A Convolution-Based Algorithm for Animated Water Waves

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Abstract

A non-linear partial differential equation solver is too sophisticated for computer graphics applications if they are only used to render effects like circular waves and ship wakes. We present an approach which simulates waves through a convolution algorithm. It handles both gravity waves and capillary waves; the latter are often neglected even though they dominate small-scale behavior. The algorithm can be integrated into a complete solution architecture: First, standard commercial 3-D software is used to prepare an animated scene with objects traveling on a water surface. Based on the movements of these objects, waves are calculated and added as bump and displacement maps to the 3-D model.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation

1. Introduction

Water waves are an everyday phenomenon and thus their simulation makes for a natural research topic in computer graphics. Despite all past efforts, both the analytical and numerical treatment of a liquid’s motion still pose considerable challenges. Fluid motion is described by the continuity equation plus the Navier-Stokes equations—a system of three non-linear partial differential equations—in conjunction with surface conditions, which typically are non-linear as well. However, a full-blown numerical solution of these equations is rarely needed to generate realistically-looking animated films.

Since the motion in the volume of the liquid remains invisible in most scenes, it is sufficient to model the surface of the liquid. Additionally, water is virtually incompressible, its viscosity can often be neglected, and non-linear effects stay small if the waves’ height is small when compared to their length. When using an approximate, linear system of partial differential equations, one arrives at the conclusion that small-amplitude surface water waves on a flat infinite ocean with infinite depth are composed of sinusoidal waves

\[(x,t) \mapsto \cos(k \cdot x - \omega(k)t + \phi),\]

where \(x \in \mathbb{R}^2\) is the position on the plane of the ocean, \(k \in \mathbb{R}^2\) the wave vector along the surface, \(\omega(k)\) the corresponding circular frequency, and \(\phi \in \mathbb{R}\) the phase. The cir-

Figure 1: A raindrop simulation by the proposed algorithm shows the characteristic multiple rings formed by the interplay of capillary and gravity waves. The results of the simulation have been used for a bump map and a caustics texture in a rendering created with Cinema 4D XL. Compare with figure 3 for the height field.
circular frequency only depends on the wave number \( k = |k| \) and is given by
\[
\omega(k) = \sqrt{gk + \frac{T}{\rho}k^3},
\]
with gravity acceleration \( g \), surface tension \( T \), and mass density \( \rho \). Under normal circumstances, their values are \( g = 9.8 \text{kg/m/s}^2 \), \( T = 0.072 \text{kg/s}^2 \), and \( \rho = 1.0 \cdot 10^3 \text{kg/m}^3 \).

The dispersion relation \( \omega(k) \) causes a number of important visual effects:

- The phase velocity \( \omega(k)/k \) reaches its minimum of 23 cm/s at a wavelength of 17 mm. Shorter waves (capillary waves dominated by surface tension) as well as longer waves (gravity waves) travel faster. Likewise, the group velocity \( d\omega(k)/dk \) possesses a minimum of 18 cm/s. A stone thrown into deep water produces complex ring waves where the innermost ring expands at this speed. In contrast to this behavior of water surface waves, waves of light or sound travel at a constant speed, regardless of their frequency. If water behaved this way, a stone thrown into water would yield a single sharp circular wave.
- A ship traveling at constant speed along a straight line on deep water leaves behind a Kelvin wake—a wedge with semiangle 19°. This angle is independent of the ship’s velocity, once more in stark contrast to sound waves: The Mach cone of a supersonic plane gets more pointed as the speed of the aircraft increases.

Therefore, an approach using the much simpler wave equation of sound and light often results in unrealistic behavior. Nonetheless, this equation is favored e.g. in computer games because of its simple solvability\(^8\). In order to realistically simulate ring waves and Kelvin ship waves for 3-D animated films without too much mathematical and computational overhead, we sought a simple solution to the linear approximation of water surface waves outlined above.

