Reasoning About Shape in Complex Datasets

Geometry, Structure and Semantics

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complex (3D) datasets

digital representations of either physically existing or designed objects that can be processed by computer applications

- single 3D models
- sets of 3D models
  - repositories
  - scientific experiments
  - ...
- aggregates
  - assemblies
  - cities/geospatial
  - medical acquisitions
  - ..
why reasoning about \textit{shape}

- the \textit{shape} is one of the most distinctive property by which we characterize complex datasets
- the \textit{shape} is realized by a \textit{geometry} (data)
- the \textit{shape} is one of the primary keys to \textit{semantics} (information)
reasoning about shape today

• gradual shift of paradigm in many scientific fields: from physical prototypes and experience to virtual prototypes and simulation
  – CAD/PLM, Bioinformatics, Medicine, Cultural Heritage, Material Science,..

• technologies today
  – graphics cards evolution
  – 3D acquisition devices are becoming more and more commonplace
  – computer networks may now rely on fast connections at low cost
  – 3D printers are now able to produce not only mock-ups but even end products

• 3D content is likely to become heavily present in tomorrow’s networked and collaborative platforms
  – in the residential domains, for networked entertainment and virtual/gaming applications
  – fabbing and personalization of 3D products
why us for « reasoning about shape »

• CNR-IMATI *gang*
  – geo/topological analysis
  – 3D and semantics
    • since 2004.. 10 years anniversary!

• Hamid Laga
  – computer vision
  – statistical shape analysis
similarity as a key to analyse 3D

• describe the content of this dataset

• use of similes
  – shaped like, looks like, has the shape of, resemble,..  

• use of descriptions referring to the functionality
  – is a, used for, could be used for,..
similarity as a key to analyse 3D

• **similarity and invariance**
  – Kendall [1977] suggests to consider invariance of the shape under Euclidean similarity transformations: “shape is all the geometrical information that remains when location, scale, and rotational effects (Euclidean transformations) are filtered out from an object”
  – ATTENTION: no default invariance group

• **similarity and the observer**
  – [Koenderink 1990] focuses on the importance of the context: “things possess a shape for the observer, in whose mind the association between the perception and the existing conceptual models takes place “

• similarity is a cognitive process which depends on the observer and the context
similarity as a key to analyse 3D

How many of these objects are similar??
similarity, invariants and context

geometric congruence

structural equivalence

functional equivalence

“natural semantics” equivalence
• congruence

  – two objects are congruent if one can be transformed into the other by rigid movements (translation, rotation, reflection – not scaling)
similarity, invariants and context

• congruence
  – two objects are congruent if one can be transformed into the other by rigid movements (translation, rotation, reflection – not scaling)

P  d  not appropriate for a text recognition system
similarity, invariants and context

• **affinity**
  
  – preserves collinearity, i.e. maps parallel lines into parallel lines and preserve ratios of distances along parallel lines
  
  – equivalent to a linear transformation followed by a translation
similarity, invariants and context

• **affinity**
  
  – preserves collinearity, i.e. maps parallel lines into parallel lines and preserve ratios of distances along parallel lines
  
  – equivalent to a linear transformation followed by a translation
• selection of invariants and development of approaches to handle them
• shape descriptions to reduce the complexity of the representation
mathematics and shape reasoning

- selection of invariants and development of approaches to handle them
- shape descriptions to reduce the complexity of the representation
- appropriate similarity measures between shape descriptions

descriptions

histograms, matrices, graphs ...

real numbers

similarity measures

dist( \ast, \& ) = d\_match( , )
• Introduction: (Michi – 20 min)
• Part I: Geometric - topological analysis (Silvia - 50 min)
  – basics spaces, functions, manifolds and metrics
  – from rigid (Euclidean spaces) to intrinsic geometry (geodesic and theorema Aegregium) to topology (Erlangen' paradigm)
  – metrics between spaces
  – applications
• Part II: Statistical Shape Analysis (Hamid - 50 min)
  – Statistical Shape Analysis on linear spaces
  – Statistical Shape Analysis on non-linear spaces
  – Applications
• Part III: Structural Analysis of Shapes (Michela - 50 min)
  – feature extraction, segmentation, graphs and skeletons
  – from geometry and structure to semantics
    • semantic annotation
    • priors for shape correspondenc
    • learning 3D mesh segmentation & labeling
    • functionality recognition
• Conclusions: (Michi – 5min)
Acknowledgements

• Shape Modeling Group @ IMATI
  – and Daniela Giorgi

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  – FP7 IP ICT 2012-2016, grant 318787
  – modelling and analysing geo-spatial data sets

• VISIONAIR: Vision Advanced Infrastructure for Research
  – FP7 Infrastructure 2011-2015, grant 262044
  – re-design of the AIM@SHAPE repository and services
Reasoning About Shape in Complex Datasets
Geometry, Structure and Semantics

Silvia Biasotti  Hamid Laga  Michela Mortara  Michela Spagnuolo

Part II. Geometric-topological Shape Analysis
Outline

• Basic notions from geometry and topology
• Isometries and intrinsic shape properties
• Basic concepts of differential (and computational) topology
• Applications
• Summary
Outline

• Basic notions from geometry and topology
  – Spaces, functions, manifolds and shape deformations
• Isometries and intrinsic shape properties
• Basic concepts of differential (and computational) topology
• Applications
• Summary
Why topological spaces?

- to represent the set of observations made by the observer (e.g., neighbor, boundary, interior, projection, contour);
- to reason about stability and robustness
Why functions?

- to characterize shapes
- to measure shape properties
- to model what the observer is looking at
- to reason about stability
- to define relationships (e.g., distances)
Continuous and smooth functions

- Let $X, Y$ topological spaces, $f : X \to Y$ is continuous if for every open set $V \subseteq Y$ the inverse image $f^{-1}(V)$ is an open subset of $X$

- Let $X$ be an arbitrary subset of $\mathbb{R}^n$; $f : X \to \mathbb{R}^m$ is called smooth if $\forall x \in X$ there is an open set $U \subseteq \mathbb{R}^n$ and a function $F : U \to \mathbb{R}^m$ such that $F = f|_X$ on $X \cap U$ and $F$ has continuous partial derivatives of all orders
Why manifolds?

• to formalize shape properties
• to ease the analysis of the shape
  – measuring properties walking on the shape
  – look at the shape locally as if we were in our traditional euclidean space
  – to exploit additional geometric structures which can be associated to the shape
Examples

• 3-manifolds with boundary:
  – a solid sphere, a solid torus, a solid knot

• 2-manifolds:
  – a sphere, a torus

• 2-manifold with boundary:
  – a sphere with 3 holes, single-valued functions (scalar fields)

• 1 manifold:
  – a circle, a line
Which shape transformation?

