Abstract
These latest years witnessed an impressive improvement of graphics hardware both in terms of features and in terms of computational power. This improvement can be easily observed in computer games, where effects which, until few years ago, could only be achieved with expensive CPU computation are now shown interactively. Although the GPU has been designed for implementing graphics effects, it is still it basically a processing unit with its own memory, and, being specialized for algebraic tasks, supplies a number of floating point operations per second which is orders of magnitude greater than the CPU.
This suggested to the graphics community that the GPU could also be used for general purpose computation and a number of papers have been published on how to hack the GPU to this target.
Following this trend we propose a framework for using GPU for implementing techniques for deformable objects represented as generic meshes. The framework only assumes than the global computation is the union of local computations, which is true for all the explicit methods.

Categories and Subject Descriptors (according to ACM CCS): I.3.1 [Computer Graphics]: Hardware Architecture I.3.5 [Computational Geometry and Object Modeling]: Physically based modeling

1. Introduction and previous work

It is well known that modeling the behavior of a deformable object is a time consuming task, because of the amount of calculations necessary to compute the reaction of a deformable object under external forces. Without going into details (please refer to [NMK*05] for an excellent survey) the basic parts for defining a model for deformable objects are:

**Forces computation.** Given the current shape of the object and the force acting on it, compute the forces produced by the deformation energy stored in the body.

**Discretization over space.** The object must be partitioned in a, possibly large, set of simple elements. Triangle or tetrahedral meshes are popular choices.

**Integration through time.** Given a description of the dynamic state of the object (e.g the position and velocity of all its points), compute the description of next state after a small time step (generally smaller than 1/20 second).

For the sake of this discussion is convenient to split the existent methods in two main classes: the **explicit methods** using **explicit integration scheme** and the others, i.e. the methods requiring an **implicit scheme** either for Force Computation of for Time Integration. This distinction is fundamental because the methods of the first class share the important property that the total computation is given by the union of local (to the elements of the discretization) and independent computations.

In other words, implicit methods require the multiplication of a sparse matrix having at least as many columns and rows as three times the number of vertices of the mesh for each simulation step. With an explicit methods this matrix is block diagonal and the multiplication can be performed locally on each finite element and summed up in the vertices.

Explicit methods (and explicit integration step) are broadly used for they are easier to implement and generally more flexible. For example they do not require preprocessing and modifications to the mesh can be done on-the-fly to simulate cuts and lacerations [BMG99, GCMS00, NvdS00]. However, both explicit and implicit methods burden the CPU...
with a large amount of computation.

To evaluate the kind of algorithm involved, consider the explicit mass spring system for cloth simulation, which is by far the easiest and most straightest method. The cloth is modeled with a net of masses and springs connecting the spring to form a lattice. One step of simulation is implemented as in figure 1:

where \( s_{rl} \) is the rest length of the spring, \( k_s \) its elasticity constant, \( s_f \) the masses to which \( s \) is connected. This is probably the most seen and the most naive code snippet about deformable objects. Still, it useful to us because it shows three things: 1) even a simple method requires many floating point operations 2) the memory requirement is modest 3) there is no branching. Clearly there are explicit models for which these observations do not hold so strictly.

Why GPU?
Programmable Graphical Processing Units have characteristics that meet those of the explicit methods: 1) They provide a higher number of floating point operations per second than the CPU (e.g. nVidia 7800 Gtx’s pixel shader performs around 165 GFLOPS against the 8 of a Pentium IV), which is why the introduction of GPUs has spawned a large number of papers pursuing the goal of using them for general purpose (see [OLG*05] for a recent survey); 2) Memory transfer is an expensive task and memory itself is limited (512 MB on nVidia 7800); 3) Branching, introduced with the latest GPU generations, it’s still a costly operation.

GPUs are essentially designed for rendering, so it is not surprising that most of the work is about speeding up rendering techniques such as ray tracing [PBMH02, WSE04, LL04], photon mapping [CSKSN05,LC03, PDC*03, Hac04], radiosity [CHH03], just to cite a few examples.

A different research trend aims to develop general purpose algorithms, such as solver for linear system [GGHM05, BFGS03], for database management [GLW*04], for sorting algorithms [Gov05] and so on for a number of non graphics applications.

