3D shape description and matching based on properties of real functions

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Abstract

This tutorial covers a variety of methods for 3D shape matching and retrieval that are characterized by the use of a real-valued function defined on the shape (mapping function) to derive its signature. The methods are discussed following an abstract conceptual framework that distinguishes among the three main components of these class of shape matching methods: shape analysis*, via the application of the mapping function, shape* description*, via the construction of a signature, and* comparison*, via the definition of a distance measure.*
Goal of the tutorial is to facilitate the understanding of the performance of the various methods by a methodical analysis of the properties of various methods at the three different stages.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Line and Curve Generation

1. Introduction

3D shape matching and retrieval are key aspects in the current panorama of search engines. Shape models carry a high value with them, and search engines able to retrieve this type of visual media would be surely useful to speed-up content design, re-use and processing. Keyword-based searching is simply not sufficient to achieve the necessary capability of resource exploration for 3D. Therefore, a variety of methods have been proposed in the literature to tackle the problem with different approaches that span from coarse filters suited to browse very large 3D repositories on the web, to domain-specific approaches.

Generally speaking, shape matching methods rely on the computation of a shape *description*, also called signature, that effectively captures some essential features of the object. The shape descriptions are then compared using an appropriate computational technique able to translate the similarity between objects into some distance between descriptors. The majority of the methods proposed in the literature mainly focus on geometric aspects, that is, the description characterizes the spatial distribution or extent of the object in the 3D space [NK01, OFCD02, KFR03]. From a practical point of view, the main advantage of these methods is that they do not make specific assumption on the topology of the digital models and the computational efficiency. Conversely, these methods generally fail in supporting more elaborate shape comparisons, such as partial matching or sub-part correspondence where the similarity has to be evaluated in terms of presence and similarity of features in the shapes. In this case, more sophisticated descriptions should be used, in order to properly characterize the essential features and store them in an efficient and salient structure. Several approaches to shape characterization have been adopted in the literature (e.g. curvature, level-sets, enclosed spheres), yielding to different structuring methods (e.g. patch segmentation, Reeb graph, skeletons, medial axis).

Given the complexity of the problem, understanding and evaluating the performance of methods for 3D matching is not an easy task: first of all, there is neither a single *best* shape characterization nor a single *best* similarity measure, and the solution largely depends on the type of shapes to be analyzed and on the application domains. Recently, a 3D shape retrieval contest has been proposed – SHREC – whose general objective is to evaluate the performances of 3D-shape retrieval algorithms http://www.aimatshape.net/event/SHREC/ . The initial results of the contest provided the first opportunity to analyze the various algorithms, their strengths, as well as their weaknesses, using a common test collection which allows a direct comparison of algorithms. A single test collection necessarily delivers only a partial view of the whole picture, and for this reason the contest quickly moved towards a multi-track organization, for partial and whole matching, polygon soup and watertight model matching, as well as a number of context-specific benchmarks, for example for mechanical part matching, molecule matching, or 3D face matching.

2. Tutorial focus and contribution

While the performance of retrieval can be evaluated in quantitative terms using appropriate benchmarks and ground truth, it is not easy to understand the contribution to the results of the various components of the retrieval system. The results, indeed, depend both on the shape descriptions and the comparison tools, which are very often quite intertwined. Moreover, existing surveys [BKS*05, TV04, BP06] mainly focus on a classification and discussion of geometry-oriented methods, which target the conversion of statistical and geometric shape analysis into feature vectors or histograms. The comparison among methods usually addresses properties of admissible input representations and formats, invariance of the description with respect to a transformation class, and retrieval performance.

The goal of the tutorial is to facilitate the understanding of the performance of the various methods by a methodical analysis of the properties of various methods at the three different stages of an abstract conceptual framework which distinguishes among the three main components of these class of shape matching methods: shape *analysis*, via the application of some mathematical technique, shape *description*, via the construction of a signature, and *comparison*, via the definition of a distance measure. More precisely, we will analyze in depth methods that approach the analysis phase by making use of the properties provided by some real function f, called the mapping function, defined on the surface $\mathcal{M}$ representing the 3D object. Therefore, the underlying conceptual framework is structured in three-steps:

1. choice and evaluation of the real functions f_i on 3D shapes $\mathcal{M}_i$;
2. construction of *high-level descriptors* $\mathcal{G}_i$ of $\mathcal{M}_i$, using f_i;
3. choice of the comparison techniques to be used for the set of shapes and descriptors $\{(\mathcal{M}_i, \mathcal{G}_i)\}_i$.

We believe that the discussion of the properties at the three levels will facilitate the evaluation of theoretical and practical performances of the methods, will indicate more precisely the strength and weaknesses of the methods, and will also suggest a way for adopting different shape descriptors according to the properties and invariants that one wishes to investigate. The choice of the real function and the nature of the descriptor play indeed the role of the "lens" through which we look at the properties of the shape. The generality and flexibility of the framework is of interest for a wide research community with applications to visualization and topological modeling. In this tutorial, we will overview and analyze a large set of solutions, evaluate their effectiveness, and discuss perspectives, open issues, and future developments.

3. Outline

The proposed tutorial relies on recent survey work of the authors in related fields, see [BFF*06, Mar05, BAB*07].

The updated version of the slides presented at Eurographics 2007 will be made available at the following URL: http://www.ge.imati.cnr.it/ima/smg/training.html

In the following, we outline the main items that we plan to discuss in the tutorial, by giving for each group a synthetic description of the methods and a summary of the most relevant references, which will be discussed in detail and with examples and emphasis on shape matching applications.

3.1. Shape matching: motivations and challenges

The first part of the tutorial will provide an introduction to the tutorial, explain the rationale of the presentation, and introduce some of the main challenges of the topic area and its perspective impact in a number of crucial applications.

3.2. Properties of the real functions

A variety of different functions have been used in the shape matching literature for characterizing relevant features of objects. In general, the availability of *a-priori* information on the classes of the input database can be used to select the mapping functions which are best suited to identify specific shape features (e.g., protrusions), thus constraining the retrieval to match them with a higher degree of importance with respect to other features. This par of the tutorial will provide some introductory definitions on the basic concepts that will be discussed, concerning critical points, Morse function, level sets and briefly introduce their discretization [Ban70, Ban67, GP74, Mil63]. Following, a variety of real-valued functions will be presented and discussed, grouped into four main categories according to their definition, domain and properties:

- the *height* [SKK91, FK97] function is among the most intuitive and simple choices for analysing the shape of an object; since it depends on the direction considered, its usage is preferred for applications in which objects have a natural predefined direction (Figure 1(a)). A more elaborate characterization of the shape according to differences in the elevation value is provided by the *elevation* [AEHW06] function, which derives from the traditional height function but aims at a rotation invariant analysis. The notion of elevation captured by this function measures how much a point is relevant in its normal direction with respect to its neighbourhood. The elevation function is defined by pairing the critical points of the height function in all directions.

- Shape properties can be effectively characterized by measuring distances between feature points or by evaluating the elongation of the shape. In this broad class, the analysis approaches based on the *geodesic distance* generally provide and isometry invariant characterization of a shape [BBK06a]. Geodesic distance has been applied in several settings, in particular for the evaluation the geodesic distance of mesh vertices from selected feature points [MP02, EK03], and for averaging all geodesic distances among the vertices [HSKK01, KT03, GSCO07] . The *Euclidean distance from a point* $\mathbf{p} \in \mathbb{R}^3$ [FK97, SV01] (e.g., the barycentre of $\mathcal{M}$, Figure 1(b)) has also been used, as it is invariant to the shape embedding and detects protrusions (resp. hollows) of $\mathcal{M}$ with respect to $\mathbf{p}$ as regions of influence of maxima (resp. minima) f.
- *curvature-based analysis* have been frequently used to characterize the shape of 3D objects; generally, curvature-based analysis are rather sensible to noise or small features and to the quality of the shape discretization in terms of sampling density and tiny triangles. More robust computation is achieved either using variations of the curvature evaluation function (e.g. [GCO06]), polynomial surface fitting [ZP01], or with a multi-scale curvature evaluation where details are discarded [MPS*04].
- The *local diameters* function [GSCO07] aims at measuring the shape by computing the *diameter* of the volume enclosed by the surface. Therefore, it provides a volumetric rather than a boundary characterization, similarly to the *distance tranforms* [DS06] which is more focused on the medial axis radius.
- If the shapes to be compared do not exhibit a uniform structure, *harmonic* [NGH04, Flo97, PP93] and *Laplacian-based* functions [RWP06, DBG*06] may provide a new and powerful set of descriptors for shape analysis as they are intrinsically defined by the Laplacian matrix of the shape (see Figure 1(c-d)).
  We will discuss the numerical (in)stability of extraction of this type of functions from the Laplacian matrix of the shape $\mathcal{M}$, a very relevant aspect that has to be considered to understand at which extent this instability affects the descriptor of $\mathcal{M}$, and eventually the matching algorithm [GV89].

The presentation and discussion of the above-cited functions will be carried out considering:

- the *saliency* of f, as its ability to identify relevant shape features of $\mathcal{M}$;
- the *smoothness* degree of f, meant as its behaviour with respect to the number, nature and properties of its critical points;

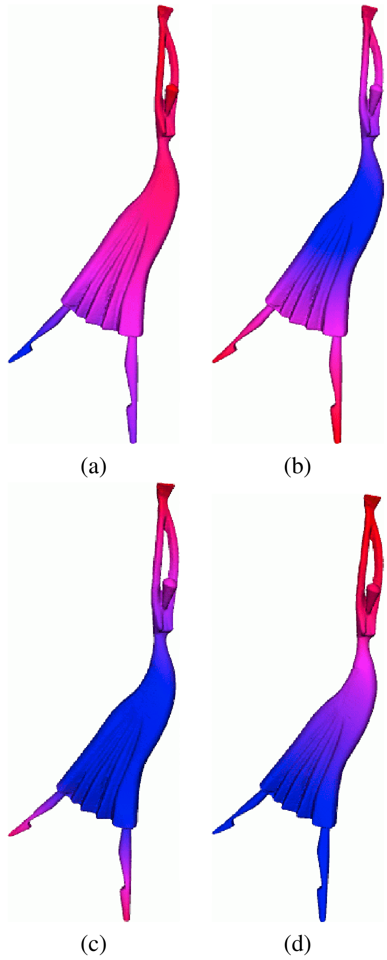

Figure 1: *(a) Height function, (b) Euclidean distance from the center of mass, (c) harmonic function, (d) first eigenfunction of the Laplacian matrix of the model.*

- the *stability* of f with respect to its discretization and computation on $\mathcal{M}$;
- the *robustness* of f, that is, the variation of f with respect to small geometric changes of the shape $\mathcal{M}$;
- the *degree of freedom* (DoF) and the number of *heuristics* used in the definition and evaluation of f.
- the *efficiency* of f in terms of the computational cost required by its evaluation on $\mathcal{M}$;
- the *invariance* of f to transformation groups;
- the hypotheses or restrictions on the *input*.

The analysis of the properties and the potentialities of the fs will provide an insight into the formalization of function suites, beyond a generic best-practice or rule-of-thumbs.

3.3. Properties of the shape descriptors

In the literature, it is quite common that functions used to analyse the shape are directly associated to a corresponding signature, or shape descriptor. For some of the methods this association is exclusive, meaning that no other function can be used to produce the same descriptor, while for other methods the descriptor is *parametric* with respect to the choice of the function.

Among shape descriptors that are parametric with respect to the choice of f, we will present:

- Reeb graphs [Ree46, CMEH*03, HSKK01, ABS03, Bia04, TS05, BFS00], size theory [Fro90, FL01, FL99, dFL06, FM99, BCF*07] and persistent homology tools [ELZ02, CZCG04, CZCG05, WAB*05, ZC05, CSEH05, CSEH07] are topological descriptors that root in Morse theory. When the function f varies, a collection of descriptors may be obtained. For any f, these descriptors code the shape by the configuration of elements or properties that characterize the topological evolution of level sets or lower level sets of f, see Figure 2;
- descriptors that decompose a function f given over simpler basis functions; examples are the spherical harmonic shape decompositions [KFR03, Vra04, VSR01] and wavelets-based methods [LTN06].

Among shape descriptors that exclusively linked to a specific choice of f, we will present:

- descriptors based on quantities extracted by intrinsic shape functions, such as the spectrum of the Laplace-Beltrami operator [RWP06, RWP07, NRW*07];
- descriptors built on isometry invariant quantities, as for example the geodesic function [JZ06, JZ05, EK03, BBK05, BBK06b, BBK06a] or the curvature [ZP01, GCO06];
- the pose-oblivious shape signature [GSCO07], that associate to $\mathcal{M}$ histograms of the distribution over the shape of two real functions, the first related to surface and the second to volume information;
- the centerline skeleton that connects feature points through the geodesic distance [MP02])

The shape descriptors will be presented from a theoretical and computational point of view, providing examples and results to assess different aspects, in particular:

- the *saliency* of the descriptor, that is its ability to capture the structure of the shape in terms of its features;
- the *concisness* of the descriptor, that is its ability to minimize the memory needed to store the descriptor while maximizing the amount of information represented; this property is related also to the type of output produced;
- the *robustness* with respect to small changes of the shape;
- the *unicity* of the descriptor: once the theoretical methodology for extracting the descriptor, the algorithm, and possible parameters have been chosen, the descriptor is unique;

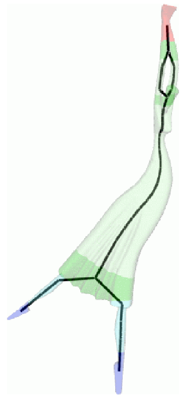

Figure 2: *(a) Reeb graph of the first eigenfunction of the Laplacian matrix of the model.*

- the *completness* in the sense that the same descriptor cannot be associated to different shapes;
- the *invariance* of the descriptor to transformation groups;
- the *degree of freedom* (DoF) and the number of *heuristics* used in the construction of the descriptor;
- the hypotheses or restrictions on the *input*;
- the *efficiency* of the descriptor in terms of the computational cost required by its construction.

3.4. Comparison methodologies

Although the surveyed descriptors are inspired by the same idea of quantifying geometric properties conveyed by f, there are substantial differences in the shape interpretation they provide and in the structures used to encode the shape information. In particular, the type of structure produced strongly influences the choice of the methods adopted for the final shape comparison step. The methodologies will be presented following a logical grouping according to the type of coding of the shape descriptor:

- the similarity between descriptors encoded as *histograms, feature vectors, or matrix structures* is evaluated by linear algebraic or statistical techniques [KFR03, Vra04, LTN06];
- the similarity among descriptors stored as *graphs* is generally evaluated by graph-matching techniques [HSKK01, SSGD03, LK03, CDS*05, BSRS04, ZSm*05, BRS06, BMSF06] (see Figure 3).
- the similarity between combinatorial descriptors is measured by friendly and computationally efficient tools, such as persistence diagrams and formal series [dFL06, BCF*07, CSEH07].

The methodologies will be presented and discussed highlighting their properties in terms of the following characteristics:

- the *properties* of the similarity measure that characterize

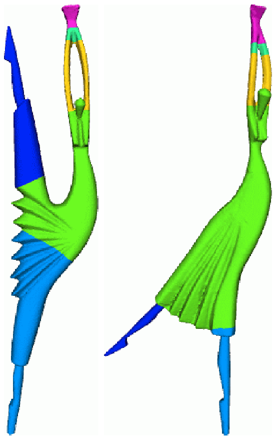

Figure 3: *Sub-part correspondence obtained using the graph comparison method defined in [BMSF06].*

it as a metric, semi-metric, or pseudo-distance [VH01, Tve77, SJ99];

- the *robustness* of the measure with respect to small changes of the shape;
- the *type of comparison* provide by the measure, in terms of supporting global, partial or sub.part correspondence;
- the *type of information*: according to the type of information stored and the way it is coded in the descriptor, the measure of similarity may take into account geometric, topological or structural information;
- the *efficiency* in terms of computational complexity required to evaluate the measure;
- the *application scenario* in which the comparison is performed.

3.5. Conclusions and future perspectives

In the conclusive part of the tutorial, we will try to provide a coherent comparison of the various techniques at the three levels of the framework, based on the analysis provided for all the aspects discussed. Obviously, the tutorial does not claim either to be an exhaustive survey of the wealth of existing methods for 3D matching or to examine all technical details of each single method. Rather, the objective of the comparison is to give a structured presentation of the methods in terms of the several properties of the descriptors and comparison tools, that are often not discussed in details in existing surveys. We believe that the presentation and discussions organized in this manner should serve as a basis for extending the performance analysis beyond standard precision-recall diagrams and help the user to understand if the reasons of good or bad retrieval results depend, for instance, on an insufficient efficacy of the descriptor, on an intrinsic instability of the function, or also on an inappropriate comparison tool.

Finally, we will list a series of topics deserving further research, such as the role of invariance with respect to transformation groups, the concurrent use of more than a single characterizing function, and the need to balance the use of geometrical and topological information for accurate shape descriptions. Last but not least, we will also address issues related to the emerging use of semantic indicators to perfom matching and retrieval, based either on (semi)-automatic annotation of shapes or in supervised classification and prototype extraction.

4. Authors' CVs

Two research groups are involved:

The *Shape Modeling Group at CNR-IMATI-GE* works since years on topics related to geometric modelling with the main aim to describe the shape of objects through geometric and topological reasoning techniques. Lately, the research themes focus on broadening the role of traditional modelling with the definition of new representations, encapsulating also knowledge technologies methodologies, able to express also the semantic level at which the perception of shape is encoded. In this field, CNR-IMATI-GE is leading the FP6 European Project NoE AIM@SHAPE.

The team *Vision Mathematics of the Univ. of Bologna*, Dept. of Mathematics, works at the use of topology and geometry in robotic applications since 1988. Mainly, the team deals with computer vision by means of a shape descriptor (the Size Functions) conceived and developed by P. Frosini. But the group interests cover a fairly wide area reaching from the abstractions of manifold topology to robot navigation and to concrete application projects.

Bianca Falcidieno is Research Director at CNR and head of the Shape Modelling Group, working in the field of Applied Mathematics and Computer Science, with applications in Computer Graphics, Geographic Information Systems, and Industrial Design. She is Editor in chief of the International Journal Shape Modelling, member of the Steering Committee of Shape Modeling International (SMI), and author of more than 200 scientific refereed papers and books. Bianca Falcidieno is the coordinator of the FP6 NoE AIM@SHAPE.

Patrizio Frosini is assistant professor in the Faculty of Engineering at the Univ. of Bologna. He is a member of the ARCES group at the Univ. of Bologna. He received the PhD degree in Mathematics from the Univ. of Florence (1991). His research interests include the study of geometrical-topological methods for shape comparison and related applications in Computer Vision.

Claudia Landi is assistant professor at the Univ. of Modena and Reggio Emilia in Reggio Emilia (Italy). She obtained a PhD in Mathematics in 2000, at the University of Pisa. Since 1994 she is member of the Vision Mathematics Group of the University of Bologna. Her main research interest is shape description via geometry and topology.

Michela Spagnuolo is senior researcher at CNR-IMATI-GE and received the Ph.D. in Computer Science Engineering, at the INSA, Lyon, France (1997). Her research interests are related to shape-based approaches to modeling digital shapes, computational topology techniques for shape analysis, geometric reasoning for the extraction of shape features from discrete surface models, and geometric models for coding uncertainty in data samples (fuzzy-based modelling). She is a member of the Steering Committee of Shape Modeling International (SMI).

Silvia Biasotti is researcher at CNR-IMATI-GE and received a Ph.D. in Mathematics and Applications at the Univ. of Genoa (2004). Her research interests include computational topology, shape abstraction and skeleton representation of polyhedral surfaces.

Daniela Giorgi is research fellow at CNR-IMATI-GE and received a Ph.D. in Applied Mathematics at the Univ. of Padua (2006). Her research interests are in Pattern Recognition and topological methods for shape analysis.

Simone Marini is researcher at CNR-IMATI-GE and received a Ph.D. in Electronic and Computer Engineering at the Univ. of Genova (2005). His main interests concern evaluation of 3D shape similarity, graph comparison, and ontological representation of scientific concepts.

Giuseppe Patané is researcher at CNR-IMATI-GE and received a Ph.D. in Mathematics and Applications at the Univ. of Genova (2005). His research interests include numerical analysis (implicit surfaces), shape analysis, computational geometry (topological graphs, local and global parameterization).

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References

[ABS03] ATTENE M., BIASOTTI S., SPAGNUOLO M.: Shape understanding by contour-driven retiling. *The Visual Computer 19*, 2-3 (2003), 127–138.

[AEHW06] AGARWAL P. K., EDELSBRUNNER H., HARER J., WANG Y.: Extreme elevation on a 2-manifold. *Discrete Comput. Geom. 36*, 4 (2006), 553–572.

[BAB*07] BIASOTTI S., ATTALI D., BOISSONNAT J.-D., EDELSBRUNNER H., ELBER G., MORTARA M., DI BAJA G. S., SPAGNUOLO M., TANASE M., VELTKAMP R.: Skeletal structures. In *Shape Analysis and Structuring*, Floriani L. D., Spagnuolo M., (Eds.). Springer-Verlag, 2007.

[Ban67] BANCHOFF T.: Critical points and curvature for embedded polyhedra. *Journal of Differential Geometry 1* (1967), 245–256.

[Ban70] BANCHOFF T. F.: Critical points and curvature for embedded polyhedral surfaces. *Am. math. Monthly 77* (1970), 475–485.

[BBK05] BRONSTEIN A. M., BRONSTEIN M. M., KIMMEL R.: On isometric embedding of facial surfaces into $\mathbb{S}^3$. In *Proceedings International Conference on Scale Space and PDE Methods in Computer Vision* (2005), vol. 3459 of *Lecture Notes in Computer Science*, pp. 622–631.

[BBK06a] BRONSTEIN A. M., BRONSTEIN M. M., KIMMEL R.: Efficient computation of isometry-invariant distances between surfaces. *SIAM Journal on Scientific Computing 28*, 5 (2006), 1812–1836.

[BBK06b] BRONSTEIN A. M., BRONSTEIN M. M., KIMMEL R.: Generalized multidimensional scaling: A framework for isometry-invariant partial surface matching. *Proceedings National Academy of Science 103*, 5 (2006), 1168–1172.

[BCF*07] BIASOTTI S., CERRI A., FROSINI P., GIORGI D., LANDI C.: *Multidimensional size functions for shape comparison.* Tech. Rep. 4, IMATI, 2007.

[BFF*06] BIASOTTI S., FLORIANI L. D., FALCIDIENO B., FROSINI P., GIORGI D., LANDI C., PAPALEO L., SPAGNUOLO M.: *Geometrical-topological properties of real functions for describing shapes.* Tech. Rep. 5, IMATI, 2006.

[BFS00] BIASOTTI S., FALCIDIENO B., SPAGNUOLO M.: Extended Reeb Graphs for surface understanding and description. In *Discrete Geometry for Computer Imagery Conference* (2000), Borgefors G., di Baja G. S., (Eds.), vol. 1953 of *Lecture Notes in Computer Science*, Springer, pp. 185–197.

[Bia04] BIASOTTI S.: *Computational Topology Methods for Shape Modelling Applications*. PhD thesis, Universitá degli Studi di Genova, May 2004.

[BKS*05] BUSTOS B., KEIM D. A., SAUPE D., SCHRECK T., VRANIĆ D. V.: Feature-based similarity search in 3D object databases. *ACM Computing Surveys 37*, 4 (December 2005), 345–387.

[BMSF06] BIASOTTI S., MARINI S., SPAGNUOLO M., FALCIDIENO B.: Sub-part correspondence by structural descriptors of 3D shapes. *Computer-Aided Design 38*, 9 (2006), 1002–1019.

[BP06] BIMBO A. D., PALA P.: Content-based retrieval of 3D models. *ACM Trans. on Multimedia Computing, Communications and Applications 2*, 1 (2006), 20–43.

[BRS06] BESPALOV D., REGLI W. C., SHOKOUFANDEH A.: Local feature extraction and matching partial objects. *Computer-Aided Design 38*, 9 (2006), 1020–1037.

[BSRS04] BESPALOV D., SHOKOUFANDEH A., REGLI W. C., SUN W.: Local feature extraction using scale-space decomposition. In *ASME Design Engineering Tech-*

nical Conferences, Computers and Information in Engineering Conference Conference (DETC 2004-57702) (Sep 2004), ASME Pres.

[CDS*05] CORNEA N. D., DEMIRCI M. F., SILVER D., SHOKOUFANDEH A., DICKINSON S. J., KANTOR P. B.: 3D object retrieval using many-to-many matching of curve skeletons. In *Proc. of Shape Modeling and Applications 2005 (SMI' 05)* (2005), IEEE Computer Society, pp. 368–373.

[CMEH*03] COLE-MCLAUGHLIN K., EDELSBRUNNER H., HARER J., NATARAJAN V., PASCUCCI V.: Loops in Reeb graphs of 2-manifolds. In *Proc. of the ACM Symposium on Computational Geometry* (2003), ACM Press, pp. 344–350.

[CSEH05] COHEN-STEINER D., EDELSBRUNNER H., HARER J.: Stability of persistence diagrams. In *Proceedings of SoCG 2005: ACM Symposium on Computational Geometry* (2005), pp. 263–271.

