Efficient Spherical Harmonics Lighting with the Preetham Skylight Model

Ralf Habel[1],Bogdan Mustata [1] and Michael Wimmer[1]

[1]Institute of Computer Graphics and Algorithms
Vienna University of Technology, Austria

Abstract

We present a fast and compact representation of a skylight model for spherical harmonics lighting, especially for outdoor scenes. This representation allows dynamically changing the sun position and weather conditions on a per frame basis. We chose the most used model in real-time graphics, the Preetham skylight model, because it can deliver both realistic colors and dynamic range and its extension into spherical harmonics can be used to realistically light a scene. We separate the parameters of the Preetham skylight model's spherical harmonics extension and perform a polynomial two-dimensional non-linear least squares fit for the principal parameters to achieve both negligible memory and computation costs. Additionally, we execute a domain specific Gibbs phenomena suppression to remove ringing artifacts.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Natural Phenomena, Spherical Harmonics, Skylight

1. Introduction

Spherical Harmonics lighting is an established technique in real-time graphics to achieve interactive global illumination for static scenes under low-frequency dynamic lighting environments [SKS02]. Several advanced techniques are based on spherical harmonics precomputed radiance transfer [KSS02, RWS*06].

To dynamically calculate the lighting coefficients, simple analytic functions or environment maps are usually used. These methods can only provide simple lighting or vary the lighting through rotation. Especially for outdoor scenes lit by a skylight model, calculating the spherical harmonics coefficients on the fly is time consuming and introduces objectionable errors if undersampled. Therefore, a fast, accurate and still compact method to calculate the lighting coefficients for a parameterized lighting environment is presented, which allows to change all parameters on a frame by frame basis, including the number of spherical harmonics bands.

The presented method is not limited to a particular skylight model, but we use the Preehtam model since it can be displayed in real time [HP03], so that full consistency and physical plausibility of the displayed sky and the used spherical harmonic lighting can be achieved. Being the most used skylight model in real-time graphics, the Preetham skylight model delivers both realistic colors and dynamic range, capturing lighting influences of the sun's halo, sky color and their intensity distribution. All of these effects can be captured when lighting a scene using spherical harmonics.

2. Related Work

Dobashi et al. [DNKY95] proposed to use a discrete cosine basis to speed up the evaluation of a skylight model by determining the optimal number of basis functions and evaluating the tabulated weights to reconstruct the hemispherical function. Compared to this previous work, the requirements of spherical harmonics lighting have to be met and the resulting coefficients are used for the radiance transfer calculations instead of evaluating them in euclidian space.

2.1. Spherical Harmonics Lighting

In spherical harmonics lighting, the radiance transfer from an environment to a surface or volume is precomputed as the weights $c_{l,m}$ of spherical harmonics basis functions (SH).

Figure 1: *Scene lit with daylight configuration*

Taking advantage of their orthogonality, the radiance transfer is evaluated using the inner product of the spherical harmonics weights calculated from a dynamic lighting environment and the precomputed weights on the surface or volume in real time. We would like to refer to [SKS02] and [DV04] for a detailed description of the method. The proposed method is concerned with the efficient reconstruction of a parameterized environment in its spherical harmonics representation.

2.2. Preetham Skylight Model

The Preetham skylight model [PSS99] approximates the full spectrum daylight for various atmospheric conditions. It uses spectral calculations and the results are verified against standard literature from atmospheric science and therefore delivers realistic colors and dynamic range. The parameters for this model are the sun's angle to the zenith θ, azimuthal angle ϕ and turbidity τ, which represents the cloudiness and haziness of the atmosphere. We reduce the parameter range of the turbidity to $[2.5, 6]$ since, as shown by Zotti et al. [ZWP07], values below 2.5 produce too high intensities at the horizon and the model is not usable for lighting below this parameter value. Since the skylight model is used for PRT rather than displaying a sky, we consider its SH representation as the ground truth.

3. Overview

The goal of this work is to represent the spherical harmonic weights as functions of the skylight model parameters: $c_{l,m}(\theta,\phi,\tau)$. A naive approach would be a full tabulation in all three parameters. However, this would needlessly waste memory (which can be an important issue on platforms such as consoles). Furthermore, it is difficult to maintain a low error over a large dynamic range, which is important since postprocessing steps such as tone mapping remap intensity values strongly.