- Not only congruence, translation, rotation, scaling but also shrinking and non uniform stretching
Shape transformations

- **Affine transformation**
- **Isometric transformation**
- **"Locally-affine" transformation**
- **Elastic deformations and gluing**
• Basic notions from geometry and topology
• Isometris and intrinsic shape properties
  – Gaussian curvature, geodesics and diffusion geometry
• Basic concepts of differential (and computational) topology
• Applications
• Summary
The evolution of geometry

- Till ‘700: Cartesian coordinates, Euclidean distances
  - Extrinsic geometry

- 1825: Theorema Aegregium
  - Intrinsic geometry

- 1872: Erlangen’s program -> topology
  - Generic deformations
Metric space

- a metric space is a set where a notion of distance (called a metric) between elements of the set is defined

- formally,
  - a metric space is an ordered pair \((X, d)\) where \(X\) is a set and \(d\) is a metric on \(X\) (also called distance function), i.e., a function
  - \(d: X \times X \to \mathbb{R}\)
  - such that \(\forall x, y, z \in X\):
    - \(d(x, y) \geq 0\); (non-negative)
    - \(d(x, y) = 0\) iff \(x = y\); (identity)
    - \(d(x, y) = d(y, x)\); (symmetry)
    - \(d(x, z) \leq d(x, y) + d(y, z)\) (triangle inequality)
What properties and invariants?

- how far are $p, q$ on $X$ and $p', q'$ on $Y$?
Isometries

• an isometry is a bijective map between metric spaces that preserves distances:
  \[ f : X \to Y, \quad d_Y(f(x_1), f(x_2)) = d_X(x_1, x_2) \]

• looking for the right metric space...
  – the Euclidean distance \( d(x, y) = \sum_{i=1}^{n} \sqrt{(x_i - y_i)^2} \)
  – geodesic distances, diffusion distances, ...
Invariance and isometries

• a property invariant under isometries is called an intrinsic property

• examples:
  – The Gaussian curvature $K$
  – The geodesic distance
  – Diffusion geometry
Principal curvatures

- the principal curvatures measure the maximum and minimum bending of a surface at each point along lines defined by the intersection of the surface with planes containing the normal
Gaussian and mean curvature

- given $k_1$ and $k_2$ the principal curvatures at a point surface
  - Gaussian curvature $K = k_1 k_2$
  - mean curvature $H = (k_1 + k_2)/2$

- according to the behavior of the sign of $K$, the points of a surface may be classified as
  - elliptic
  - hyperbolic
  - parabolic or planar
Examples

- $K > 0$

- $K < 0$

- $K = 0, H \neq 0$
Conformal structure

- A conformal structure is a structure assigned to a topological manifold such that angles can be defined.
  - In the parameter plane, the definition of angles is easy.
  - To cover a manifold, it could be necessary to consider many local coordinate systems with overlapping.
  - If the transition function from one local coordinate to another is angle preserving, the angle value is independent of the choice of the local chart.
Conformal structure & Riemann surface

- A topological surface with a conformal structure is called a Riemann surface.
- A 2-manifold (real) can be turned into a Riemannian surface iff it is:
  - Orientable
  - Metrizable
- A Möbius strip, Klein bottle, projective plane do not admit a conformal structure.
Geodesic distance

- the arc length of a curve $\gamma$ is given by $\int_{\gamma} \, ds$
- minimal geodesics: shortest path between two points on the surface
- geodesic distance between P and Q: length of the shortest path between P and Q
- geodesic distances satisfy all the requirements for a metric
- a Riemannian surface carries the structure of a metric space whose distance function is the geodesic distance
Diffusion geometry

• The diffusion distance measures
  – The heat diffusion on the shape between two points
  – The probability of arriving from one point to another in a random walk with a fixed number of steps

• The computation of diffusion is related to on the Laplace operator:

\[ \Delta f := \text{div}(\text{grad } f) = \nabla \cdot \nabla f = \nabla^2 f \]

• The Laplace-Beltrami operator generalizes the Laplace operator to Riemannian manifolds
Laplace-Beltrami problem

- \( \Delta f = -\lambda f \)
- orthonormal eigensystem

\[ \mathcal{B} := \{ (\lambda_i, \psi_i) \}_{i} \quad \Delta \psi_i = \lambda_i \psi_i \]

\[ \lambda_0 \leq \lambda_1 \leq \ldots \leq \lambda_i \leq \lambda_{i+1} \leq \ldots \leq +\infty \]

- Discrete Laplace-Beltrami operator

\[
\Delta f(p_i) := \frac{1}{d_i} \sum_{j \in N(i)} w_{ij} \left[ f(p_i) - f(p_j) \right]
\]

- \( N(i) \) index set of 1-ring of vertex \( p_i \)
- \( f(p_i) \) function value at vertex \( p_i \)
- \( d_i \) mass associated with vertex \( p_i \)
- \( w_{ij} \) edge weights
Discrete geometric Laplacian

- Desbrun et al. (1999)
  \[ w_{ij} := \frac{\cot(\alpha_{ij}) + \cot(\beta_{ij})}{2} \quad d_i := a(i)/3 \]
  - the cotangent weights take into account the angles opposite to edges,
  - the masses take into account the area around vertices

- Meyer et al. (2002)
  \[ d_i := a_V(i) \]
  - cotangent weights, masses considering the Voronoi area

- Belkin et al. (2003, 2008)
  - weights constructed using heat kernels

- Reuter et al. (2005, 2006)
  - weak formulation of the eigenvalue problem
    \[ \langle \Delta f, \varphi_i \rangle_{L^2(M)} = -\lambda \langle f, \varphi_i \rangle_{L^2(M)} \]
    with \( \varphi_i \) cubic form functions
Heat equation

- The heat kernel $h_t(x, y)$ represents the amount of heat transferred from $x$ to $y$ in time $t$

$$h_t(x, y) = \sum_{i \geq 0} e^{-\lambda_i t} \psi_i(x)\psi_j(y)$$

- Heat kernel (autodiffusion) function [Sun et al 2009, Gebal et al 2009]

$$HKF_t(x) = h_t(x, x)$$
Outline

• Basic notions from geometry and topology
• Isometries and intrinsic shape properties
• Basic concepts of differential (and computational) topology
  – Homeomorphisms, topology invariants and basic concepts of Morse theory
• Applications
• Summary
Which mathematics?

- differential (and computational) topology
  - formal definition of the domain (topological spaces)
  - invariants and properties (functions)

... but not only!
Homeo- & diffeo- morphisms

• a homeomorphism between two topological spaces $X$ and $Y$ is a continuous bijection $h: X \to Y$ with continuous inverse $h^{-1}$

• given $X \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{R}^m$, if the smooth function $f: X \to Y$ is bijective and $f^{-1}$ is also smooth, the function $f$ is a diffeomorphism
several transformations $f: X \rightarrow X'$ that can be applied to a space $X$

are these transformations enough to describe a shape and its relation to other shapes? what else? can we define other invariants?
Why algebraic topology?

- algebraic topology associates algebraic invariants to each space so that two spaces are homeomorphic if they have the same invariants

- approach: to decompose a topological space into simple pieces that are easier to study (e.g. to decompose a polyhedron into faces, edges, vertices or a surface into triangles)
Many invariants

• algebraic topology
  – Invariants: homeomorphisms

• what if we want to reason about shapes under more invariants?

