Tangible Heritage: Production of Astrolabes on a Laser Engraver

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Abstract
The astrolabe, an analog computing device, used to be the iconic instrument of astronomers during the Middle Ages. It allowed a multitude of operations of practical astronomy which were otherwise cumbersome to perform in an epoch when mathematics had apparently almost been forgotten. Usually made from wood or sheet metal, a few hundred instruments, mostly from brass, survived until today and are valuable museum showpieces. This paper explains a procedural modelling approach for the construction of the classical kinds of astrolabes, which allows a wide variety of applications from plain explanatory illustrations to 3D models, and even the production of working physical astrolabes usable for public or classroom demonstrations.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling–Geometric algorithms, languages, and systems; I.3.8 [Computer Graphics]: Applications

1. Introduction
The Planispheric Astrolabe, an ingenious computing device which origin lies somewhere in the Near East of the later 4th century, used to be the representative instrument of astronomers and astrologers alike in the later medieval and renaissance period, and is frequently seen on contemporary paintings of scholarly environments and scientific activity. Unlike the Armillary Sphere, which was a three-dimensional model of the (geocentric) universe, the astrolabe maps the sky onto a flat disk that rotates over another disk with an intricate pattern of arcs that represents local horizontal coordinates, so that the sky situation for a specific date can be seen. On the reverse side, the instrument provides an altitude meter, and this combination allows many practical astronomical operations, of which the determination of time from a single altitude measurement of the sun or a star appears to have been the most common.

After further development by early Arab mathematicians and astronomers, the astrolabe saw wide distribution in the cultural region of early Islam, where the practical determination of prayer times depending on solar altitude was important for every believer. It was then introduced to Europe via Moorish Spain around the year 1000. Rare until about 1300, it has seen a sharp production peak in Europe in the late 16th century, but was quickly thereafter replaced by the

Figure 1: Front of a Planispheric Astrolabe. The parts are described in section 3.
telescope as observing instrument, while its quick, but not overly accurate results were abandoned in favour of those available by the then rapidly emerging mathematical procedures. In Europe, it was still described in books on astronomical instrument making well into the 18th century. In Islam countries, it was widely used until the late 19th century, and has been manufactured there until only a few decades ago. Today tourists are sometimes offered instruments which are sold as astrolabes, but which are unfortunately usually badly constructed and therefore useless.

Astrolabes are frequently shown in exhibitions as masterpieces of early Islam or European Renaissance instrument making. In all its strict construction and functionality, there was some artistic freedom for ornaments, dedications, or adorations. Original instruments are typically unaffordable, however, and also good brass reproductions are difficult to make and therefore costly.

For most of the applications the astrolabe has been used for, it has been superseded by more modern instruments, and some of the questions of everyday life it helped to answer in the medieval period are no longer even posed today. So visitors in a museum usually are confronted with an instrument that looks appealing to most, but is hard to understand because the basic astronomical applications are no longer commonly used in today’s mechanized world.

The seemingly complicated arrangement of lines and circles on most astrolabes is mostly derived directly from the stereographic projection. Traditional construction manuals typically contain long tables of precomputed values like circle centers and radii, but sometimes even omitted the description of basic construction principles in the understanding that the executing craftsman is not interested in the latter. Of course, these manuals can still be followed to manually add up all the lines and circles in a CAD drawing, a process which however appears rather tedious and repetitive. Also, a manually drawn instrument will only show one instance of the construction process will be immediately clear from figure in the same style as the finished drawings. The resulting astrolabe figures can be printed, or used as basis for a working virtual 3D instrument for cultural heritage applications in the medieval setting, and even physical models have been created from the same code by controlling a laser engraver/cutter. These real-world instruments are usable for, e.g., museums, where visitor groups could be demonstrated hands-on how to operate these instruments. The year 2009 has been declared “International Year of Astronomy” by UNESCO, and we expect a series of exhibitions and seminars on the history of astronomy, where these instruments can be used.