In our method, we first eliminate one parameter by exploiting the fact that the azimuthal angle ϕ represents a z-rotation of the hemispherical function. Since a rotation is a linear transformation in spherical harmonics, we can defer its evaluation and thus reduce the parameter space to the two dimensions (θ,τ) without introducing an error. For these remaining two parameters, the key observation is that all SH-weights show a largely polynomial behavior in (θ,τ). Therefore the SH-weights can be well compressed by performing a two-dimensional polynomial non-linear least square fit in θ and τ, making a very fast reconstruction possible. We have tried other bases such as a discrete Fourier basis, but they showed higher errors with the same number of coefficients.

3.1. Polynomial Fitting and Reconstruction

The goal is to represent each SH weight as a polynome of degree (d_i, d_j) with coefficients $(p_{l,m})_{i,j}$:

$$c_{l,m}(\theta,\tau) = P_{l,m}(\theta,\tau) = \sum_{i,j} (p_{l,m})_{i,j}\theta^i\tau^j,\ i = 0..d_i,\ j = 0..d_j \tag{1}$$

In order to determine the coefficients, we evaluate the SH weights $c_{l,m}$ on a dense grid in (θ,τ). The convolution of the SH basis function with the skylight function required at each grid point is calculated by sampling until the result converges. This effectively treats the skylight model as a black box so that any parameterizable signal can be used. This data is then used for a polynomial non-linear least square fit in (θ,τ), resulting in the polynomial coefficient matrix [Atk88] $(p_{l,m})_{i,j}$ for each SH weight $c_{l,m}$. This calculation has to be done only once for each model because the complete model is reparameterized.

To reconstruct the SH weights in real time, we exploit the fact that the polynomial parameter matrix $(\theta^i\tau^j)_{i,j}$ for a given set of parameters (θ,τ) is the same for all SH weights. Therefore, for each SH weight, this matrix is multiplied and summed component wise with $(p_{l,m})_{i,j}$ to evaluate equation 1 and obtain $c^{rec}_{l,m}(\theta,\tau) = P_{l,m}(\theta,\tau)$. The reconstruction is required only once per frame and can therefore be easily carried out on the CPU.

3.2. Error Measurement

To measure the error of the reconstruction, we have to take into account that the signal is used in an inner product with the vector of surface SH weights, thus adding up the error contributions of the reconstructed SH weights $c^{rec}_{l,m}$, therefore allowing only very small errors in the reconstruction of each $c^{rec}_{l,m}$. Since the surface weights are not known and vary, the ΔL^∞ and ΔL^2 norms related to the minimum intensity $L_{\min}$ of the original data give a relative upper bound of the maximum and mean error for each parameter set:

$$E^u_{\max}(\theta,\tau) = \frac{1}{L_{\min}} \sum_{l,m} \|c^{rec}_{l,m} - c_{l,m}\| \tag{2}$$

and respectively

$$E^u_{\text{mean}}(\theta,\tau) = \frac{1}{L_{\min}} \sqrt{\sum_{l,m} (c^{rec}_{l,m} - c_{l,m})^2}\,. \tag{3}$$

Considering normalized surface SH weights, equations 2 and 3 are overestimates of the upper bound since a surface point never sees the whole environment. In our implementation we chose the maximum number of reconstructed SH bands as $N_{max} = 7$, which lies well above the number of bands commonly used. By fitting a polynomial with degree $d_i = 13$ in θ and degree $d_j = 7$ in τ, the highest mean error drops below 1.5% for 7 bands, introducing virtually no error ($E^u_{\text{mean}} < 0.005\%$) up to 5 bands, ensuring an accurate reconstruction over the complete parameter range. A comparison of the worst case reconstruction can be seen in Figure 2, and Figure 3 shows the highest upper bound error dependent on the number of bands used. In the case that higher band numbers are required, using a polynomial of higher degrees decreases the error. After the reconstruction, the SH weights are rotated around the zenith to take the azimuthal angle ϕ into account. The rotation matrix does not need to be fully constructed since

