[ x = \begin{bmatrix} e_1 \\ e_2 \\ e_3 = t \end{bmatrix} \]

- Space
- Time
- Background
- Foreground
Other “informative” models

- Low-rank models in high dimensions

Example:
Video
(PETS DB)

Low-rank matrix:
Many columns are (almost)
linear combination of others

Video (PETS DB)
Other “informative” models

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Example:
Video (PETS DB)

Low-rank matrix:
Many columns are (almost) linear combination of others

Given \( X = U \Sigma V \) (SVD)
\[ \Sigma = \text{diag}(\sigma_1, \cdots, \sigma_N) \]
small \( \|\sigma\|_0 \), or small \( \|\sigma\|_1 = \|X\|_* \)
General Sparsity Applications

1. **Data Compression/Transmission** (by definition);
2. **Data restoration**: 
   - Denoising,
   - Deblurring,
   - Inpainting, ...
3. **Simplified model and interpretation** (*e.g.*, in ML)
General Sparsity Applications

1. **Data Compression/Transmission** (by definition);

2. **Data restoration**:  
   - Denoising,
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More generally,  
For regularizing (stabilizing) inverse problems  
+ Impact on data sampling philosophy! (*see after*)
First Part:

- Sparsity, low-rankness and relatives:
  “From information to structures”

- Compress while you sample:
  “From structure to scrambled sensing”

- and Reconstruct!
  “From scrambled sensing to information”
Sampling with Sparsity

- Paradigm shift:
  - “Computer readable” sensing
  + prior information (structures)
Sampling with Sparsity

- Paradigm shift:
  "Computer readable" sensing
  + prior information (structures)

- Examples:
  Radio-Interferometry, Compressed Sensing, MRI, Deflectometry, Seismology, ...

Optimized setup: sampling rate \( \propto \) information
Sampling with Sparsity

but ... non-linear reconstruction schemes!

Regularized inverse problems:

Reconstruct $x \in \mathbb{R}^N$ from $y = \text{Sensing}(x) \in \mathbb{R}^M$

given a sparse model on $x$.

Examples: Tomography, frequency/partial observations, ...

$x^* = \arg\min_{u \in \mathbb{R}^N} S(u)$ s.t. $\text{Sensing}(u) \approx \text{Sensing}(x)$

Sparsity metric:

- e.g., small $S(\alpha) = \|\alpha\|_1$ if $u = \Psi \alpha$,
- small Total Variation $S(u) = \|\nabla u\|$

Noise: Gaussian, Poisson, ...
Compressed Sensing

CS in a nutshell:

“Forget” Dirac, forget Nyquist,
ask few (linear) questions
about your informative (sparse) signal,
and recover it differently (non-linearly)”
Compressed Sensing

\[ y = \Phi x \]

CS in a nutshell:

“Forget” Dirac, forget Nyquist, ask few (linear) questions about your informative (sparse) signal, and recover it differently (non-linearly)”
Compressed Sensing

If $x$ is K-sparse and if $\Phi$ well “conditioned” then:

$$x^* = \arg \min_{u \in \mathbb{R}^N} \|u\|_0 \quad \text{s.t.} \quad y = \Phi u$$

$$\|u\|_0 = \#\{j : u_j \neq 0\} \quad \text{Non-linear reconstruction}$$
If $x$ is $K$-sparse and if $\Phi$ well “conditioned” then:

$$x^* = \arg \min_{u \in \mathbb{R}^N} \|u\|_1 \quad \text{s.t.} \quad y = \Phi u$$

(relabel.)

$$\|u\|_1 = \sum_j |u_j|$$

(Basis Pursuit) [Chen, Donoho, Saunders, 1998]
Compressed Sensing

∃ δ ∈ (0, 1) \quad \text{Restricted Isometry Property} \quad \sqrt{1 - δ} \|v\|_2 \leq \|\Phi v\|_2 \leq \sqrt{1 + δ} \|v\|_2

for all 2K sparse signals v.

any subset of 2K columns is an isometry

If \( x \) is K-sparse and if \( \Phi \) well “conditioned” then:

\[
x^* = \arg \min_{u \in \mathbb{R}^N} \|u\|_1 \quad \text{s.t.} \quad y = \Phi u
\]