[CSEH07] COHEN-STEINER D., EDELSBRUNNER H., HARER J.: Stability of persistence diagrams. *Discrete Computational Geometry 37* (2007), 103–120.

[CZCG04] COLLINS G., ZOMORODIAN A., CARLSSON A., GUIBAS L. J.: A barcodes shape descriptor for curve point cloud data. *Computers and Graphics 28* (2004), 881–894.

[CZCG05] CARLSSON G., ZOMORODIAN A., COLLINS A., GUIBAS L. J.: Persistence barcodes for shapes. *Intern. Journal of Shape Modeling 11*, 2 (2005), 149–187.

[DBG*06] DONG S., BREMER P.-T., GARLAND M., PASCUCCI V., HART J.: Spectral surface quadrangulation. *ACM SIGGRAPH 25*, 3 (August 2006). Proceedings SIGGRAPH 2006.

[dFL06] D'AMICO M., FROSINI P., LANDI C.: Using matching distance in size theory: a survey. *International Journal of Imaging Systems and Technology 16*, 5 (2006), 154–161.

[DS06] DEY T. K., SUN J.: Defining and computing curve-skeletons with medial geodesic function. In *Proc. of Symposium on Geometry Processing* (2006), pp. 143–152.

[EK03] ELAD A., KIMMEL R.: On bending invariant signatures for surfaces. *IEEE Trans. on Pattern Analysis and Machine Intelligence 25*, 10 (2003), 1285–1295.

[ELZ02] EDELSBRUNNER H., LETSCHER D., ZOMORODIAN A.: Topological persistence and simplification. *Discrete Computational Geometry 28* (2002), 511–533.

[FK97] FOMENKO A., KUNII T. L.: *Topological Modelling for Visualization*. Springer Verlag, 1997.

[FL99] FROSINI P., LANDI C.: Size theory as a topological tool for computer vision. *Pattern Recognition and Image Analysis 9* (1999), 596–603.

[FL01] FROSINI P., LANDI C.: Size functions and formal series. *Appl. Algebra Engrg. Comm. Comput. 12* (2001), 327–349.

[Flo97] FLOATER M. S.: Parametrization and smooth approximation of surface triangulations. *Computer Aided Geometric Design 14*, 3 (1997), 231–250.

[FM99] FROSINI P., MULAZZANI M.: Size homotopy groups for computation of natural size distances. *Bull. Belg. Math. Soc. 6* (1999), 455–464.

[Fro90] FROSINI P.: A distance for similarity classes of submanifolds of a Euclidean space. *Bull. Austral. Math. Soc. 42* (1990), 407–416.

[GCO06] GAL R., COHEN-OR D.: Salient geometric features for partial shape matching and similarity. *ACM Transactions on Graphics 25*, 1 (2006), 130–150.

[GP74] GUILLEMIN V., POLLACK A.: *Differential Topology.* Englewood Cliffs, NJ: Prentice-Hall, 1974.

[GSCO07] GAL R., SHAMIR A., COHEN-OR D.: Pose-oblivious shape signature. *IEEE Transactions on Visualization and Computer Graphics 13*, 2 (2007), 261–271.

[GV89] GOLUB G., VANLOAN G.: *Matrix Computations*. John Hopkins University Press, 2nd. edition, 1989.

[HSKK01] HILAGA M., SHINAGAWA Y., KOHMURA T., KUNII T. L.: Topology matching for fully automatic similarity estimation of 3D shapes. In *ACM SIGGRAPH* (2001), pp. 203–212.

[JZ05] JAIN V., ZHANG H.: A spectral approach to shape-based retrieval of articulated 3d models. *CAD 37*, 5 (2005), 509–530.

[JZ06] JAIN V., ZHANG H.: Robust 3d shape correspondence in the spectral domain. In *Proc. of Shape Modeling International* (2006), pp. 118–129.

[KFR03] KAZHDAN M., FUNKHOUSER T., RUSINKIEWICZ S.: Rotation invariant spherical harmonic representation of 3D shape descriptors. In *Proc. of Symposium on Geometry Processing* (2003), pp. 156–165.

[KT03] KATZ S., TAL A.: Hierarchical mesh decomposition using fuzzy clustering and cuts. *ACM SIGGRAPH* (July 2003), 954–961.

[LK03] LEYMARIE F. F., KIMIA B. B.: Computation of the shock scaffold for unorganized point clouds in 3*d*. In *Computer Vision and Pattern Recognition* (June 2003), vol. 1, pp. 821–827.

[LTN06] LAGA H., TAKAHASHI H., NAKAJIMA M.: Spherical wavelet descriptors for content-based 3d model retrieval. In *Proc. of Shape Modeling and Applications* (2006), pp. 15–25.

[Mar05] MARINI S.: *3D Shape Similarity Through Structural Descriptors*. PhD thesis, University of Genova, April 2005.

[Mil63] MILNOR J.: *Morse Theory.* Princeton University Press, New Jersey, 1963.

[MP02] MORTARA M., PATANÉ G.: Shape-covering for skeleton extraction. *Int. Journal of Shape Modelling 8*, 2 (2002), 245–252.

[MPS*04] MORTARA M., PATANE G., SPAGNUOLO M., FALCIDIENO B., ROSSIGNAC J.: Blowing bubbles for multi-scale analysis and decomposition of triangle meshes. *Algorithmica 38*, 1 (2004), 227–248.

[NGH04] NI X., GARLAND M., HART J. C.: Fair Morse functions for extracting the topological structure of a surface mesh. *ACM SIGGRAPH 23*, 3 (2004), 613–622.

[NK01] NOVOTNI M., KLEIN R.: A geometric approach to 3D object comparison. In *Proc. of Shape Modelling and Applications* (2001), pp. 167–175.

[NRW*07] NIETHAMMER M., REUTER M., WOLTER F.-E., BOUIX S., PEINECKE N., KOO M.-S., SHENTON M.: Global medical shape analysis using the laplace-beltrami spectrum. In *MICCAI07, 10th International Converence on Medical Image Computing and Computer Assisted Intervention* (2007).

[OFCD02] OSADA R., FUNKHOUSER T., CHAZELLE B., DOBKIN D.: Shape distributions. *ACM Trans. Graph. 21*, 4 (2002), 807–832.

[PP93] PINKALL U., POLTHIER K.: Computing discrete minimal surfaces and their conjugates. *Experimental Mathematics* (1993), 15–36.

[Ree46] REEB G.: Sur les points singuli'ers d'une forme de Pfaff complètement intégrable ou d'une fonction numèrique. *Comptes Rendu Hebdomadaires des Séances de l'Académie des Sciences 222* (1946), 847–849.

[RWP06] REUTER M., WOLTER F.-E., PEINECKE N.: Laplace-Beltrami spectra as Shape-DNA of surfaces and solids. *Computer-Aided Design 38*, 4 (2006), 342–366.

[RWP07] REUTER M., WOLTER F.-E., PEINECKE N.: Laplace spectra as fingerprints for image recognition. *Computer-Aided Design 39* (2007), 460–476.

[SJ99] S S., JAIN R.: Similarity measures. *IEEE Transactions on Pattern Analysis and Machine Intelligence 21*, 9 (September 1999), 871–883.

[SKK91] SHINAGAWA Y., KUNII T. L., KERGOSIAN Y. L.: Surface coding based on Morse theory. *IEEE Computer Graphics and Applications 11* (1991), 66–78.

[SSGD03] SUNDAR H., SILVER D., GAGVANI N., DICKINSON S.: Skeleton based shape matching and retrieval. In *Proc. of Shape Modelling and Applications* (2003), pp. 130–139.

[SV01] SAUPE D., VRANIC D. V.: 3d model retrieval with spherical harmonics and moments. In *Proceedings of the 23rd DAGM-Symposium on Pattern Recognition* (London, UK, 2001), Springer-Verlag, pp. 392–397.

[TS05] TUNG T., SCHMITT F.: The Augmented Multiresolution Reeb Graph approach for content-based retrieval of 3D shapes. *Int. Journal of Shape Modelling 11*, 1 (June 2005), 91–120.

[TV04] TANGELDER J., VELTKAMP R.: A survey of content based 3d shape retrieval methods. In *Proceedings of Shape Modeling Applications, 2004* (2004), pp. 145–156.

[Tve77] TVERSKY A.: Features of similarity. *Psychological Review 84* (1977), 327–352.

[VH01] VELTKAMP R. C., HAGENDOORN M.: State-of-the-Art in Shape Matching. In *Principles of Visual Information Retrieval*, Lew M., (Ed.). Springer-Verlag, 2001, pp. 87–119.

[Vra04] VRANIC D.: *3D model retrieval*. PhD thesis, University Leipzig, June 2004.

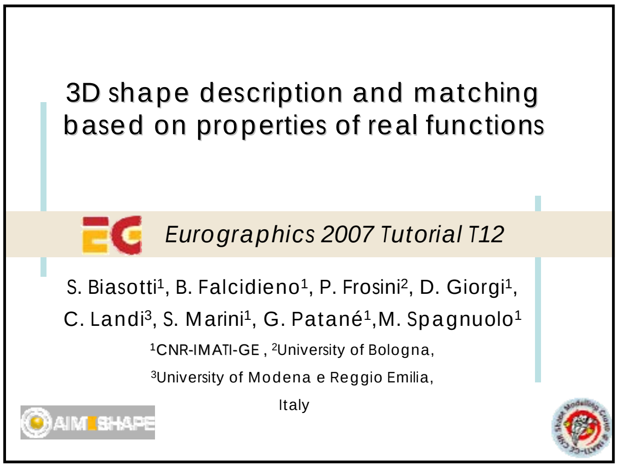
[VSR01] VRANIC D., SAUPE D., RICHTER J.: Tool for 3D-object retrieval: Karhunen-Loeve transform and spherical harmonics. In *Proc. of IEEE 2001 Workshop on Multimedia Signal Processing* (2001), Dugelay J.-L., Rose K., (Eds.), pp. 293–298.

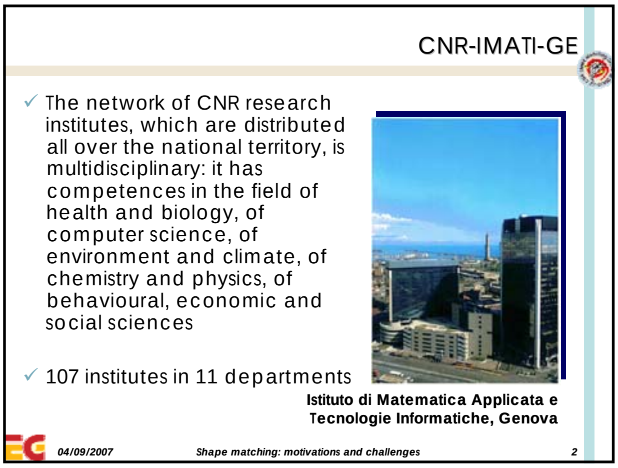
[WAB*05] WANG Y., AGARWAL P. K., BROWN P., EDELSBRUNNER H., RUDOLPH J.: Coarse and reliable geometric alignment for protein docking. In *Proceedings of Pacific Symposium on Biocomputing* (2005), pp. 64–75.

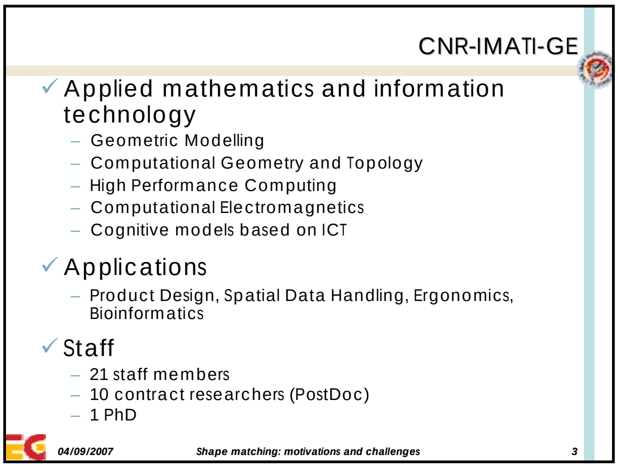
[ZC05] ZOMORODIAN A., CARLSSON G.: Computing persistent homology. *Discrete Computational Geometry 33* (2005), 249–274.

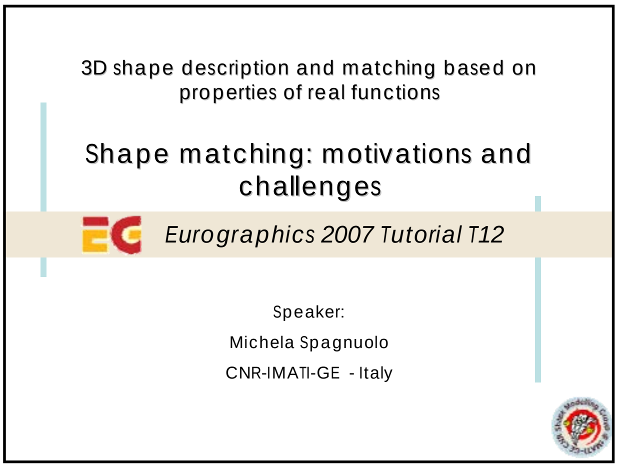
[ZP01] ZAHARIA T., PRETEUX F. J.: 3D-shape-based retrieval within the MPEG-7 framework. In *Nonlinear Image Processing and Pattern Analysis XII, Edward R. Dougherty; Jaakko T. Astola; Eds.* (May 2001), Dougherty E. R., Astola J. T., (Eds.), vol. 4304 of *Proc. of the Society of Photo-Optical Instrumentation Engineers (SPIE) Conference*, pp. 133–145.

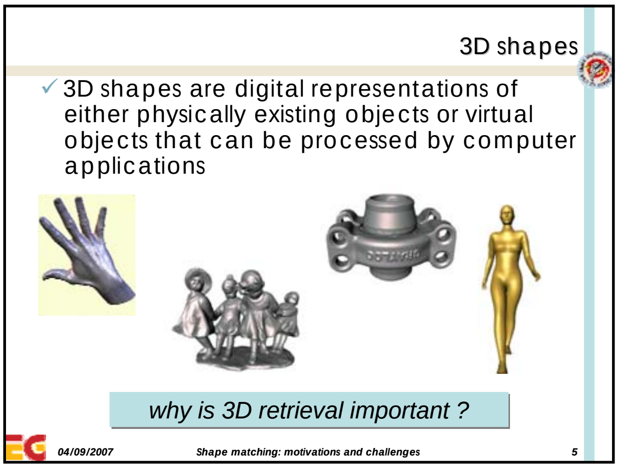
[ZSm*05] ZHANG J., SIDDIQI K., MACRINI D., SHOKOUFANDEH A., DICKINSON S.: Retrieving articulated 3-D models using medial surfaces and their graph spectra. In *Proc. of Int. Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition* (2005).

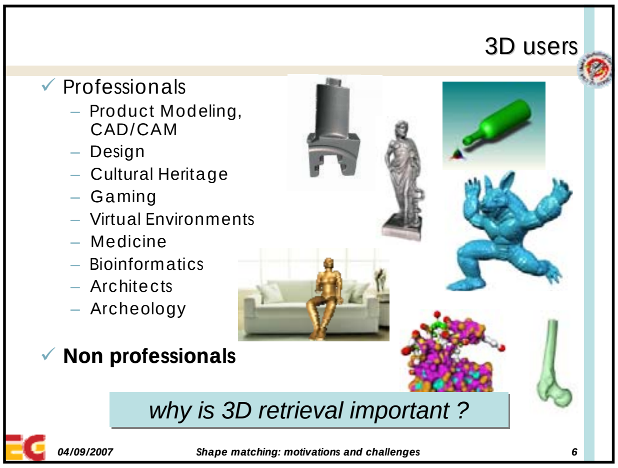

3D shape description and matching based on properties of real functions
Eurographics 2007 Tutorial T12
S. Biasotti[1], B. Falcidieno[1], P. Frosini[2], D. Giorgi[1], C. Landi[3], S. Marini[1], G. Patané[1], M. Spagnuolo[1]
[1]CNR-IMATI-GE , [2]University of Bologna, [3]University of Modena e Reggio Emilia, Italy
AIM@SHAPE


CNR-IMATI-GE
The network of CNR research institutes, which are distributed all over the national territory, is multidisciplinary: it has competences in the field of health and biology, of computer science, of environment and climate, of chemistry and physics, of behavioural, economic and social sciences
107 institutes in 11 departments
Istituto di Matematica Applicata e Tecnologie Informatiche, Genova
04/09/2007
Shape matching: motivations and challenges
2


CNR-IMATI-GE
Applied mathematics and information technology
Geometric Modelling
Computational Geometry and Topology
High Performance Computing
Computational Electromagnetics
Cognitive models based on ICT
Applications
Product Design, Spatial Data Handling, Ergonomics, Bioinformatics
Staff
21 staff members
10 contract researchers (PostDoc)
1 PhD
04/09/2007
Shape matching: motivations and challenges
3


3D shape description and matching based on properties of real functions
Shape matching: motivations and challenges
Eurographics 2007 Tutorial T12
Speaker:
Michela Spagnuolo
CNR-IMATI-GE - Italy


3D shapes
3D shapes are digital representations of either physically existing objects or virtual objects that can be processed by computer applications
why is 3D retrieval important ?
04/09/2007
Shape matching: motivations and challenges
5


3D users
Professionals
Product Modeling, CAD/CAM
Design
Cultural Heritage
Gaming
Virtual Environments
Medicine
Bioinformatics
Architects
Archeology
Non professionals
why is 3D retrieval important ?
04/09/2007
Shape matching: motivations and challenges
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3D retrieval: why ?

- Due to great technological advances, 3D content is poised to become the 4th wave of multi-media

  - 3D shapes can be easily digitized
  - modelling and processing of 3D shapes are mature research fields (geometric modelling & computer graphics)
  - 3D content can be delivered easily as most of the PCs connected to the Internet are now equipped with high-performance 3D graphics hardware

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3D retrieval : why ?

- plenty of online stores selling 3D models
- target customers
  - gaming industry, entertainment, simulation
- industrial impact foreseen at short/medium-term
  - Product Design, 3D-TV, medical sector, gaming, ..
  - SecondLife, EverQuest II, Sony Exchange Station
- everyday life impact foreseen at long-term
  - *shapes*.google.com
  - 3D acquired and *streamed* on mobile devices

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3D media: why ?

04/09/2007 Shape matching: motivations and challenges 9

3D media: why ?

04/09/2007 Shape matching: motivations and challenges 10

3D retrieval

- 3D models carry a high value with them
- Search & retrieval is necessary for supporting content creation and re-use :
  - Designers
  - Product models
  - Drug design
  - Gaming assets
- Search & retrieval is necessary for supporting analysis and classification (*understanding*):
  - Medicine
  - Bioinformatics

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3D media: problems and peculiarities

- Data deluge for 3D content is even larger than for 2D content
  - Storage problems
  - Delivery problems
- Complete shape description
  - Recognition/classification of 2D content has to handle occlusion problems, while 3D content is based on the full representation of the shape
- Consistency
  - The representation space has the same dimension of the object space