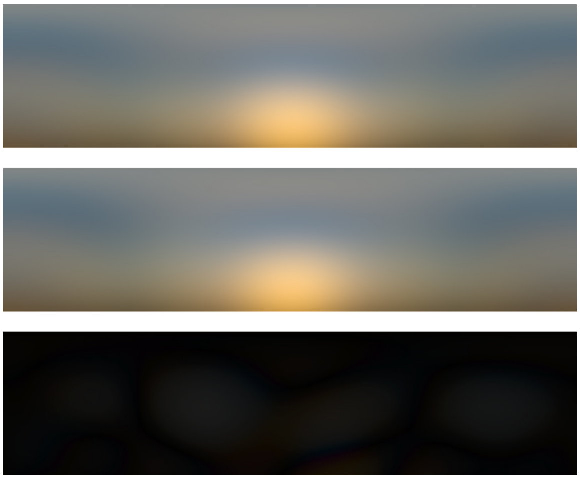

Figure 2: *Worst case reconstruction ($N = 7$, $\theta = \pi/2, \tau = 6$) with the correct extension (top), the reconstructed signal (center) and the difference (bottom).*

$$c_{l,m}(\phi) = c_{l,m}\cos(\|m\|\phi) - \text{sgn}(m)c_{l,-m}\sin(\|m\|\phi), m \neq 0 \tag{4}$$

implicitly rotates the weights [KSS02], only correlating $c_{l,m}$ with their counterpart $c_{l,-m}$, where the so-called zonal harmonic (i.e., $m = 0$) weights are excluded. No error is introduced by rotating the SH weights.

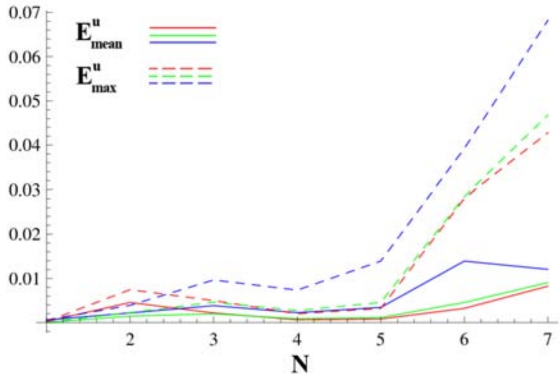

Figure 3: *Highest upper bound mean and maximal relative error dependent on the number of bands N used for all color channels.*

3.3. Gibbs Phenomenon Suppression

Skylight models are only defined on a hemisphere, whereas spherical harmonics require a spherical environment. Thus, a jump discontinuity occurs at the horizon. As in every finite frequency extension of a discontinuity, the Gibbs phenomenon appears and causes severe ringing in the lower hemisphere. There are ways to reconstruct a spherical harmonics signal without ringing [Gel97], but another basis change into Gegenbauer polynomials is required and therefore cannot be combined for spherical harmonics lighting. In our case, a domain-specific solution can be applied since the discontinuity is not dependent on ϕ and is therefore separated in θ, only affecting zonal harmonics. To suppress the ringing artifacts from the horizon, only the zonal harmonics are filtered with

$$c'_{l,0} = c_{l,0}\,\text{sinc}(\frac{\pi l}{N}), \tag{5}$$

which is equivalent to a one-dimensional box filter in θ [Ant93]. This causes a slight smoothing in θ but does not change the general appearance, and the weights are only filtered where necessary. The ringing artifacts in the lower hemisphere are strongly reduced without introducing significant smoothing, leaving only slight artifacts from non-zonal harmonic contributions as seen in Figure 4. As an optimization, for a fixed number of used bands, the zonal polynomial coefficient matrices $(p_{l,0})_{i,j}$ can be pre-scaled since equation 5 only depends on N.

4. Results

The proposed method only requires a very small memory footprint: 131 KB for 7 bands and 28KB for 4 bands respectively. This small amount of memory fits into the L2 or even L1 cache of any modern processor, resulting in about 80 microseconds of maximum computation time for 7 bands (measured on a P4, 3.2 GHz). Both computation time and

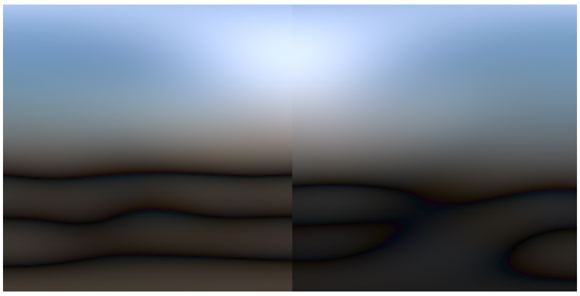

Figure 4: *Original (left) and filtered (right) spherical harmonic reconstruction. The ringing artifacts from the horizon are strongly reduced.*

memory resources needed are negligible compared to any other part of a graphics pipeline (the computation is done only once per frame).