The rest of the paper is structured as follows: the section on related work focuses on procedural modelling in the domain of cultural heritage applications, followed by some pointers to construction descriptions for astrolabes. Then follows a functional description of the instrument which explains the various features that may be available and the advantages of using a programmed approach. Some construction details and their implementation are described afterwards, followed by a section on practical production of astrolabes with a laser engraver/cutter. The discussion concludes with potential other applications and future work.

2. Related Work

2.1. Procedural Modelling

Procedural modelling is frequently used in large scenes like urban visualization, when (mostly artificial) structures are too complex to be modelled completely in detail, and where on the other hand lots of buildings share similar structure. A limited set of rules for building construction can yield a wide variety of acceptable buildings that overall results in an acceptably realistic model of a large urban area.

In the area of cultural heritage, Birch et al. [BBJ*01] describe such a system that allows the rapid generation of simple house models. These can be used as “vernacular” buildings that fill the scene between more interesting buildings which have to be modelled in detail.

Wonka et al. [WWSR03] describe the modelling of buildings and Müller et al. [MWH*06] of complete large urban areas with the help of shape grammars. These allow the generation not only of buildings and cities with simple ground plans, but Müller et al. also show a large model reconstruction of ancient Pompeii that can be encoded in a limited set of rules.

Another application of procedural modelling is the encoding of regularly structured architectural detail. The traditional way of creating a detailed model in multiple resolutions would start with a highly detailed mesh, and automatic polygon reduction schemes would provide different levels of detail, probably losing the architectonic intent and providing a non-optimal mesh. Havemann and Fellner [HF04] explain the procedural construction of gothic church windows and their efficient implementation in their Generative Modeling Language (GML). Later, from the same group, Berndt et al. [BFH05] present their overall solution for an extremely compact shape representation that encodes 3D shapes with only a few control polygons plus functions in GML, which is capable of creating the final mesh in the required resolution by Catmull/Clark subdivision. Their solution is thus optimal for transferring complex geometric 3D
models also in a web application if bandwidth is limited, and if the models are reasonably regular to be described by geometric procedures. Berndt et al. present again windows of gothic cathedrals as excellent examples where this approach works very well. The GML syntax is very similar to Adobe PostScript [Ado90], a stack-based programming language mostly used for printers or as graphics file format best suited for vector graphics.

The current role of procedural modelling for digital cultural heritage is also described in [AG07, section 5.1].

2.2. Astrolabes

From medieval times, several manuscripts and printed texts on astrolabe construction survived and are topic of high interest by linguists and historians of mathematics and astronomy alike (e.g. [Lor05, Cha91, St613]).

The first modern large study on surviving astrolabes, and still a valuable reference, is [Gun32]. [Mic47] is often cited as best 20th-century source of construction details, although some issues remain open. [Sau84] explains construction and use also of other variants of the astrolabe. Good photographs of old instruments can be found in [WW98].

Recently, the Web allows access to images of collections of old instruments [Oxf06], general information [Mor05] and even a Java application [Pow]. An online catalogue has been under construction for years [Ko02]. Also, there are several individuals who build brass replica of certain instruments, or metal astrolabes in classical style without copying a specific model.

3. Description of the Astrolabe

The astrolabe consists of several distinct parts which must be present on any usable instrument. Their fabrication allows some artistic freedom, although their construction must adhere to strict mathematical rules. The construction of the astrolabe is based on the stereographic projection which was invented by Hipparchos (2nd ct. B.C.).

The front side (Figure 1) is a predecessor of the planispheric star map which is still widely used by today’s amateur astronomers. On a modern planisphere, a transparent plastic sheet with a horizon mask rotates over a map of the sky visible for a certain geographical latitude, with the celestial pole of the respective hemisphere as map center. Scales on the borders of both disks allow the use to align a time of day with the date of interest, and the horizon mask then shows the sky visible at that date and time. In contrast, on the astrolabe, the horizon table is on the lower, fixed disk, while the star map rotates above. Durable transparent materials were not available for astrolabes, so the star map was constructed in a grid-like fashion, hence its name rete (lat. “(spider-)net”; yellow in Fig. 1). The star map must include the annual path of the sun, i.e. the ecliptic, which by the stereographic projection is mapped onto a circle that lies ex-centric on the rete and shows degree marks for the signs of the zodiac. The stars are marked as end points of pointers, arrows, flames, leaves or similar shapes which grow from the struts of the rete. A disadvantage of the stereographic projection is the high degree of distortion far from the center of projection, so the star map on the astrolabe was almost always limited to the range of declinations the sun can reach, and the (usually) southern part of the sky was missing.