• critical points of functions may give good characterizations of shape properties which reflect different invariants
Morse theory & shape similarity

- to combine the topology of $X$ with the quantitative measurement provided by $f$
  - $f$ is the lens to look the properties of $(X, f)$
  - different choices of $f$ provide different invariants
- to construct a general framework for shape characterization which if parameterized wrt the pair $(X, f)$
Comparing shapes

- to assess how far two shapes \((X, f)\) and \((Y, g)\) — a distance between topological spaces equipped with functions is needed

natural pseudo-distance:
- shapes are similar if there exists a homeomorphism between the spaces that preserves the properties conveyed by the functions
Scalar functions & shape descriptions

- **Persistent topology** (Ferri, Frosini 1990, Edelsbrunner et al. 2000)
- **Morse and Morse-Smale complexes** (Edelsbrunner 2001, Edelsbrunner et al. 2003)
- ...

- **applications**
  - shape segmentation/abstraction
  - shape retrieval and classification
  - ...

Biasotti S. et al.: *Describing shapes by geometric-topological properties of real functions. ACM Computing Surveys, 2008*
Scalar functions & shape comparison

• Multi-variate functions (e.g. textures) [Biasotti et al 2008, Biasotti et al, CGF, 2013]

• Functional maps [Ovsjanikov et al 2012, Rustamov et al 2013]

• Automatic selection of expressive functions (e.g. using a clustering approach) [Biasotti et al, CAG, 2013]

• Learning descriptions (e.g. from kernels of Reeb graphs or spectral properties) [Barra&Biasotti, Patt. Rec., 2013, Litman&Bronstein 2013]

• Feature selection [Bonev et al, CVIU 2013]
Outline

• Basic notions from geometry and topology
• Isometris and intrinsic shape properties
• Basic concepts of differential (and computational) topology
• Applications
  – Shape correspondence
  – Attribute transfer
  – Shape matching
• Summary
Application to 3D shape analysis

- **Shape correspondence**
  - Finding correspondences between a discrete set of points on two surface meshes

- **Shape matching**
  - Quantifying the similarity between couples of objects
  - Indexing a database
  - Identifying an object as belonging to a class
Intrinsic correspondence [LF2009]

• looking for an intrinsic correspondence means finding corresponding points such that the mapping between them is close to an isometry

• idea:

any genus zero surface can be mapped conformally to the unit sphere

1-1 and onto conformal map of a sphere to itself (Mobius map): uniquely defined by three corresponding points
Intrinsic correspondence [LF2009]

• Algorithm
  1. sampling points: local maxima of Gauss curvature & (geodesically) farthest point algorithm
  2. discrete conformal flattening to the extended complex plane
  3. compute the Möbius transformation that aligns a triplet in the common domain
  4. evaluate the intrinsic deformation error and build a fuzzy correspondence matrix
  5. produce a discrete set of correspondences

• pay attention to...
  – what about higher genus surfaces?
  – drawbacks of the discrete (linear) flattening technique
Comparing textured 3D shapes [BC*13]

• retrieval of textured models
Comparing textured 3D shapes [BC*13]

- photometric description
  - the multidimensional persistence spaces and CIELab coordinates

S. Biasotti, A. Cerri, D. Giorgi, M. Spagnuolo, PHOG: Photometric and geometric functions for textured shape retrieval, CGF 2013
Comparing textured 3D shapes [BC*13]

- hybrid geometric-photometric description
  - the geodesic distance weighed with respect to the Riemannian and CIELab spaces

S. Biasotti, A. Cerri, D. Giorgi, M. Spagnuolo, PHOG: Photometric and geometric functions for textured shape retrieval, CGF 2013
Comparing textured 3D shapes [BC*13]

- geometric description
  - the intra-distance matrix of geometric functions defined on the shape

S. Biasotti, A. Cerri, D. Giorgi, M. Spagnuolo, PHOG: Photometric and geometric functions for textured shape retrieval, CGF 2013
Examples

- **SHREC’13 dataset**
  - 10 classes of 24 textured models each
  - two level classification
    - highly relevant: models with same shape and texture
    - marginally relevant: models with same shape
Performances

Average precision recall graphs

- A
- Gi
- G1
- V2
- PHOG

Precision vs. Recall Graph

- X-axis: Recall
- Y-axis: Precision
Summary

• ... the right space
  – rigid transformations (rotations, translations)
    • Euclidean distances
  – isometries/symmetries
    • Riemannian metric
    • curvature (but unstable to local noise/perturbations)
    • geodesics, diffusion geometry, Laplacian operators, etc
  – local invariance to shape parameterizations
    • conformal geometry
  – similarities (i.e. scale operations)
    • normalized Euclidean distances
  – affinity (and homeomorphisms)
    • Morse theory
    • persistent topology
    • size theory
• ... a suitable shape description
  – coarse coding (but fast)
    • histograms
    • matrices
  – articulated shapes
    • medial axes
    • Reeb graphs
  – overall global appearance
    • silhouettes
  – if shape loops are relevant
    • graph-based descriptions
    • persistent topology
Open issues

• **Geometry, structure, similarity, context**
  – is it possible to understand something about functionality?
  – machine learning vs geometric-reasoning
  – 3D query modalities
  – what if shape is influenced/modified by the context?


10. V. Barra, S. Biasotti 3D shape retrieval using Kernels on Extended Reeb Graphs, Pattern Recognition, 2013,


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Part III. Statistical Shape Analysis
Outline

• Introduction and motivations
• Statistical Shape Analysis on linear spaces
• Statistical Shape Analysis on non-linear spaces
  – Kendall’s shape space
  – Square-Root Velocity representations
• Applications
• Summary
Modeling the continuous variability in shape collections

- Comparing pairs or collections of shapes
  - Ability to say whether two (collections of) shapes are similar or not
  - Localize similarities and differences

- Computing summary statistics
  - Shape atlas: mean shapes, covariances, and high-order statistics.

- Stochastic modeling of shape variations
  - Provide probability distributions, thus generative models, associated with shape classes.

- Exploration of the shape space
  - Interpolations and extrapolations
  - Random generation of valid instances of shapes
  - Statistical inferences, regressions and hypothesis testing.
Feature or descriptor-based analysis

A mapping of the shape space into a (finite) (low) dimensional feature space

Shape space

Feature space

$d(x_1, x_2)$
# Feature or descriptor-based analysis

A mapping of the shape space into a (finite) (low) dimensional feature space

## 2D
- Morphological properties (size, area, aspect ratio, symmetry, ...)
- Fourier / wavelet descriptors
- Zernike moments
- Shape context (SC)
- Inner Distance-based Shape Context
- Shape distribution
- Curvature Scale Space
- ......

