Tutorial

Symmetry in Shapes
Theory and Practice

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"The branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space."
Charles and Ray Eames

*Powers of Ten*, 1977
Symmetry

συμμετρία

1. “similarity, correspondence, or balance among systems or parts of a system"

2. “an exact correspondence in position or form about a given point, line, or plane”

3. “beauty or harmony of form based on a proportionate arrangement of parts”

Collins English Dictionary
Symmetry
Symmetry

Group Theory
  • Mathematical language of symmetry

Transformations

Translation

Scale

Rotation
Symmetry as *invariance to transformations*

Rotation by \( \frac{360^\circ}{5} = 72^\circ \)

\[
2 \cdot \frac{360^\circ}{5} = 144^\circ
\]

\[
3 \cdot \frac{360^\circ}{5} = 216^\circ
\]

\[
4 \cdot \frac{360^\circ}{5} = 288^\circ
\]

\[
5 \cdot \frac{360^\circ}{5} = 360^\circ = 0^\circ
\]

\[
\text{Cyclic Group } C_5
\]
Symmetry Groups

Symmetry as \textit{invariance to transformations}

Rotation by $\frac{360^\circ}{5} = 72^\circ$

$2 \cdot \frac{360^\circ}{5} = 144^\circ$

$3 \cdot \frac{360^\circ}{5} = 216^\circ$

$4 \cdot \frac{360^\circ}{5} = 288^\circ$

$5 \cdot \frac{360^\circ}{5} = 360^\circ = 0^\circ$

Reflection

Dihedral Group $D_5$ \hspace{1cm} Cyclic Group $C_5$
Symmetry Groups

**Group Generators**

Dihedral Group $D_5$

Generating transformations

- $\star \cdot \star = \star^2 = \star^3 = \star^4 = \star^5 = \star$
Symmetry Groups

**Group Axioms**

- **Closure**  
  \( a, b \in G \rightarrow a \cdot b \in G \)

Dihedral Group \( D_5 \)

- \( a = \text{Ref. A} \)
- \( b = \text{Ref. B} \)
- \( a \cdot b = \text{Ref. A} \cdot \text{Ref. B} = \text{Rot. 288°} \)
Symmetry Groups

Group Axioms

- **Closure** \( a, b \in G \rightarrow a \cdot b \in G \)

- **Associative** \( a, b, c \in G \rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c) \)

Dihedral Group \( D_5 \)
Symmetry Groups

**Group Axioms**

- **Closure** \( a, b \in G \rightarrow a \cdot b \in G \)
- **Associative** \( a, b, c \in G \rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c) \)
- **Identity** \( \exists 1 \in G \rightarrow \forall a \in G : 1 \cdot a = a \cdot 1 = a \)

**Dihedral Group** \( D_5 \)
Symmetry Groups

**Group Axioms**

- **Closure** \( a, b \in G \rightarrow a \cdot b \in G \)
- **Associative** \( a, b, c \in G \rightarrow (a \cdot b) \cdot c = a \cdot (b \cdot c) \)
- **Identity** \( \exists 1 \in G \rightarrow \forall a \in G : 1 \cdot a = a \cdot 1 = a \)
- **Inverse** \( \forall a \in G \exists b \rightarrow a \cdot b = b \cdot a = 1 \)
Symmetry Groups

dihedral group $D_5$

cyclic group $C_3$

infinite group $O(2)$
Symmetry Groups

Group Generators

- Trans
- Scale
- Rot
- Rot + Trans
- Rot + Scale
- Rot × Trans
- Trans × Trans
- Rot × Scale
Patterns

1D - Frieze Groups

- Translation (T)
- Glide reflection (GR)
- 180° rotation (R)

- Vertical reflection (VR)
- GR + horizontal reflection (HR)
- VR + GR

- R + HR + VR + GR

2D - Wallpaper Groups
Symmetry Groups?

- Metal Foam
- Antibody
- Roof Construction
- Human Brain
- Spiral Galaxy
- Design by F. Gehry
Classification

Global vs. Partial

(a) complete symmetry group on parts of a shape

(b) partial translational symmetry

(c) partial rotational symmetry
Classification

Global vs. Partial

Exact vs. Approximate
Classification

Global vs. Partial
Exact vs. Approximate
Intrinsic vs. Extrinsic
Understanding Geometry

Physical Object  

Digital Measurement

Geometry Representation

Acquisition

Reconstruction
Understanding Geometry

Symmetry encodes Redundancy

Symmetry Analysis  →  Reconstruction
Symmetry is *Absence* of information.

“100 Random Points”

“A 10x10 Regular Grid of Points”
Symmetry is **Absence** of information

→ structure discovery by **minimizing** representation cost
Symmetry is Absence of information

→ structure discovery by minimizing representation cost