Due to the complicated dispersion relation, there is no simple finite-difference formulation for this approximation. However, one can describe the propagation of waves with help of convolution. Let the elevation of the wave field be \( \eta_t(x) \) with \( \eta_0 = 0 \) representing the surface at rest. Store the momentary state of the ocean at time \( t \) as a complex-valued function \( x \mapsto \eta_t(x) \in \mathbb{C} \) and use the real value of this function for display. Given \( \eta_t \), one can calculate the elevation at any time \( t + \Delta t \) by convolution with a certain kernel function \( \Psi_{\Delta t}(x) \):
\[
\eta_{t+\Delta t}(x) = \int \int_{\mathbb{R}^2} d^2 y \Psi_{\Delta t}(y) \eta_t(x-y).
\]

Formally, the kernel \( \Psi \) is given by the mathematically ill-defined expression
\[
\Psi_{\Delta t}(x) = \int \int_{\mathbb{R}^2} d^2 k \exp \left( i(k \cdot x - \omega(k)\Delta t) \right).
\]

Like in Huygen’s principle, this is the circular wave caused by a disturbance at time 0 at the origin, observed after a time span of \( \Delta t \). In order to use this kernel with a lattice-based numerical approximation, it is necessary to introduce cutoffs for long and short waves. These cutoffs have the additional benefit of eliminating the mathematical problems with this expression.

The convolution scheme allows to calculate each frame without intermediate time steps. Sudden disturbances, such as a stone dropped into a pool, can be incorporated by altering \( \eta_t \) at the time \( t \) of this incident, for example by adding a function to \( \eta_t \) which is zero outside the cross section of the stone. In order to model continuous disturbances such as traveling ships, however, one has to integrate over the time-span \( \Delta t \).

We have developed a prototype system for water wave animation with this algorithm. The solution is based on Maxon Cinema 4D XL, but could use virtually any other commercial 3-D software package. This program is used to model and animate a flat ocean and objects traveling on the water, which still is flat. These pictures are rendered black and white in a view form above and then fed into our special add-on software. It computes the corresponding wave elevation fields, optionally separates the results into a blurred, low-pass version and a corresponding high-pass version. In addition, the software calculates the caustics pattern on the ground. These parallel sequences of pictures are fed into the 3-D software as displacement maps, bump maps, and texture maps for the final rendering—now including waves and seen from an arbitrary point of view. The division into displacement maps and bump maps allows for a relatively coarse tessellation resulting in short rendering times.
The next section outlines previous work done for simulating water waves in 3-D computer graphics. Section 3 describes our convolution-based algorithm for time evolution. Section 4 explains how objects which move on the water surface are incorporated into this approach. A discussion of the results follows in section 5. The final section 6 closes with a summary and an outlook on possible extensions.

2. Previous Work

The first two major works about water wave animation rooted in physics have been published in 1986: Fournier and Reeves as well as Peachey use wave trains with the shape of a trochoid respectively an approximation to a trochoid shape. Waves running onto a shore are subjected to refraction and therefore compressed. Gonzato carries the idea of “wave tracing” further to calculate refraction due to varying water depth and the diffraction at obstacles. Here, capillary waves are introduced, even though they are simply modeled by fractal noise. In their recent work, Gamito and Musgrave offer a more detailed theory of wave tracing.

Thon and Ghazanfarpoor use wave spectra measured e.g. from photographs to synthesize ocean waves. Using quite a different approach, Schneider and Westermann simulate waves using fractal noise and then render these in real time with light reflection, refraction, and caustics with modern consumer graphics hardware.

Goss presents an particle-system algorithm tailored to display ship waves in real-time. Kahn superimposes circular waves from points along a ship’s path to simulate its wake.

To allow a more complete interaction of the water with disturbances as well as changing boundaries, there have been several approaches based on partial differential equations. Kaas and Miller use a linearized model of a shallow water limit, which is equivalent to the wave equation of light and sound. This is solved by a finite-difference-method. Recently, Layton and van de Panne have improved these ideas by incorporating non-linear advection.

O’Brien and Hodgins model the water volume as an array of square columns which exchange fluid. If the content of a column moves upward faster than a threshold velocity, their algorithm generates spray. To achieve real-time response, Chen and Lobo replace the full three-dimensional Navier-Stokes equations by a two-dimensional version. The local pressure calculated in this manner is used to steer a three-dimensional height field which represents the water surface.

Foster and Metaxas simulate solutions of the full Navier-Stokes equations by using a voxel representation for the volume and marker particles to trace the surface. Thanks to this representation, their algorithm can produce e.g. breaking waves and overturns. Surface tension is simulated by altering pressure gradients. To stabilize the method for large time steps, a damping term has to be introduced. Taking a different road, Stam proposes an intrinsically stable algorithm for the full Navier-Stokes equations. It can be simplified using Fourier transform. However, the presented algorithm does not allow free boundaries such as water surfaces. Foster and Fedkiw combine Stam’s solver with marker particles to trace the fluid surface.

Weimer and Warren note that certain multi-grid partial differential solvers can be formulated with help of vector subdivision operations. The method they describe assumes a slow flow, i.e. viscosity dominates inertial behavior, and is therefore not well-suited for water simulation.