## 3D
- Morphological properties (size, volume, aspect ratio, symmetry, ...)
- 3D Fourier / wavelet descriptors
- Zernike moments
- Spherical harmonics and spherical wavelets
- Shape context (SC)
- Shape distribution
- Spin images,
- Heat Kernel signatures
- Reeb graphs
- ......
Feature or descriptor-based analysis

The mapping is often **not invertible**

– Problem: Cannot compute summary statistics or perform statistical inferences

What is \( \frac{1}{2} (x_1 + x_2) \)?
Statistical shape analysis – a warm up

Landmark-based shape representations

A shape as a set of $n$ anatomical landmarks

$$P = \{ p_i = (x_i, y_i) \in \mathbb{R}^2, i = 1, \ldots, n \}$$
Statistical shape analysis – a warm up

• A shape as a set of $n$ ordered landmarks

$$P = \{ p_i = (x_i, y_i) \in \mathbb{R}^2, i = 1, \ldots, n \}$$

• Shape is a property that is invariant to translation, scale, and rotation
  – Remove translation by centering shapes to their center of mass
  – Rescale the shapes such that $\|p\|^2 = \sum_{i=1}^{n} |p_i|^2 = 1$

• Pre-shape space

$$\mathcal{D} = \{ p = (p_i, i = 1 \ldots n) \mid \sum p_i = 0, \|p\| = 1 \}.$$
Invariance to rotation

- Given two shapes P and Q, rotate Q such that the SSD between the corresponding landmarks is minimized

  - Compute the Singular Value Decomposition (SVD) of the matrix \( M = P \times Q' \). That is, \( M = U\Sigma V^* \).
  - The optimal rotation matrix that aligns \( Q \) to \( P \) is given by \( O = UV' \), with \( O \in SO(2) \).
  - Rotate \( Q \) with \( O \). That is \( Q \leftarrow OQ \).

Shape space becomes \( S = \mathbb{D} / SO(d) \), where

- \( d = 2 \) for 2D shapes
- \( d = 3 \) for 3D shapes

Perform statistical analysis in this space
Linear methods for statistical shape anal.

Assume that $S$ is a vector space equipped with the Euclidean distance

Mean shape \[ \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i. \]

Covariance matrix $K$ \[ K = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})(x_i - \bar{x})^T. \]

Statistically feasible shapes \[ x = \bar{x} + \sum_{i=1}^{d} \alpha_k v_k, \quad \alpha_k \in \mathbb{R} \]

Gaussian distribution on the parameters \[ -\log \Pr(\alpha) = \frac{1}{2} \sum_{i=1}^{d} \frac{\alpha_i^2}{\lambda_i} + \text{const.} \]

Eigen decomposition of $K$ \[ \lambda_k, v_k \]

Leading eigenvalues

Leading eigenvectors

Shape parameterization \[ \overrightarrow{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_d) \]
Application to 3D face analysis

3D morphable model for face analysis and synthesis

Image courtesy of Blanz and Vetter 1999

Pipeline

• Database
  – Laser scans of 200 faces (100 males, 100 females)

• A 3D face is represented by
  – A shape vector \( \mathbf{X} = (x_1, y_1, z_1, \ldots, x_n, y_n, z_n)^T \)
  – An appearance vector \( \mathbf{T} = (r_1, g_1, b_1, \ldots, r_n, g_n, b_n)^T \)

• Use N examplar faces to train the morphable model
  – Normalize all the faces for translation, scale and rotation
  – Put all the faces in one-to-one correspondences
  – Run PCA on the shape and on the appearance vectors

\[
\mathbf{x} = \bar{\mathbf{x}} + \sum_{i=1}^{d} \alpha_k \mathbf{v}_k, \text{ where } \alpha_k \in \mathbb{R}
\]
Application to 3D face analysis

Face shape space exploration

Image courtesy of Blanz and Vetter 2003
Application to human body shape analysis

Exploration of the space of human body shapes

Image courtesy of Allen et al. 2003
Some facts ....

- **Correspondence**
  - Assume that the landmarks are given and that they are in correspondence

- **Invariance**
  - Translation, scale
  - Rotations – depends also on the quality of the correspondences
  - How about re-parameterization?

- **Statistical analysis**
  - Assume that the population of shapes follows a Gaussian distribution.
  - Is the distribution really Gaussian?
  - Can we fit distributions from the parametric or non-parameteric families?
Outline

• Introduction and motivations
• Statistical Shape Analysis on linear spaces
  • Statistical Shape Analysis on non-linear spaces
    – Kendall’s shape space
    – Square-Root Velocity representations
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Back in time to .... 1528

In: The Four Books of Human Proportions by Albrecht Durer (1528)

In: On Growth and Evolution by D’Arcy W. Thompson (1917)

• Ulf Granendar’s pattern theory (1976)
  – Shape is not represented as such but as a deformation of another, called template.
And to 1984 ...

• David G. Kendall (1984) – statistics into shape analysis

**Shape is what is left when differences which can be attributed to translations, rotations and dilations have been quotiented out**

Shapes as Landmarks in correspondence  →  Normalization (translation, scale, rotation)  →  Shape space

$\mathcal{D} = \{ p = (p_i, i = 1 \ldots n) \}
\sum p_i = 0, \| p \| = 1$

Assume a vector space (PCA-based analysis)

• Active Shape Models [Cootes et al. ]

Non-linear manifold
Kendall’s shape space

Statistics directly on the manifold

- Sample (Karcher) mean

\[ \mu = \arg \min_{X \in \mathcal{S}} \sum_{i=1}^{n} d_{\mathcal{S}}(X, X_i) \]

(1) \( v_i = \exp_{\mu}^{-1}(X_i) \)  
(2) \( v = \frac{1}{k} \sum_{i=1}^{k} v_i \)  
(3) \( \mu \leftarrow \exp_{\mu}(\epsilon v) \)

*Slide adapted from A. Srivastava, ICIP2013 Keynote talk.*
Kendall’s shape space

Intrinsic covariance matrix

- Work on the tangent space $T_\mu(S)$ to the manifold at the mean

\[
(1) \quad v_i = \exp^{-1}_\mu(X_i) \quad (2) \quad C = \frac{1}{k - 1} \sum_{i=1}^k (v_i - \mu)(v_i - \mu)^t
\]

- Statistical analysis on $T_\mu(S)$
  - Tangent PCA (TPCA)
  - Probability models on $T_\mu(S)$ (e.g., Multivariate normal, GMM)

- Project back the statistics on the manifold using exponential map

*Slide adapted from A. Srivastava, ICIP2013 Keynote talk.*
Issues with Kendall’s approach

• Landmarks selection and registration
  – How to select landmarks on shapes?
  – Different selections may lead to different results
  – Pre-defined sampling forces a specific registration

From landmarks to continuous objects

Assume continuous objects and discretize only at the implementation stage

Parameterize Shapes on a continuous domain $D$

$D = S^1$

Closed curves

$s = (\theta, \varphi)$

$D = S^2$

Genus-0 surfaces
Parameterization provides registration

Initial parameterization

After re-parameterization of $f_2$

\[
\int_D \|f_1(s) - f_2(s)\|^2 ds \neq \int_D \|f_1(s) - f_2(\gamma(s))\|^2 ds
\]
Parameterization provides registration