The horizon tablet had to be constructed for the geographic latitude of the observer, so most astrolabes have several interchangeable plates, or tympana, which are all stored inside the flat cylindrical body, or mater, of the instrument. Only one tympan is visible at a time, and can show an — at first glance — exceedingly rich assortment of circle segments, which are explained in section 4.2 below.

On European instruments, above the rete there was a rule, or index, helping to read the various marks.

On the back, astrolabes showed a variety of additional scales and nomograms. The only compulsory scale was an altitude scale on the outer border, which together with the backsight, or alhidade, allowed measurements of stellar or solar altitudes. Other scales are described in section 4.4.

To determine time, a stellar or solar altitude was measured, then the rete was rotated until the star pointer or solar position on the zodiac was on the correct almucantar. The index was placed on the solar position on the ecliptic and pointed to the correct time on the limbus.

The original astrolabes were typically made from wood or brass plate. The plain drawings were transferred and engraved or etched on the plate, then the outlines had to be cut, and the parts were assembled on a central axis, so that rete, index and alhidade could be rotated.

4. Construction and Implementation

From today’s point of view, the construction is simple and requires only secondary-school mathematical capabilities, so due to space limitations we can show only the explanatory geometric figures (which are also result of the modelling effort) and few formulae, and must refer the reader to e.g. [Mic47] or [Sau84] for more construction details.

All parts of the astrolabe have been implemented as a set of native EPS (Encapsulated PostScript) graphic files. Aside from still being a de-facto standard in the printing domain, PostScript is a Turing-complete programming language in its own right, and is reasonably simple to be used directly, without the help of another program that would precompute auxiliary values, for the generation of 2D graphics when nothing more is required and the final image is mostly prepared for print. The astrolabe is in its essence a flat model of the heavens and is entirely constructed of straight lines and circle segments, which are also available as native PostScript...
operators. Moreover, PostScript provides clip and fill operators, so that an area bounded by a defined path can be either excluded from further change, or filled with a single colour (or a colour gradient, or even a pattern). Native PostScript programming is in fact everything that is required to create the intricate line art typical for this kind of instrument.

In our case, the EPS files consist of three parts: at the start of the file, a configuration block allows the setting of a wide variety of parameters that activate the required features and influence their final appearance. For example, geographic latitude is the key parameter that controls the final result of the horizon plate (see Fig. 4). Or, depending on whether the user wants to create a “Western” (European) or “Eastern” (Islamic) instrument, different diagrams can be selected for the back, with some more switches that influence details in the finally drawn diagrams. The pastel colouring, if activated, emulates the 17th century style of coloured copperplate-printed astrolabes. Also linear distortions caused by some printers can be compensated by prescaling factors, so that circles will be printed correctly.

Then follows the part where numerous procedures are defined. Some are purely general astronomical computations after [Mee98], others fulfill generalized high-level PostScript drawing operations like typesetting of letters along a circle, and finally there are plotting functions that draw the required diagrams and make use of the previously defined functions.

The final part is again the actual page description, which, by evaluation of the configuration variables, calls the required plotting functions with the correct parameters.

4.1. The star map, or rete

The construction is performed with the aid of a sphere (the celestial sphere) and a tangential plane, usually on the North Celestial Pole (Fig. 2(a)); the rare southern projection uses a tangent plane on the South Celestial Pole, and such an instrument can be created as well. Medieval astronomical clocks usually used this kind of projection. The celestial sphere is seen from the outside, and from the point where the sun is placed at the beginning of spring (vernale equinox). The ecliptic (annual path of the sun) therefore is shown projected as an inclined line. The angle of inclination is the obliquity of the ecliptic, about \( \varepsilon = 23.44^\circ \), but slowly changing in time, with a value closer to 24° seen in antique sources.