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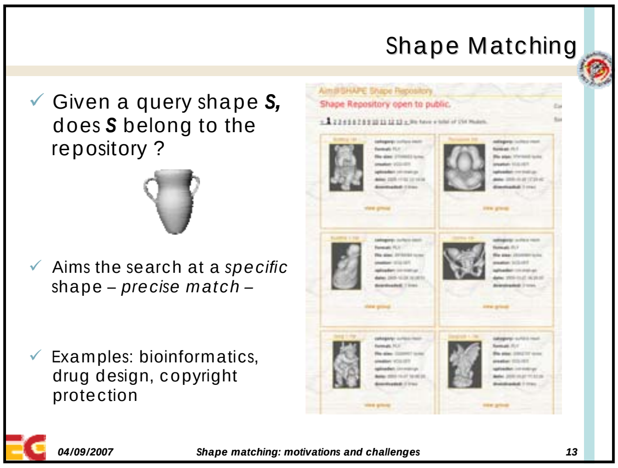
Shape Matching
Given a query shape S, does S belong to the repository ?
Aims the search at a specific shape – precise match –
Examples: bioinformatics, drug design, copyright protection
AIM@SHAPE Shape Repository
Shape Repository open to public.
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Shape matching: motivations and challenges
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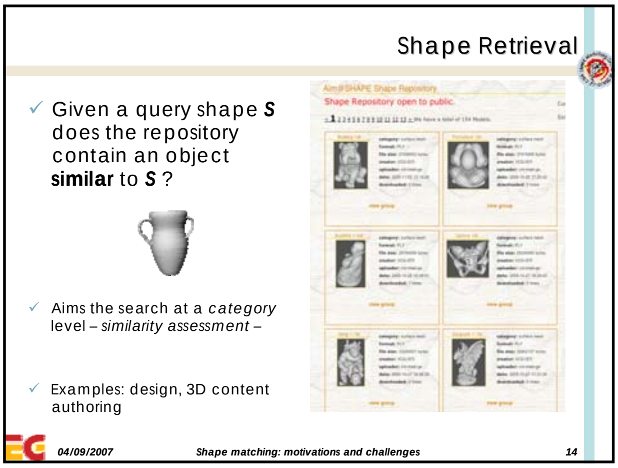
Shape Retrieval
Given a query shape S does the repository contain an object similar to S ?
Aims the search at a category level – similarity assessment –
Examples: design, 3D content authoring
AIM@SHAPE Shape Repository
Shape Repository open to public.
04/09/2007
Shape matching: motivations and challenges
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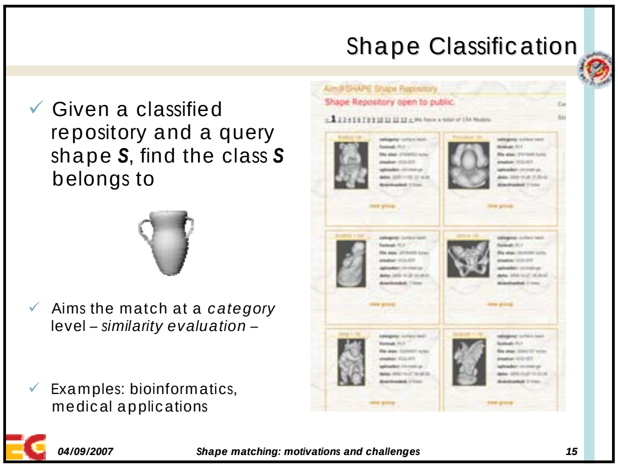
Shape Classification
Given a classified repository and a query shape S, find the class S belongs to
Aims the match at a category level – similarity evaluation –
Examples: bioinformatics, medical applications
AIM@SHAPE Shape Repository
Shape Repository open to public.
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Shape matching: motivations and challenges
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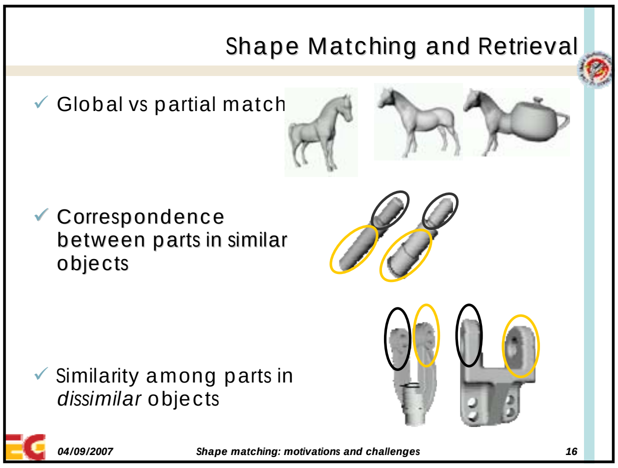
Shape Matching and Retrieval
Global vs partial match
Correspondence between parts in similar objects
Similarity among parts in dissimilar objects
04/09/2007
Shape matching: motivations and challenges
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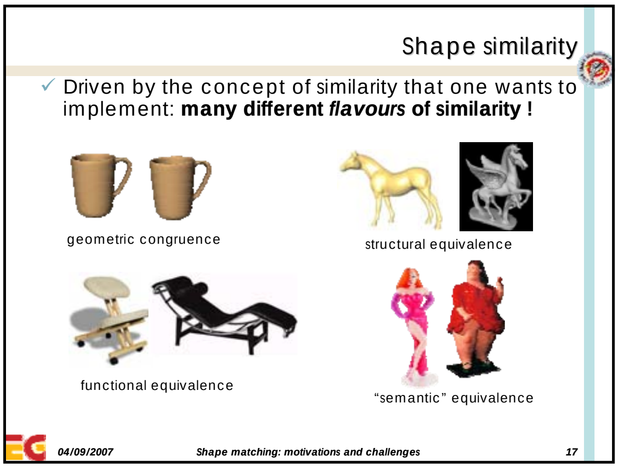
Shape similarity
Driven by the concept of similarity that one wants to implement: many different flavours of similarity !
geometric congruence
structural equivalence
functional equivalence
"semantic" equivalence
04/09/2007
Shape matching: motivations and challenges
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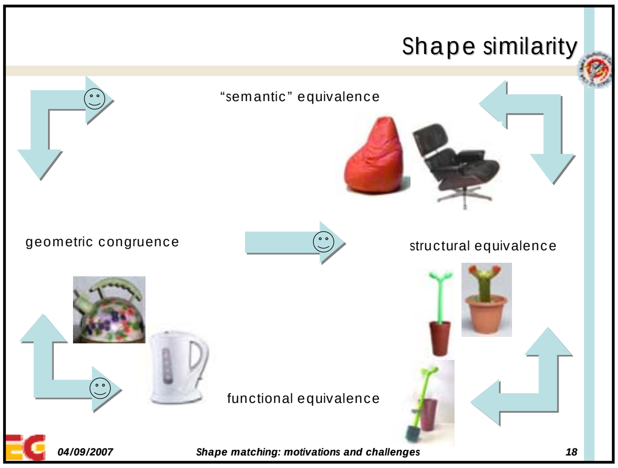
Shape similarity
"semantic" equivalence
geometric congruence
structural equivalence
functional equivalence
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Shape matching: motivations and challenges
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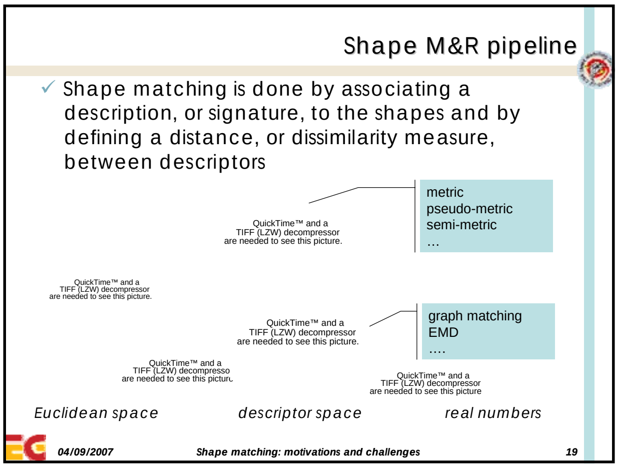
Shape M&R pipeline
Shape matching is done by associating a description, or signature, to the shapes and by defining a distance, or dissimilarity measure, between descriptors
metric
pseudo-metric
semi-metric
….
graph matching
EMD
….
Euclidean space
descriptor space
real numbers
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Shape matching: motivations and challenges
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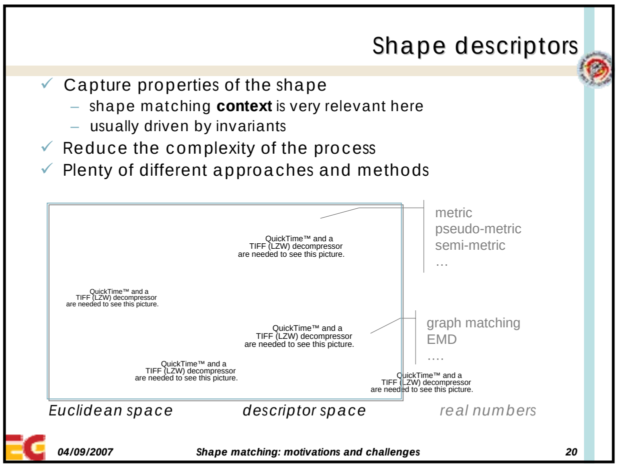
Shape descriptors
Capture properties of the shape
– shape matching **context** is very relevant here
– usually driven by invariants
Reduce the complexity of the process
Plenty of different approaches and methods
metric
pseudo-metric
semi-metric
…
graph matching
EMD
….
Euclidean space
descriptor space
real numbers
04/09/2007
Shape matching: motivations and challenges
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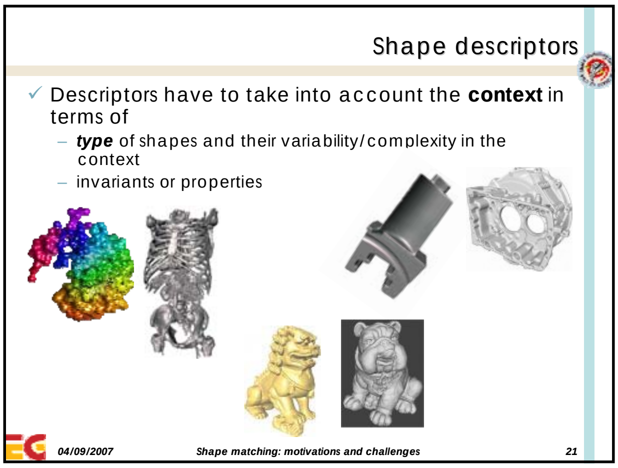
Shape descriptors
Descriptors have to take into account the **context** in terms of
– ***type*** of shapes and their variability/complexity in the context
– invariants or properties
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Shape matching: motivations and challenges
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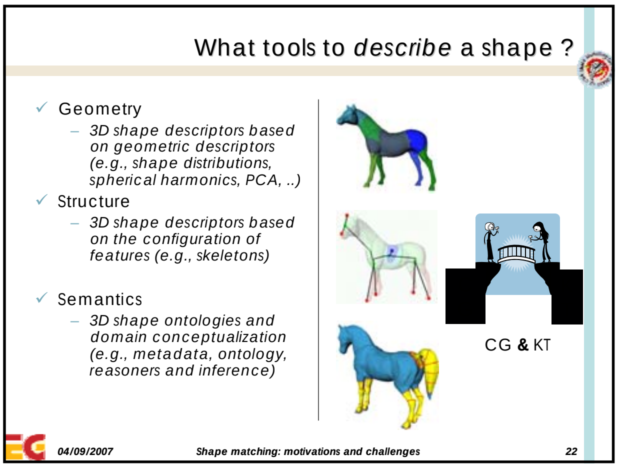
What tools to *describe* a shape ?
Geometry
– 3D shape descriptors based on geometric descriptors (e.g., shape distributions, spherical harmonics, PCA, ..)
Structure
– 3D shape descriptors based on the configuration of features (e.g., skeletons)
Semantics
– 3D shape ontologies and domain conceptualization (e.g., metadata, ontology, reasoners and inference)
CG & KT
04/09/2007
Shape matching: motivations and challenges
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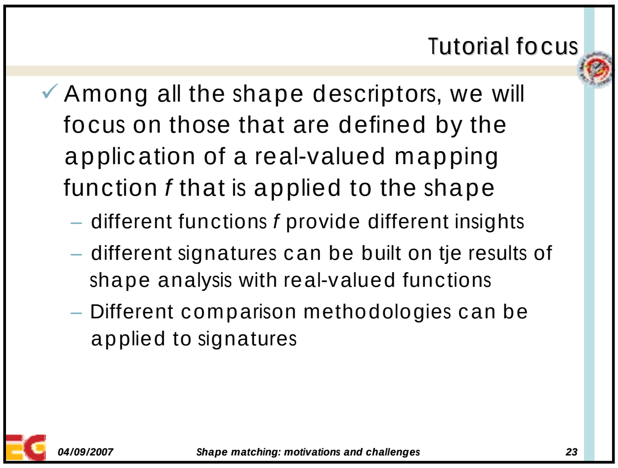
Tutorial focus
Among all the shape descriptors, we will focus on those that are defined by the application of a real-valued mapping function *f* that is applied to the shape
– different functions *f* provide different insights
– different signatures can be built on tje results of shape analysis with real-valued functions
– Different comparison methodologies can be applied to signatures
04/09/2007
Shape matching: motivations and challenges
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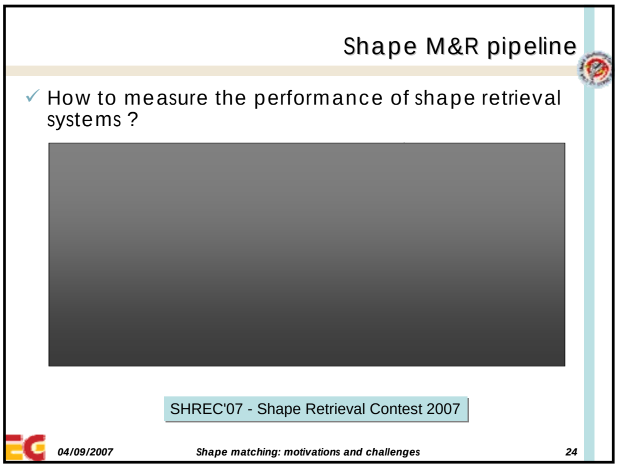
Shape M&R pipeline
How to measure the performance of shape retrieval systems ?
SHREC'07 - Shape Retrieval Contest 2007
04/09/2007
Shape matching: motivations and challenges
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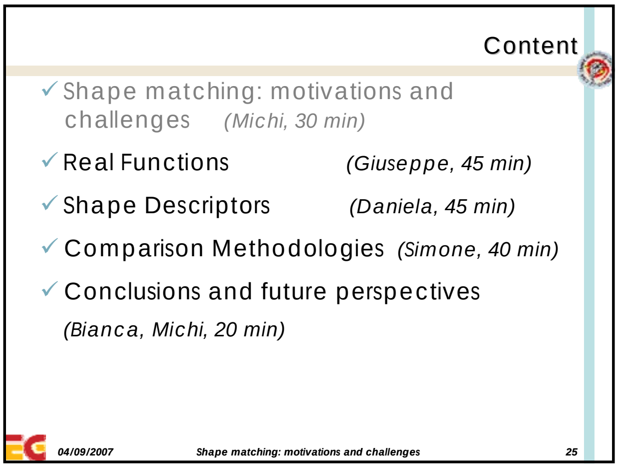
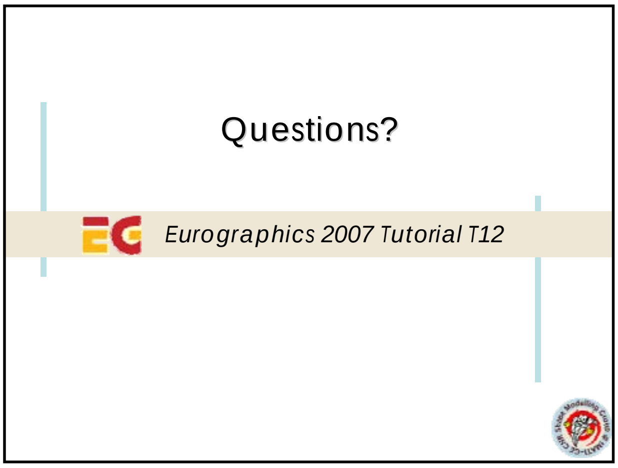

Content
✓Shape matching: motivations and challenges (Michi, 30 min)
✓Real Functions (Giuseppe, 45 min)
✓Shape Descriptors (Daniela, 45 min)
✓Comparison Methodologies (Simone, 40 min)
✓Conclusions and future perspectives (Bianca, Michi, 20 min)
04/09/2007
Shape matching: motivations and challenges
25


Questions?
Eurographics 2007 Tutorial T12

3D Shape Description and Matching Based on Properties of Real Functions

Real functions

Eurographics 2007 Tutorial T12

Speaker
Giuseppe Patanè
CNR-IMATI-GE - Italy

Outline

- Differential properties of real functions defined on 3D shapes.
- Discrete case:
  - critical points and level sets: definition and properties
  - functions:
    - definition (ie., height and elevation, Euclidean-, curvature- and geodesic-based functions, local diameters, harmonic function, Laplacian eigenfunctions)
    - properties (ie., saliency, smoothness, stability, robusteness, degrees of freedom and heuristics, efficiency, invariance).

Real functions 2

Critical points [GP76,Mil63]

- Given a smooth function *f* defined on a manifold:
  - a point *x* is called ***regular*** if the differential df_x is surjective
  - a point *x* is called ***critical*** is the differential df_x is the zero map
  - a critical point *x* is called ***non-degenerate*** if the Hessian matrix *H* of *f* is non-singular at *x*
  - if *x* is a non-degenerate critical point of *f*, then the number λ of negative eigenvalues of *H* is called the ***index*** of *x*.

Real functions 3

Morse functions and critical points

- In other words:
  - a point *p* is ***critical*** for *f* if:

$$\frac{\partial f}{\partial x_1}(p)=0, \frac{\partial f}{\partial x_2}(p)=0, \ldots, \frac{\partial f}{\partial x_k}(p)=0$$

  - *f* is ***Morse*** at *p* if:

$$|H_f(p)| = \left|\frac{\partial^2 f}{\partial x_i \partial x_j}(p)\right| \neq 0.$$

- The definition of critical points is *local* and *sensitive* to small perturbations of the surface.
- The function *f* is called ***simple*** if any pair *x*, *y* of distinct critical points verifies *f(x) ≠f(y)*.

Real functions 4

Morse functions

- On any smooth compact manifold there exist Morse functions.
- Morse functions are everywhere dense in the space of all smooth functions on the manifold.
- On a compact manifold, any Morse function has only a finite number of critical points .
- The set *S* of all simple Morse functions is everywhere dense in the set of all Morse functions.
- Examples are: *height function, distance functions, geodesic distance, etc*.

Real functions 5

Morse theory & critical points

- **Morse Lemma.**
  In a neighbourhood of each non-degenerate critical point *p*, the function *f* can be expressed as:

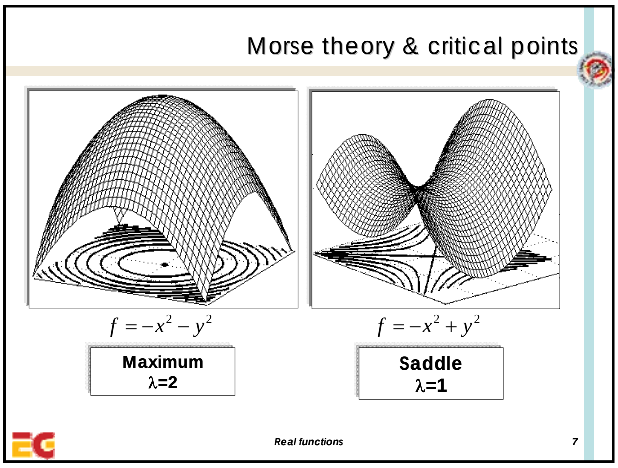
$$f = f(p) - (y_1)^2 - \ldots - (y_\lambda)^2 + (y_{\lambda+1})^2 + \ldots (y_n)^2$$

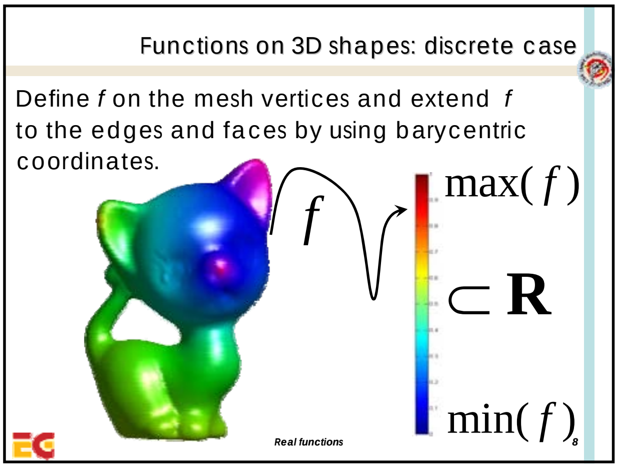
  where λ is the *index* of the critical point.

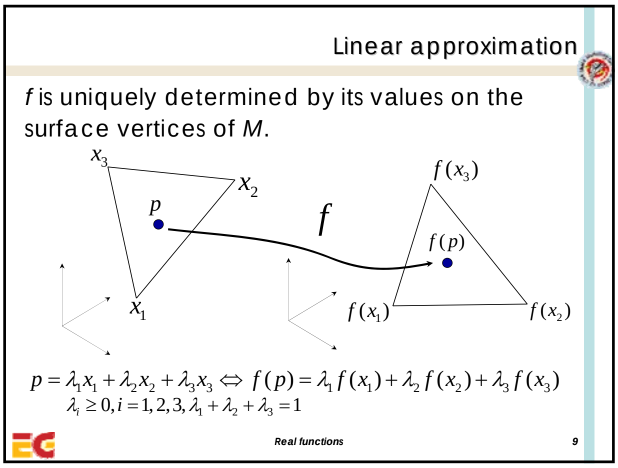
- **Euler formula.**

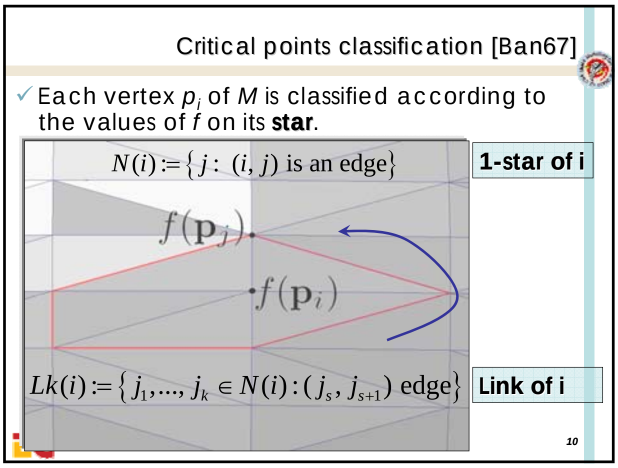
  #maxima - #saddles + #minima= χ(S).

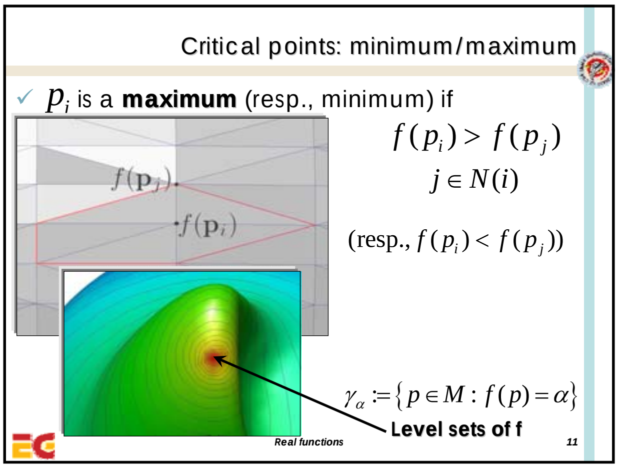
Real functions 6

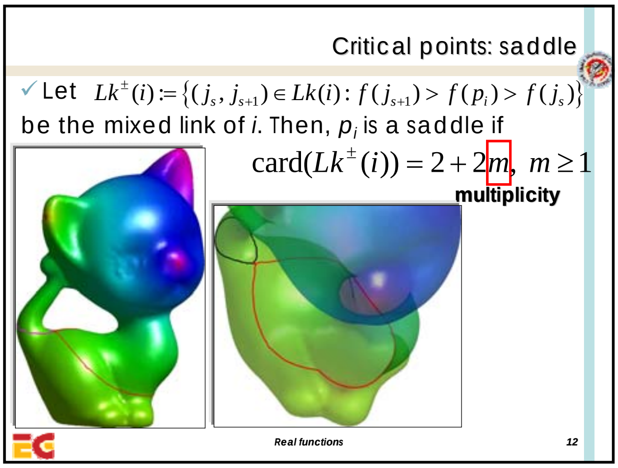

Morse theory & critical points
$f = -x^2 - y^2$
Maximum
λ=2
$f = -x^2 + y^2$
Saddle
λ=1
Real functions
7


Functions on 3D shapes: discrete case
Define f on the mesh vertices and extend f to the edges and faces by using barycentric coordinates.
f
max(f)
⊂ R
min(f)
Real functions
8


Linear approximation
f is uniquely determined by its values on the surface vertices of M.
x_3
x_2
x_1
p
f
$f(x_3)$
$f(p)$
$f(x_1)$
$f(x_2)$
$p = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \Leftrightarrow f(p) = \lambda_1 f(x_1) + \lambda_2 f(x_2) + \lambda_3 f(x_3)$
$\lambda_i \geq 0, i = 1,2,3, \lambda_1 + \lambda_2 + \lambda_3 = 1$
Real functions
9


Critical points classification [Ban67]
Each vertex p_i of M is classified according to the values of f on its star.
$N(i) := \{ j : (i, j) \text{ is an edge} \}$
1-star of i
$f(\mathbf{p}_j)$
$f(\mathbf{p}_i)$
$Lk(i) := \{ j_1, ..., j_k \in N(i) : (j_s, j_{s+1}) \text{ edge} \}$
Link of i
10


Critical points: minimum/maximum
p_i is a maximum (resp., minimum) if
$f(p_i) > f(p_j)$
$j \in N(i)$
(resp., $f(p_i) < f(p_j)$)
$f(\mathbf{p}_j)$
$f(\mathbf{p}_i)$
$\gamma_\alpha := \{ p \in M : f(p) = \alpha \}$
Level sets of f
Real functions
11


Critical points: saddle
Let $Lk^{\pm}(i) := \{ (j_s, j_{s+1}) \in Lk(i) : f(j_{s+1}) > f(p_i) > f(j_s) \}$
be the mixed link of i. Then, p_i is a saddle if
$\text{card}(Lk^{\pm}(i)) = 2 + 2m, \; m \geq 1$
multiplicity
Real functions
12

Critical point properties: discrete case [Ban67,Ban70]

- If f is **general** (ie., $f(x) \neq f(y)$, whenever x and y are distinct vertices of M), then the critical points of (M,f)
  - satisfy the Euler formula

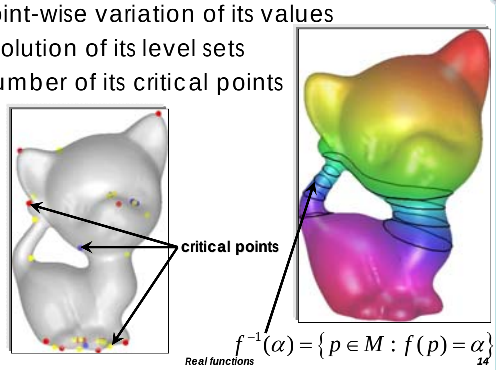
$$\chi(M) = \#\text{minima-}\#\text{saddles+}\#\text{maxima},$$

  where saddles are counted with their multiplicity m.
  - are located where the topological changes of (M,f) happen.

Real functions 13

How can we study the behavior of functions on M?

- Point-wise variation of its values
- Evolution of its level sets
- Number of its critical points
- …



$$f^{-1}(\alpha) = \{p \in M : f(p) = \alpha\}$$

Real functions 14

Evaluating the properties of f

- **Saliency:** ability to measure the shape features we are focusing on.
- **Smoothness:** behavior of f wrt the *nature* of its critical points.
- **Stability** wrt discretization and computation.
- **Robustness:** low variation of the f values wrt small changes of the shape.
- **DoF and heuristics:** number and type of parameters involved in the definition and/or computation of f.
- **Efficiency:** computation cost.
- **Invariance** wrt a group of transformations.

Real functions 15

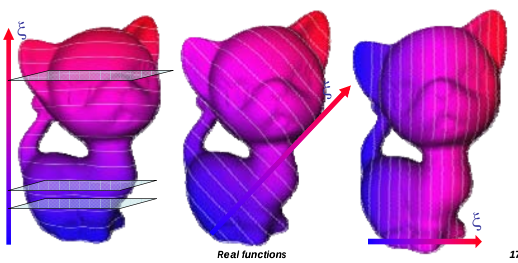
Examples

- Common choices of f are:
  - Height and elevation
  - Euclidean-based
  - curvature-based
  - geodesic-based
  - local diameters
  - harmonic functions
  - Laplacian eigenfunctions
  - …

Real functions 16

Height function [Ban70,FK97]

- Given a direction ξ, the height function value at $p \in M$ with respect to ξ is defined as $f_\xi(p) := \langle p, \xi \rangle$.
- The level sets correspond to the intersection of the surface with planes orthogonal to the direction ξ.



Real functions 17

Height function

- **Saliency:** f is able to identify the shape features of M along the direction ξ.
- **Smoothness:**
  - Critical points are points whose normal is parallel to the direction ξ.
  - In the continuous case, almost all height functions are Morse (ie, the critical points are non degenerate).
- **Stability:** exact evaluation/computation of f and interpolation on the faces/edges of M.

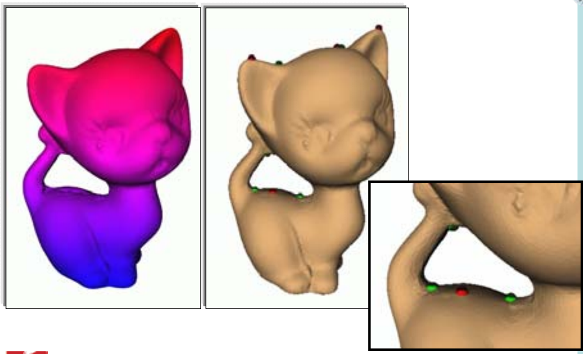
Real functions 18

Height function

- **Robustness:** the computation of *f* is robust, while its critical points aren't.
- **DoF and heuristic:** the choice of ξ.
- **Efficiency:** the computational cost is *O(1)*.
- **Invariance**: the function *f* is
  - invariant to translations
  - dependent on rotations: the recognized properties depend on the chosen direction.