Initial parameterization

After re-parameterization of $f_2$

Problem: $\| f_1 \circ \gamma - f_2 \circ \gamma \| \neq \| f_1 - f_2 \|$
Re-parameterizations do not act by isometry under the $\mathbb{L}^2$ metric

$$\| f_1 \circ \gamma - f_2 \circ \gamma \| = \left( \int_D |f_1(\gamma(s)) - f_2(\gamma(s))|^2 ds \right)^{1/2} = \left( \int_D |f_1(\tilde{s}) - f_2(\tilde{s})|^2 J_{\gamma}(s)^{-1} d\tilde{s} \right)^{1/2} \neq \| f_1 - f_2 \|$$

Often different from one
( $\gamma$ often is not area preserving)

Euclidean metric is not invariant to re-parameterization of the shapes
Invariance

• Re-parameterization is an additional nuisance group
  – It needs to be removed in same way as translation, scale and rotation

Compare surfaces using a Riemannian metric that is invariant to scale, translation, rotation, and re-parameterization
Formulation

- A shape space $\mathcal{F}$ and a metric on this space
  - Shapes become points on this space
  - Pathes $F$ are deformations (bending & stretching) that align one shape to another
  - Shortest pathes $F^*$ (geodesics) are optimal deformations
  - Geodesic distance (length of $F^*$) is a measure of dissimilarity

$$F^* = \arg \min_{F : [0, 1] \to \mathcal{F}} \left( \int_0^1 \langle F_t(t), F_t(t) \rangle^{(1/2)} \, dt \right)$$

Which shape space? Which metric on this space?
Optimize over all possible rotations and diffeomorphisms

\[
\min_{\gamma \in \Gamma, \quad O \in SO(3)} \left( \min_{F : [0, 1] \to \mathcal{F}} \left( F(0) = f_1, F(1) = O(f_2 \circ \gamma) \right) \right)
\]

Shortest path (geodesic) between \( F(0) \) and \( F(1) \) under fixed rotation and re-parameterization

Registration of \( f_2 \) onto \( f_1 \) (finds optimal rotation and re-parameterization)
Step 1 - Representation

A 3D Shape as a continuous surface

\[ f(s) = (x, y, z) \]

- Normalize for translation
  \[ f_{\text{centered}}(s) = f(s) - \frac{\int_{s^2} f(s) \|a(s)\| ds}{\int_{s^2} \|a(s)\| ds} \]

- Normalize for scale
  \[ f_{\text{scaled}}(s) = \frac{f(s)}{\sqrt{\int_{s^2} \|a(s)\| ds}} \]

Genus-0 surfaces

Preshape space \( \mathcal{F} \) is the space of all normalized surfaces
Step 2 - Q-maps: Square Root Representation

Q-map of a surface $f$

$Q(f)(s) = q(s) = \sqrt{|a(s)|} f(s)$

area of $f$ at $s \in S^2$

Action of the re-parameterization

$q = \sqrt{|a|} f$

$(q, \gamma) = (q \circ \gamma) \sqrt{J_\gamma}$

$\|q_1 - q_2\| = \|(q_1, \gamma) - (q_2, \gamma)\|$
Riemannian metric on the space of Q-maps

The space of normalized surfaces

\[ T_f(\mathcal{F}) \]

\[ \langle \langle v_1, v_2 \rangle \rangle_f = \langle Q^*, f(v_1), Q^*, f(v_2) \rangle \]

\[ Q^*, f(v) = \frac{1}{2|a|^3} (a \cdot a_v) f + \sqrt{|a|} v \]

The space of Q-maps

\[ \mathbb{L}^2 \]

\[ w_1 = Q^*, f(v_1) \]

\[ w_2 = Q^*, f(v_2) \]

Dot product on the space of Q-maps

\[ \langle w_1, w_2 \rangle = \int_D \langle w_1(s), w_2(s) \rangle ds \quad \text{for } w_1, w_2 \in T_q(\mathbb{L}^2) \]

Under this metric, the action of \( \Gamma \) on \( \mathcal{F} \) is by isometries

Pre-shape and shape space

- **Pre-shape space**
  - Center and re-scale all surfaces
  - Pre-shape space $\mathcal{F}$ is the space of all normalized surfaces

- **Shape space**
  - Rotation group $SO(3)$: $SO(3) \times \mathcal{F} \rightarrow \mathcal{F} : (O, f) = Of$
  - Reparameterization group: $\mathcal{F} \times \Gamma \rightarrow \mathcal{F} : (f, \gamma) = (f \circ \gamma)$
  - Equivalence classes represent each shape uniquely
    \[ [f] = \text{closure}\{O(f \circ \gamma) | O \in SO(3) \text{ and } \gamma \in \Gamma\} \]
  - Shape space is the set of all equivalence classes
    \[ \mathcal{I} = \{[f] | f \in \mathcal{F}\} \]
Geodesics in shape space

\[
\min_{\gamma \in \Gamma, \quad O \in SO(3)} \left( \min_{F : [0, 1] \to \mathcal{F}} \right.
\begin{align*}
F(0) &= f_1, \\
F(1) &= O(f_2 \circ \gamma)
\end{align*}
\left( \int_0^1 \langle F_t(t), F_t(t) \rangle^{(1/2)} \, dt \right)
\]

Geodesic in pre-shape space

Geodesic in shape space
Step 3 – Solving the optimization problem

\[
\min_{\gamma \in \Gamma, \ O \in SO(3)} \left( \min_{F : [0, 1] \to \mathcal{F}} \right)
\begin{align*}
F(0) &= f_1, \\
F(1) &= O(f_2 \circ \gamma)
\end{align*}
\left( \int_0^1 \langle F_t(t), F_t(t) \rangle^{(1/2)} \ dt \right)
\]

Geodesic in pre-shape space

Geodesic in shape space

- **Step 3.1.**
  - Solve the inner optimization for fixed rotation and re-parameterization (path straightening algorithm)

- **Step 3.2.**
  - Solve the outer optimization over SO(3) and $\Gamma$
Step 3.1. Solving the inner optimization

\[
\min_{\gamma \in \Gamma_0, \ O \in SO(3)} \left( \min_{F: [0, 1] \to \mathcal{F}} \right.
\]
\[
F(0) = f_1, \ F(1) = O(f_2 \circ \gamma)
\]

Path straightening

- Energy of a path
  \[
  E[F] = \int_0^1 \langle \langle F_t, F_t \rangle \rangle_F dt
  \]

- Critical point of \( E \) is a geodesic
- Use gradient descent

Step 3.2. Solving the outer optimization

\[
\min_{\gamma \in \Gamma_0, \quad O \in SO(3)} \left( \min_{F : [0, 1] \rightarrow \mathcal{F}} F(0) = f_1, \quad F(1) = O(f_2 \circ \gamma) \right)
\]

Fix the parameterization, optimize over SO(3)

Standard Procrustes analysis

(1) \( A = \int_{S^2} q_1(s)q_2(s)^T \, ds \) \quad (2) \( A = U\Sigma V^T \) \quad (3) \( O^* = UV^T \)
Step 3.2. Solving the outer optimization

\[
\min_{\gamma \in \Gamma_0,\ O \in SO(3)} \left( \min_{F: [0, 1] \to \mathcal{F}} F(0) = f_1, F(1) = O(f_2 \circ \gamma) \right)
\]

\[
\left( \int_0^1 \langle F_t(t), F_t(t) \rangle^{1/2} \, dt \right)
\]

\[
\gamma^* = \arg \min_{\gamma \in \Gamma_1} \| q_1 - (q_2, \gamma) \|^2
\]

\[
H_2(\gamma)
\]