With the eye point on the South Celestial Pole (SCP), draw lines through the equator endpoints to mark radius \( r_0 \), and to the extrema of the ecliptic diameter to mark radii \( r_1 \) and \( r_2 \) on the tangential plane. These three radii are the most important sizes on the astrolabe, and their ratio depends on the exact value of \( \varepsilon \). When the projection sphere has radius \( r = 1 \), the other dimensions can be derived from triangular relations as \( r_0 = 2 \), \( r_1 = \frac{2\cos\varepsilon}{1+\sin\varepsilon} \) and \( r_2 = \frac{2\cos\varepsilon}{1-\sin\varepsilon} \).

Next, we draw the projection of the ecliptic, i.e., a circle of radius \( r_0 = \frac{2\varepsilon}{r} \) around point \((0, -\varepsilon)\) where \( \varepsilon = \frac{23.44^\circ}{2} \).

Along this circle, the sun moves during the year by about one degree per day. The ecliptic has to be partitioned into the 12 signs of the zodiac and marked for each degree of ecliptical longitude \( \lambda \). Because the stereographic projection causes strong distortions for areas on the sphere far from the tangential point, the star map usually is cut off at the southern solstice radius \( r_2 \).

A star map would be incomplete without stars. While their right ascension \( \alpha \) maps directly to polar angle counterclockwise from the First Point of Aries \( \Phi \), their declination (latitudinal coordinate) \( \delta \) maps to radius \( r_2 = \frac{2\cos\delta}{1+\sin\delta} \). If we just want a functional model and print the rete on transparent material, we can now draw a star map in today’s usual style, representing the constellations with stick figures and star
symbols of different sizes, depending on their magnitudes (brightness), like in the top half of figure 2(a).

However, the classical astrolabe was made from metal or wood, so we must construct a rete as fretwork or tracery that allows to see the underlying tympan for operation. Depending on instrument size, about 10 to 30 stars should be included on such a rete. This is the part that allows most artistic freedom but needs most manual decisions, however, so that many different styles of rete artwork are possible.

As a design aid, we first can print the modern star map that includes the ecliptic circle. As mechanical requirement, we have to find a way to connect the ecliptic circle to the rotational center (pole). Most classical instruments therefore provide a horizontal strut from $^\circ \text{P}$ to the First Point of Libra $\Omega$. Also frequently seen are an outer circle along $e_3$, and a part of a circle that follows the equator. Apart from these elements, we can now freely add struts that should run through parts of the sky map so that no bright stars are occluded. The struts can be straight or form a complicated tracery, but they should be balanced so that the center of gravity is still the center of rotation, because a disbalance will cause errors when the astrolabe is used for an altitude measurement and should hang vertically. From the auxiliary struts or tracery, we can now grow the star pointers which, depending on the overall style, are usually formed as flames or leaves.

Stellar positions are not fixed for eternity, but over decades and centuries change, mostly due to precession, caused by the wobble of the Earth’s axis and visible as slow shift (1 degree in about 72 years) of all stars parallel to the ecliptic. Therefore, any rete will be outdated within a few decades after production. Here another critical advantage of the procedural modeling approach comes into play: The star positions are computed from stellar coordinates which are stored for a standard epoch inside the EPS file, and a configuration variable in the file header is set to the year of interest, so that coordinates for that year are computed by the PostScript interpreter, from which the points on the rete are finally derived. The pointers on the instrument presented here are defined as pairs of native PostScript spline curves, where the roots are at fixed locations on the rete struts, the endpoints are the star positions which are subject to precession, and the control points, which are chosen with artistic freedom so that the whole star pointer shows an elegant sweep, are subject to half the precession effect.