3. Time Evolution by Convolution

To treat the problem numerically, we discretize the elevation field \( \eta \) to a finite lattice of size \( L \times L \) with \( N \times N \) points. It consists of the \( (p \Delta x, q \Delta x) \) with \( p, q = 0, 1, \ldots, N - 1 \) and \( \Delta x = L/(N - 1) \). In our experiments, \( N = 256 \) or, in extreme cases, \( 512 \) proved to be sufficiently large. The real part of the computed \( \eta \) will be delivered as a bump map and/or displacement map to the 3-D rendering software. Therefore, such maps will then consist of e.g. \( 256 \times 256 \) pixels.

To calculate the elevation field in an animated film from one picture frame to the next, we use the time evolution given by equation 2 with \( \Delta t \) being the time between frames. The double integral is approximated using the double sum

\[
\tilde{\eta}_{t+\Delta t}(p\Delta x, q\Delta x) = \frac{(\Delta x)^2}{(\Delta t)^2} \sum_{r=-M}^{M-1} \sum_{s=-M}^{M-1} \Phi_M(r\Delta x, s\Delta x) \eta_t((p-r)\Delta x, (q-s)\Delta x) \tag{4}
\]

Here, \( \eta_t \) is treated as zero outside of \( [0, L] \times [0, L] \). \( M \) measures the half size of a sliding window which is used for the convolution. We use \( M = 40 \), so that the window contains \( 80 \times 80 \) lattice points. Instead of the ill-defined \( \Psi_M \) of equation 3 a regularized version \( \Phi_M \) is used. Especially, \( \Phi_M \) has to smoothly approach zero on the boundary of the sliding window (see below).

The \( \tilde{\eta}_{t+\Delta t}(p\Delta x, q\Delta x) \) resulting from equation (4) cannot be used directly, because the resulting waves would reflect strongly at the borders. This reflection can be suppressed reliably by cutting off the elevation field near the borders in linear fashion:

\[
\eta_{t+\Delta t}(p\Delta x, q\Delta x) = h(p)h(q)\tilde{\eta}_{t+\Delta t}(p\Delta x, q\Delta x) \tag{5}
\]

with

\[
h(p) := \begin{cases} p/M & \text{if } p < M \\ 1 & \text{if } M \leq p < N - M \\ (N - 1 - p)/M & \text{if } p \geq N - M. \end{cases}
\]

The resulting elevation field \( \eta_{t+\Delta t} \) will be used to calculate the next frame’s field \( \eta_{t+2\Delta t} \), which in turn will serve as base for the succeeding one, etc.

The remaining task is to construct a regularized kernel.

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\[ \Psi_{x+t}(x) = \int_{\mathbb{R}^2} d^2y \Psi_t(y) \Psi_t(x-y). \]  

In order to preserve this property, we have chosen to modify the dispersion relation given in equation (1) by replacing

\[ \omega(k) \rightarrow \omega(k) - ic(k), \]  

where \( c(k) \geq 0 \) is a damping function depending on the wave number. The well-defined analogue of equation 3 then is a discrete Fourier transform of a function with fast decay.

To define \( c(k) \), one has to identify the waves which are to be suppressed. Using the stationary phase method\(^2\) to approximate equation (3), one finds that the major contributions to \( \Psi_{\Lambda t}(x) \) arise from wave vectors \( k \) at which the exponent has zero gradient, i.e. \( x/\Delta = \text{group velocity vector} \)

\[ v_G(k) := \frac{\partial \omega(k)}{\partial k} = \frac{g + 3T|k|^2/\rho}{2\omega(k)} k^0. \]  

We want to cutoff contributions to \( x \) which lie near the border of the \( 2M \times 2M \) window, or, to be specific, where \( |x| > \frac{3}{4} M \Delta \). Hence, the \( k \) which follow the following criterion have to be suppressed:

\[ |v_G(k)| > \frac{3 M \Delta}{4}. \]  

To this end, one can choose \( c(k) \) in the modified dispersion relation (7) as follows:

\[ c(|k|) := \frac{5}{M \Delta} \left( |v_G(k)| - \frac{3 M \Delta}{4} \right) ^+ \]  

where the pre-factor is somewhat arbitrary (this choice proved to be useful in our experiments) and where

\[ (a)^+ := \begin{cases} a & \text{if } a > 0 \\ 0 & \text{else.} \end{cases} \]