GPU based techniques for simulation have also been proposed. The boundary value problems on a regular grid, like the solution of the Navier-Stokes equations used in fluids, are particularly suitable for being solved on the GPU [GWL*03, HCSL02, WLL04, Har04]. Fewer solutions have been proposed for simulation with meshes. In [JHT05] a mass spring system is implemented in the GPU by placing particles on a regular grid and using implicit connectivity, so that each internal particle has 18 neighbors. Each particle is associated with a pixel in the PBuffer (referred as position-buffer) that stores the particle position. At each simulation step, a pixel shader computes the total forces acting on the corresponding particle, by summing up the force contributions given by the neighbors (i.e. the adjacent texels).

In order to visualize the object, a mesh is associated to the non internal particles of the mass spring system. Each particles of the surface mesh fetches its position from the position-buffer (shader model 3.0), while the normal, needed to shade the surface, is approximated as the normalized sum of all normalized difference vectors from particle neighbors to the particle itself.

The advantage of implicit connectivity is that they need only one texture look up per particle. On the other hand, the amount of texture memory is directly proportional to the accuracy of the representation. An improved version of this approach [JHT05] uses a fine grained mesh for the surface connected with a coarser mass spring system in GPU. Irregular tetrahedral meshes are used in [GEW05]. In this work, each vertex stores, for each tetrahedron it belongs to, three references to the other three vertices and the corresponding springs’ rest lengths. For each vertex, the force contributions of all the tetrahedra in the one-ring neighborhood are summed. In this case more texture fetches are involved: for each vertex and for each tetrahedron they need three texture fetches to find out the indices of the other three vertices and the three dependent fetches to find out their position.

2. Problem’s analysis
Since we want to define an as general as possible framework, we will consider a mesh as set of vertices and a set of relations among vertices. Relation is a general term that we use to refer to a line, a triangle, a quadrilateral, a tetrahedron and so on, every primitive that refer to a constant number of vertices.

Both vertices and relations may have a number of attributes (e.g. each vertex needs to store its velocity). The only requirement is that the relations have constant arity.

Then, we need a way to store connectivity because, as will
3. GPU’s bonds

As aforementioned the most seen snippet of code about deformable object the algorithm in figure 1 cannot directly implemented on the graphics hardware. Due to the architectural bonds, modern GPU do not expose a texture write instruction at a computed address (scatter operation). A common way to overcome this limitation (when it is possible) is to convert a scatter operation in a gather operation. For example the previous snippet of code may be translated as reported in figure 2.

The textures in a GPU’s program represent the analogous of arrays in the CPU’s programming paradigm. Typically in a physical simulation there are some vectorial quantities that have to be continuously updated in function of a small time step $dt$. In the code in figure 2, for example, the vertex(mass) position and the vertex speed are integrated through time.

As an example of the framework we will consider the management of deformable tetrahedral meshes (simplicial complexes of order 3). In this case the relations are tetrahedra. The only variable attribute that we will put on the relations will be the four indices of the vertices of the tetrahedron. In the constant attributes texture (RelConstTexture) instead, there will be the six lengths of the springs at rest shape. In this example we will consider the elastic constant $k$ common to each spring in the system. Using the texture coordinates defined at the four vertices of the activation quad (whose size is equal to the dimension of a vertex variable attribute texture) is possible to access to each attributes in the $\text{VerVar}\_\text{Tex}$. Knowing the dimension of one of these textures and the number of constant attributes for each vertex, the address of the attributes in the $\text{VerConst}\_\text{Tex}$ can be easily calculated. In this texture, other than offset and the number of relations per vertex $\text{nelem}$, we will insert, for a more plain exposition, the vertex identification serial. Using offset and $\text{nelem}$ is then

\begin{algorithm}
Alg OneStep($dt$) {
  for each $m$ in Masses {
    $m->f = m->\text{left\_s.k}$ * $\left\langle \frac{m->m1.p - m->p}{|m->m1.p - m->p|}\right\rangle$;
    $m->f += m->\text{right\_s.k}$ * $\left\langle \frac{m->m2.p - m->p}{|m->m2.p - m->p|}\right\rangle$;
    $m->v += m->f/m->\text{mass}$ * $dt$;
    $m->p += m->v$ * $dt$;
  }
}
\end{algorithm}

Figure 2: GPU compliant implementation of the mass spring system (chain of springs)

be clear soon, a vertex needs to have a pointer to the relations that include it.

