Tutorial 6

Eurographics 2009

Schedule

12:00 – 12:15 Introduction  
  • Prof. Nadia Magnenat-Thalmann

12:15 – 13:05 Anatomical modelling from medical data  
  • Prof. Nadia Magnenat-Thalmann and Jérôme Schmid

13:05 – 13:30 Physically-based simulation of biological tissues (Part 1)  
  • Dr. Hervé Delingette

15:00 – 15:25 Physically-based simulation of biological tissues (Part 2)  
  • Dr. Hervé Delingette

15:25 – 16:15 Medical visualisation and applications  
  • Dr. Marco Agus and J.A. Iglesias Guitián

16:15 – 16:30 Conclusion and discussion

Physically-based simulation of biological tissues

Dr. Hervé Delingette – INRIA, Asclepios, France
Overview

Measuring Soft Tissue Deformation
Continuum Models of Soft Tissue
Discretization Methods
Interactive Simulation : SOFA Platform
Examples

Soft Tissue Characterization

Biomechanical behavior of biological tissue is very complex

Most biological tissue is composed of several components :
  • Fluids : water or blood
  • Fibrous materials : muscle fiber, neuronal fibers, …
  • Membranes : interstitial tissue, Glisson capsule
  • Parenchyma : liver or brain
Soft Tissue Characterization

To characterize a tissue, its stress-strain relationship is studied.

Stress \( \sigma = \frac{F}{\pi r^2} \)

Strain \( \varepsilon = \frac{h^*-h}{h} \)

In stress-strain relationships there are:

- **Hysteresis phenomenon**
- **Visco-elasticity phenomenon**
- **Non-linearity**
- **Anisotropy**

Loading \( \rightarrow \) Unloading

Slope = Young Modulus

Linear Domain
Parameter estimation

Complex for biological tissue:
- Heterogeneous and anisotropic materials
- Tissue behavior changes between in-vivo and in-vitro
- Ethics clearance for performing experimental studies
- Effect of preconditioning
- Potential large variability across population

Soft Tissue Characterization

Different possible methods
- In vitro rheology
- In vivo rheology
- In silico rheology
- Elastometry
Soft Tissue Characterization

**In vitro** rheology

- can be performed in a laboratory.
  Technique is mature
- Not realistic for soft tissue (perfusion, …)

![Diagram of rheology setup]

Source: Cimit, Boston USA

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Soft Tissue Characterization

**In vivo** rheology

- can provide stress/strain relationships at several locations
- Influence of boundary conditions not well understood

Source: Cimit, Boston USA
Soft Tissue Characterization

**In silico** rheology (Inverse Problems)
- well-suited for surgery simulation (computational approach)
- require the geometry before and after deformation

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**Elastometry** (MR, Ultrasound)
- measure property inside any organ non invasively
- validation ? Only for linear elastic materials

Source Echosens, Paris
Soft Tissue Characterization

May be difficult to find “reliable” soft tissue material parameters

Example: Liver soft tissue characterization

<table>
<thead>
<tr>
<th>First Author</th>
<th>Experimental Technique</th>
<th>Liver Origin</th>
<th>Young Modulus (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yamashita [111]</td>
<td>Image-Based</td>
<td>Human</td>
<td>Not Available</td>
</tr>
<tr>
<td>Carter [17]</td>
<td>in-vivo</td>
<td>Human Liver</td>
<td>≈ 170</td>
</tr>
<tr>
<td>Dan [27]</td>
<td>ex-vivo</td>
<td>Porcine Liver</td>
<td>≈ 10</td>
</tr>
<tr>
<td>Lin [62, 61]</td>
<td>ex-vivo</td>
<td>Bovine Liver</td>
<td>Not Available</td>
</tr>
<tr>
<td>Nava [76]</td>
<td>in-vivo</td>
<td>Porcine Liver</td>
<td>≈ 90</td>
</tr>
<tr>
<td>Miller [74]</td>
<td>in-vivo</td>
<td>Porcine Liver</td>
<td>Not Available</td>
</tr>
<tr>
<td>Sakuma [92]</td>
<td>ex-vivo</td>
<td>Bovine Liver</td>
<td>Not Available</td>
</tr>
</tbody>
</table>

Table 2: Must use a “proper” model to estimate its parameters.