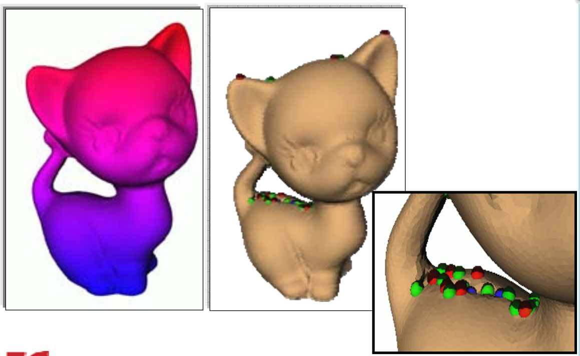
Real functions 19

Height function: robustness example



Real functions 20

Height function: robustness example

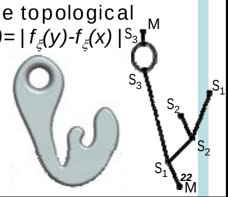


Real functions 21

Elevation [AEHW04]

- For any point *x* of *M*, there exists at least one direction ξ such that *x* is a critical point of the height functions f_ξ and $f_{-\xi}$.
- Then, for every $\xi \in S^2$
  - let *x*, *y* be two critical points of the height function wrt the direction ξ,
  - if *x*, *y* are paired according to the topological persistence, then $pers(x)=pers(y)=|f_\xi(y)-f_\xi(x)|$
  - the elevation is defined as

    Elevation(x)=pers(x).



Real functions 22

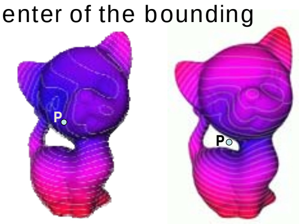
Elevation

- **Saliency:** *f* identifies the depression and protrusions of *M* wrt any normal direction.
- **Smoothness:** *f* is continuous and smooth almost everywhere.
- **Efficiency:** the overall computational cost for
  - finding the persistence pairs is $O(n \log^2 n)$
  - classifying critical points is $O(n^5 \log^2 n)$.
- **Invariance:** *f* is invariant to translations and rotations.

Real functions 23

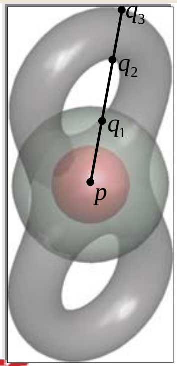
Euclidean distance from a point [FK97]

- $f(x) := \| x - p \|_2$
- The level sets correspond to the intersection of the surface with a set of spheres centered at the point *p*.
- Common choices of the point are the barycenter, the center of the bounding sphere, etc.



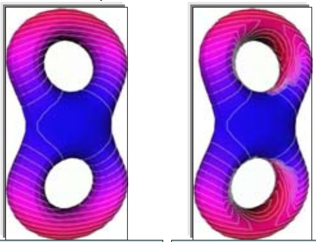
24

Radial distance from a point [SV01]



$$f(q) := \max\{r \geq 0 : r(q-p) \in M\}$$

In an analogous way, *f* can be defined on the unit sphere and used to define the spherical harmonics of *f*.



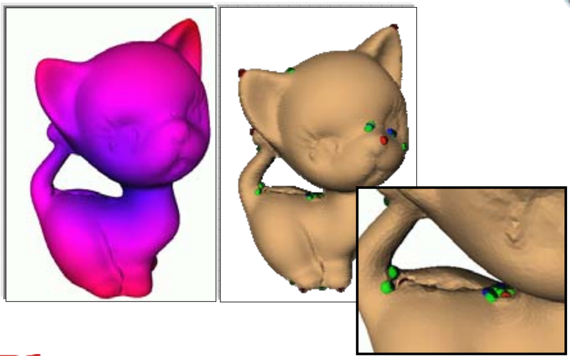
Euclidean distance from *p* | Radial distance from *p*

Euclidean distance from a point

- **Saliency:** maxima and minima are located on protrusions and concavities wrt *p*.
- **Smoothness:** in the continuous case, almost all distance functions from a point are Morse.
- **Stability:** in the discrete case, exact computation at the mesh vertices.
- **Robustness:** the computation of *f* is robust, while its critical points aren't.
  - For instance, the distance from the barycenter: due to its dependence on all the vertices, the barycenter is not affected by small perturbations of *M*.

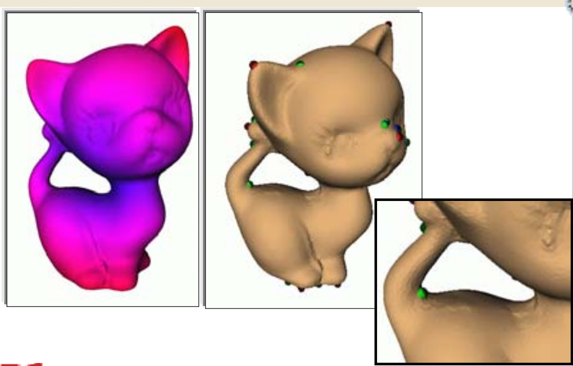
Real functions 26

Euclidean distance from a point: robustness example



Real functions 27

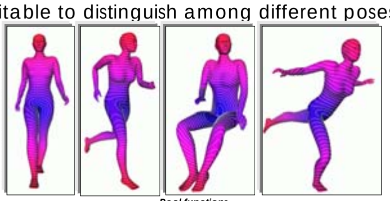
Euclidean distance from a point: robustness example



Real functions 28

Euclidean distance from a point

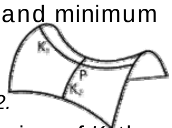
- **DoF and heuristics:** the point *p*.
- **Efficiency:** *f* is computed in *O(n)* time.
- **Invariance:**
  - *f* is invariant to translations and rotations of the coordinate system
  - *f* is suitable to distinguish among different poses.



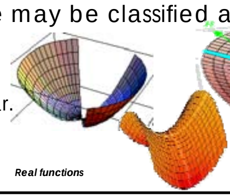
Real functions 29

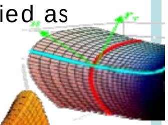
Curvature-based function [GCO06,MPS*04, ZP01]

- The principal curvatures k_1 and k_2 at a point $p \in M$ measure the maximum and minimum bending of a surface at *p*:
  - the Gaussian curvature $K=k_1 k_2$
  - the Mean curvature $H=(k_1+k_2)/2$.



- According to behavior of the sign of *K*, the points of a surface may be classified as
  - elliptic
  - hyperbolic
  - parabolic or planar.





Real functions 30

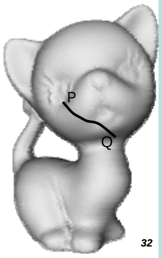
Curvature-based function

- **Saliency:** is provided by the characterization of the local shape as elliptic, hyperbolic, parabolic/planar.
- **Smoothness:** related to the differentiability degree of *M*.
- **Stability:** a coarse surface sampling and an irregular connectivity affect the discretization of the curvature.
- **Robustness:** low degree.
- **DoF and heuristics:** the size of the neighborhood used to compute *K* and *H*.
- **Efficiency:** depends on the size of the neighborhood; at least O(*n*) wrt the 1-star.
- **Invariance:**
  - K is intrinsic, ie it is invariant wrt isometries
  - H is extrinsic and depends on the surface embedding.

Real functions 31

Geodesic distance: definition and properties

- Given two points $p,q \in M$, the geodesic distance *g(p,q)* is the length of the shortest path between *p* and *q*.
- The geodesic distance is invariant to isometric transformations.
- The shortest path is not unique.
- Exact computation in $O(n^2 \log n)$.
- Approximations:
  - Dijkstra [VL99]: $O(n \log n)$
  - [SSK*05]: $O(n \log n)$
  - Fast marching [KS98]: $O(n)$.



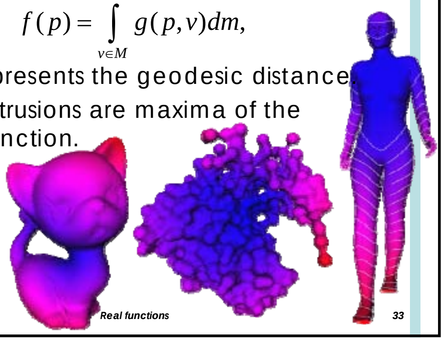
Real functions 32

Average geodesic distance [HSKK01]

- The mapping function is defined as

$$f(p) = \int_{v \in M} g(p,v)\,dm,$$

where *g* represents the geodesic distance.
- Surface protrusions are maxima of the mapping function.



Real functions 33

Average geodesic function

- Discretized using a set of base points $\{b_1,\ldots, b_n\}$ instead of all mesh vertices:

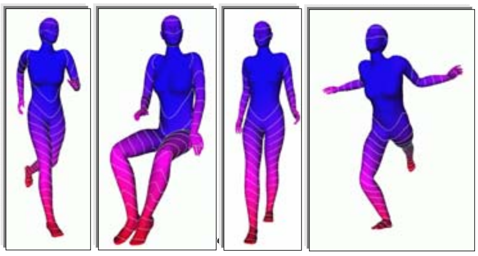
$$f(p) = \sum_i g(p,b_i)\,area(b_i)$$

where $area(b_i)$ is the influence region of b_i.
- It has been extended to consider also the angle variation along a path [ST03].

Real functions 34

Average geodesic distance

- **Saliency:** *f* discriminates protrusions of *M*.
- **Invariance:** *f* is invariant to isometries, that is, it does not distinguish among different poses of the same surface.

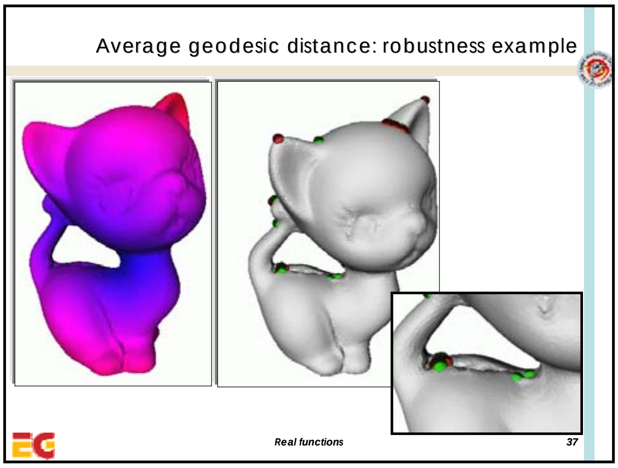


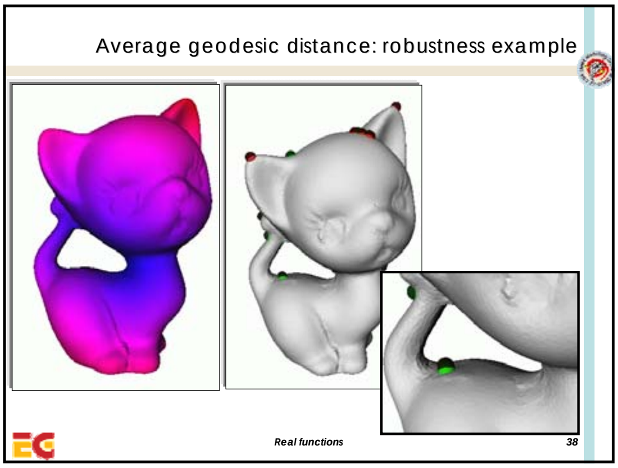
35

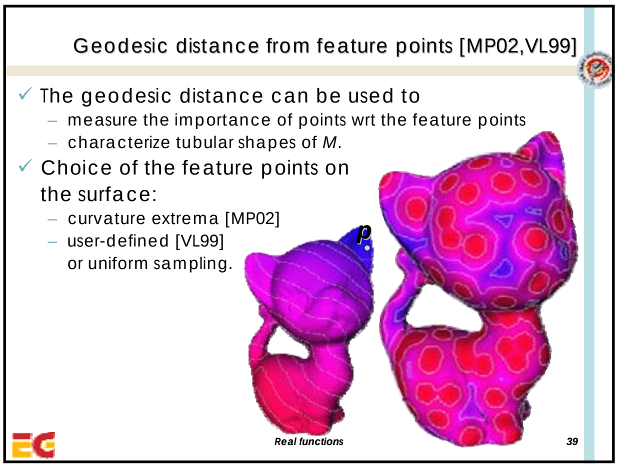
Average geodesic distance

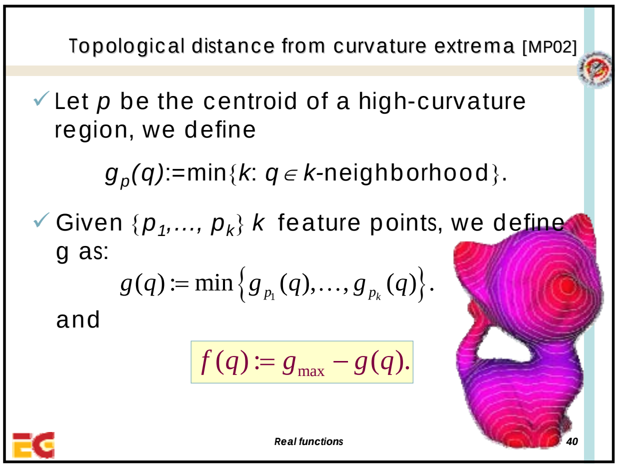
- **Smoothness:** *f* is smooth.
- **Stability:**
  - the discretization and computation depend on the chosen algorithm, eg., Dijkstra [VL99], [SSK*05], fast marching [KS98]
  - generally, a coarse surface sampling and an irregular connectivity affect the discretization of the geodesic distance
  - the instabilities are averaged by the integral in the definition of *f*.
- **Robustness:** *f* is robust to local shape changes.
- **DoF and heuristics:** choice of the base points used to discretize the integral.
- **Efficiency:** depends on the discretization and number of base points. It is computationally expensive using the Dijkstra's algorithm with al vertices as base points: $O(n^2 \log n)$.

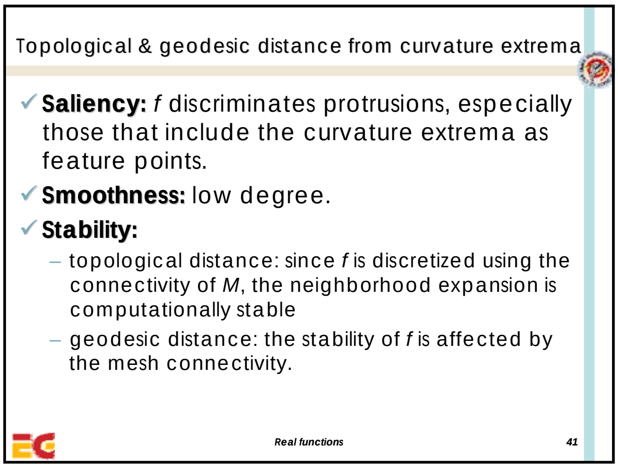
Real functions 36

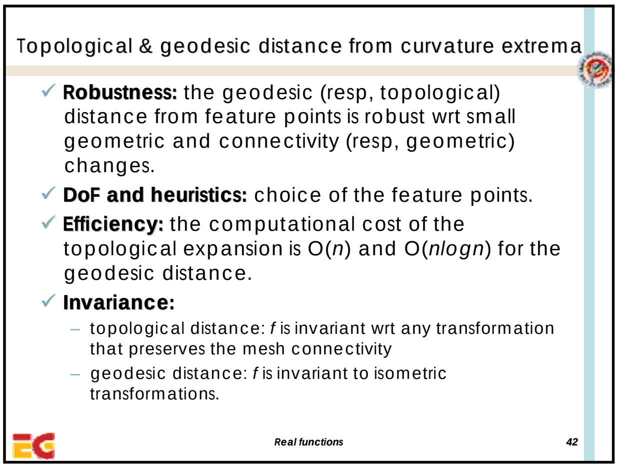

Average geodesic distance: robustness example
Real functions
37


Average geodesic distance: robustness example
Real functions
38

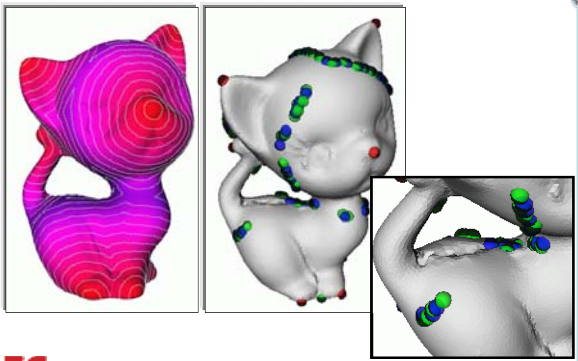

Geodesic distance from feature points [MP02,VL99]
The geodesic distance can be used to
– measure the importance of points wrt the feature points
– characterize tubular shapes of M.
Choice of the feature points on the surface:
– curvature extrema [MP02]
– user-defined [VL99] or uniform sampling.
p
Real functions
39


Topological distance from curvature extrema [MP02]
Let p be the centroid of a high-curvature region, we define
$g_p(q)$:=min{k: $q \in$ k-neighborhood}.
Given {p_1,…, p_k} k feature points, we define g as:
$g(q) := \min\{g_{p_1}(q), \ldots, g_{p_k}(q)\}.$
and
$f(q) := g_{\max} - g(q).$
Real functions
40


Topological & geodesic distance from curvature extrema
Saliency: f discriminates protrusions, especially those that include the curvature extrema as feature points.
Smoothness: low degree.
Stability:
– topological distance: since f is discretized using the connectivity of M, the neighborhood expansion is computationally stable
– geodesic distance: the stability of f is affected by the mesh connectivity.
Real functions
41

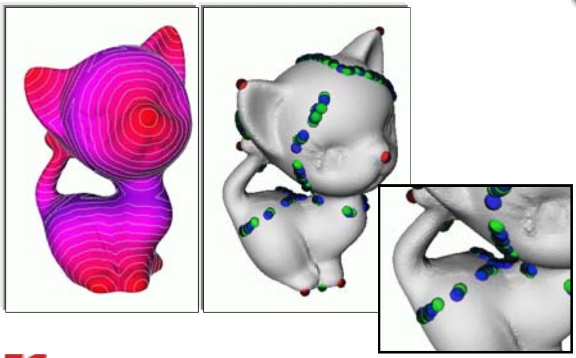

Topological & geodesic distance from curvature extrema
Robustness: the geodesic (resp, topological) distance from feature points is robust wrt small geometric and connectivity (resp, geometric) changes.
DoF and heuristics: choice of the feature points.
Efficiency: the computational cost of the topological expansion is O(n) and O(nlogn) for the geodesic distance.
Invariance:
– topological distance: f is invariant wrt any transformation that preserves the mesh connectivity
– geodesic distance: f is invariant to isometric transformations.
Real functions
42

Topological distance from curvature extrema: robustness example



Real functions 43

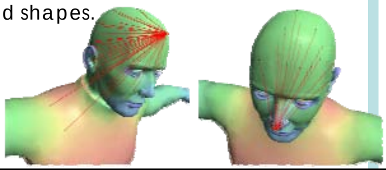
Topological distance from curvature extrema: robustness example



Real functions 44

Local diameter shape function [GSC07]

- On a smooth surface, the exact diameter of a shape at a point *p* is the distance to the antipodal point of *p* wrt the direction opposite to the normal at *p.*
- The local diameter function at *p*
  - is a statistical measure of the diameters in a cone around the direction opposite to the normal at *p*.
  - requires closed shapes.



Local diameter shape function

- **Saliency:** morphological characterization of the shape in terms of relative size of its parts.
- **Smoothness:** no guarantees of smoothness for the local shape diameter: it may fail at sites of branching or in particular visibility cones.
- **Stability:** yes.
- **Robustness:** robust to deformations that do not locally alter the shape volume.
- **DoF and heuristics:** no DoF and several heuristics to drive the statistical sampling of the diameters.
- **Efficiency:** O(n^2).
- **Invariance:**
  - invariant to translations and rotations
  - Invariance to pose changes is forced by averaging the values of *f* at the vertices of *M* wrt the values of neighbors.

Real functions 46

Harmonic functions [DBG*06]

Smooth functions with a (generally) low number of critical points are achieved by solving the Laplace equation with Dirichlet boundary conditions.

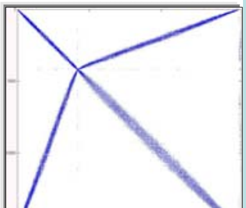
$$\begin{cases} \Delta f(p_i) = 0, i \in I^C & \text{Laplace equation} \\ f(p_i) = \alpha_i, i \in I & \text{Dirichlet boundary conditions} \end{cases}$$

Real functions 47

Laplacian matrix of a triangle mesh

$$\Delta f(p_i) = \sum_{j \in N(i)} [f(p_j) - f(p_i)] w_{ij} = 0$$

$$L_{ij} = \begin{cases} \sum_{j \in N(i)} w_{ij} & i = j \\ -w_{ij} & (i,j) \text{ edge} \\ 0 & \text{else} \end{cases}$$

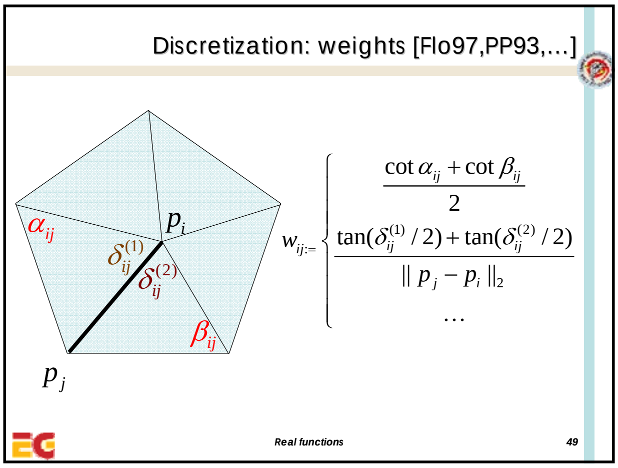


$L \in \mathbf{R}^{n \times n}, rank(L) = n-1$

$$\bar{L}\,\bar{f} = b$$

Sub-matrix of *L* | Unknown values of *f* | Know right-hand vector

Real functions 48



Discretization: weights [Flo97,PP93,…]
α_{ij}
p_i
$\delta_{ij}^{(1)}$
$\delta_{ij}^{(2)}$
β_{ij}
p_j
$w_{ij}:= \begin{cases} \frac{\cot\alpha_{ij}+\cot\beta_{ij}}{2} \\ \frac{\tan(\delta_{ij}^{(1)}/2)+\tan(\delta_{ij}^{(2)}/2)}{\|p_j-p_i\|_2} \\ \dots \end{cases}$
Real functions
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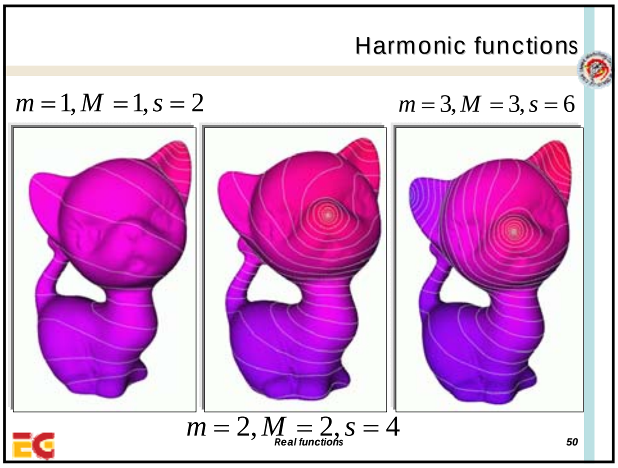
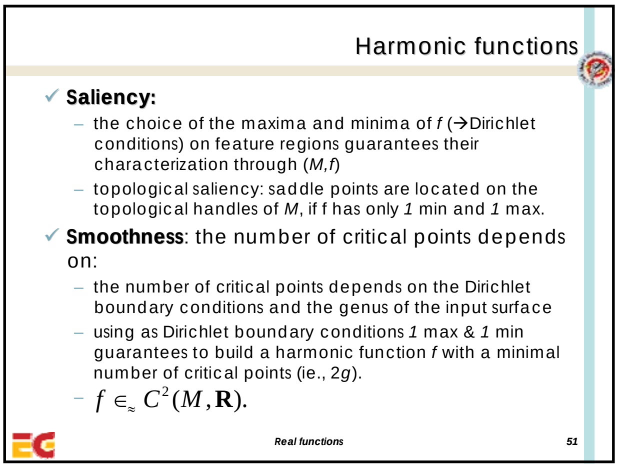
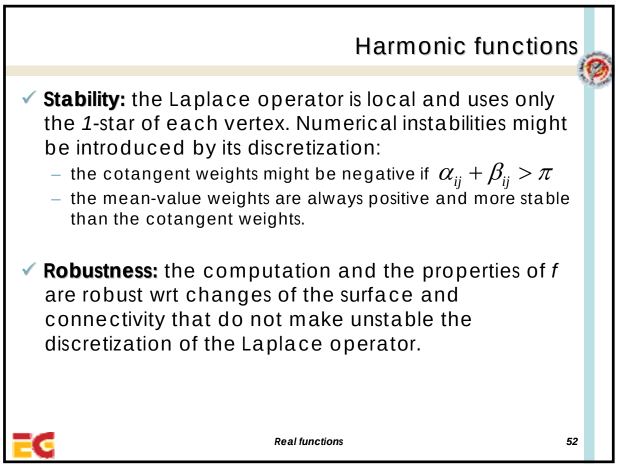


Harmonic functions
$m=1, M=1, s=2$
$m=3, M=3, s=6$
$m=2, M=2, s=4$
Real functions
50



Harmonic functions
Saliency:
– the choice of the maxima and minima of f (→Dirichlet conditions) on feature regions guarantees their characterization through (M,f)
– topological saliency: saddle points are located on the topological handles of M, if f has only 1 min and 1 max.
Smoothness: the number of critical points depends on:
– the number of critical points depends on the Dirichlet boundary conditions and the genus of the input surface
– using as Dirichlet boundary conditions 1 max & 1 min guarantees to build a harmonic function f with a minimal number of critical points (ie., 2g).
– $f \in_{\approx} C^2(M,\mathbf{R})$.
Real functions
51



Harmonic functions
Stability: the Laplace operator is local and uses only the 1-star of each vertex. Numerical instabilities might be introduced by its discretization:
– the cotangent weights might be negative if $\alpha_{ij}+\beta_{ij}>\pi$
– the mean-value weights are always positive and more stable than the cotangent weights.
Robustness: the computation and the properties of f are robust wrt changes of the surface and connectivity that do not make unstable the discretization of the Laplace operator.
Real functions
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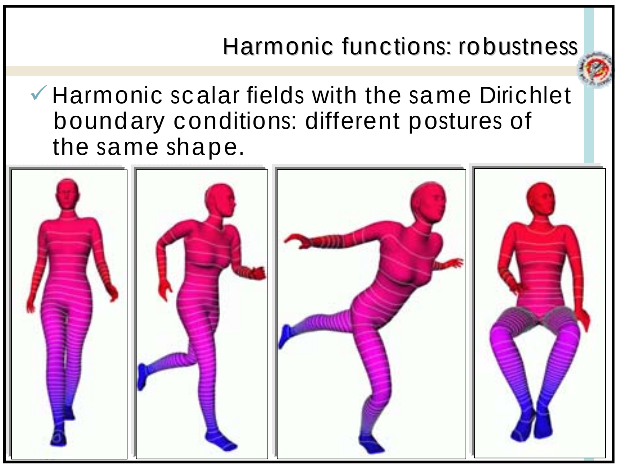
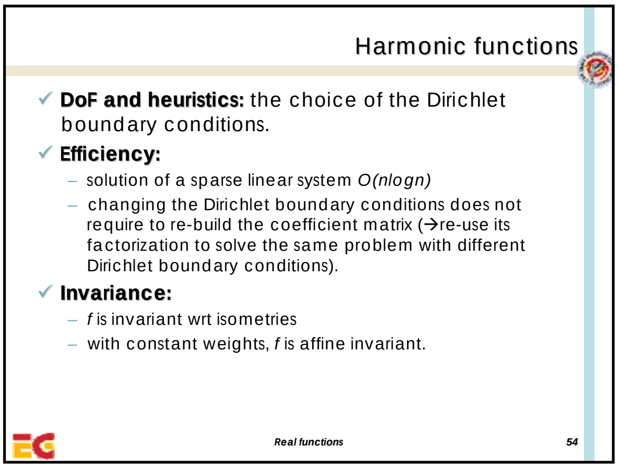


Harmonic functions: robustness
Harmonic scalar fields with the same Dirichlet boundary conditions: different postures of the same shape.