Fix the rotation, optimize over \( \Gamma \)

(1) Cost function
\[
H_2(\gamma) = \| q_1 - (q_2, \gamma) \|^2 = \| q_1 - \phi(\gamma) \|^2
\]

(2) Mapping and differential
\[
\phi(\gamma) = (q_2, \gamma) = \sqrt{J_\gamma} (q_2 \circ \gamma)
\]
\[
\phi_{*, \gamma id}(b) = (1/2) (\nabla \cdot b) q_2 + \nabla q_2 \cdot b
\]

(3) Gradient of energy
\[
d \gamma = \sum_{i=1}^\infty \langle q_1 - q_2, \phi_{*, \gamma id}(b_i) \rangle b_i
\]
Construction of the orthonormal basis

- Basis for $T_{\gamma id}(\Gamma)$
  - Fourier-type basis (boundary constraints)
  - Gradients of spherical harmonics
  - Monomials (boundary constraints)
- Use Gramm-Schmidt to orthonormalize
• Hemispherical surfaces (e.g. Human Faces)
Results – computing geodesics

• Closed surface (biomedical applications)
Results – correspondences and geodesics

Isometric deformations

Correspondences are color-coded

$L[F^*] = 0.1609$

$L[F^*] = 0.1369$
Results – correspondences and geodesics

Isometric deformations

$L[F^*] = 0.2183$
Results – correspondences and geodesics

Elastic deformations

S Kurtek et al. (EG2013), Landmark-Guided Elastic Shape Analysis of Spherically-Parameterized Surfaces
Results – correspondences and geodesics

Elastic deformations
Results – correspondences and geodesics

Missing parts

$L[F^*] = 0.0997$

$(L[F^*] = 0.1977)$
Results – correspondences and geodesics

Missing parts

$L[F^*] = 0.1983$

S Kurtek et al. (EG2013), Landmark-Guided Elastic Shape Analysis of Spherically-Parameterized Surfaces
Statistical summaries

Mean shape (the Karcher Mean)

- Given a set of surfaces $\{f_1, f_2, \ldots, f_n\} \in \mathcal{F}$
- Karcher mean

$$\bar{f} = \arg \min_{[f] \in \mathcal{S}} \sum_{i=1}^{n} d([f], [f_i])$$

1) Start with an initial guess $\bar{q}$. This can be chosen as one of the elements of $\mathcal{F}$
2) Compute the geodesic $\xi_i$ between $\bar{q}$ and $q_i$ for every $i = 1, \ldots, n$.
3) Let $v_i \in T_{\bar{q}}(\mathcal{C})$ be a tangent vector to $\xi_i$ at $\bar{q}$.
4) The gradient of $\mathcal{V}$ at $\bar{q}$ is proportional to $\vartheta = \sum_{i=1}^{n} v_i$.
5) Update $q$ with a small step in the direction of the gradient $\vartheta$ and project back on the hypersphere.
6) Repeat steps 2 to 5 until convergence.
Covariance

1. Compute shooting vectors:
   \[ \mathbf{v}_i = F^*_t(0) \] where \( F^* \) is a geodesic between \( \bar{f} \) and \( O^*_i(f_i \circ \gamma_i^*) \)

2. Use Gram-Schmidt to compute orthonormal basis of shooting vectors in under.___

3. Project each of the shooting vectors onto this basis.

4. Use singular value decomposition to perform PCA.
Results – Statistical summaries

Mean shape

Shape atlas

S Kurtek et al. (EG2013), Landmark-Guided Elastic Shape Analysis of Spherically-Parameterized Surfaces
Results – statistical summaries

Kurtek et al. (IPMI 2011), Parameterization-Invariant Shape Statistics and Probabilistic Classification of Anatomical Surfaces
### Results – statistical summaries

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{F}$</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-\sigma \rightarrow \sigma$</td>
<td>$-\sigma \rightarrow \sigma$</td>
</tr>
<tr>
<td>PC1</td>
<td><img src="image1" alt="PC1_F" /></td>
<td><img src="image2" alt="PC1_S" /></td>
</tr>
<tr>
<td>PC2</td>
<td><img src="image3" alt="PC2_F" /></td>
<td><img src="image4" alt="PC2_S" /></td>
</tr>
<tr>
<td>PC1</td>
<td><img src="image5" alt="PC1_F" /></td>
<td><img src="image6" alt="PC1_S" /></td>
</tr>
<tr>
<td>PC2</td>
<td><img src="image7" alt="PC2_F" /></td>
<td><img src="image8" alt="PC2_S" /></td>
</tr>
</tbody>
</table>
• Shape symmetrization and measure of asymmetry

\[ \tilde{f} = H(v)f \]

(Reflection of \( f \) with respect to arbitrary plane)
Results - Symmetry

• Shape symmetrization and measure of asymmetry

\[ L(F^*) = 0.1535 \]

(Reflection of \( f \) with respect to arbitrary plane)

Length of the path is a measure of asymmetry

\[ \tilde{f} = H(v)f \]

S Kurtek et al. (EG2013), Landmark-Guided Elastic Shape Analysis of Spherically-Parameterized Surfaces
Results - symmetry

- Shape symmetrization and measure of asymmetry

\[ L(F^*) = 0.0963 \]

\[ L(F^*) = 0.1189 \]
Results - symmetry

S Kurtek et al. (EG2013), Landmark-Guided Elastic Shape Analysis of Spherically-Parameterized Surfaces
Application to 3D shape analysis

- **Shape differences**
  - Simultaneous correspondence (registration) and geodesics (optimal deformations) and dissimilarity without descriptors!! (isometric as well as elastic deformations, and missing parts)

- **Summary statistics**
  - Compute *mean shapes*, covariances, and high-order statistics of a collection of shapes.

- **Stochastic modeling**
  - Develop models that capture the variability in shape classes

- **Statistical inference**
  - Study hypothesis testing, likelihood ratios, etc.
Limitations

• Limited to genus-0 manifold surfaces
  – Lack of proper (and efficient) parameterization of high genus surfaces

• Correspondence
  – When deformations are drastic, the correspondence may fail
    (issues with semantically similar but geometrically very different)

• Extensions
  – High genus
  – Non-manifold shapes
Open issues
Related tutorials


Acknowledgement

- Anuj Srivastava
  Florida State University, US
- Sebastian Kurtek
  Ohio State University, US
Reasoning About Shape in Complex Datasets
Geometry, Structure and Semantics

Silvia Biasotti
Hamid Laga
Michela Mortara
Michela Spagnuolo

Part IV. Structural analysis of shapes
Outline

• Shape understanding: from geometry and structure to semantics
  – Shape segmentation
  – Structural representations
  – Methods:
    • Tailor, Plumber, Fitting Primitives, Fuzzy clustering, core extraction (comparison), others (SDF, nearly convex approximation, co-hierarchical analysis of shape structures, consistent segmentation ...)