Overview

Measuring Soft Tissue Deformation

Continuum Models of Soft Tissue

Discretization Methods

SOFA Platform

Examples
Continuum Mechanics

Fluid Mechanics

Structural Mechanics

Linear Elasticity (Anisotropic, heterogeneous)

Hyperelasticity Elasticity

Visco-Elasticity

1D Elasticity

Point X is deformed into point $\phi(X)$

How much deformation around point X?
1D Elasticity: stretch ratio

Rest length: 2\, dx   New length: \phi(x+dx) - \phi(x-dx)

Stretch ratio at X is

\[
\frac{d\phi}{dX}\bigg|_{X} = \frac{\phi(X)}{X}
\]

1D Elasticity: strain energy

Deformation energy W depends “how stretched” the bar is

W depends on strain \( \varepsilon = \) distance between s and 1

What is the energy necessary to deform the bar?
1D Elasticity: strain

Different choices of strain

\[ \varepsilon(s) = \frac{1}{\alpha} (s^\alpha - 1) \quad \text{For } \alpha > 0 \]
\[ \varepsilon(s) = s - 1 \quad \text{For } \alpha = 1 \quad \text{Engineering strain} \]
\[ \varepsilon(s) = \frac{1}{2} (s^2 - 1) \quad \text{For } \alpha = 2 \quad \text{Green-Lagrange strain} \]
\[ \varepsilon = \log s \quad \text{For } \alpha = 0 \quad \text{Henky strain} \]

1D Elasticity: stress

Stress is the energy conjugate of strain

\[ \sigma = \frac{\partial W}{\partial \varepsilon} \quad \varepsilon = \frac{\partial W}{\partial \sigma} \]

<table>
<thead>
<tr>
<th>Extensive Variable</th>
<th>Intensive Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>Force</td>
</tr>
<tr>
<td>Angle</td>
<td>Torque</td>
</tr>
<tr>
<td>Volume</td>
<td>Pressure</td>
</tr>
<tr>
<td><strong>Strain</strong></td>
<td><strong>Stress</strong></td>
</tr>
</tbody>
</table>

For \( \alpha = 1 \) \quad \text{(First Piola-Kirchhoff) nominal stress}
For \( \alpha = 2 \) \quad \text{Second Piola-Kirchhoff stress}
For \( \alpha = 0 \) \quad \text{Cauchy stress}
St Venant Kirchhoff Material

**Basic Material:**
- \( W \) is a quadratic function of strain
- Stress is proportional to strain \( \sigma = \frac{\partial W}{\partial \varepsilon} \)

**1D case:** \( \lambda \) is the material stiffness

\[
W = \int_{\Omega} \frac{1}{2} \sigma \varepsilon = \int_{\Omega} \frac{\lambda A}{2} \varepsilon^2 dX = \int_{\Omega} \frac{\lambda A}{2 \alpha^2} \left( \left( \frac{d\phi}{dX} \right)^{\alpha} - 1 \right)^2 dX
\]

**1D Elasticity : discretization**

Represent the bar with a single segment

- **Rest Configuration**
- **Deformed Configuration**

- Stretch Ratio \( s = \frac{L}{L_0} \)
- Strain \( \varepsilon = \frac{1}{\alpha} \left( \frac{L}{L_0}^{\alpha} - 1 \right) \)

- Strain Energy \( W = \frac{\lambda A L_0^{1-2\alpha}}{2\alpha^2} \left( L^{\alpha} - L_0^{\alpha} \right)^2 \)
1D Elasticity: discretization

Represent the bar with a single segment

\[ \Phi(X) = 0 \quad \Phi(X) = L \]

For \( \alpha = 1 \)
\[ W = \frac{\lambda A}{2L_0} (L - L_0)^2 \] (Quadratic Spring Energy)

For \( \alpha = 2 \)
\[ W = \frac{\lambda A}{8L_0^3} (L^2 - L_0^2)^2 \] (Biquadratic Spring Energy)