To implement the cutoff, we found two other promissing options which appear to merit further investigation: First, equation (9) could be reformulated using the smoother expression

\[ \frac{100 \Delta}{(M \Delta)^2} \left( |v_G(k)| - \frac{1}{2} \frac{M \Delta}{\Delta} \right) ^2. \]

Secondly, the group velocity could be kept below \( \nu_{\max} := \frac{3 M \Delta}{4} \) by approximating the dispersion relation \( \omega(k) \) of equation (1) with

\[ \sqrt{gk + \frac{\rho}{2} k^2 a \arctan(k/a) + b^2}, \]  

where

\[ a := \frac{2\nu_{\max}^2}{\pi T} \quad \text{and} \quad b := \frac{g}{2\nu_{\max}}. \]
4. Wave Generation by Swimming Objects

Ships and other objects in the water surface generate waves through their motion and/or form changes. Our solution uses the motion between two picture frames to calculate how the elevation field \( \eta_{t+\Delta t} \) is to be altered for the following frame. For each swimming object, an additive contribution is computed.

For each object, we render the animation as a picture sequence where the scene is shown from straight above, with the ocean in black and the cross-section formed by the object with the water surface in white. Such a rendering can easily be obtained through Boolean operations available in standard 3-D animation software.

\[ \Delta \eta_{t+\Delta t}(x) = \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} d^2 y \left( f_t(x-y) \Phi_{\Delta_t}(y) + f_{t+\Delta t}(x-y) \Phi_{\Delta t}^2(y) \right) \]

where

\[ \Phi_{\Delta t}^1(y) := \int_{\mathbb{R}^2} \frac{d^2 k}{4\pi^2} \exp(i(k \cdot y - \omega(k)\Delta t)) \]

\[ \times \left( -1 + i\omega(k)\Delta t e^{-\phi(k)} + i\phi(k) - 1 \right) \]

\[ \Phi_{\Delta t}^2(y) := \int_{\mathbb{R}^2} \frac{d^2 k}{4\pi^2} \exp(i(k \cdot y)) \]

\[ \times \left( 1 + i\omega(k)\Delta t e^{\phi(k)} - i\phi(k) - 1 \right) \]

with \( \phi(k) := (k \cdot v - \omega(k))\Delta t \).

The integral over \( s \) can be eliminated by partial integration:

\[ \Delta \eta_{t+\Delta t}(x) = \int_{t}^{t+\Delta t} ds \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} d^2 k \int_{\mathbb{R}^2} d^2 y \]

\[ \times \exp(i(k \cdot y - \omega(k)(t+\Delta t - s))) g_s(x-y) \]

\[ g_s(x) := \frac{t+\Delta t - s}{\Delta t} f_t(x - (s-t)v) + \frac{s-t}{\Delta t} f_{t+\Delta t}(x + (t+\Delta t - s)v) \]

where \( v \) is the object's momentary velocity vector. It is calculated from the trajectory of the center of gravity of \( f_t \).

Waves are generated by changes of \( f_t \), so we use \( g_s := \frac{df_t}{ds} \) as disturbance. In analogy to equations (2) and (3), acting over the interval \( s \in [t, t+\Delta t] \) leads to the following additive change to the elevation field \( \eta_{t+\Delta t}(x) \):

\[ \Delta \eta_{t+\Delta t}(x) = \int_{t}^{t+\Delta t} ds \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} d^2 k \int_{\mathbb{R}^2} d^2 y \]

\[ \times \exp(i(k \cdot y - \omega(k)(t+\Delta t - s))) g_s(x-y) \]

\[ g_s(x) := \frac{t+\Delta t - s}{\Delta t} f_t(x - (s-t)v) + \frac{s-t}{\Delta t} f_{t+\Delta t}(x + (t+\Delta t - s)v) \]

\[ \Delta \eta_{t+\Delta t}(x) = \int_{t}^{t+\Delta t} ds \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} d^2 k \int_{\mathbb{R}^2} d^2 y \]

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\[ g_s(x) := \frac{t+\Delta t - s}{\Delta t} f_t(x - (s-t)v) + \frac{s-t}{\Delta t} f_{t+\Delta t}(x + (t+\Delta t - s)v) \]
The resulting expression for $\Delta \eta_{\ell+kT}(x)$ is discretized in the same way as in equation (4) and cut-off at the border in analogous form to equation (5). It has to be evaluated for every swimming object and for every picture frame except for the first one.

5. Results and Discussion

Figure 1 displays a simulation of raindrops, rendered with Maxon Cinema 4D XL. At pseudo-random moments, the elevation field will be offset at single, pseudo-randomly placed lattice points. Our system reproduces the typical ripples preceding the relatively large inner ring.

