1 Hermite Splines

A Hermite Spline is defined by an array of control points \( \mathcal{V} \). Hence, each segment \( S_i \) is defined by two control points \( V_i \) and \( V_{i+1} \), each of which consists of a position \( p \) and a slope \( m \), respectively. A segment is evaluated as a cubic hermite polynomial defined by the two control points and is defined as

\[
\begin{align*}
\mathbf{p}(t) &= (2t^3 - 3t^2 + 1)p_0 + (t^3 - 2t^2 + t)m_0 + \\
&\quad (-2t^3 + 3t^2)p_1 + (t^3 - t^2)m_1, \\
\frac{\mathbf{p}(t)}{\partial t} &= (6t^2 - 6t)p_0 + (3t^2 - 4t + 1)m_0 + \\
&\quad (-6t^2 + 6t)p_1 + (3t^2 - 2t)m_1,
\end{align*}
\]

with \( \frac{\mathbf{p}(t)}{\partial t} \) being the first derivative. If we make sure, that neighbouring segments share both position and slope, we can achieve \( C^1 \) continuity for the spline. This allows us to conduct analysis tasks, that rely on an analytical representation for the trajectory plus its first derivative.