Harmonic functions
DoF and heuristics: the choice of the Dirichlet boundary conditions.
Efficiency:
– solution of a sparse linear system O(nlogn)
– changing the Dirichlet boundary conditions does not require to re-build the coefficient matrix (→re-use its factorization to solve the same problem with different Dirichlet boundary conditions).
Invariance:
– f is invariant wrt isometries
– with constant weights, f is affine invariant.
Real functions
54

Eigensystem of the Laplacian matrix [NGH04,RWP06]

- The spectrum of the Laplacian matrix associated to *M* enables to define a set of functions "intrinsically" defined by the input shape.
- Since *L* is symmetric, it has a real eigensystem

$$Lx_i = \lambda_i x_i, \quad i = 1, \dots, n$$

and

$$\forall y \in \mathbf{R}^n, \quad y = \sum_{i=1}^{n} \alpha_i x_i.$$

Real functions 55

Spectrum of the Laplacian Matrix

- Eigenvalues:

$$0 = \lambda_1 \le \lambda_2 \le \dots \le \lambda_n$$

- Eigenvectors:

$$(x_i, \lambda_i), \quad Lx_i = \lambda_i x_i$$

- i-th function

$$f_i : M \to \mathbf{R}$$

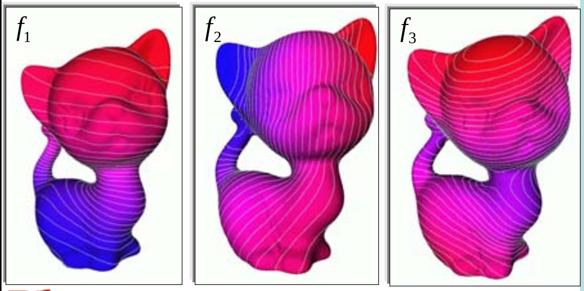
$$f_i(p_k) = \sqrt{\lambda_i}[x_i]_k$$

$$i = 2, \dots, n$$

Real functions

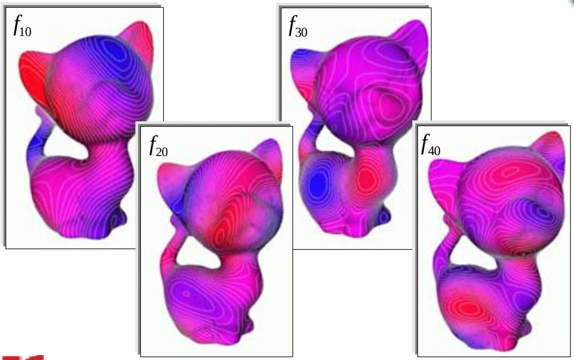
Laplacian-based functions: examples

- Large set of smooth eigenfunctions with a "generally" low number of critical points.



Real functions 57

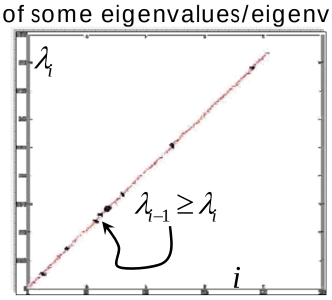
Laplacian-based functions: examples



Real functions 58

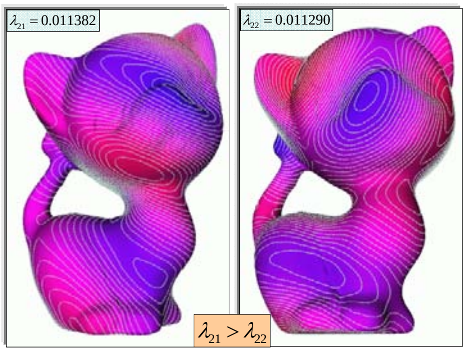
Eigenfunction switch

- Generally, the numerical computation of the Laplacian spectrum may switch the order of some eigenvalues/eigenvectors.

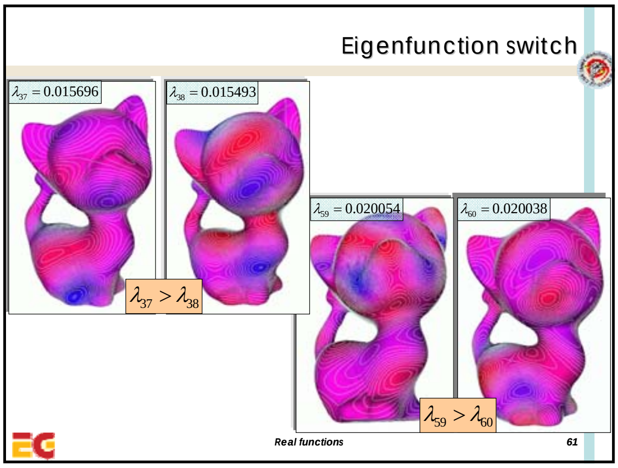


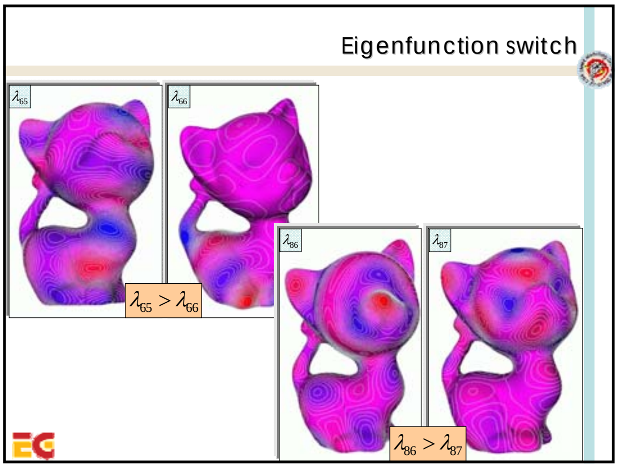
Real functions 59

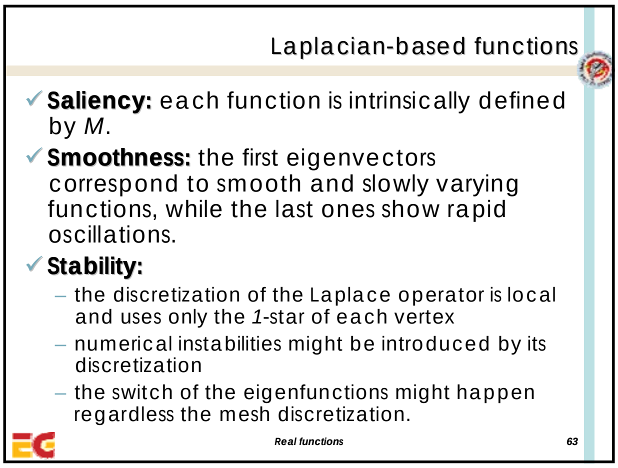
Eigenfunction switch

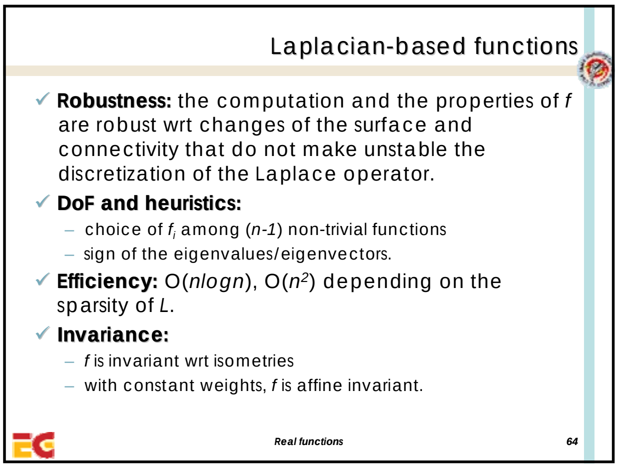


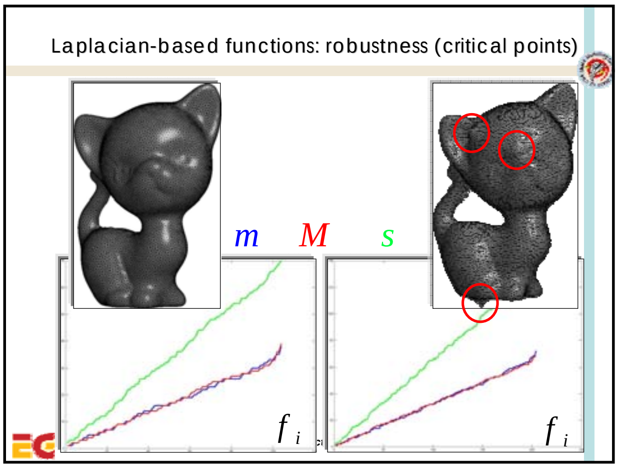
Real functions 60

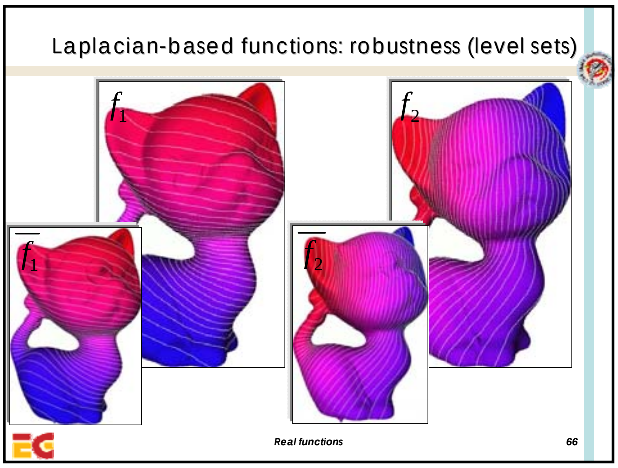

Eigenfunction switch
$\lambda_{37} = 0.015696$
$\lambda_{38} = 0.015493$
$\lambda_{37} > \lambda_{38}$
$\lambda_{59} = 0.020054$
$\lambda_{60} = 0.020038$
$\lambda_{59} > \lambda_{60}$
Real functions
61


Eigenfunction switch
λ_{65}
λ_{66}
$\lambda_{65} > \lambda_{66}$
λ_{86}
λ_{87}
$\lambda_{86} > \lambda_{87}$

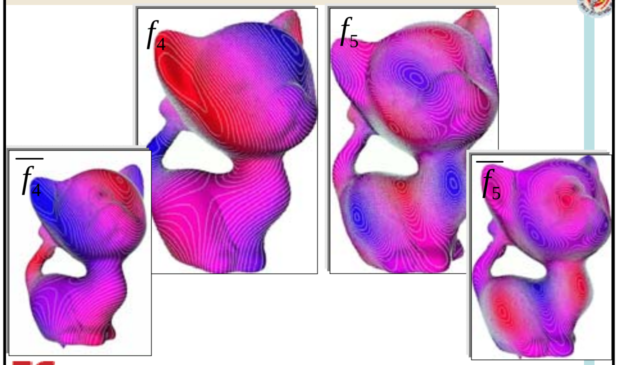

Laplacian-based functions
✓ **Saliency:** each function is intrinsically defined by *M*.
✓ **Smoothness:** the first eigenvectors correspond to smooth and slowly varying functions, while the last ones show rapid oscillations.
✓ **Stability:**
– the discretization of the Laplace operator is local and uses only the *1*-star of each vertex
– numerical instabilities might be introduced by its discretization
– the switch of the eigenfunctions might happen regardless the mesh discretization.
Real functions
63


Laplacian-based functions
✓ **Robustness:** the computation and the properties of *f* are robust wrt changes of the surface and connectivity that do not make unstable the discretization of the Laplace operator.
✓ **DoF and heuristics:**
– choice of f_i among (*n-1*) non-trivial functions
– sign of the eigenvalues/eigenvectors.
✓ **Efficiency:** O(*nlogn*), O(n^2) depending on the sparsity of *L*.
✓ **Invariance:**
– *f* is invariant wrt isometries
– with constant weights, *f* is affine invariant.
Real functions
64


Laplacian-based functions: robustness (critical points)
m
M
s
f_i
f_i


Laplacian-based functions: robustness (level sets)
f_1
f_2
$\bar{f}_1$
$\bar{f}_2$
Real functions
66

Laplacian-based functions: robustness (level sets)



Real functions 67

Questions?

Eurographics 2007 Tutorial T12

Appendix: Perturbation Theory for Eigenvalues and Eigenvectors [GV89]

Eurographics 2007 Tutorial T12

Speaker

Giuseppe Patanè

CNR-IMATI-GE - Italy

Perturbation theory: general case

- Right eigenvector $Ax = \lambda x$
- Left eigenvector $y^* A = \lambda y^*$
- If *A* is diagonalizable, $y_i^* x_j = 0, i \neq j$
- Consider the matrix

$$A_\varepsilon := A + \varepsilon B, \qquad |b_{ij}| \leq 1$$

with right eigenvector $x_i(\varepsilon)$ and eigenvalue $\lambda_i(\varepsilon)$.

Pb: Which relation exists between the eigensystem of A, A_ε ?

Real functions 70

Eigenvalue perturbation: general case

- For each eigenvalue, the following relation holds

$$\left| \frac{\lambda_i(\varepsilon) - \lambda_i}{\varepsilon} \right| \rightarrow \frac{\| B \|_2}{s(\lambda_i)}, \qquad s(\lambda_i) := |y_i^* x_i|.$$

- Then, the above estimation is:
  - proportional to the l_2-conditioning number of the perturbation matrix *B*
  - inversely proportional to the angle between the left and right eigenvectors.

Real functions 71

Eigenvalue perturbation: general case

- We note that

$$s(\lambda_i) = \underbrace{|y_i^* x_i|}_{\geq 0} \leq \|y_i\|_2 \|x_i\|_2 = 1.$$

- The term $s(\lambda_i)^{-1}$ is called **conditioning number** of the eigenvalue λ_i.
- Then, an eigenvalue is **well-conditioned** iff its conditioning number is not close to zero.

Real functions 72

Eigenvalue perturbation: Laplacian matrix

- If the input surface is closed (or with boundary + virtual edges), the Laplacian matrix is symmetric and

$$y_i \equiv x_i, \quad s(\lambda_i) = 1, \quad i = 1, \ldots, n.$$

- Each eigenvalue is well-conditioned and

$$\left| \frac{\lambda_i(\varepsilon) - \lambda_i}{\varepsilon} \right| \to \| B \|_2, \quad \varepsilon \to 0.$$

- The variation of the eigenvalues depends only on the l_2-norm of the perturbation matrix *B*.

Real functions 73

Eigenvector perturbation: general case

- For the *i*-th eigenvector, we have

$$\|x_i(\varepsilon) - x_i\|_2 \leq \varepsilon \sum_{j \neq i}^{n} \left| \frac{y_j^* B x_i}{(\lambda_i - \lambda_j) s(\lambda_j)} \right| + O(\varepsilon^2).$$

- Then, the bound depends on:
  - the conditioning number of **each** eigenvalue $s(\lambda_i)$
  - the differences $\lambda_i - \lambda_j$
  - the factors $\beta_{ij} := y_j^* B x_i$.

Real functions 74

Eigenvector perturbation: general case

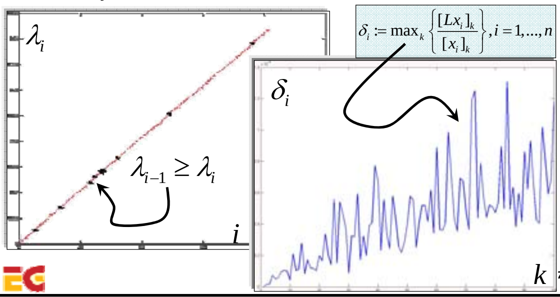
- The perturbation in the eigenvector is proportional to the conditioning number of the whole set of eigenvalues.
- If the eigenvalues are close to one another, we may have difficulties in computing the eigenvectors.
- Let *A* have distinct eigenvalues. If for some eigenvalue $s(\lambda) < 1$, then there exists a matrix *E* such that λ is a repeated eigenvalue of (*A*+*E*) and

$$\frac{\|E\|_2}{\|A\|_2} \leq \frac{s(\lambda)}{\sqrt{1 - s(\lambda)^2}}.$$

Real functions 75

The perversity theorem does not hold

- Then, even if the eigenvalues are distinct, if one eigenvalue is ill-conditioned, the computation of the eigenvalues, and especially the eigenvectors, may be very difficult.



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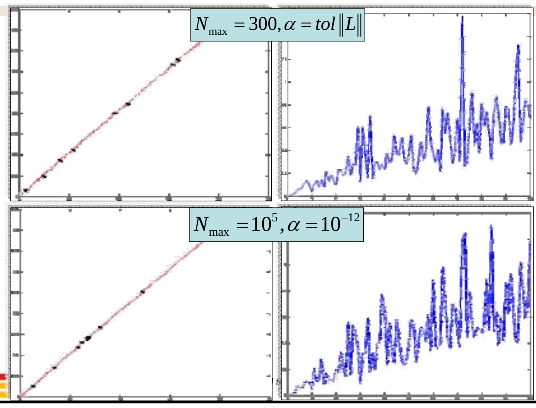
Jacobi iterations and stop criteria

- The (first or last) elements of the eigensystem of the input matrix are evaluated by using the Jacobi method with *2* stop criteria:
  - max. number of iteration $N_{\max}$
  - approximation threshold α.
- Increasing $N_{\max}$ and reducing α do not avoid the switching of eigenvalues and eigenvectors.

Real functions 77

Jacobi iterations and stop criteria



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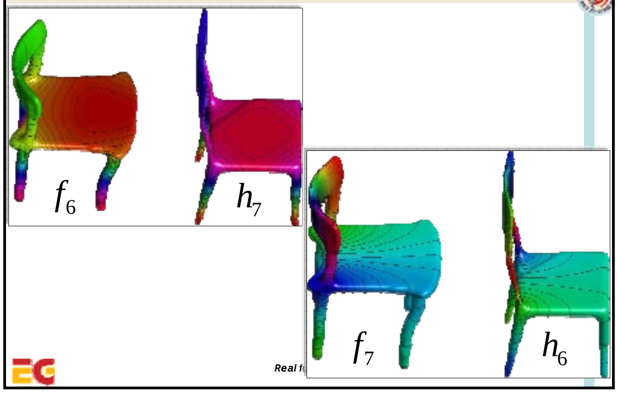
Eigenfunction “switch” on different shapes



Real f...

Eigenfunction switch: discussion

- The switch of the eigenfunctions
  - can happen among the eigenfunctions of the same surface;
  - is strictly correlated to the computation of the eigensystem;
  - a “good” geometry and connectivity (wrt the computation of the entries of L) do not guarantee to avoid the switch of the eigenfunctions.

Real functions 80

Questions?