• From geometry to semantics in the context of Virtual Humans

• Knowledge-driven shape annotation

• Prior knowledge for shape correspondence
  – Semantic correspondence & functionality recognition
Knowledge about 3D shapes

- Knowledge related to the geometry
- Knowledge related to the application domain
- Knowledge related to the context
From geometry to knowledge: Analysis

• Pb: **extract** and **associate** knowledge to 3D shapes

• Shape Analysis: **extracts** knowledge **implicitly encoded** in the geometry

• **How?**

  – “**Analysis** is the process of observing and breaking down a complex topic or substance into smaller parts to gain a better understanding of it, describing such parts and their relations with the whole.”

  – From geometry to structure
From geometry to knowledge: Analysis

• From geometry to structure
  – From geometric measures (volume, area, spatial distributions ...)
  – To Structural Shape descriptors (feature recognition, segmentation, skeleton extraction)
From geometry to knowledge: Understanding

• **Shape Understanding:** *recognize the object or its part in a specific context (semantics, functionality)*

• **How?**
  – Propagating labels from annotated models
  – using a priori knowledge about the context
  – Using supervised methods
From geometry to knowledge: Annotation

• Shape Annotation: associates knowledge to digital shapes and their components in a formal manner
  – context-driven annotation
  – support reasoning
Structural Analysis

**Characterization:**
Evaluation of scalar functions over the surface

**Segmentation:**
Identification of regions having homogeneous properties (main components or features of interest)

**Structuring:**
Extraction of subparts and their spatial arrangement
Segmenation

• Studies on perception state that humans recognize shapes by mentally segmenting them into their (simpler) constituting parts

• Segmenting a digital model in parts with homogeneous properties is needed in many applications about shape:
Shape segmentation

• Approximation/compression
• Collision detection
• Modelling
• Comparison
• Morphing/Animation
• Understanding
Shape segmentation

- Approximation/compression
- Collision detection
- Modelling
- Comparison
- Morphing/Animation
- Understanding
Shape segmentation

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Shape segmentation

- Approximation/compression
- Collision detection
- Modelling
- Comparison
- Morphing/Animation
- Understanding
Segmentation

• Typically builds on low-level characterization and may be coded as a structural representation

Definition

• $M = \{V,E,F\}$ a mesh.
• $S = V, E$ or $F$ (typically $F$).
• A Segmentation

$$S = \{M_0, M_1, ..., M_{k-1}\}$$

is the set of sub-meshes induced by a partition of $S$ in $k$ (disjoint) subsets.
Criteria

• What are the features of a «good» segmentation?
  – Planar / curved segments?
  – Smooth boundaries?
  – Big vs small patches?
  – Few / many segments?
  – …

• Depends on the application
Two main kinds of segmentation

**Patch-based**

Segments are surface patches having specific geometric properties (e.g. geodesic distance, curvature, ...)

**Part-based**

More intuitive, parts have a volumetric nature and a specific **meaning**.
Segmentation as an optimization pb

- Given a mesh $M = \{V,E,F\}$ and $S \in \{V,E,F\}$, find a disjoint partition of $S$ into $S_1,...,S_k$ such that the function

$$J = J(S_1, ..., S_k)$$

is minimised (or maximised) according to a set of constraints $C$.

[Shamir2008]
Constraints and Attributes

• Constraints describe the properties that the partition (or the induced submeshes, i.e. the segments) must satisfy
  – Ex: max number of segments
  – Ex: connectedness of submeshes
  – The set of constraints might be empty

• Attributes pertain to elements (vertices, edges, faces) and are evaluated during the optimization process.
Constraints

- Constraints describe the properties that the partition (or the induced submeshes, i.e. the segments) must satisfy
  - Cardinality
    - Elements in a segment
    - Number of segments
    - ...
  - Geometry
    - dimension: area, diameter, radius,...
    - Convexity, curvature
    - Smooth boundary
    - ...
  - Topology
    - Connectedness
    - Disc-like
    - ...


Attributes

• Attributes pertain to elements (vertices, edges, faces) and are evaluated during the optimization process.
  – Distances (euclidean, geodesic)
  – Planarity, normal direction
  – Curvature, smoothness
  – Similarity with primitives
  – Symmetry
  – Shape diameter function
  – ...

Structural Analysis
Techniques

- Region growing
- Iterative clustering
- Hierarchical clustering
- Spectral clustering
- Graph cut
- Interactive methods
- Co-segmentation
- Supervised methods
- ...
• No optimal solution in general

• Some examples (focusing on “part-type” for shape understanding)
Metamorphosis of Polyhedral Surfaces using Decomposition.

Hierarchical mesh decomposition using fuzzy clustering and cuts

Mesh Segmentation using Feature Point and Core Extraction

Hierarchical Mesh Segmentation based on Fitting Primitives


Reverse engineering, feature recovery, denoising... → Distance to primitives → Hierarchical clustering → HFP
Hierarchical Convex Approximation of 3D Shapes for Fast Region Selection

Consistent mesh partitioning and skeletonisation using the shape diameter function

From geometric to semantic VH

- **All the pipeline: Tailor-Plumber-VH Annotation**
- Geometry: Triangulated mesh of a body model
- Characterization: “Tailor”
- Segmentation: “Plumber”
- Structuring: “Shape Graph”
- Context + Annotation: “VH Annotator”
- Semantics: Annotated mesh with human body parts
Tailor

• Multi-scale morphological characterization of vertices over neighbourhoods of increasing size
<table>
<thead>
<tr>
<th>TIP</th>
<th>MOUNT</th>
<th>PIT</th>
<th>DIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLEND</td>
<td>LIMB</td>
<td>JOINT</td>
<td>FUNNEL</td>
</tr>
<tr>
<td>WELL</td>
<td>SPLIT</td>
<td>HOLLOW</td>
<td></td>
</tr>
</tbody>
</table>
Tailor results
Tailor

• Skeleton
Plumber

- Segmentation into tubular features and “bodies”
- Based on the Tailor characterization
- Works in a multi-scale fashion wrt tube section size
- Computes axis and sections of each tubular feature

- Selection of the scale $R$
- Classification of vertices and identification of seed limb region
- Tubular feature extraction
- Increase $R$ and repeat
Results
Shape graph

• **Nodes**: Geometric attributes of segments
  – Tubes: axis length & max turning, section size
  – Blobs: volume

• **Edges**: type of junction
  – Tube-tube
  – Tube-body (cap)
  – Handle (Tube on Body or Tube on Tube)
Virtual Humans

- Plumber is particularly suitable to locate human limbs consistently.
Automatic annotation


- In specific domains it is possible to assign to each segment a semantic annotation automatically.