But even with this generalisation, a single rete tracery design can only work for a few centuries at best: if we leave the roots of the star pointers at fixed positions on the struts, precession would for epochs too far from the design epoch bend the star pointers into an unelegant shape (Fig. 2(c)), and sometimes the star positions cross the rete struts. For an instrument far from the design epoch, the star pointers must therefore have to be rooted at different positions on the struts, and even some struts may have to be relocated to avoid occlusion of a star. Obviously, selection of the respective root points depends on the epoch parameter.

4.2. The horizon plates, or tympana

The astrolabe’s second essential part is the horizon tablet or tympan, which has to be constructed depending on the geographical latitude $\phi$ where the instrument shall be used for. All lines on classical tympana are constructed geometrically, so individual instruments differ only in the selection of line categories and their densities, which can also be selected in the file header. To increase usability, most astrolabes contain several exchangeable plates constructed for different latitudes. For classical instruments, the execution of this drawing required mathematical skills from the craftsman, and has also been very time consuming and therefore costly. In contrast, when the line art is encoded in constructional procedures, the program will immediately “draw itself” to provide a correctly constructed plate for any latitude required.

4.2.1. Horizon, altitude and azimuth circles

The basic construction of the horizon plate starts in the same way as the construction of the rete to find circles $r_0, e_1, e_2$.

Because the Earth is reduced into the center of the projection sphere, the observer’s horizon, a tangential plane on the earth’s surface, is reduced to an inclined plane through the sphere’s center (Fig. 3). Again, we draw lines through the extreme points of this horizon diameter on the sphere to reach points $a_0$ and $a_0^\circ$, the north and south points, resp., of the horizon circle. Similarly, to draw a circle, or almucantar, for altitude $h$ above the horizon, we first draw its projection in the celestial sphere, then project its extreme points from the SCP towards the projection plane to find the intersections from which the circle can be drawn.

Traditionally, up to three twilight lines can be found below the horizon circle. In PostScript, it is easy to not only draw these circles as lines, but we can fill the regions with approximate twilight colours (Fig. 4).

The construction of the azimuth arcs is sketched in Figure 6.
4.2.2. Unequal Hour Lines

In antiquity and the Middle Ages, several systems of reckoning time were used in parallel. The most important was the system of unequal hours, where the bright day (from sunrise to sunset) was split in twelve parts of equal length, as was the night. In summer, daylight hours were thus longer than nighttime hours, and vice versa in winter. At the equinoxes, day and night hours were of equal length, hence the names equal hours or equinoctial hours for the hours that we use today. The unequal hours were counted from sunrise or sunset, respectively. The astrolabe is capable of determining at least these two systems of time. Only when mechanized clocks were introduced and were usually capable of displaying only the equal hours, the other traditional systems were abandoned within a few decades [Zin67, p.17].

Although the sun is needed to determine the unequal hour, the lines are usually drawn not in the part of the tympan above the horizon, but in the lower part, to avoid further cluttering of the already dense mesh of arcs in the area representing the sky. To read the daytime, the solar nadir (antisolar point) was found on the ecliptic opposite the real position with the use of the index.

The traditional way of construction for the unequal hour lines was as follows: first, the parts of the circles $e_1$, $r_0$ and $e_2$ that lie below the horizon were split in twelve parts of equal length. Then, through each triplet of split points was connected with a circular arc. Although this is just an approximation of the correct solution where the diurnal circle for every declination should be split in twelve parts, the error is in the range of the line width up to about latitude $\varphi = 50^\circ$.

This similarity can be best shown by comparing approximate and correct lines on the same tympan (see Fig.4).

Two other time reckoning styles counted equal hours from sunrise, or sunset, respectively. Such hour lines are just rotated copies of the horizon and can be seen in the lower part of Figure 1.

4.3. The case, or mater

This part consists of a flat disk and a thicker ring (limbus) with degree or time scales mounted on its outside. The flat disk serves as carrier for the engravings of the back, see section 4.4. The limbus is circular, but with a decorated protrusion (throne) where a shackle is attached, so that the astrolabe can be hung vertically for altitude measurements.

