Interactive Diffraction from Biological Nanostructures

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Abstract
We describe a technique for interactive rendering of diffraction effects produced by biological nanostructures such as snake skin surface gratings. Our approach uses imagery from atomic force microscopy that accurately captures the nanostructures responsible for structural coloration, that is, coloration due to wave interference, in a variety of animals. We develop a rendering technique that constructs bidirectional reflection distribution functions (BRDFs) directly from the measured data and leverages precomputation to achieve interactive performance. We demonstrate results of our approach using various shapes of the surface grating nanostructures. Finally, we evaluate the accuracy of our precomputation-based technique and compare to a reference BRDF construction technique.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture

1. Introduction
In biology, structural coloration is the production of color through the interaction of light with nanoscale surface structures or intra-cellular submicron photonic crystals. Color production is due to wave interference with quasiperiodic structures whose periodicity leads to interaction with visible light. Spectacular examples of structural colors include the vivid and sometimes iridescent colors of some insects’ wings, bird feathers, and reptile scales. Stam [Sta99] pioneered diffraction shaders for rendering structural colors for analytically known nanostructures. This was followed by a number of efforts to model wave effects in computer graphics. Cuypers et al. [CHB∗12] proposed “wave-based BSDFs” (WBSDFs) using Wigner Distribution Functions (WDFs). Other approaches include Optical Path Differences (OPD) [ZCG08] and Augmented Light Fields [OKG∗10], which also employ WDFs. All these methods involve complex computations and are unsuitable for interactive rendering. Lindsay and Agu [LA06] focus on interactive rendering using low order spherical harmonics. This approach is too simplistic to model complex nanostructures. Thus, most existing techniques are either too slow, or rely on simplifying assumptions to achieve real-time performance.

In this paper, we instead derive reflectance models suitable for interactive rendering directly from physical measurements of biological structures. In particular, our approach applies to surfaces with quasiperiodic nanostructures that can be represented as heightfields. We develop a technique to precompute look-up tables that allow us to efficiently evaluate the BRDF and achieve real-time performance. We demonstrate our approach using nanostructures acquired from the sheds of two snake species, Elaphe guttata and Xenopheltis unicolor, which exhibit moderate and intense iridescence of the skin, respectively. We also verify our technique by comparing with their measured reflectances.

2. Overview
Our method is based on the pioneering work of Stam [Sta99] which shows how to directly formulate BRDFs in terms of nanoscale heightfields. The BRDF of a nanostructure $h(x,y)$ can be expressed in terms of the Fourier transform $P$ of the function $p(x,y) = e^{i\phi h(x,y)}$ as,

$$\text{BRDF}_\lambda(\omega_r, \omega_i) = \frac{F^2 G}{\lambda^2 Aw} \langle |P\left(\frac{u}{\lambda}, \frac{v}{\lambda}\right)|^2 \rangle. \quad (1)$$

Here, $\lambda$ represents a specific wave-length, and $\omega_i$ and $\omega_r$ are the incident and reflected unit direction vectors pointing away from the surface. $F$ represents the Fresnel term and $u, v, w$ are computed from the incident and reflected directions as $(u, v, w)^T = -\omega_i - \omega_r$. Please refer to Stam’s work [Sta99] for details about the other terms in Equation 1.
Stam makes the simplifying assumptions that the heightfields are homogeneous random functions. He then provides implementations for two special cases, (a) Gaussian processes and (b) periodic-like structures of regularly or randomly distributed bumps. Unfortunately, complex biological structures do not conform with these assumptions. We provide a practical extension to Stam’s method for general discrete heightfields. Our main contributions are:

- A method to render structural colors due to diffraction gratings directly based on physical measurements.
- An algorithm for interactive rendering leveraging precomputed look-up tables.

3. Efficient Evaluation for Discrete Nanostructures

For a sufficiently sampled, band-limited continuous heightfield, its discrete time Fourier transform (DTFT) consists of non-overlapping replicas of the continuous spectrum. We thus propose to replace the continuous Fourier transform term in Equation 1 with the DTFT term. However, a naive implementation using DTFT coefficients involves computing them for each rendered pixel and is impractical.

The BRDF formulation in Equation 1 requires integrating ‘coherent’ electro-magnetic radiation at the point of reflection. Since most sources emit light with limited spatial coherence, we introduce a Gaussian window \( g(x, y) \) to limit the surface area of integration. Note that the DTFT of a windowed signal is equivalent to the convolution of the DFT of that signal and the Fourier transform of the truncating window. This allows us to replace the DTFT with the DFT. However, since \( p(x, y) \) contains an optical geometry dependent term \( w \), using DFT does not alleviate the problem of view-dependence of the transform coefficients. We thus employ a Taylor series expansion of \( p(x, y) \) to separate the optical geometry dependent terms from the transform coefficients. With \( G = \mathcal{F}(g) \), the Fourier term in Equation 1 becomes:

\[
P \left( \frac{\mu}{X}, \frac{\nu}{Y} \right) \propto \sum_{n=0}^{\infty} \frac{(wk)^n}{n!} \left[ \text{DFT} \{W^n\} \ast G \right] \left( \frac{\mu}{X}, \frac{\nu}{Y} \right).
\]

Our final contribution is to provide a precomputation framework that encompasses the integration over the color (wavelength) spectrum to speed-up the computation of CIE XYZ colors using the standard rendering equation. With our proposed simplifications and by limiting the Taylor series, the X channel of the reflected light in the viewing direction \( \omega_k \) becomes:

\[
x \propto \sum_{n=0}^{N} \frac{(wk)^n}{n!} Ip(u, v), \quad Ip(u, v) \text{ incorporates the integration over the wavelength-spectrum for the Fourier terms.}
\]

For the brevity of discussion, the mathematical form of the \( Ip(u, v) \) term is not presented here. Note that similar derivations can be applied for the Y and Z channels. We precompute and tabulate \( Ip(u, v), p \in [0, N] \). The resulting lookup tables are loaded and used by our shader to compute RGB colors on-the-fly. Computation of \( Ip(u, v) \) is the most time-consuming task for our shader and precomputing it allows us to achieve interactive performance.

4. Results

We validate our method qualitatively against real-world snake-skin diffraction under controlled conditions. Figure 1a shows our experimental setup and Figure 1b shows reflections from an Elaphe sample. Our rendering under similar conditions produces color patterns which are qualitatively similar to those observed, see Figure 1c. We typically use 40 – 80 lookup tables of size 501 \( \times \) 501. We also render diffraction colors on a real snake surface geometry using a Xenopeltis nanostructure, which exhibits typical highly iridescent patterns for Xenopeltis, see Figure 2. The run-time performance of our shader for a screen of 1024 \( \times \) 1024 pels is consistently above 120 fps on an NVidia GTX680 GPU.

References