3D Elasticity

Deformation Function
\[ X \in \Omega \mapsto \phi(X) \in \mathbb{R}^3 \]

Displacement Function
\[ U(X) = \phi(X) - X \]
Deformation Gradient

The local deformation is captured by the deformation gradient:

$$ F = \frac{\partial \phi}{\partial X} $$

$$ F_{ij} = \frac{\partial \phi_i}{\partial X_j} = \begin{bmatrix} \frac{\partial \phi_1}{\partial X_1} & \frac{\partial \phi_1}{\partial X_2} & \frac{\partial \phi_1}{\partial X_3} \\ \frac{\partial \phi_2}{\partial X_1} & \frac{\partial \phi_2}{\partial X_2} & \frac{\partial \phi_2}{\partial X_3} \\ \frac{\partial \phi_3}{\partial X_1} & \frac{\partial \phi_3}{\partial X_2} & \frac{\partial \phi_3}{\partial X_3} \end{bmatrix} $$

F(X) is the local affine transformation that maps the neighborhood of X into the neighborhood of φ(X).

Stretch Tensor

Distance between point may not be preserved

Distance between deformed points

$$ (ds)^2 = \|\phi(X + dX) - \phi(X)\|^2 \approx dX^T (\nabla \phi^T \nabla \phi) dX $$

Right Cauchy-Green Deformation tensor

$$ C = \nabla \phi^T \nabla \phi $$

Measures the change of metric in the deformed body.
Strain Tensor

Example: Rigid Body motion entails no deformation
\[ \phi(X) = RX + T \]
\[ F(X) = \nabla \phi(X) = R \quad C = R^T R = Id \]

Strain tensor captures the amount of deformation
- It is defined as the “distance between C and the Identity matrix”

\[ E = \frac{1}{2} (\nabla \phi^T \nabla \phi - Id) = \frac{1}{2} (C - Id) \]

Diagonal Terms: \( \varepsilon_i \)
- Capture the length variation along the 3 axis

Off-Diagonal Terms: \( \gamma_i \)
- Capture the shear effect along the 3 axis
## Analogy 1D-3D Elasticity

<table>
<thead>
<tr>
<th>1D Elasticity</th>
<th>3D Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deformation Gradient</strong></td>
<td>$\frac{d\phi}{dX}$</td>
</tr>
<tr>
<td><strong>Square Stretch Ratio</strong></td>
<td>$s^2 = \left( \frac{d\phi}{dX} \right)^2$</td>
</tr>
<tr>
<td><strong>Green Strain</strong></td>
<td>$\varepsilon(s) = \frac{1}{2}(s^2 - 1)$</td>
</tr>
<tr>
<td><strong>SVK Strain Energy</strong></td>
<td>$w(X) = \frac{\lambda A(\varepsilon(s))^2}{4}$</td>
</tr>
</tbody>
</table>

## Linearized Strain Tensor

Use displacement rather than deformation:

$$\nabla \phi (X) = Id + \nabla U (X)$$

$$E = \frac{1}{2} \left( \nabla U + \nabla U^T + \nabla U^T \nabla U \right)$$

Assume small displacements:

$$E_{Lin} = \frac{1}{2} \left( \nabla U + \nabla U^T \right)$$
Hyperelastic Energy

The energy required to deform a body is a function of the invariants of strain tensor $E$:

- Trace $E = I_1$
- Trace $E^*E = I_2$
- Determinant of $E = I_3$

$$W(\phi) = \int \omega(I_1, I_2, I_3) dX$$

Total Elastic Energy

Linear Elasticity

Isotropic Energy

$$w(X) = \frac{\lambda}{2} (\text{tr} E_{\text{Lin}})^2 + \mu \text{tr} E_{\text{Lin}}^2$$

$(\lambda, \mu)$: Lamé coefficients

$w(X)$: density of elastic energy

Hooke’s Law

Advantage:
- Quadratic function of displacement

$$w = \frac{\lambda}{2} (\text{div} U)^2 + \mu \lVert \nabla U \rVert^2 - \frac{\mu}{2} \lVert \text{rot} U \rVert^2$$