(relex.)

\[
\|u\|_1 = \sum_j |u_j|
\]

(Basis Pursuit) \quad \text{[Candes 08]}

[Candes 08]

[Chen, Donoho, Saunders, 1998]
Compressed Sensing

∃ δ ∈ (0, 1) 

Restricted Isometry Property

\[ \sqrt{1 - \delta} \| v \|_2 \leq \| \Phi v \|_2 \leq \sqrt{1 + \delta} \| v \|_2 \]

for all 2K sparse signals v.

any subset of 2K columns is an isometry

Examples:
+ Gaussian
+ Bernoulli
+ Random Fourier
+ ....

\[ M = O(K \ln N/K) \ll N \]
\[ \Phi \in \mathbb{R}^{M \times N}, \Phi_{ij} \sim_{iid} \mathcal{N}(0, 1) \]

If x is K-sparse and if Φ well “conditioned” then:

\[ x^* = \arg \min_{\| u \|_1 = \sum_j |u_j|} \| u \|_1 \] s.t. \( y = \Phi u \) (relax.)

(Canades 08)

[Basis Pursuit] [Chen, Donoho, Saunders, 1998]
Compressed Sensing

If $x$ is $K$-sparse and if $\Phi$ well "conditioned" then:

$$x^* = \arg \min_{u} \|u\|_1 \quad \text{s.t.} \quad y = \Phi u$$

$$\|u\|_1 = \sum_j |u_j|$$

(Basis Pursuit) [Chen, Donoho, Saunders, 1998]

Solvers:
Linear Programming,
Interior Point Method,
Proximal Methods,
... Tons of toolboxes ...

Laurent Jacques - “Compressive sensing: how to sample data from what you know!”
Compressed Sensing

If $x$ is $K$-sparse and if $\Phi$ well “conditioned” then:

$$x^* = \arg \min_u \|u\|_1 \quad \text{s.t. } y = \Phi u$$

$$(\text{Basis Pursuit})$$

$$\|u\|_1 = \sum_j |u_j|$$

Robustness: vs sparse deviation + noise.

$$\|x - x^*\| \leq C \frac{1}{\sqrt{K}} \|x - x_K\|_1 + D\epsilon$$

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Linear Programming,
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... in summary, CS is

\[ M = O(K \log N/K) \]

\[ x \]

or \( x^* \approx x \)

if noise, quantization, non-linearities

**Ask few (linear) questions**

about your informative (sparse) signal, and recover it **differently** (non-linearly)’’

Candès, Romberg, Tao, 2006
Donoho, 2006
1-Pixel Camera, Wakin et al., 2006
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- Sparsity, low-rankness and relatives:
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- Compress while you sample:
  “From structure to scrambled sensing”

- and Reconstruct!
  “From scrambled sensing to information”
  (very broad and active field ... just one slide)
Reconstruct? (just one slide)

- For solving:

\[ x^* = \arg\min_{u \in \mathbb{R}^N} S(u) \text{ s.t. } \text{Sensing}(u) \approx \text{Sensing}(x) \]

- e.g., sparsity, low-rank, TV, ...
- e.g., L2/L1 distance, robust to Gaussian/Poisson noise, ...

many possibilities/solvers ...
Reconstruct?

- For solving:
  \[ x^* = \arg\min_{u \in \mathbb{R}^N} S(u) \text{ s.t. } \text{Sensing}(u) \approx \text{Sensing}(x) \]

  - many possibilities/solvers ...
  - Convex optimization: tons of toolboxes
    - SPGL1, L1Magic, (F)ISTA, ADMM, ...
    - Proximal algorithms (see also B. Goldluecke’s part)
  - Iterative (greedy) methods:
    - matching pursuit and relatives (OMP)
    - iterative hard thresholding, CoSAMP, SP, smoothed L0, ...
    - Approximate Message Passing Algorithms, Bayesian, ...

  e.g., sparsity, low-rank, TV, ...
  e.g., L2/L1 distance, robust to Gaussian/Poisson noise, ...
Second Part:

- Compressive imaging appetizer: The Rice single pixel camera
- Other case studies:
  - Radio-interferometry and aperture synthesis
  - Hyperspectral CASSI imaging
  - Highspeed Coded Strobing Imaging
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Rice Single-pixel Camera

\[ x \]