- Virtual Human Context

- Shape graph + Geometric attributes of segments

- Annotation function

  $a: S (\text{segments}) \rightarrow L (\text{labels})$

  $L = \{ \text{head, neck, trunk, arm, hand, palm, finger, fingertip, leg, foot} \}$
Algorithm

- Segmented model

- Blob with max volume ← TRUNK

- How many tubes are adjacent to TRUNK?
  - 5

- Tubes adjacent to caps with max length \* section ← LEG
  - Adjacent caps ← FOOT

- tubes left ← ARM

- Tube with max length/section ← NECK
  - adjacent cap ← HEAD

- Cap ← HAND
  - yes
  - Adjacent blob ← PALM; next adjacent tubes ← FINGER; next adjacent caps ← FINGERTIP
  - no

- Adjacent to a cap?

- END
Body of index 3 is adjacent to tubes: 2, 3, 4, 5, 6, created Shape Graph

Body #6 has volume = 8.62936e+06
Body #5 has volume = 529545
Body #4 has volume = 276659
Body #3 has volume = 4.69445e+07
Body #2 has volume = 177698
Body #1 has volume = 1.06276e+06
Body #3 has maximum volume = 4.69445e+07

body #3 annotated as TRUNK
computeAdjacentTubes, NumBodies 6
Body of index 6 is adjacent to tubes: 6,
Body of index 5 is adjacent to tubes: 2,
Body of index 4 is adjacent to tubes: 3,
Body of index 3 is adjacent to tubes: 2, 3, 4, 5, 6,
Body of index 2 is adjacent to tubes: 4,
Body of index 1 is adjacent to tubes: 5,

Trunk has 5 adjacent tubes
score for tube 2 for being the NECK: 0.83781
score for tube 3 for being the NECK: 0.855791
score for tube 4 for being the NECK: 0.65594
score for tube 5 for being the NECK: 0.743126
score for tube 6 for being the NECK: 4.01759

tube #6 got maximum score: annotated as NECK
body #6 annotated as HEAD

score for tube 2 for being a LEG: 186311
score for tube 3 for being a LEG: 190452
score for tube 4 for being a LEG: 634311
score for tube 5 for being a LEG: 654423
score for tube 6 for being a LEG: 0

tube #5 got maximum score: first LEG annotated
-body #1 annotated as FOOT

tube #4 got second maximum score: second LEG annotated
-body #2 annotated as FOOT

tube #2 and tube #3 annotated as ARMS
body #5 annotated as HAND

body #4 annotated as HAND
Interactive Annotation

The key question is:

- Is it possible to devise a segmentation algorithm that captures all the shape features which have a meaning within a given context?
- NOT IN GENERAL !!!
- Some contexts are too large to be exhaustively formalized, and the “meaning” of a geometric feature must rely on a priori knowledge of the observer
- Some features are far too complex to be described in formal mathematical terms (e.g. the “face” of an animal)
- One segmentation is not enough!
Multi-Segmentation

- **Solution:** *Pick* the interesting features from different shape segmentations

(b) Morse-based, (c) Plumber, (d) fitting primitives
Once relevant features have been (geometrically) identified, how should we tag them?
We map computable geometric measures with values of the semantic attributes.

Concepts formalized within the input ontology can be inspected and instantiated through a graphical browser.
Resulting Knowledge Bases

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Instances</th>
<th>Domain Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Girl Head</strong>&lt;br&gt;size = 7.23</td>
<td><strong>Human body</strong>&lt;br&gt;Part of</td>
</tr>
<tr>
<td></td>
<td><strong>Girl Arm1</strong>&lt;br&gt;length = 58.00</td>
<td><strong>Body part</strong>&lt;br&gt;Is a</td>
</tr>
<tr>
<td></td>
<td><strong>Girl Foot1</strong>&lt;br&gt;</td>
<td><strong>Head</strong>&lt;br&gt;Is a</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Arm</strong>&lt;br&gt;Is a</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Leg</strong>&lt;br&gt;Is a</td>
</tr>
</tbody>
</table>


Semantic Annotation of 3D Surface Meshes based on Feature Characterization
Semantic correspondence & functionality recognition

• Shape as a graph
• Structural relationships btw parts
• Parts have geom. descriptors
• Context and context-aware similarity
• Unsupervised semantic correspondence
• Supervised functionality recognition
Structural relationships

- Structural similarity: $K_{rel}(R_i, R_j) = 1 \text{ iff } R_i = R_j$
  $0 \text{ otherwise}$
Geometric descriptors

- Shape Distribution [Osada et al. 2002]
- Size – radius of bounding sphere
- Aspect – eigenvalues of PCA

- Geometric similarity: $K_{geo} (K_d, K_s, K_a)$
Part similarity

• Two parts are similar if their geometry and context are similar

• Model context using graph kernels

\[
K^p(G_1, G_2, P_A, P_B) = K_{\text{geo}}(P_A, P_B) \times \sum_{P_S \in \mathcal{N}_{G_1}(P_A) \atop P_Q \in \mathcal{N}_{G_2}(P_B)} K_{\text{rel}}(e, f)K^{p-1} (G_1, G_2, P_S, P_Q)
\]

• Compare two nodes by comparing all walks of length \(p\)
  – Geometry of nodes and type of relationships
Functionality recognition

• Supervised learning algorithm
  – Support Vector Machine (SVM)
    • Use of non-linear kernels to model feature dependencies
    • Flexibility (wrt the choice of the kernels)
    • Decision function
      \[ f(X) = \text{sign} \left( \sum_i \alpha_i t_i K(X_i, X) + b \right) \]
      • where \(X_i\) are the selected support vectors, and \(\alpha_i\) are positive weights, \(K(x, y)\) is a nonlinear kernel that quantify the similarity between \(x\) and \(y\)
• Best matches using part context
Reasoning About Shape in Complex Datasets

Geometry, Structure and Semantics

Silvia Biasotti
Hamid Laga
Michela Mortara
Michela Spagnuolo
where did we start from?

• reasoning about shape is important
  – computational theories for shape analysis
  – application domains pose challenging issues

“Applied computer science is now playing the role which mathematics did from the seventeenth to the twentieth centuries providing an orderly, formal framework and exploratory apparatus for other sciences”

*Virtual Astronomy, Information Technology and the New Scientific Methodology*

George Djorgovski (2005)
what did we learn?

• reasoning about shape is not an easy task
  – role of the observer and context
  – difficult to capture in formal rules

• reasoning about shape relies on advanced mathematics
  – geometric-differential approaches
  – statistical shape analysis
  – structure as a road to reach semantics
what do we need more?

• **Derive symbolic representations of 3D data**
  - creating symbolic and editable representations out of “sensed” data
  - high-level editing independent of the underlying geometric representation
what do we need more?

• **Goal-oriented synthesis of 3D models**
  
  – Acquisition and capture of knowledge contributing to the “goal”
  
  – Methodologies for model generation (semantics-oriented modeling of 3D objects)
  
  – Creation of libraries of models in the form of shape/function models
what do we need more?

- Documentation of 3D content
  - annotation of single objects, scenes, and workflows: the annotation is content, context and user dependent;
  - methodologies for annotation
    - classification, propagation of the annotation via similarity assessment and matching, ...
    - massive annotation tasks: 3D city models?
  - how to maintain the annotation across workflows that act on the representation?
  - standards
did you enjoy the tutorial?!

• if not, well, good news....

.. this is the end !!