Drawback:
- Not invariant with respect to global rotation

Extension for anisotropic materials
Shortcomings of linear elasticity

Non valid for «large rotations and displacements»

St-Venant Kirchoff Elasticity

Isotropic Energy

\[ w(X) = \frac{\lambda}{2} \left( \text{tr} \ E \right)^2 + \mu \text{tr} \ E^2 \]

\((\lambda, \mu) : \text{Lamé coefficients}\)

Advantage:
- Generalize linear elasticity
- Invariant to global rotations

Drawback:
- Poor behavior in compression
- Quartic function of displacement

Extension for anisotropic materials
St Venant Kirchoff vs Linear Elasticity

Rest Position

Linear

St Venant Kirchoff

Other Hyperelastic Material

Neo-Hookean Model

\[ w(X) = \frac{\mu}{2} trE + f(I_3) \]

Fung Isotropic Model

\[ w(X) = \frac{\mu}{2} e^{\nu E} + f(I_3) \]

Fung Anisotropic Model

\[ w(X) = \frac{\mu}{2} e^{\nu E} + \frac{k_1}{k_2} \left( e^\gamma (I_3 e^{-\gamma}) - 1 \right) + f(I_3) \]

Veronda-Westman

\[ w(X) = c_1 (e^{\gamma trE}) + c_2 trE^2 + f(I_3) \]

Mooney-Rivlin

\[ w(X) = c_{10} trE + c_{01} trE^2 + f(I_3) \]
Overview

Measuring Soft Tissue Deformation
Continuum Models of Soft Tissue
**Discretization Methods**
Interactive Simulation: SOFA Platform
Examples

Discretisation techniques

Four main approaches:
- Volumetric Mesh Based
- Surface Mesh Based
- Meshless
- Particles
Different types of meshes

Surface Elements:
- Triangle
  - 3, 12 nodes and more

Volume Elements:
- Tetrahedron
  - 4, 10 nodes
- Prismatic
  - 6, 15 nodes and more
- Hexahedron
  - 8, 20 nodes and more
- Quad
  - 4, 8 nodes and more

Structured vs Unstructured meshes

Example 1: Liver meshed with hexahedra
- 3 months work
  - (courtesy of ESI)

Example 2: Liver meshed with tetrahedra
- Automatically generated (1s)
Volumetric Mesh Discretization

Classical Approaches:
- Finite Element Method (weak form)
- Rayleigh Ritz Method (variational form)
- Finite Volume Method (conservation eq.)
- Finite Differences Method (strong form)

FEM, RRM, FVM are equivalent when using linear elements

Rayleigh-Ritz Method

Step 1: Choose
- Finite Element (e.g. linear tetrahedron)
- Mesh discretizing the domain of computation
- Hyperelastic Material with its parameters
- Boundary Conditions

\[ w(X) = \frac{\lambda}{2} \left( \text{tr} \, E \right)^2 + \mu \text{tr} \, E^2 \]

Young Modulus
Poisson Coefficient
Rayleigh-Ritz Method

Step 2
- Write the elastic energy required to deform a single element

\[ W_{T_i} = \sum_{jk} U_j^T [K]_{jk}^T U_k \]

\[ [K]_{jk} = \frac{1}{36V(T)} (\nu M_j M_j + \rho \nu M_j M_j + \nu (M_j \cdot M_j) [I_{d_{jk}}]) \]

\[ u(P_i) = Q_j - P_i = U_i \]

\[ u(X) = \sum_{i=1}^{4} \lambda_i(X) u(P_i) \]

\[ \nabla \lambda_i(X) = -\frac{M_i}{6V(T)} \]

\[ trE = -\sum_i \frac{M_i \cdot U_i}{6V(T)} \]

Rayleigh-Ritz Method

Step 3
- Sum to get the total elastic energy

\[ W(U) = \int_{\Omega_k} w(I_1, I_2, I_3) dX = \sum_{T_i} W_{T_i} = U^T K U \]