[Duarte, Davenport, Takbar, Laska, Sun, Kelly, Baraniuk, 2008]
Rice Single-pixel Camera

\[ \varphi_{ji} x_i, \quad \varphi_{ji} \in \{0, 1\} \]

\( j^{th} \) random pattern \( \varphi_j \in \mathbb{R}^N \)

[Duarte, Davenport, Takbar, Laska, Sun, Kelly, Baraniuk, 2008]
Rice Single-pixel Camera

\[ y_j = \sum_i \varphi_{ji} x_i \]

Optical sum

\[ y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{pmatrix} \]

\[ \varphi_{ji} x_i, \quad \varphi_{ji} \in \{0, 1\} \]

\[ j^{th} \text{ random pattern } \varphi_j \in \mathbb{R}^N \]

[Duarte, Davenport, Takbar, Laska, Sun, Kelly, Baraniuk, 2008]
Rice Single-pixel Camera

$sparse \text{ in } \Psi$

(e.g., wavelets)

$x^* = \arg\min ||\alpha||_1 \text{ s.t. } \Phi \Psi \alpha = y$

[Duarte, Davenport, Takbar, Laska, Sun, Kelly, Baraniuk, 2008]
Rice Single-pixel Camera

$x^* = \text{argmin} \| \alpha \|_1 \text{ s.t. } \Phi \Psi \alpha = y$

Proof of concept but PD is unique!
Specific imaging application: high energy photons with single costly photomultiplier
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Aperture Synthesis in Radio-Astronomy

Observation of the sky in radio frequency

$E_1(\omega, t)$

$E_2(\omega, t)$

$I_\omega(x, y)$

Astronomical radio sources

Planar approximation
Aperture Synthesis in Radio-Astronomy

Observation of the sky in radio frequency

“Imagine a swimming pool with two swimmers, and you want to detect their positions from the waves they produce ... ”
Aperture Synthesis in Radio-Astronomy

Observation of the sky in radio frequency

\( \mathbf{I}(x, y) \)

Planar approximation

Astronomical radio sources

\( \mathbf{E}_1(\mathbf{\tilde{\omega}}, t) \)

\( \mathbf{E}_2(\mathbf{\tilde{\omega}}, t) \)

\( \mathbf{B} \)

baseline
Aperture Synthesis in Radio-Astronomy

Observation of the sky in radio frequency

Van Cittert Zernike Theorem:

$$\hat{I}_\omega(\vec{B}^\perp) = \langle E_1 E_2^* \rangle_{\Delta t}$$
Aperture Synthesis in Radio-Astronomy

Observation of the sky in radio frequency

Van Cittert Zernike Theorem:

\[ \hat{I}_{\omega}(\vec{B}^\perp) = \langle E_1 E_2^* \rangle_{\Delta t} \]

Time correlation:

\[ \hat{I}_{\omega}(\vec{B}^\perp) \]

\[ \vec{B} \]

baseline

Astronomical radio sources

Planar approximation

Acts as a larger telescope
E pur si muove! “And yet it moves!”

- using $N$ telescopes, $\binom{N}{2}$ possible Fourier observations
- and baselines undergo Earth rotation!
E pur si muove! “And yet it moves!”

- using $N$ telescopes, $\binom{N}{2}$ possible Fourier observations and baselines undergo Earth rotation!
- Example:
  
  Each telescope pair = 1 elliptic path
  
  Earth rotation
  
  1 observation = 1 telescope pair at 1 instant
  
  Fourier domain
  
  4 - 20m
  
  18 - 120m
  
  Other configurations:
  - Very Large Array VLA
  - Square Kilometer Array SKA
  
  Arcminute Microkelvin Imager AMI
E pur si muove! “And yet it moves!”

- using $N$ telescopes, $\binom{N}{2}$ possible Fourier observations
- and baselines undergo Earth rotation!
- Example:

Partial Fourier Sampling Problem!

$$y = SFx + n$$

Selection of a few frequencies
2-D Fourier Transform
E pur si muove! “And yet it moves!”