Eurographics 2007 Tutorial T12

3D Shape Description and Matching Based on Properties of Real Functions

Shape Descriptors

Eurographics 2007 Tutorial T12

Speaker

Daniela Giorgi

CNR-IMATI-GE - Italy

Outline

- Descriptors parametric with respect to *f:*
  - Reeb graph
  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 2

Properties to be discussed

- **Saliency**: ability to capture the essential features of the shape
- **Conciseness**: ability to minimize the memory needed to store the descriptors while maximizing the amount of information
- **Robustness** wrt small changes of the shape
- **Uniqueness** and **completeness**
- **Invariance** to transformation groups
- **DoF and Heuristics** used in the construction of the descriptor
- **Input:** hypotesis and restrictions
- **Efficiency**: computational cost

Shape Descriptors 3

Outline

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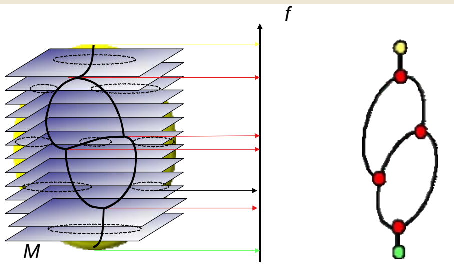
  - Salient geometric features

Shape Descriptors 4

Reeb graph [Ree46]



Reeb graphs are used to store the evolution of the level sets of the mapping function ***f***

Shape Descriptors 5

Reeb graph [Ree46]

Let M be a compact 2-manifold and *f*: M→R a simple Morse function;

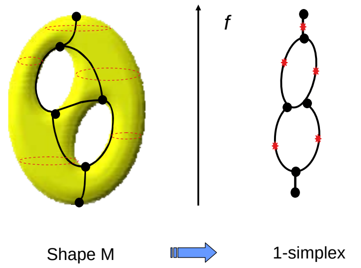
Let "~" be the equivalence relation:

(P, f(P)) ~ (Q, f(Q)) ⇔ *f(P) = f(Q)* and *P* and *Q* are in the same connected component of $f^{-1}(f(P))$

The quotient space on MxR is a finite and connected simplicial complex K of dimension 1, such that the counter-image of each vertex of dim 0 of K is a singular connected component of the level sets of f, and the counter-image of the interior of each simplex of dim 1 is homeomorphic to the topological product of one connected component of the level sets by R

Shape Descriptors 6

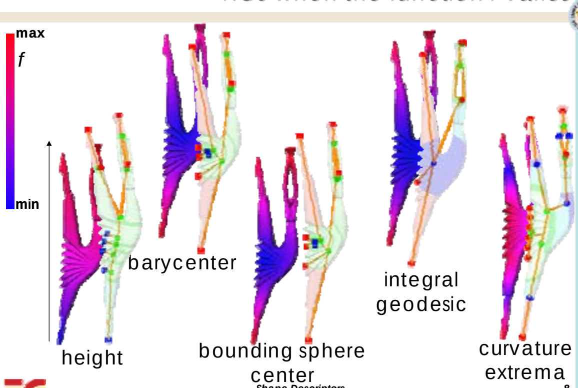
Reeb graph



The 1-simplex is often associated to a geometric embedding (*centerline skeleton*), or used to store additional geometric data

Shape Descriptors 7

RGs when the function *f* varies



Shape Descriptors 8

Reeb graph based representations

- Different proposals for descriptors induced by the Reeb graph:
  - Multiresolution Reeb graph (MRG) [HSKK01, BSR06]
  - Augmented Multiresolution Reeb graph (aMRG) [TS05]
  - Extended Reeb graph (ERG) [BFS00,Bia04,BMSF06]

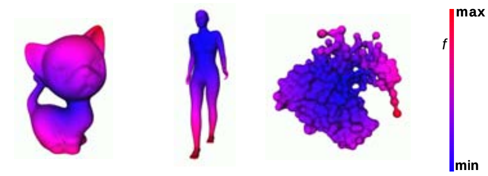
Shape Descriptors 9

Multiresolution Reeb graph [HSKK01]

- It is defined on the basis of the function:

$$f(p)=\int_{v \in S} g(p,v)dS$$

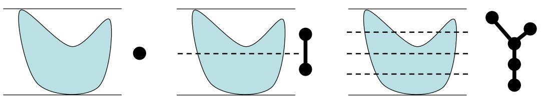
where *g* represents the geodesic distance
- Surface protrusions are maxima of the function *f*



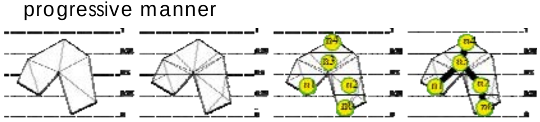
Shape Descriptors 10

Multiresolution Reeb graph [HSKK01]

- Provides a hierarchical graph encoding



- The graph is extracted inserting contours in a progressive manner



- The area *A* of a region and the relative size *L* of the interval of *f* are associated as attributes to nodes

Shape Descriptors 11

Augmented Multiresolution Reeb graph [TS05]

- The descriptor is the same but the nodes are enriched with attributes storing also other geometric measures related to the spatial extent of the regions associated to the nodes
  - Relative volume
  - Statistic measure of the chords
  - Koenderink shape index
  - Statistic orientation of the triangle normals
  - Statistic on the texture (when available)

Shape Descriptors 12

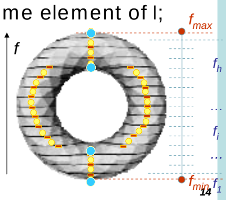
Multiresolution Reeb graphs [HSKK01,TS05]

- **Saliency**: captures protrusions of the shape
- **Conciseness**: very good conciseness properties due to the synthesis of the information (geometry and topology) in an attributed graph
- **Robustness**: no theoretical results
- **Uniqueness**: fixed the resolution, the MRG is unique
- **Completeness**: no
- **Invariance**: inherited by *f*
- **DoF and heuristics**: the resolution has to chosen
- **Input**: manifold, closed triangle meshes
- **Efficiency**: the cost of the graph extraction is O(*n*+*k*), where *k* is the number of vertices added during the construction

Shape Descriptors 13

Extended Reeb Graph [Bia04]

- Founds on an extended Reeb equivalence
  - let f:M→R be a real valued function;
  - let I=$\{(f_{min}, f_1),(f_h,f_{max}),(f_i,f_{i+1}),i=1\ldots h-1\} \cup \{f_{min},f_1,\ldots f_h, f_{max}\}$ be a partition of $[f_{min}, f_{max}]$;
  - the an extended Reeb equivalence between *P*, $Q \in M$ is given by:
    - *f(P)*, *f(Q)* belong to the same element of I;
    - *P*, *Q* belong to the same connected component of $f^{-1}(f(t)), t \in I$.



Shape Descriptors 14

Geometric embedding of the ERG [BMSF06]

- Each arc can be oriented using the growing direction of the mapping function: the ERG is a direct acyclic graph
- Store with each ERG node *n* attributes measuring properties of regions or subparts associated to *n* (eg, using spherical harmonics)
- Store for each ERG arc *e* the number of slices traversed by the arc (arc length)

Shape Descriptors 15

Extended Reeb Graph [Bia04]

- **Saliency**:
  - captures the structure among the features characterized by the critical points of the mapping function
  - the geometric embedding influences the saliency
  - it preserves the topology of the manifold
- **Conciseness**: very good conciseness properties due to the synthesis of the information (geometry and topology) in an attributed graph
- **Robustness**: no theoretical results
- **Uniqueness:** fixed the partition, the ERG is unique
- **Completeness:** no

Shape Descriptors 16

Extended Reeb Graph [Bia04]

- **Invariance**: inherited by *f*
- **DoF and heuristics**: the partion has to be chosen
- **Input:** orientable 2-manifold represented by triangle meshes
- **Efficiency**: the cost of the graph extraction is *O(n+k)* (where *k* is the number of vertices added during the construction)

Shape Descriptors 17

Outline

- Descriptors parametric with respect to *f:*
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  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 18

Size Theory and Size Functions [Fro90]

- Size Theory proposes an approach where Shapes are toplogical spaces endowed with real functions, and comparing shapes means comparing the properties expressed by the real functions
- If two shapes are similar, a homeomorphism between the shapes almost preserving the function values must exist
- How can we measure how well a homeomorphism can preserve the values taken by the considered function?
- In Size Theory two shapes are similar if their the natural pseudo-distance is small

Shape Descriptors 19

Size Theory and Size Functions [Fro90]

- M, N topological spaces

H a (subset of) the set of all homeomorphisms

$$\gamma : M \to N$$

Consider two continuous measuring functions

$$f : M \to \Re^k, \quad g : N \to \Re^k$$

Define the natural pseudo-distance

$$\Theta(\gamma) = \max_{P \in M} \| f(P) - g(\gamma(P)) \|_\infty$$

$$d((M, f),(N, g)) = \begin{cases} \inf_{\gamma \in H} \Theta(\gamma) & \text{if } H \text{ is not empty} \\ +\infty & \text{if } H \text{ is empty} \end{cases}$$

Shape Descriptors 20

Size Theory and Size Functions [Fro90]

- The natural pseudo-distance is a powerful tool to compare shapes, but difficult to compute (we have to study all the homeomorphisms between two spaces)
- We need a tool to study the natural pseudo-distance
- We can get information from size functions, a mathematical tool providing a lower bound for the natural pseudo-distance

Shape Descriptors 21

(Multidimensional) Size Functions [FM99,FL99,BCF*07]

$(\mathcal{M}, \vec{\varphi})$ size pair, with $\vec{\varphi} : \mathcal{M} \to \mathbb{R}^k$;

for every $\vec{x} = (x_1, \ldots, x_k)$, $\vec{y} = (y_1, \ldots, y_k) \in \mathbb{R}^k$

- $\vec{x} \preceq \vec{y}$ $(\vec{x} \prec \vec{y}) \overset{def}{\Longleftrightarrow} x_i \leq y_i$ $(x_i < y_i)$, $i = 1, \ldots, k$;
- $\mathcal{M}_{\vec{x}} = \{P \in \mathcal{M} : \vec{\varphi}(P) \preceq \vec{x}\}$;

$\Delta^+ = \{(\vec{x}, \vec{y}) \in \mathbb{R}^k \times \mathbb{R}^k : \vec{x} \prec \vec{y}\}$;

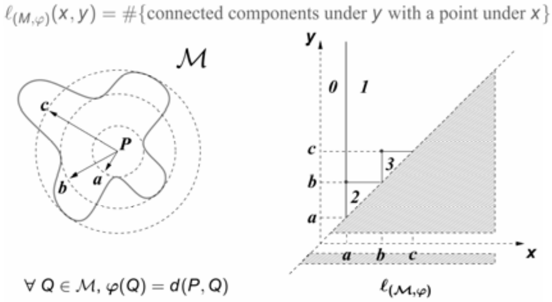
Definition

The (multidimensional) size function of the size pair $(\mathcal{M}, \vec{\varphi})$ is the function $\ell_{(\mathcal{M},\vec{\varphi})} : \Delta^+ \to \mathbb{N}$ that takes each $(\vec{x}, \vec{y})$ to the number of connected components of $\mathcal{M}_{\vec{y}}$ that contain at least a point of $\mathcal{M}_{\vec{x}}$.

Shape Descriptors 22

Example with k=1

$\ell_{(M,\varphi)}(x,y) = \#\{\text{connected components under } y \text{ with a point under } x\}$



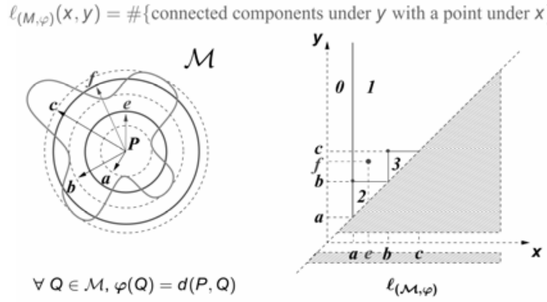
$\forall\, Q \in \mathcal{M}$, $\varphi(Q) = d(P, Q)$

(Symbols $\vec{x}, \vec{y}, \vec{\varphi}$ are replaced by x, y, φ).

Shape Descriptors 23

Example with k=1

$\ell_{(M,\varphi)}(x,y) = \#\{\text{connected components under } y \text{ with a point under } x\}$



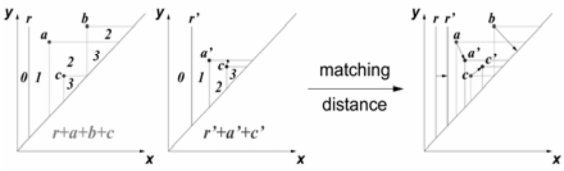
$\forall\, Q \in \mathcal{M}$, $\varphi(Q) = d(P, Q)$

(Symbols $\vec{x}, \vec{y}, \vec{\varphi}$ are replaced by x, y, φ).

Shape Descriptors 24

K=1: Representation and matching [FL01,dAFL06]

Each 1-dimensional size function can be represented by a formal series of points representing vertices of triangular region in Δ^+.



K=1: Stability of matching distance [dAFL06]

- ✓ Matching Stability Theorem:
  The matching distance satisfy the following stability condition:

$$\max_{P \in \mathcal{M}} |\varphi(P) - \psi(P)| \leq \epsilon \Rightarrow d_{match}(\ell_{(\mathcal{M},\varphi)}, \ell_{(\mathcal{M},\psi)}) \leq \epsilon.$$

- ✓ Lower bound for the natural pseudo-distance:
  Let δ be the matching distance between the two size functions $\ell_{(M,\varphi)}$ e $\ell_{(N,\psi)}$. Then

$$d((M,\varphi),(N,\psi)) \geq \delta.$$

Multidimensional size functions [BCF*07]

- ✓ PROBLEMS WITH k>1:
  - How to extend the representation as formal series of points and lines?
  - How to compare size functions with k>1? A direct approach involves working in a domain of $\Re^{2k}$
  - How to obtain stability in computation?
- ✓ SOLUTION: there exists a foliation in half-planes of the domain of multidimensional size functions s.t. on each leaf of the foliation the multidimensional size function coincides with a particular 1-dimensional size function; this allow to difine a stable multidimensional matching distance

Multidimensional size functions [BCF*07]

- Define a foliation of $\Delta^+ = \{(\vec{x},\vec{y}) \in \mathbb{R}^k \times \mathbb{R}^k : \vec{x} \prec \vec{y}\}$ by a parameterized family of half-planes:

$$\pi_{(\vec{l},\vec{b})} : \vec{x} = s\vec{l} + \vec{b},\ \vec{y} = t\vec{l} + \vec{b},\ s,t \in \mathbb{R}, s < t$$

  with
  $\vec{l} = (l_1,\ldots,l_k),\ \vec{b} = (b_1,\ldots,b_k) \in \mathbb{R}^k,\ \|\vec{l}\| = 1,\ l_i > 0,\ \sum b_i = 0.$
- Given a size pair $(\mathcal{M},\vec{\varphi})$, $\vec{\varphi} : \mathcal{M} \to \mathbb{R}^k$, define a 1D-measuring function for each $(\vec{l},\vec{b})$:

$$F^{\vec{\varphi}}_{(\vec{l},\vec{b})}(P) = \max_{i=1,\ldots,k} \left\{ \frac{\varphi_i(P) - b_i}{l_i} \right\}.$$

Reduction Theorem

For every $(\vec{x},\vec{y}) = (s\vec{l}+\vec{b}, t\vec{l}+\vec{b}) \in \pi_{(\vec{l},\vec{b})}$, it holds that

$$\ell_{(\mathcal{M},\vec{\varphi})}(\vec{x},\vec{y}) = \ell_{(\mathcal{M},F^{\vec{\varphi}}_{(\vec{l},\vec{b})})}(s,t).$$

Multidimensional size functions [BCF*07]

- ✓ on each leaf of the foliation, size functions can be represented by formal series of points and lines;
- ✓ the induced 1D matching distance on each leaf of the foliation is stable wrt small changes of the leaves;
- ✓ a multidimensional matching distance can be defined

$$D_{match}\left(\ell_{(\mathcal{M},\vec{\varphi})}, \ell_{(\mathcal{N},\vec{\psi})}\right) = \sup_{(\vec{l},\vec{b})} \min_{i=1,\ldots,k} l_i \cdot d_{match}(\ell_{(\mathcal{M},F^{\vec{\varphi}}_{(\vec{l},\vec{b})})}, \ell_{(\mathcal{N},F^{\vec{\psi}}_{(\vec{l},\vec{b})})})$$

- ✓ theorems about the stability of the matching distance and the lower bound for the natural pseudo distance can be stated also in the case k>1

Size Functions

- ✓ **Saliency**:
  - captures the structure among the features characterized by the critical points of the mapping function
  - connection with the comparison of topological spaces in terms of natural pseudo-distance
- ✓ **Conciseness**: very concise combinatorial description
- ✓ **Robustness**: theoretically proven robustness wrt small shape changes
- ✓ **Unique** but not **complete**

Size Functions

- **Invariance**: inherits invariance properties from the underlying measuring functions
- **DoF** and **heuristics**: none
- **Input**: representation of shapes as size graphs (from discrete or discretized objects)
- **Efficiency**: the computational complexity is $O(n\log n + m\alpha(2m+n,n))$

Shape Descriptors 31

Outline

- Descriptors parametric with respect to *f:*
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  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 32

Persistent Homology [ELZ02]

- The idea of Persistent Homology is to control the placement of topological events in a growing space and assess their relevance according to their life-time
- Given a growing complex ***K***, represented by a filtration

$$\{K^i\}_{i=,\dots,n},\ K^n = K,\ K^i \text{subcomplex of } K^{i+1}$$

the ***j***-persistent ***k***-th homology group of ***K^i*** is a group isomorphic to the image of the homomorphism

$$\eta_{ij}^{\ k} : H_k(K^i) \rightarrow H_k(K^{i+j})$$

induced by the inclusion of ***K^i*** into ***K^{i+j}***

- Persistence represents the life-time of cycles in the growing filtration

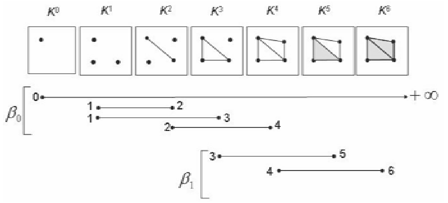
Shape Descriptors 33

Persistent Homology [ELZ02]

- The persistent homology of a growing complex can be represented by a set of intervals, called persistence intervals: a $\mathscr{P}$-interval is a pair

$$(i, j),\quad i, j \in Z \cup +\infty,\ \ 0 \le i < j$$

such that there exists a cycle that is completed at the level ***i*** of the filtration and remains non-bounding until the level ***j***



Shape Descriptors 34

Persistence Homology and Barcodes [CGCZ05]

- The shape of a complex *K* can be described by filtering the complex by the increasing values of a real function
- Idea: construct a new complex strictly related to *K*, namely the tangent complex *T(K)* (closure of the space of all tangents to all points in K), and filter it with the function computing the curvature at a point along a tangent direction
- The barcode of the shape is the set of $\mathscr{P}$-intervals for the filtered tangent complex

Shape Descriptors 35

Barcodes and Persistence Diagrams [CSEH07]

- More recently $\mathscr{P}$-intervals have been described as point sets in the extended plane, and named persistence diagrams: barcodes are essentially a different representation of persistence diagrams for the tangent complex with the curvature function
- The Bottleneck Stability Theorem has been proved:

Let X be a triangulable space with continuous tame functions

$$f, g : X \rightarrow \Re$$

Then the persistence diagrams $D(f), D(g)$ satisfy

$$d_B(D(f), D(g)) \le \| f - g \|_\infty$$

with d_B the Bottleneck distance (also true for the Hausdorff distance)

Shape Descriptors 36

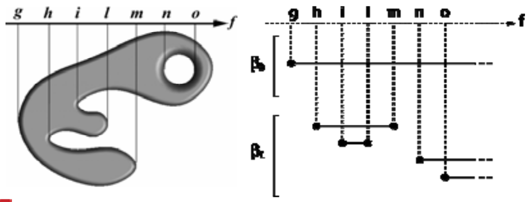
Persistent homology [ELZ02]

- Recent research directions:
  - Vines and Vineyards [CSEM06]
  - Multidimensional Persistence [CZ07a]
  - Localized Homology [CZ07b]
  - Persistence Intervals [DW07]
  - Extended Persistence[CSEH07]

Shape Descriptors 37

Persistence barcodes and diagrams [ELZ02,CGCZ05]

- **Saliency**:
  - captures the structure among the features characterized by the critical points of the mapping function
  - the authors have confined themselves to compute barcodes using a specific space and a specific function, but the underlying theory is valid for more general classes of shapes and functions



Shape Descriptors 38

Persistence barcodes and diagrams [ELZ02,CGCZ05]

- **Conciseness**: very concise combinatorial description
- **Robustness**: theoretically proven robustness wrt small shape changes
- **Unique** but not **complete**
- **Invariance**: inherits invariance properties from the underlying measuring functions
- **DoF** and **heuristics**: none
- **Input**: Barcodes computed on curve PCD and mathematical surfaces, but triangulations and a more general input are admissible
- **Efficiency**: Computing persistent homology requires at most O(m^3), with m the number of simplices

Shape Descriptors 39

Outline

- Descriptors parametric with respect to *f:*
  - Reeb graph
  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 40

Spherical Harmonics [VSR01]

- Idea: build multi-resolution feature vectors using the Fourier expansion of a function defined on the sphere
- Represent the spherical function $f: S^2 \rightarrow \mathbf{R}$ (eg. the spherical extent function, measuring the extent of the object in given directions) as

$$f(\theta,\varphi)=\sum_{l\geq 0}\sum_{|m|\leq l} a_{l,m} Y_l^m(\theta,\varphi)$$

- Feature vectors can be extracted from the first rows of coefficients, thereby providing a multiresolution approach

Shape Descriptors 41

Spherical Harmonics [KFR03]

- Represent a function *f* defined on the sphere through its spherical harmonics and consider the vector of energies (i.e. frequency norms)

$$SH(f)=\{\|f_0(\theta,\varphi)\|,\|f_1(\theta,\varphi)\|,\ldots\}$$

with f_l the frequency components

$$f_l(\theta,\varphi)=\sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\varphi)$$

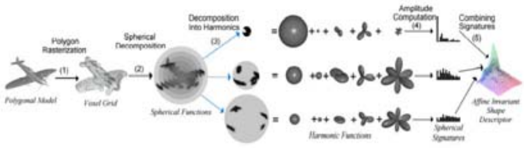
- Let R be a rotation; then it holds:

$$SH(R(f)) = SH(f)$$

Shape Descriptors 42

Spherical Harmonics [KFR03]

- Extension to voxel description:
  - Restrict the voxel grid to a collection of concentric spheres
  - Represent each spherical restriction in terms of the energy of its frequency decomposition, thus obtaining a 1D descriptor
  - The final descriptor resulting from the analysis of spheres with different radii is a 2D grid indexed by radius and frequency



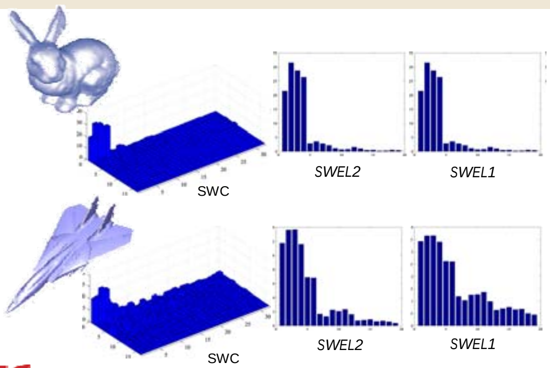
Shape Descriptors 43

Spherical Wavelets [LTN06]

- The problem of the sensitivity of the sampling of the spherical function to latitude-longitude parametrization of the sphere is addressed
- A rotation invariant sampling is proposed, relying on the flat octahedron parametrization of the sphere
- A Spherical Wavelet Transform is applied to the spherical shape function
- Resulting descriptors:
  - Matrix of wavelet coefficients (SWC)
  - L1 energy-based feature vector (SWEL1)
  - L2 energy-based feature vector (SWEL2)

Shape Descriptors 44

Spherical wavelet descriptors [LTN06]



Shape Descriptors 45

Spherical Harmonics and Wavelets [VSR01,KTR03,LTN06]

- **Saliency**: captures the geometrical properties expressed by the spherical function
- **Conciseness**: very concise descriptors (feature vectors or matrices)
- **Robustness**: robustness wrt small changes of the spherical function derived from the decomposition properties
- **Unique,** but not **complete**, since a finite number of coefficients or energies is taken into account
- **Invariance**:
  - Wrt translation
  - Wrt rotation: [VSR01] requires alignment, [KTR03] is invariant only to rotations applied to the sampled input, in [LTN06] SWC requires alignment, while SWEL1 and SWEL2 are rotation invariant

Shape Descriptors 46

Spherical Harmonics and Wavelets [VSR01,KTR03,LTN06]

- **DoF** and **heuristics**:
  - Sampling and voxelization
  - number of frequency components
  - [LTN06] requires the choice of the wavelet transform
- **Input**: meshes (also poligon soups) and grids
- **Efficiency**: the computational complexity in [KTR03] is $O(n^3)$ with n size of the voxel grid

Shape Descriptors 47

Outline

- Descriptors parametric with respect to *f:*
  - Reeb graph
  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 48

Shape DNA [RWP06]

- The shape DNA is the beginning of the spectrum of the Laplace – Beltrami operator, defined for real valued functions on Riemannian manifolds:

Given a Riemannian *n*- manifold M and $f: M \to \Re$ the Laplace – Beltrami operator is

$$\Delta f := div(grad\ f)$$

(different from the discrete Laplacian on graphs)

Shape DNA = $\{\lambda_0 \leq \lambda_1 \leq \ldots \leq \lambda_m\} \in \Re^m{}_{\geq 0}$

with λ_i eigenvalues of the Helmholtz equation $\Delta f = -\lambda f$

Shape Descriptors 49

Shape DNA [RWP06]

- Numerical computation:

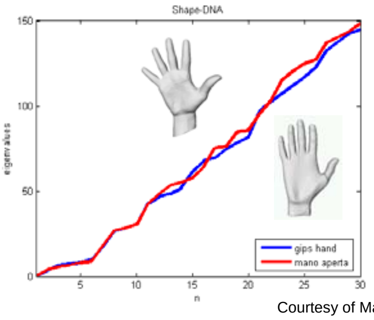
Translate the Helmotz equation into a variational problem and employ the Finite Elements Method (FEM) with m form functions, leading to the generalized eigenvalue problem:

$$AU = \lambda BU$$

with A, B sparse positive (semi-)definite symmetric matrices and $U = (U_1, U_2, \ldots, U_m)$ eigenfunctions with corresponding eigenvalues λ

Shape Descriptors 50

Shape DNA [RWP06]



Courtesy of Martin Reuter

Shape Descriptors 51

Shape DNA [RWP06]

- **Saliency**:
  - description of geometrical and topological properties intrinsic to the object
  - scalable amount of captured shape information, related to the dimension of the cropped spectrum
- **Robustness**: continuously dependent on shape deformations
- **Unique** but not **complete**, since there exist isospectral but not isometrical shapes
- **Invariance** wrt isometry

Shape Descriptors 52

Shape DNA [RWP06]

- **DoF and heuristics**:
  - choice of the number of eigenvalues in the cropped spectrum
  - choice of the form functions for FEM
- **Input**: parametric surfaces, polygonal meshes, solid polyhedra, but conversion to a dataform supported by the FEM engine is required; independent w.r.t. parametrization
- **Efficiency**: the eigenvalue computation is the most time consuming step

Shape Descriptors 53

Outline

- Descriptors parametric with respect to *f:*
  - Reeb graph
  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 54

Bending Invariant Surface Signatures [EK03]

- Geodesic distances between surface points are invariant to surface bending
- Idea: use geodesic distances to define an isometrical embedding of a surface in a small dimensional Euclidean space, in which geodesic distances are approximated by Euclidean ones
- Method: apply a MultiDimensional Scaling (MDS) procedure on a geodesic distance matrix, with geodesics computed via the Fast Marching on Triangulated Domains (FMTD) algorithm

Shape Descriptors 55

Bending Invariant Surface Signatures [EK03]