- Write the conservation of energy

\[ W(U) = F^T U + \int_{\Omega} \rho(X) (X \cdot g) dX \]

Internal Energy

Nodal Forces

Gravity Potential Energy
Rayleigh-Ritz Method

Step 3

- Write first variation of the energy:

**Linear Elasticity**

\[ KU = R \]  \hspace{1cm} Static case

\[ M\ddot{U} + C\dot{U} + KU = R(t) \]  \hspace{1cm} Dynamic case

**HyperElasticity=NonLinear Elasticity**

\[ K\left(U\right) = R \]  \hspace{1cm} Static case

\[ M\ddot{U} + C\dot{U} + K\left(U\right) = R(t) \]  \hspace{1cm} Dynamic case

Surface-Based Methods

Possible approaches:

- **Boundary Element Models (BEM)**
  - Based on the Green Function of the linear elastic operator
  - Requires homogeneous material

- **Matrix Condensation**
  - Full Matrix inversion

- **Iterative Precomputed Generation**
  - Solve 3*Ns equations F=KU
Other Methods

Meshless Methods
• Use only points inside and specific shape functions
• Can better optimize location of DOFs
• Can cope with large deformations
• Deformation accuracy unknown

Particles
• Smooth Particles Hydrodynamics that interact based on a state equation

Time Integration Scheme

Explicit Schemes:
• Euler, Runge Kutta
• Conditionally Stable: time step must be lower than a critical time step
• Fast update but not suitable for stiff materials

Implicit Schemes:
• Euler, Newmark
• Require solving a linear system of equations
Some Bibliography References

<table>
<thead>
<tr>
<th>Reference</th>
<th>Description</th>
</tr>
</thead>
</table>

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Interactive Simulation : SOFA Platform

Examples
Towards Realistic Interactive Simulation

Surgery Simulation must cope with several difficult technical issues:

- Soft Tissue Deformation
- Collision Detection
- Collision Response
- Haptics Rendering

Real-time Constraints:

- 25Hz for visual rendering
- 300-1000 Hz for haptic rendering

SOFA :: Objectives

Provide a common software framework for the medical simulation community

Enable component exchange to reduce development time

Promote collaboration among research groups

Enable validation and comparison of new algorithms

- www.sofa-framework.org
## SOFA :: Targeted Users

**Non-Technical end users**
- Rapid prototyping with XML scene descriptions
- Text editing – no compiling necessary
- Plug n’ play interface (Maya plug-in)

**Researchers and developers**
- Develop new application procedurally
- Add functionalities by writing new modules in C++

## SOFA :: a flexible and efficient framework

**Component Abstraction**
- Minimize inter-dependencies between components

**Objects have multi-modal representation**
- Visual, Behavior, Collision, Haptic, etc.

**Physics-based objects can be further decomposed**
- Degrees of Freedom
- Force Fields
- Integration Schemes
- Solvers
SOFA :: a flexible and efficient framework

Scene graph representation
• Common in computer graphics
• Dynamic hierarchy is useful for collision management
• New objects or complete scenes can be added easily

Transparent support for parallel computing
• GPU optimized computation
• Cluster-based computing

SOFA :: Current Results

Create complex and evolving simulations by combining new algorithms with algorithms already included in SOFA

Modify most parameters of the simulation by simply editing an XML file

Efficiently simulate the dynamics of interacting objects using abstract equation solvers

Reuse and easily compare various deformable models
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Examples

Example 1: Simulation of knee joint

Cruciate ligaments segmented from MRI
Collateral ligaments determined from geometry
Simulation in SOFA

Simulation of Liver Surgery

Gliding
Gripping
Cutting (pliers)
Cutting (US)
Example 3: Cardiac Simulation

4 Cardiac Phases:
- Filling
- Isovolumetric Contraction
- Ejection
- Isovolumetric Relaxation

2 Volumetric Conditions:
- Pressure Field in the endocardium
- Isovolumetric Constraint of myocardium

Fiber Tracking on the Average Cardiac DTI

Use cardiac fiber orientation based on Diffusion Tensor MRI

http://www.inria.fr/asclepios/software/MedINRIA