4. Our Framework

Although each texel contains a RGBA color information from a GPGPU point of view the four coordinates of a texture’s pixel may assume other meanings. In our framework a single entry of a texel may be interpreted in three different ways: a scalar value, a single vector’s component, a pointer (an index) to a texel or to an entry in another texture.

As previously seen the GPU architecture create a substantial division between the attributes that remain constant during the simulation and the attributes that assume variable values (please refer to Figure 3).

The textures are mainly divided in three classes:

- Vertex attributes textures
- Relation attributes textures
- Vertex - Relations Connectivity texture

Both vertex and relation textures are split in two subclasses, one storing the constant attributes (e.g. elasticity constants) and one referring non constant attributes (e.g. position). The textures for the vertex variable attributes ($\text{VerVar}$) has all the same dimension (proportional to the vertices’ number and influenced by the GPU’s characteristics). In the other hand, typically, there is a single texture containing the constant vertex values($\text{VerConst}$); the dimension of this texture is roughly:

$$\text{dim(VerConst)} = \text{dim(VerVar)} \times \left(\text{floats\_per\_vertex}\right)/4$$

The $\text{VerConst}$ always exists because at least contains necessary two value related to the connectivity texture ($\text{Conn}$): an offset and number of relations incident in a single vertex.

In each $\text{Conn}$’s texel there are four indexes referred to the textures containing the relation’s variable and constant attributes; with this information a vertex can access at the relations that influence him state.

5. A case of study: Mass - Spring System

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\begin{algorithm}
Alg OneStep($dt$) {
  for each $m$ in Masses {
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    $m->f += m->\text{right\_s.k}$ * $\left\langle \frac{m->m2.p - m->p}{|m->m2.p - m->p|}\right\rangle$;
    $m->v += m->f/m->\text{mass}$ * $dt$;
    $m->p += m->v$ * $dt$;
  }
}
\end{algorithm}

\footnote{In each texel is possible to store four different scalar attributes.}
### 6. Real-time update of a relation’s attribute

The distinction between constant attributes texture and variable attributes texture allow to update the state of a vertex. In the same manner the framework allows to change dynamically the relation’s characteristic directly on GPU; without the need for a costly communications between the main system memory and the graphic hardware. It will be sufficient to draw a quad composed of only one fragment. The texture coordinates defined at the quad’s vertices will refer to the particular relation that we want to update. To update more than one texel for each attribute’s relation, as in the vertex case, we maintain the single attribute in different textures of the same dimension. If in mass-spring system we would eliminate a tetrahedron it will be sufficient to mark it in some convenient way (for example we can set the vertices indexes at -1 in the RelVar in order to indicate that this element must be ignored during the physical simulation).

### 7. Results

Tables 4 and 5 shows some results when running triangle and tetrahedral meshes. From this table it can be seen that for average sized meshes the GPU implementation is twice as fast that the GPU ones on a NVidia GeForce 6800GT. This is quite a low gain considering the large gap in terms of FLOPS, but it is not too surprising, since our framework uses several dependant fetches per iteration. On the other hand, the mass spring system is somewhat the worst case because it performs few floating point operations per element (triangle, tetrahedron). Table 5 shows the fps for a ill conditioned tetrahedral mesh, where all the tetrahedra share a common vertex. As a consequence, the fragment program for this vertex will have to iterate over all the tetrahedra while all the other per fragment computation (for vertices connected to 6 tetrahedra each) will have to wait.

### 8. Conclusions and future work

We presented a general framework to handle irregular meshes entirely on the GPU. The framework has been tested...
by implementing mass spring systems for simplicial complexes of order 1, 2 and 3 (chains, triangle meshes and tetrahedral meshes respectively), but it supports with no modifications other types of structure (e.g. quadrilateral or hexahedral meshes) as long as it is expressed as a set of vertices and a set of relations among groups of vertices, with the only assumption the all the relations connected the same number of vertices. With respect to previous approaches, the framework requires more dependent fetches, but on the other hand it saves texture memory and, differently from [GEW05] does not cause replication of data.

This will allow us to implement accurate methods that require much more memory per element that the mass spring systems, such as the so called explicit FEM [OH99], which is the next step of this work.

References