- Sample with *n* vertices a given triangulated surface, via iterative Voronoi sampling, and build an *n* x *n* dissimilarity matrix *D*

$$D_{ij} = (\delta_{ij})^2$$

with δ_{ij} the geodesic distance between vertices i,j computed following the FMTD algorithm

- Define a dimension *m* for the Euclidean embedding space and apply MDS on the matrix *D*, yelding an *n* x *m* matrix whose rows define the coordinates in $\Re^m$ of the points of the signature surface

Shape Descriptors 56

Bending Invariant Surface Signatures [EK03]

- These two steps define a bending invariant descriptor, that allows to translate the problem of matching non-rigid objects in various posture into a simpler problem of matching rigid objects

Shape Descriptors 57

Bending Invariant Surface Signatures [EK03]

- Drawback: embedding in the Euclidean space may introduce metric distortions

⇩

- Extension to non-Euclidean embeddings (such as embedding on the sphere [BBK05]) and introduction of Generalized MDS [BBK07]
- In [BBK06] partial surface matching is also addressed, introducing the Partial Embedding distance

Shape Descriptors 58

Bending Invariant Surface Signatures [EK03]

- **Saliency**:
  - metric properties are captured by the geodesic distance
  - scalable amount of captured shape information, related to the dimension of the embedding
- **Invariance** wrt isometry
- Not **unique,** due to the randomly chosen starting point in the sampling stage, and not **complete**

Shape Descriptors 59

Bending Invariant Surface Signatures [EK03]

- **Input**: triangulated surfaces
- **DoF and heuristics**:
  - choice of the dimension of the sampling and the embedding
  - choice of the specific MDS algorithm (classical, least squares, fast)
- **Efficiency**:
  - Computing the matrix requires O(n^2), with n the number of sampled vertices
  - the MDS algorithm is at most O(nN), with N number of iterations

Shape Descriptors 60

Outline

- Descriptors parametric with respect to *f:*
  - Reeb graph
  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

Shape Descriptors 61

Spectral Embedding [JZ07]

- Ideas similar to [Elad and Kimmel 2003] are developed, introducing a descriptor suitable to compare articulated objects
- The matrix D is an affinity matrix involving a Gaussian of width σ

$$D_{i,j} = e^{-\frac{\delta_{ij}^2}{\sigma}}$$

with geodesic distances approximated through an heuristic
- The embedding in $\Re^m$ is given by the first m eigenvectors of the matrix, computed via Nyström approximation

Shape Descriptors 62

Spectral Embedding [JZ07]

- The descriptor is given by the embedded surface or by the matrix first eigenvalues

Shape Descriptors 63

Spectral Embedding [JZ07]

- **Saliency**:
  - metric properties captured by the geodesic distances
  - the possibility to use affinity matrices based on different functions (e.g. Euclidean or combined distances) is suggested
  - Scalable amount of shape information, related to the embedding dimension
- **Stability**: problems related to eigenmode switching and eigenmode sign assignment have to be faced
- **Robustness**: sensitiveness to outliers in the data
- **Invariance** wrt isometries

Shape Descriptors 64

Spectral Embedding [JZ07]

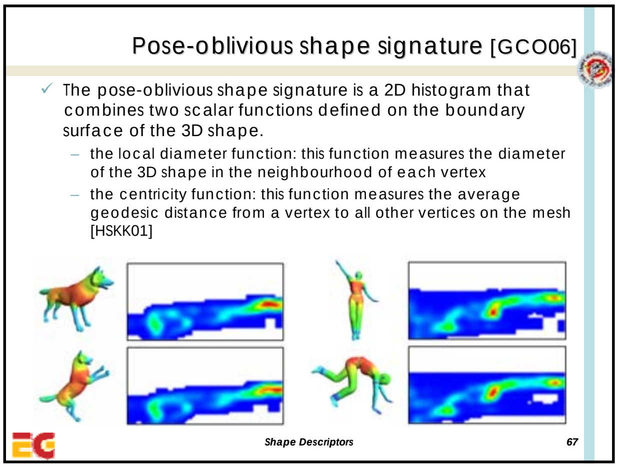
- **Unique** but not **complete**
- **Input**: triangulated surfaces (possibly to be repaired)
- **DoF and heuristics**:
  - sampling rate
  - embedding dimension
  - Gaussian width
  - heuristic to compute the geodesic distance
- **Efficiency**: $O(Nnlogn+N^3)$ operations required to compute and eigen-decompose the affinity matrix, with n the number of vertices of the mesh and N the number of sampled points

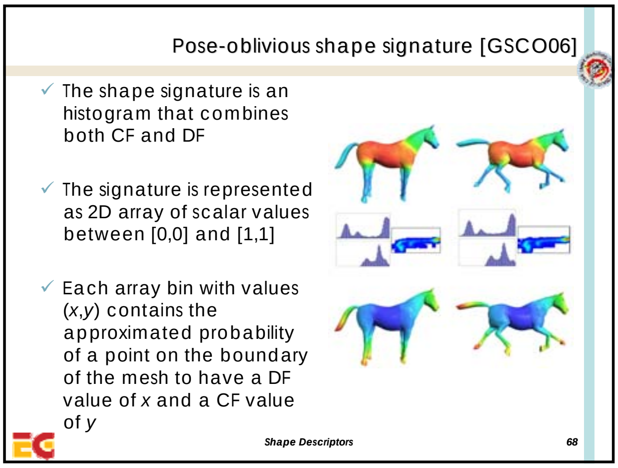
Shape Descriptors 65

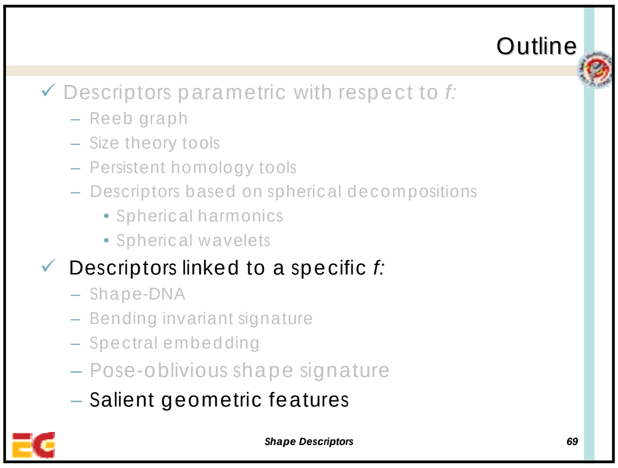
Outline

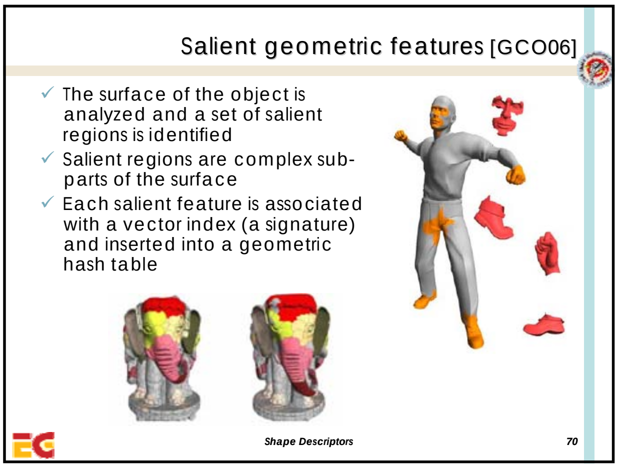
- Descriptors parametric with respect to *f:*
  - Reeb graph
  - Size theory tools
  - Persistent homology tools
  - Descriptors based on spherical decompositions
    - Spherical harmonics
    - Spherical wavelets
- Descriptors linked to a specific *f:*
  - Shape-DNA
  - Bending invariant signature
  - Spectral embedding
  - Pose-oblivious shape signature
  - Salient geometric features

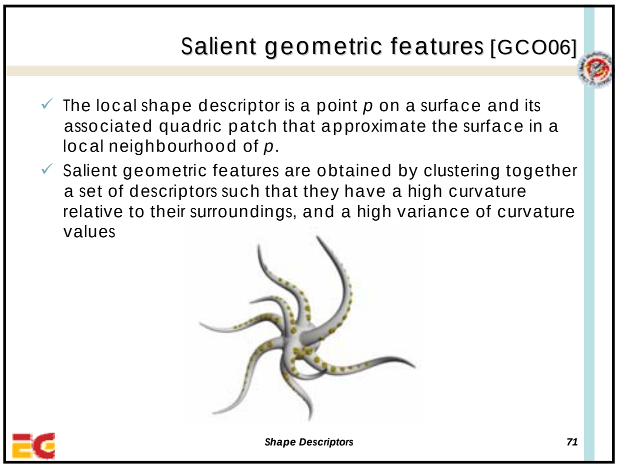
Shape Descriptors 66

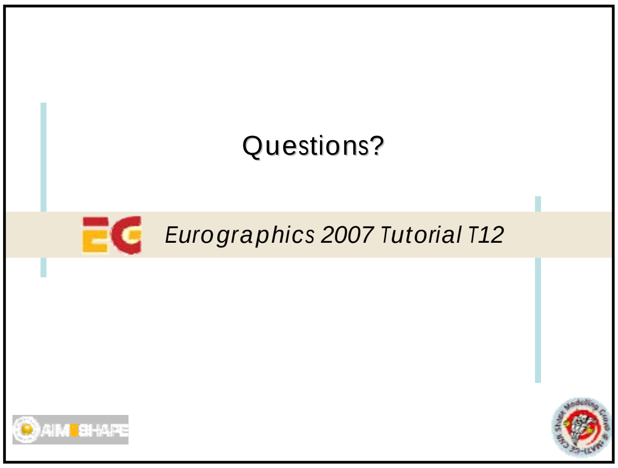

Pose-oblivious shape signature [GCO06]
The pose-oblivious shape signature is a 2D histogram that combines two scalar functions defined on the boundary surface of the 3D shape.
– the local diameter function: this function measures the diameter of the 3D shape in the neighbourhood of each vertex
– the centricity function: this function measures the average geodesic distance from a vertex to all other vertices on the mesh [HSKK01]
Shape Descriptors
67


Pose-oblivious shape signature [GSCO06]
The shape signature is an histogram that combines both CF and DF
The signature is represented as 2D array of scalar values between [0,0] and [1,1]
Each array bin with values (x,y) contains the approximated probability of a point on the boundary of the mesh to have a DF value of x and a CF value of y
Shape Descriptors
68


Outline
Descriptors parametric with respect to f:
– Reeb graph
– Size theory tools
– Persistent homology tools
– Descriptors based on spherical decompositions
• Spherical harmonics
• Spherical wavelets
Descriptors linked to a specific f:
– Shape-DNA
– Bending invariant signature
– Spectral embedding
– Pose-oblivious shape signature
– Salient geometric features
Shape Descriptors
69


Salient geometric features [GCO06]
The surface of the object is analyzed and a set of salient regions is identified
Salient regions are complex sub-parts of the surface
Each salient feature is associated with a vector index (a signature) and inserted into a geometric hash table
Shape Descriptors
70


Salient geometric features [GCO06]
The local shape descriptor is a point p on a surface and its associated quadric patch that approximate the surface in a local neighbourhood of p.
Salient geometric features are obtained by clustering together a set of descriptors such that they have a high curvature relative to their surroundings, and a high variance of curvature values
Shape Descriptors
71


Questions?
Eurographics 2007 Tutorial T12

3D Shape Description and Matching Based on Properties of Real Functions

Comparison Methodologies

Eurographics 2007 Tutorial T12

Speaker
Simone Marini
CNR-IMATI-GE - Italy

AIM@SHAPE

Evaluating the matching characteristics

- **Properties** of the similarity measure
- **Robustness** of the similarity measure
  - *Low* variation of the measure wrt *small* variations of the shape descriptor
- **Type of comparison**
  - global and/or partial matching
- **Type of information** taken into account
  - geometrical, topological, structural
- **Computational complexity** of the matching algorithm
- **Application context**

Comparison methodologies 2

Properties of similarity measures

- Let **S** be the set of shape descriptors, the distance measure **d** between two shapes descriptors is defined as:

$$d : S \times S \rightarrow \Re$$

- Properties:
  - $d(x,x) = 0$ (self identity)
  - $d(x,y) > 0, \; x \neq y$ (positivity)
  - $d(x,y) = d(y,x)$ (symmetry)
  - $d(x,z) \leq d(x,y) + d(y,z)$ (triangular inequality)
  - $d(x,z) \leq \max\{d(x,y), d(y,z)\}$ (strong t. i.)

Comparison methodologies 3

Properties of similarity measures

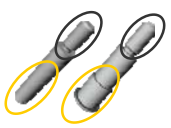
- The measure properties are grouped as in the following:
  - **semi-metric**: self-identity, positivity, symmetry
  - **pseudo-metric**: self-identity, symmetry, triangular inequality
  - **metric**: a pseudo-metric that satisfies the positivity
  - **ultra-metric**: a metric satisfying strong triangular inequality
- The perceptual space can be approximated by the metric properties ? [Tve77,SJ99]
  - symmetry and triangular inequality should not holds for partial matching
- Metrics can be used for indexing purposes

Comparison methodologies 4

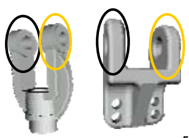
Type of comparison

- Global Matching:
  - Overall shape comparison
  - Real number representing the similarity estimation between the two objects

- Sub-Part Correspondence:
  - Real number as similarity estimation
  - Mapping among similar sub-parts



- Partial Matching:
  - Real number as similarity estimation
  - Similar sub-parts between objects having different overall shape



Comparison methodologies 5

Type of information taken into account

- according to the type of information stored and the way it is coded in the descriptor, the measure of similarity may take into account:
  - geometric information
  - topologic information
  - structural information

Comparison methodologies 6

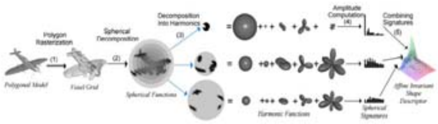
Comparison methodologies

- for descriptors represented by matrices and vectors
  - Spherical Harmonic representation [KFR03]
  - Shape DNA [RWP06]
  - Bending Invariant Surface Signatures[EK03,BBK06]
  - Spectral Embedding [JZ07]
  - Pose-oblivious shape signature [GSCO07]
  - Salient geometric features [GCO06]
- for descriptors represented by formal series
  - Size functions [dFL], [dAFL05]
  - Barcodes and persistence diagrams [CZCG05]
- for descriptors represented by graphs
  - Multiresolution Reeb Graphs [HSKK01]
  - Extended Reeb Graphs [BMSF06]

Comparison methodologies 7

Spherical Harmonic representation [KFR03]

- Represent a function *f* defined on the sphere through its spherical harmonics and consider the vector of energies (i.e. frequency norms)
- Extension to voxel description:
  - Restrict the voxel grid to a collection of concentric spheres
  - Represent each spherical restriction in terms of the energy of its frequency decomposition, thus obtaining a 1D descriptor
  - The final descriptor resulting from the analysis of spheres with different radii is a 2D grid indexed by radius and frequency
- 2D descriptors are compared by using the L_2 norm



Comparison methodologies 8

Spherical Harmonic representation: matching characteristics [KFR03]

- **Properties** of the similarity measure
  - metric
- **Robusteness** of the similarity measure
  - induced by the properties of metrics
- **Type of comparison**
  - global matching
- **Type of information** taken into account:
  - geometric information
- **Computational complexity** of the matching algorithm
  - linear in the number of entries stored in the 2D array
- **Application context**
  - retrieval of 3D objects, not suitable for articulated objects

Comparison methodologies 9

Shape DNA [RWP06]

- Shape DNA signatures are *m*-dimensional feature vectors that can be compared using any metric between vectors, e.g. the Euclidean *p*-norm

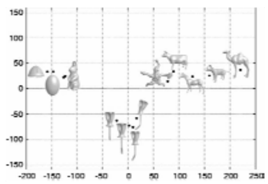
$$d_p(u,v)=\left(\sum_{i=1}^{m}|u_i-v_i|^p\right)^{\frac{1}{p}}$$

the Hausdorff distance, the Pearson correlation distance
- according to empirical evidence, d_2 yields good results while being easy to compute

Comparison methodologies 10

Shape DNA [RWP06]

- Matching results on a small database of meshes, including different classes of deformed models, show a nice clustering of objects



- Other experiments on collections of grey-scale and colour images [RWP07]
- Medical applications on brain surfaces [NRW07], using statistical methods to distinguish populations; extention to 3D brain data

Comparison methodologies 11

Shape DNA: matching characteristics [RWP06]

- **Properties** of the induced similarity measure
  - metric (using the Euclidean *p*-norm)
- **Robustness** of the similarity measure:
  - induced by the robustness of metrics
- **Type of comparison**: global matching
- **Type of compared information**
  - the descriptor stores geometric and topological information, but it is difficult to control them in a differentiated manner in the definition of the measure
- **Computational complexity** of the matching algorithm
  - p-norms are linear in the number of vertices of the model
- **Application context**:
  - medical applications, suitable for articulated objects

Comparison methodologies 12

Bending Invariant Surface Signatures [EK03]

- Given the surface signatures, any algorithm to evaluate the similarity of rigid objects can be involved in the comparison step
- Example: Compute the vectors of the first few moments of the surfaces and compute their Euclidean distance

Comparison methodologies 13

Bending Invariant Surface Signatures [EK03]

- **Properties** of the induced similarity measure
  - Depends on the matching method used
- **Robustness** of the similarity measure
  - Depends on the matching method used
- **Type of comparison**
  - Global or partial matching depending on the matching method used
- **Type of compared information**
  - Depends on the matching method used
- **Computational complexity** of the matching algorithm
  - Depends on the matching method used
- **Application context**
  - face recognition

Comparison methodologies 14

Spectral Embedding [JZ07]

- Compare shapes by computing existing shape descriptors (Light Field, Spherical Harmonics) on spectral embeddings
- Use the vectors of normalized eigenvalues and define:

$$D_{EVD}(Q,S) = \frac{1}{2}\sum_{i=1}^{m}\frac{\left[\left|\lambda_i^Q\right|^{\frac{1}{2}} - \left|\lambda_i^S\right|^{\frac{1}{2}}\right]^2}{\left|\lambda_i^Q\right|^{\frac{1}{2}} + \left|\lambda_i^S\right|^{\frac{1}{2}}}$$

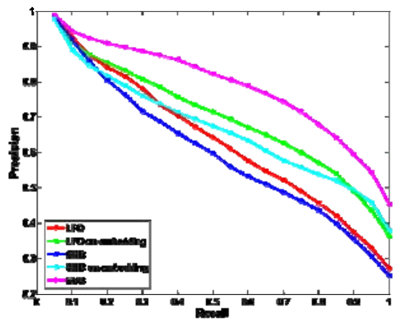
- Compute a correspondence cost derived from the correspondence between the vertices of the two shapes (possibly after a first filter using EVD)

$$D_{CCD}(Q,S) = \sum_{p\in Q}\left\| V_Q(p) - V_S(match(p)) \right\|$$

Comparison methodologies 15

Spectral Embedding [JZ07]

Precision-Recall plot for McGill database



Comparison methodologies 16

Spectral Embedding: matching characteristics [JZ07]

- **Properties** of the induced similarity measure
  - $D_{EVD}(Q,S) = \frac{1}{2}D(f,g)$;
  - $D(f,g) = \chi^2 = \int \frac{(f-g)^2}{f+g}, \quad f = \left|\lambda_i^Q\right|^{\frac{1}{2}}, g = \left|\lambda_i^S\right|^{\frac{1}{2}}$
  - X^2 is a semi-metrics if f and g are positives
- **Robustness** of the similarity measure
  - induced by the robustness of metrics

Comparison methodologies 17

Spectral Embedding: matching characteristics [JZ07]

- **Type of comparison**
  - Global matching
- **Type of compared information**
  - geometric and topological information
- **Computational complexity** of the matching algorithm
  - D_{EVD} is linear in the number of vertices of the embedded model
- **Application context**
  - suitable for articulated objects

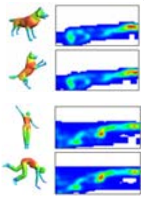
Comparison methodologies 18

Pose-oblivious shape signature[GCO06]

- The pose oblivious is a 2D histogram that combines local diameter function and centricity function both defined on the boundary surface of the 3D shape.
- Matching:
  - correlation coefficient:

$$R(P,Q) = \frac{N\sum p_i q_i - \sum p_i \sum q_i}{\sqrt{(N\sum p_i^2 - (\sum p_i)^2)(N\sum q_i^2 - (\sum q_i)^2)}}$$

  - $\chi^2 = D(f,g) = \int \frac{(f-g)^2}{f+g}$



Comparison methodologies 19

Pose-oblivious shape signature : matching characteristics [GSCO07]

- **Properties** of the similarity measure
  - X^2 is a semi-metrics if *f* and *g* are positives
  - correlation coefficient is a semi-metric
- **Robustness** of the similarity measure
  - induced by the properties of measures
- **Type of comparison**
  - global matching
- **Type of information** taken into account
  - the descriptor stores geometric information
- **Computational complexity** of the matching algorithm
  - linear in the number of entries stored in the 2D array
- **Application context**
  - retrieval of 3D objects, suitable for rticulated objects

Comparison methodologies 20

Salient geometric features [GCO06]

- Each salient feature is associated with a vector index (a signature) and inserted into a geometric hash table
- Given a query object, its salient feature are extracted and used to query the database for a list of matching features
- The returned features identify the models having larger number of matches.