The convolution-based approach produces ship wakes which form the correct angle, as can been seen in figure 5. By recomputing the kernels, curved paths can be handled as well, see figure 2. Just like in nature, the Kelvin wake is distorted along the curve.

Up to now, the system does neither reflect nor diffract waves at boundaries or swimming objects. Instead, waves move through obstacles. This remains nearly invisible in open-ocean scenes, but can be noticed when shores, islands, or pool walls come into play. In addition, the simulation lacks shallow-water effects, especially the refraction of waves hitting a shore. Non-linear effects are not yet incorporated into the algorithm. Since the water surface is modeled by an elevation field, waves cannot break.

The discretization to a grid causes little visual defects. Of course, the time step $\Delta t$ and the grid interval $\Delta x$ must be chosen in such a way that the sliding window contains the important features of a ring wave. The size $2M\Delta t$ of the window must be large enough to capture the major part of the ring wave. On the other hand, the wave must not degenerate to a few pixels in the middle.

To adjust the values of $\Delta t$, $\Delta x$, and $M$, one can estimate the size of a ring wave $\Delta t$ after a point-like disturbance has occurred. This ring has to fit into the window, but should not be much smaller. The sharply defined inner wave front expands radially at a speed set by the minimum $1.1 \sqrt{Tg/\rho}$ of the group velocity. The outer front of the ring wave decays more softly, so that its velocity is harder to define. The asymptotic expansion of gravity waves contains the term $\cos(g(\Delta t)^2/4r - \pi/4)$, where $r$ is the distance from the origin of the disturbance. The largest $r$ for which this expression attains a maximum obeys $g(\Delta t)^2/4r - \pi/4 = 0$. Therefore, an estimate for the outer size of the ring wave can be given as $g(\Delta t)^2/\pi$. In our experiments, we still got convincing results when half of the size of the sliding window was 1/8 of this size.

This leads to the following conditions on $\Delta t$, $\Delta x$, and $M$:

$$\Delta x \ll 1.1 \sqrt{Tg/\rho} \Delta t \ll M\Delta x, \quad g(\Delta t)^2/\pi \approx 8M\Delta x.$$  

Furthermore, the objects’ velocity moving across the water must not be too large: The waves they generate within $\Delta t$ have to fit into the window used for approximating the $k$-integrals of $\Phi_{\Delta t}^{(1)}$ and $\Phi_{\Delta t}^{(2)}$ in equation 10.

Because there is only one computational step per picture frame, the stability of the solution algorithm is not a very critical issue. In addition, the discretized system obeys a propagation identity corresponding to equation (6) which rules out many run-away effects. An open question remains whether the suppression of boundary reflections according

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to equation (5) can cause instability. However, in our experiments such effects were not visible.

Using the prototype software, a PC equipped with an
Pentium®-III processor running at 500 MHz computes
waves at a speed of 90 s per picture frame (L = 256, 2M =
80, one swimming object). The most time-intensive part
of the algorithm consists in the convolution, even though
we limit the evaluation of equation (10) to the neighborhood
of swimming objects and pre-compute Φ(1) and Φ(2) per object
for each frame. This pre-computation and the discretized cal-
culation of equation 3 could be implemented through a Fast
Fourier transform (FFT), but this would not yield an appreci-
able acceleration.

6. Summary and Future Work

We have presented an approach to simulate water surfaces
waves using a simple algorithm. Even though it neglects
nonlinearity and viscosity, the model believably reproduces
visually important effects such as the distinct behavior of
capillary and gravity waves. It produces realistic ring waves
and ship waves.

Typically, only a close inspection of the results reveals
shortcomings of this algorithm when compared to more
complex approaches. To further improve the simulation, one
can think of several tasks:

- Simulate ocean waves using a pseudo-random realization
  of a spectrum measured in nature18.
- Reflect waves at boundaries; diffract waves at obstacles.
- Generate foam at sharp wave crests. It could be rendered
  using a texture map or via a particle system14.
- Simulate wakes stirred by the propellers of ships.
- Calculate velocity data to render motion blurred waves.
- Simulate non-linear effects of hydrodynamics—for in-
  stance, by including self-advection19 or by deforming the
  waves in order to produce steeper peaks5.

After freezing the features of the solution, we want to
speed up the run-time performance of the algorithm using
the floating-point SIMD instructions of common micropro-
cessors to implement convolution and FFT.

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