We provide the resulting data for the case of the Preetham skylight model as C-arrays in conjunction with a full implementation of the proposed method [Hab08], which is ready to be used in any spherical harmonics setup. Figures 1 and 5 show a scene lit with the proposed method using 5 SH-bands. This combines naturally with a directional sunlight, soft shadows and a Reinhard tone mapper [Rei02] for a full sky representation and high dynamic range reproduction. We would like to refer to the accompanying video for a visualization of this combination of techniques.

Figure 5: *Scene lit at sunset.*

5. Conclusion and Future Work

We have presented an efficient method to reconstruct and filter skylight models for spherical harmonics lighting without introducing significant errors, demonstrated on the Preetham skylight model. Both memory consumption and computation time are kept at a negligible level. In the future, we would like to fit other skylight models and add parameters such as ground color or occluders like clouds and identify their correlations, optimal basis representations and corresponding filters to achieve a complex and dynamic environment representation for spherical harmonics lighting, only defined by a few high-level parameters.

6. Acknowledgements

This research was funded by the Austrian Science Fund (FWF) under contract no. P17261-N04.

References

[Ant93] ANTONIOU A.: *Digital Filters, Analysis,Design, and Applications*. McGraw-Hill, New York, NY, USA, 1993.

[Atk88] ATKINS K. E.: *Numerical Analysis*. John Wiley and Sons, 1988.

[DNKY95] DOBASHI Y., NISHITA T., KANEDA K., YAMASHITA H.: Fast display method of sky color using basis functions. In *Pacific Graphics '95* (1995).

[DV04] DEMPSKI K., VIALE E.: *Advanced Lighting and Materials With Shaders*. Wordware Publishing Inc., Plano, TX, USA, 2004.

[Gel97] GELB A.: The resolution of the Gibbs phenomenon for spherical harmonics. *Mathematics of Computation 66*, 218 (1997), 699–717.

[Hab08] //www.cg.tuwien.ac.at/research/publications/2008/Habel_08_SSH/, 2008.

[HP03] HOFFMAN N., PREETHAM A. J.: Real-time light-atmosphere interactions for outdoor scenes. *Graphics programming methods* (2003), 337–352.

[KSS02] KAUTZ J., SLOAN P.-P., SNYDER J.: Fast, arbitrary brdf shading for low-frequency lighting using spherical harmonics. In *EGRW '02: Proceedings* (Aire-la-Ville, Switzerland, Switzerland, 2002), Eurographics Association, pp. 291–296.

[PSS99] PREETHAM A. J., SHIRLEY P., SMITS. B. E.: A practical analytic model for daylight. In *Siggraph 1999, Computer Grahics Procedings* (Los Angeles, 1999), Rockwood A., (Ed.), Addison Wesley Longman, pp. 91–100.

[Rei02] REINHARD E.: Parameter estimation for photographic tone reproduction. *J. Graph. Tools 7*, 1 (2002), 45–52.

[RWS*06] REN Z., WANG R., SNYDER J., ZHOU K., LIU X., SUN B., SLOAN P.-P., BAO H., PENG Q., GUO B.: Real-time soft shadows in dynamic scenes using spherical harmonic exponentiation. In *SIGGRAPH '06: Proceedings* (New York, NY, USA, 2006), ACM Press, pp. 977–986.

[SKS02] SLOAN P.-P., KAUTZ J., SNYDER J.: Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments. In *SIGGRAPH '02: Proceedings* (New York, NY, USA, 2002), ACM Press, pp. 527–536.

[ZWP07] ZOTTI G., WILKIE A., PURGATHOFER W.: A critical review of the preetham skylight model. In *WSCG ' 2007 Short Communications Proceedings I* (Jan. 2007), Skala V., (Ed.), University of West Bohemia, pp. 23–30.