Comparison methodologies 21

Salient geometric features [GCO06]

- The vector index used in the hash table encode the following information:
  - area of the salient feature
  - curvature of the salient feature
  - number of local minimum(s) or maximum(s) curvatures in the salient feature
  - the curvature variance in the salient feature
- The similarity between objects is given by the number of correspondence among the salient features

Comparison methodologies 22

Salient geometric features : matching characteristics [GCO06]

- **Properties** of the similarity measure
  - similarity measure is not proposed by authors
- **Type of comparison**
  - Sub-part correspondence and partial matching.
- **Type of information** taken into account:
  - geometric information
- **Computational complexity** of the matching algorithm
  - depends on the geometric hashing used
- **Application context**
  - retrieval of 3D objects, object alignement

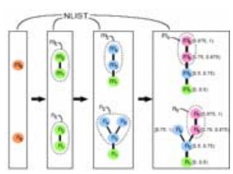
Comparison methodologies 23

Multiresolution Reeb Graph [HSKK01]

- Similarity between two nodes *P*,*Q* is weighted on their attributes:

$$sim(P,Q) = \alpha \,|\, A(P) - A(Q) \,| + (1-\alpha)\,|\, L(P) - L(Q) \,|\,,\ 0 < \alpha < 1$$

- Nodes with maximal similarity are paired if:
  - Share the same range of *f*
  - Parent nodes are matched
  - Belong to graph paths already matched
- The distance between two MRGs is the sum of all node similarities



Comparison methodologies 24

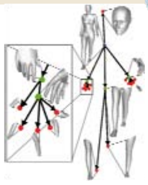
Multiresolution Reeb Graph: matching characteristics [HSKK01]

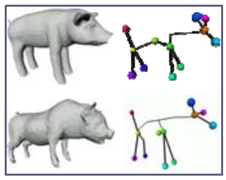
- **Properties** of the induced similarity measure
  - metric
- **Robustness** of the similarity measure
  - stability properties of metric
- **Type of comparison**
  - global matching (suitable also for sub-part correspondence and partial matching)
- **Type of compared information**
  - structural and geometric information
- **Computational complexity** of the matching algorithm
  - $O(M \cdot (M+N))$ where M and N is the number of nodes of the two multiresolution graphs
- **Application context**
  - Retrieval of free form objects

Comparison methodologies 25

Extended Reeb Graphs [BMSF06]

- Two ERGs are compared using a graph-matching approach based on the "best common subgraph" detection
- Also sub-part correspondences are recognized
- Heuristics are used to improve
  - Quality of the results
  - Computational time





Comparison methodologies 26

Extended Reeb Graphs [BMSF06]

- Given G_1 and G_2, two direct, acyclic and attributed graphs:
  - the distance d between two nodes $v_1 \in G_1$ and $v_2 \in G_2$ is

$$d(v_1, v_2) = \frac{w_1 G_S + w_2 St_S + w_3 Sz_S}{3} \qquad w_i \in [0,1] \qquad \sum w_i = 1$$

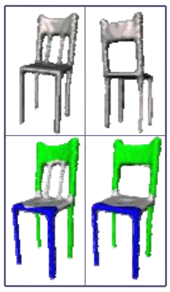
  - the distance $D(G_1,G_2)$ depends both on the geometry and the structure of the objects:

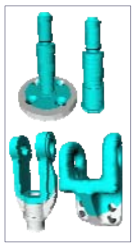
$$D(\mathcal{G}1, \mathcal{G}2) = 1 - \frac{\sum_{v \in \mathcal{G}} (1 - d(\psi_1(v), \psi_2(v)))}{max(|\mathcal{G}1|, |\mathcal{G}2|)}$$

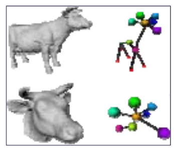
Comparison methodologies 27

Extended Reeb Graphs [BMSF06]

- Some examples of sub-part correspondence and partial matching







Comparison methodologies 28

Extended Reeb Graphs: matching characteristics [BMSF06]

- **Properties** of the Induced similarity measure
  - semi-metric
- **Robustness** of the similarity measure
  - Stability properties of semi-metrics
- **Type of comparison**
  - global matching, sub-part correspondence and partial matching
- **Type of compared information**
  - structural and geometric information
- **Computational complexity** of the matching algorithm
  - $O(n^3)$ where n is $\max\{|G_1|,|G_2|\}$
- **Application context**
  - free form objects and CAD models

Comparison methodologies 29

Matching distance between 1-dimensional size functions [dAFL06]

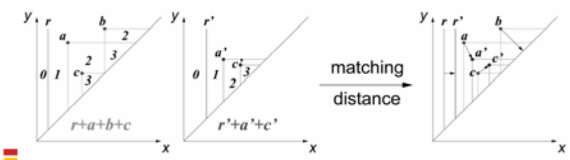
- Two size functions ℓ_1, ℓ_2, with associated formal series C_1 and C_2, can be compared by measuring the reciprocal distances of cornerpoints and cornerlines

$$\delta((x,y),(x',y')) = \min\left\{\max\{|x-x'|,|y-y'|\}, \max\left\{\frac{y-x}{2}, \frac{y'-x'}{2}\right\}\right\}$$

and choosing the matching which minimizes the maximum of these distances

$$d_{match}(\ell_1, \ell_2) = \min_{\sigma} \max_{p \in C_1} \delta(p, \sigma(p))$$

when σ varies among the bijections between C_1 and C_2



Comparison methodologies

Matching distance between 1-dimensional size functions [dAFL06]

- Matching Stability Theorem:

  The matching distance satisfy the following stability condition:

$$\max_{P\in\mathcal{M}} |\varphi(P)-\psi(P)| \le \epsilon \Rightarrow d_{match}(\ell_{(\mathcal{M},\varphi)},\ell_{(\mathcal{M},\psi)}) \le \epsilon.$$

- Lower bound for the natural pseudo-distance:

  Let δ be the matching distance between the two size functions $\ell_{(M,\varphi)}$ e $\ell_{(N,\psi)}$. Then

$$d((M,\varphi),(N,\psi)) \ge \delta.$$

Comparison methodologies 31

Matching distance between multidimensional size functions [BCF*07]

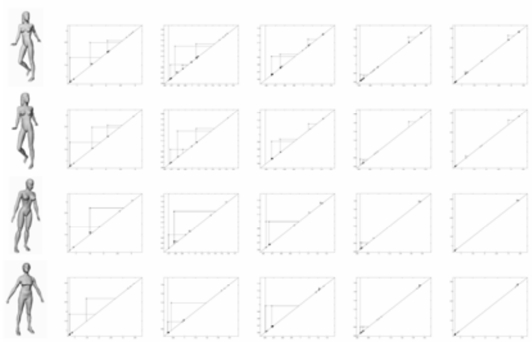
- On each leaf of a particular foliation of their domain, multidimensional size functions coincide with a particular 1-dimensional size function
- the induced 1D matching distance on each leaf of the foliation is stable wrt small changes of the leaves;
- a multidimensional matching distance can be defined

$$D_{match}\left(\ell_{(\mathcal{M},\vec{\varphi})},\ell_{(\mathcal{N},\vec{\psi})}\right) = \sup_{(\vec{l},\vec{b})} \min_{i=1,\dots,k} l_i \cdot d_{match}(\ell_{(\mathcal{M},F^{\vec{\varphi}}_{(\vec{l},\vec{b})})},\ell_{(\mathcal{N},F^{\vec{\psi}}_{(\vec{l},\vec{b})})})$$

- theorems about the stability of the matching distance and the lower bound for the natural pseudo distance can be stated also in the case k>1

Comparison methodologies 32

Multidimensional Size Functions [BCF*07]



Comparison methodologies 33

Size Functions: matching characteristics [dAFL06,BCF*07]

- **Properties** of the induced similarity measure
  - the matching distance is a metric
  - it provides a lower bound for the natural pseudo-distance
- **Robustness** of the similarity measure
  - stability theorem for the matching distance
- **Type of comparison**
  - global matching
- **Type of compared information**
  - geometric-topological
- **Computational complexity** of the matching algorithm
  - $O(n^{2.5})$, where n is the number of cornerpoints taken into account
- **Application context**
  - Medical images, trademarks recognition, 3D retrieval

Comparison methodologies 34

Barcodes [CZCG05]

- I, J intervals in a barcode, $\delta(I,J) = |I \cup J - I \cap J|$
- A matching between barcodes S_1, S_2 is the set

$$M(S_1,S_2) \subseteq S_1 \times S_2 = \{(I,J)\ s.t.\ I \in S_1,\ J \in S_2\}$$

  s.t. any interval in S_1, S_2 occurs in at most one pair (I,J)
- Distance between S_1, S_2 relative to M

$$D_M(S_1,S_2) = \sum_{(I,J)\in M} \delta(I,J) + \sum_{L\in N} |L|$$

  with N the set of non matched intervals

Comparison methodologies 35

Barcodes [CZCG05]

- Barcode pseudo-metric:

$$D(S_1,S_2) = \min_M D_M(S_1,S_2)$$

- Minimizing D_M is equivalent to maximizing the similarity

$$S_M(S_1,S_2) = \frac{1}{2}\left(\sum_{S_1}|I| + \sum_{S_2}|J| - D_M(S_1,S_2)\right)$$

- Recasting the problem as a graph problem, such minimization is equivalent to the well known maximum weight bipartite matching problem

Comparison methodologies 36

Barcodes [CZCG05]

- ✓ Examples on mathematical surfaces
- ✓ Classification results on a set of 80 hand-drawn copies of letters

Comparison methodologies 37

Persistence Diagrams [CSEH07]

- ✓ Describing $\mathcal{P}$– intervals as point sets in the extended plane, i.e. by persistence diagrams, the Bottleneck Stability Theorem has been proved
- ✓ Under conditions on the space and the functions f,g, it holds that the Bottleneck distance between persistence intervals $D(f), D(g)$ satisfies

$$d_B(D(f), D(g)) \leq \| f - g \|_\infty$$

where d_B is defined as

$$d_B(X,Y) = \inf_\gamma \sup_x \| x - \gamma(x) \|_\infty$$

with X, Y multisets of points, $x \in X$, $y \in Y$ range over all points and γ ranges over all bijections from X to Y

Comparison methodologies 38

Barcodes and Persistence Diagrams [CSEH07]

- ✓ In terms of persistence diagrams, the distance defined for barcodes can be written

$$d(D_1, D_2) = \inf_\gamma \sum_p \| p - \gamma(p) \|_1$$

with γ ranging in the set of bijections between D_1 and D_2, but this distance does not guarantee the stability property proven for persistence diagrams under the Bottleneck distance

- ✓ Under certain assumptions, the Barcode Theorem holds, guaranteeing the stability property under the Bottleneck distance

Comparison methodologies 39

Barcodes and persistence diagrams: matching characteristics [CZCG05,CSEH07]

- ✓ **Properties** of the induced similarity measure
  - pseudo-metric between barcodes
  - metric between persistence diagrams
- ✓ **Robustness** of the similarity measure
  - stability theorems for barcodes and persistence diagrams under the Bottleneck distance
- ✓ **Type of comparison**
  - global matching
- ✓ **Type of compared information**
  - geometric-topological
- ✓ **Computational complexity** of the matching algorithm
  - for the pseudo-metric between barcodes, it depends on the algorithm used to minimize $D(s_1, s_2)$
- ✓ **Application context:** 3D and curve PCD comparison

Comparison methodologies 40

Questions?

Eurographics 2007 Tutorial T12

3D shape description and matching based on properties of real functions

Conclusions and future perspectives

Eurographics 2007 Tutorial T12

Speakers

Bianca Falcidieno Michela Spagnuolo

CNR-IMATI-GE - Italy

AIM@SHAPE

Shape M&R: remarks

- Shape matching and retrieval is a complex process that involves reasoning at many different levels
  - Perception
  - Query formulation
  - Context understanding and formalization
  - Shape complexity (retrieval in broad or narrow domains)
- Crucial aspects that determine the performance of shape and retrieval systems
  - Saliency of the descriptors
  - Efficiency of the matching
  - *Indexing*

2

Shape M&R: remarks on performances

- Quantitative measures: relatively easy ..
  - Precision/recall, first and second tier, CG, ..
  - *More attention to the ground truth definition*
  - *Flexible classification tools*
- Qualitative measures :
  - more difficult, need users or scenarios and specific models
- Reproducibility of results
  - Executables should be provided
  - Benchmarks for specific shape M&R tasks

3

Evaluation and benchmarking

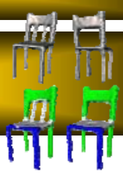
- SHREC'07 - Shape Retrieval Contest 2007 promoted by AIM@SHAPE, and coordinated by Remco Veltkamp (UU)
  - Organized **every year**, in conjunction with Shape Modeling International – SMI – (next year, Stony Brook, June 2008)
  - **Multi-track**: this year 7 tracks for watertight models, partial matching, protein models, CAD models, relevance feedback, similarity measures, 3D faces

http://www.aimatshape.net/event/SHREC

4

why SHREC

- PSB has limited benchmarking capabilities
- different representation models
  - NOT format, but MODELS: triangle meshes, NURBS, Breps, that may have different characteristics



- different “types of” similarity
  - Form (geometry or structure)
  - Function & Semantics
  - ..

5

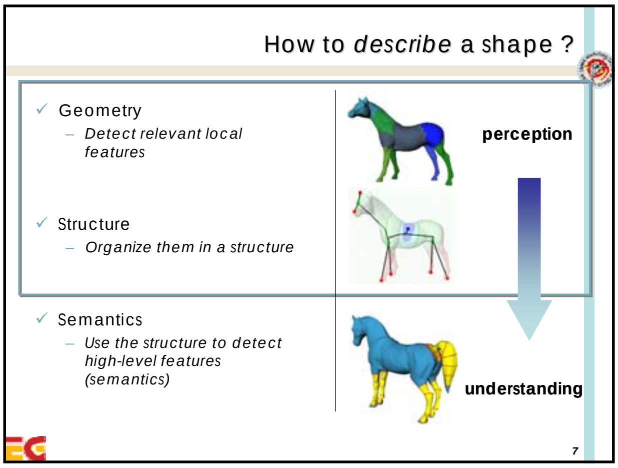
Shape M&R: remarks

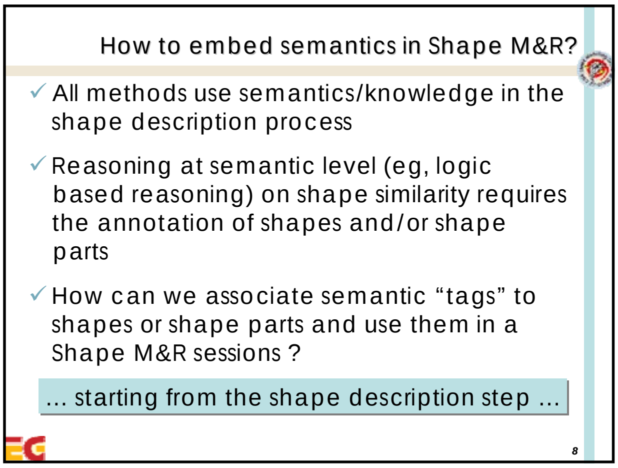
- The results did not show any method really outperforming others
- Results per single query can give insights on what method is best suited for specific shapes

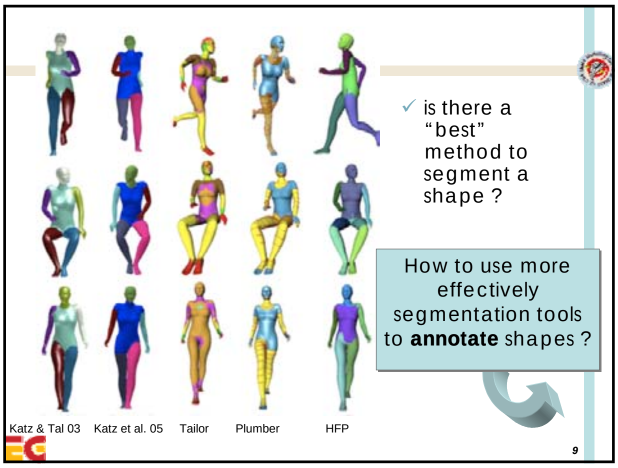
No single *best* method exists

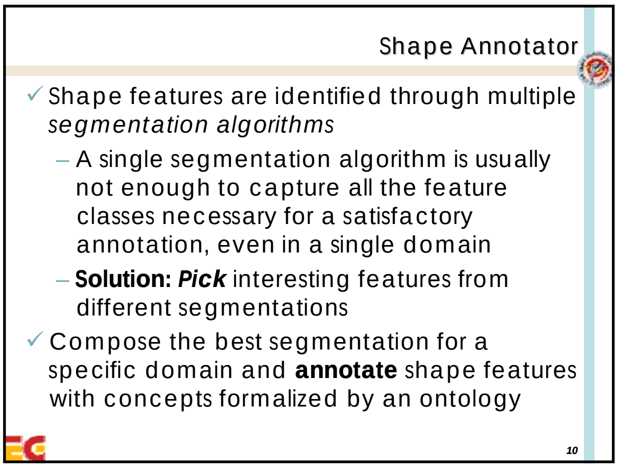
Benchmarking can help the definition of *best practices* to help users selecting the most appropriate retrieval method for the application context, shape category and complexity, type of similarity implied

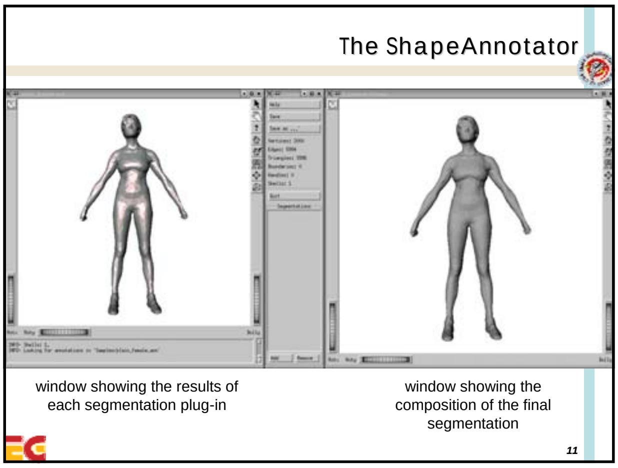
6

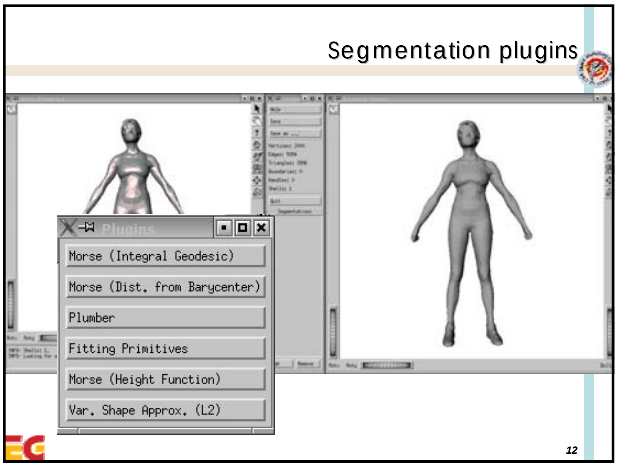

How to *describe* a shape ?
Geometry
Detect relevant local features
Structure
Organize them in a structure
Semantics
Use the structure to detect high-level features (semantics)
perception
understanding
7

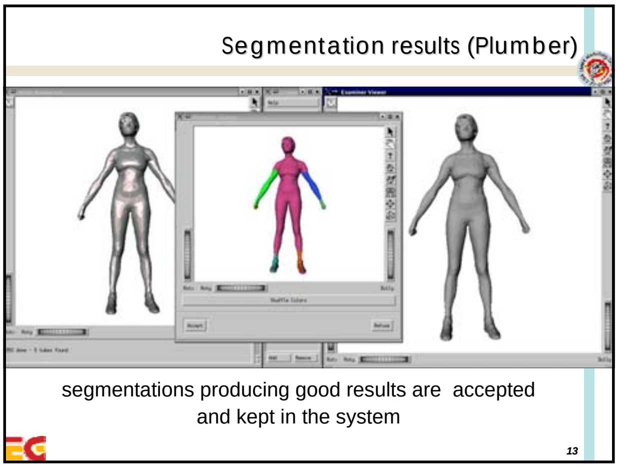

How to embed semantics in Shape M&R?
All methods use semantics/knowledge in the shape description process
Reasoning at semantic level (eg, logic based reasoning) on shape similarity requires the annotation of shapes and/or shape parts
How can we associate semantic “tags” to shapes or shape parts and use them in a Shape M&R sessions ?
… starting from the shape description step …
8

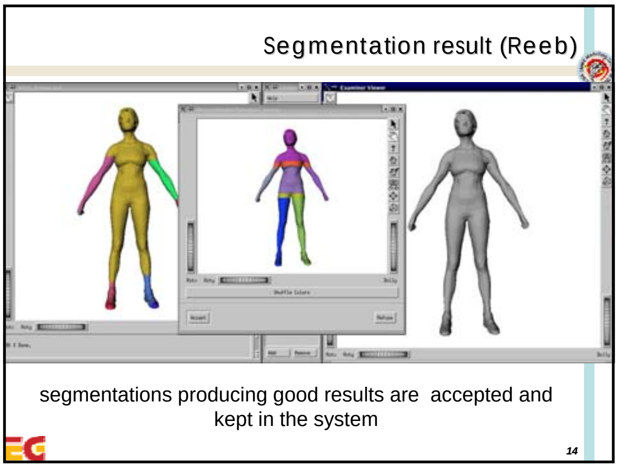

is there a “best” method to segment a shape ?
How to use more effectively segmentation tools to **annotate** shapes ?
Katz & Tal 03
Katz et al. 05
Tailor
Plumber
HFP
9

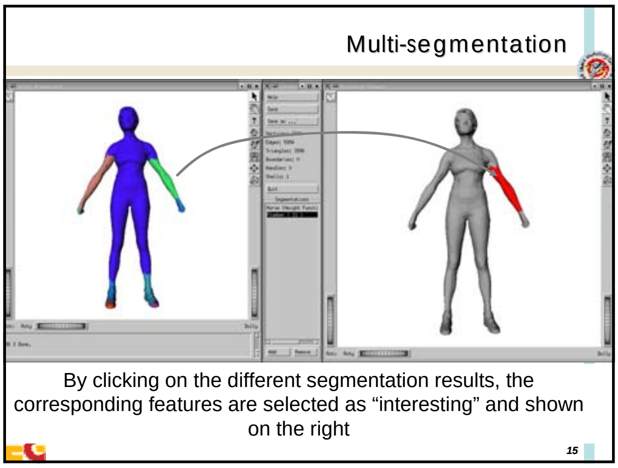

Shape Annotator
Shape features are identified through multiple *segmentation algorithms*
– A single segmentation algorithm is usually not enough to capture all the feature classes necessary for a satisfactory annotation, even in a single domain
– **Solution: *Pick*** interesting features from different segmentations
Compose the best segmentation for a specific domain and **annotate** shape features with concepts formalized by an ontology
10

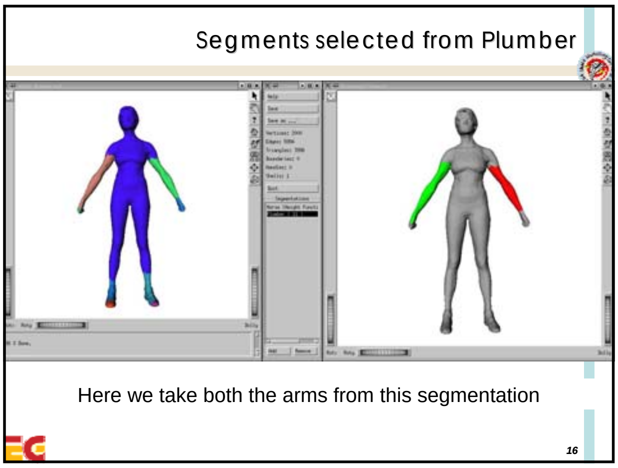

The ShapeAnnotator
window showing the results of each segmentation plug-in
window showing the composition of the final segmentation
11

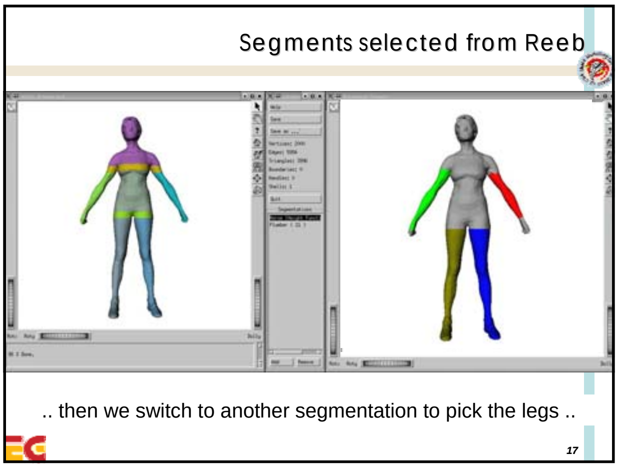

Segmentation plugins
Plugins
Morse (Integral Geodesic)
Morse (Dist. from Barycenter)
Plumber
Fitting Primitives
Morse (Height Function)
Var. Shape Approx. (L2)
12

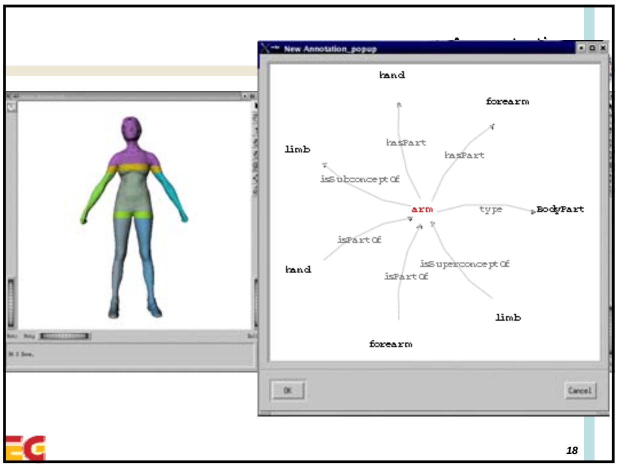

Segmentation results (Plumber)
segmentations producing good results are accepted and kept in the system
13


Segmentation result (Reeb)
segmentations producing good results are accepted and kept in the system
14

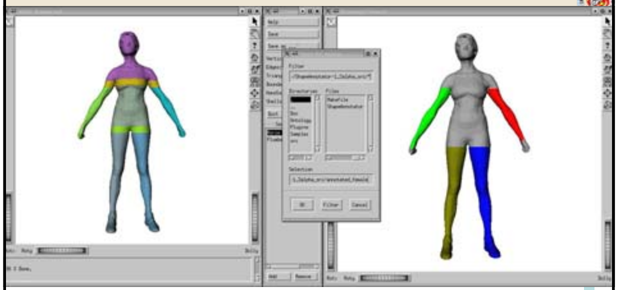

Multi-segmentation
By clicking on the different segmentation results, the corresponding features are selected as “interesting” and shown on the right
15


Segments selected from Plumber
Here we take both the arms from this segmentation
16


Segments selected from Reeb
.. then we switch to another segmentation to pick the legs ..
17


New Annotation_popup
hand
forearm
limb
hasPart
isSubconceptOf
arm
type
BodyPart
isPartOf
isSuperconceptOf
hand
limb
forearm
OK
Cancel
18

Saving the annotated mesh



The multi-segmented and annotated mesh can be saved and shared in a knowledge base

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Shape Annotator

- ✓ Submitted papers
- ✓ Technical report available (contacts: Francesco Robbiano, Marco Attene)

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Shape Annotator and 3D Search

- ✓ From geometry-based to semantics-based search engines for 3D content
  - See the presentation of Francesco, IMATI, this afternoon
  - How to formalize shape descriptions (ShaDe)
- ✓ Mixing geometric and semantic reasoning will allow for text-based query formulation to be used and mixed with geometric similarity evaluation and retrieval

21

Trends/challenges

- ✓ To make the **creation and modification** of 3D shapes, with the associated knowledge, as easy as **cut & paste** for texts
  - the development of appropriate knowledge representations and data structures for **intelligent** 3D shapes management
  - the ability to identify regions in 3D digital shapes that correspond to what humans perceive as regions of interest and tag them appropriately (**semantic annotation**)

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Trends/challenges

- ✓ To make the **creation and modification** of 3D shapes, with the associated knowledge, as easy as **cut & paste** for texts
  - To develop **3D search engines** based not only on geometry but also on semantics
  - To deliver the **appropriate 3D content in the appropriate modality**
    - development of appropriate visualization techniques including semantic rendering and semantic LODs
    - visual rendering of the relevance of retrieval results

23

… the end …

Questions?

Eurographics 2007 Tutorial T12

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