- using $N$ telescopes, $\binom{N}{2}$ possible Fourier observations
- and baselines undergo Earth rotation!
- Example:

Partial Fourier Sampling Problem!

$$y = SFx + n$$

Selection of a few frequencies
2-D Fourier Transform

Image prior: SPARSITY!

+ Additional improvements: multiple sparsity basis, reweighted L1, ...
Reconstruction results

M31

Arcminute Microkelvin Imager (AMI)

Fourier Inversion

Reconstruction results

M31

Fourier Inversion

Arcminute Microkelvin Imager (AMI)

Sparsity
Averaging
Reweighted
Analysis (SARA)

reconstruction (log)
residual

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Hyperspectral imaging

- Fusion of spectrometry and imaging
Hyperspectral imaging

- Fusion of spectrometry and imaging
- Applications:
  - material classification/segmentation
Hyperspectral imaging

- Fusion of spectrometry and imaging
- Applications:
  - material classification/segmentation
  - microscopy/spectroscopy
  - counterfeit detection
  - environmental monitoring
  - skin decease detection
  - ...

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How is it usually done?

- Single filtering
- Multiplexed filtering
- Line scanning
How is it usually done?

Issues:

- acquisition time is slow
- low spatial/spectral/temporal resolution (depending on selected sensing)
- Huge amount of data at sensing
- But “low complexity” (sparse/low-rank) signals
Compressive HS imaging

- high-dimensional data = natural field for CS!
- Coded Aperture Snapshot Spectral Imaging (CASSI)
  - Mixing dispersive element + coded aperture
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$y = DSCx = \Phi x$
Compressive HS imaging

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Sensing model: \( y = DSCx = \Phi x \) + with multiple \( c \)
Compressive HS imaging

- high-dimensional data = natural field for CS!
- Coded Aperture Snapshot Spectral Imaging (CASSI)
  - Mixing dispersive element + coded aperture

Optically:

Compressive HS imaging

- Reconstruction: solving

\[ x^* = \Psi^T \left( \arg \min_{\alpha} \tau \| \alpha \|_1 + \frac{1}{2} \| y - DSC \Psi \alpha \|_2^2 \right) \]

\[ \Psi = \Psi_1 \otimes \Psi_2 \]

2-D wavelet
Symmlet-8

Compressive HS imaging

Reconstruction:

(a) Original (several wav.)
(b) With 12 shots and random \( c \)
(c) With 12 shots and optimized \( c \)
(d) example of optimized \( c \)

Second Part:

- Compressive imaging appetizer:
  The Rice single pixel camera
- Other case studies:
  - Radio-interferometry and aperture synthesis
  - Hyperspectral CASSI imaging
  - Highspeed Coded Strobing Imaging
High Speed Imaging

- Imaging high speed object lead to blurry image if low shutter frequency

- But hardware limitation in # fps (e.g., O(20fps) )

- Solution: “Highspeed Coded Strobing Imaging”
  - keep the detector fps rate unchanged
  - and add high rate coding of the shutter!
  (Reddy, Veeraraghavan, Chellappa, ...)

(source: wikipedia)
Highspeed Coded Strobing Imaging

Highspeed Coded Strobing Imaging

Optically:

Highspeed Coded Strobing Imaging

- Reconstruction: regularized with optical flow

Conclusions
Conclusion

- **Sparsity prior involves new sensing methods:** e.g., Compressed Sensing, Compressive Imaging.

- **Future:**
  - More sensing examples: [http://nuit-blanche.blogspot.com](http://nuit-blanche.blogspot.com) hyperspectral, network, GPR, Lidar, ... (explosion)
  - Better sparsity prior:
    - structured, model-based, mixed-norm (Cevher, Bach, ...)
    - co-sparsity/analysis model (Gribonval, Nam, Davies, Elad, Candes)
  - Non-linear sensing models?
    - 1-bit CS is one instance, phase recovery (Candès), polychromatic CT, ...
Laurent Jacques - “Compressive sensing: how to sample data from what you know!”

Links (Science 2.0.)

› Rice CS Resources page:
http://www-dsp.rice.edu/cs

› Igor Carron’s “Nuit Blanche” blog:
http://nuit-blanche.blogspot.com
1 CS post/day!
Thank you!