4.4. The back, or dorsum

The back of the astrolabe may carry a wide range of diagrams, and differs strongly between European and Persian instruments. Due to space limitations we can merely name the classical European diagrams shown in figure 5 and must again refer to [Mic47] or [Sau84] for details.

The altitude scale along the outer border is used in combination with the alidade (backsight) to measure altitudes.

Inscribed, there is the zodiac with its regular partition into 12 signs of 30 degrees each. Further inside, a calendar circle is drawn, which can be either concentric with irregular day marks, or eccentric with regular day marks, to map the irregular speed of the (geocentric) sun during the year. The placement of the eccentric circle is determined from the length of
The seasons [Mic47, p.76], and is also variable over the centuries. The calendar and ecliptic have to be aligned properly so that for every day of the year the sun’s position on the zodiac can be determined. For an instrument that shows the Julian calendar (i.e., before 1582), spring begin (0° à) was not on March 21, but up to 10 days earlier. The exact amount of rotation is computed from astronomical literature [Mee98].

The Shadow Square, with its sides usually split in 12 parts, basically is a tangent scale for altitude measurements.

The Unequal Hours Diagram in the right upper quadrant provides a direct way to determine the unequal hour from a single measurement of solar altitude. The quadrant for Unequal/Equal Hours Conversion (upper left) can be found mostly on later instruments.

5. Production for Hands-On Experience: Cardboard and Laser Woodcutting

For practical demonstrations, the cheapest solution is a simple print on cardboard as has already been done in the original period, where the rete can now also be printed on an overhead slide. For higher durability, such an instrument can be laminated. Better for handling and practical experience, however, is the production of rigid instruments.

A 2kW CO₂ laser engraver/cutter with xy table proved to be an ideal tool for our purposes. It works in two modes of operation: First, the line art is burned into the material in a scanline mode, then the contours of the parts (with a certain colour code) are cut in direct xy pen plotter mode. The line style of contour lines of the astrolabe are defined in the cutlines procedure, which is redefined when the lasercut switch is active. Also, the setting of lasercut invalidates any color settings or fill operations. Unfortunately the machine is not capable of cutting metal, so it was decided to use modelling plywood as base material. All parts were imported into a standard vector graphics drawing program, and the machine was used as printer where the driver dialogue in this case also controls different laser power and pulse settings for different materials. The sequence of cutting operations is directly taken from the EPS file, so that the necessary care can be taken that the machine first cuts holes, then the outline. The engravings are done in quite high resolution (300 or 600 dpi), which takes however more than an hour for an A3 sized wood plate. The lasercut parts have to be further assembled, which is an easy weekend job for do-it-yourselfers.

The tympans are printed on a regular colour laser printer on decorative paper, so that many different tympans for different latitudes can be provided inside the mater. A decorative screw is used as polar axis to keep all parts together.
6. 3D Modelling

The EPS programs can also be used as basis for 3D models for various applications in the cultural heritage domain. The transformation into a 3D model is currently done in standard 3D modelling software, where bitmaps of the drawings can be used as textures and bump/displacement maps alike (the engraved lines were usually filled with dark ink), and the outline curves are extruded into flat objects. The components are joined along the axis, where rete, index and (on the reverse side) alhidade can rotate.

7. Discussion and Future Work

The instruments presented here are not reproductions of existing old instruments. They show an original design, which however must follow the basic construction principles of the stereographic projection. For simpler use or explanatory purposes, a modern star map on overhead transparency can replace or augment the classic rete, and a wide variety of printed paper horizons (tympans) can be included in the mater.

To test star coordinates for instruments of medieval times, old star catalogues from the literature [Ado90] have been entered, and a modern-style star map can easily be plotted for any of them, so that a researcher of old manuscripts could have an instrument at hand that provides the same functionality, coordinates and and data (albeit different appearance) as the original author probably had when writing.

An important group of instruments are Universal Astrolabes which do not need a tympan for every latitude, but are capable of solving problems for any latitude, although they are generally more difficult to operate. Also, the classical Persian astrolabe design includes several other diagrams which do not need a tympan for every latitude, but...

References


