

Topological, Geometric and Structural Approaches to Enhance Shape Information

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Abstract

Nowadays, the increasing power of hardware components has made available a huge amount of digital models, more and more complex and detailed, as needed in many innovative research fields such as medical imaging, entertainment, product modelling and design. In this scenario, both industry and academy feel an urgent need of tools to efficiently describe, recognise and retrieve shapes, which usually require to enhance the raw geometry with additional high-level shape information. Analyzing and enhancing geometry is the basis for an efficient understanding of shapes, and has been for many years one of the most challenging issues at IMATI-GE/CNR. Recent advances in the field of semantic-based knowledge systems dealing with multi-dimensional media boosted this research field significantly, and the Shape Modeling Group at IMATI-GE/CNR produced several innovative methods that are reviewed in this paper. Approaches are classified and described along with some results, and discussed with respect to applications. Open issues are outlined along with future research plans.

Categories and Subject Descriptors: I.3.5 [Computer Graphics]: *Computational Geometry and Object Modeling* – Boundary representation, Geometric algorithms, languages and systems

1. Introduction

Understanding the implicit knowledge carried by a digital shape in unspecialized contexts is not a trivial task. In general, characterising a shape amounts to construct a computational description of the most representative features of the shape, usually a few basic types, possibly along with their relationships (structural decomposition). This and other aspects of shape modeling and reasoning have been studied for many years at the department of Genova of the Institute of Applied Mathematics and Information Technologies (IMATI-GE / CNR).

Some aspects of these problems have been originally tackled in the field of product modeling, and relevant results have been achieved especially concerning free-form feature recognition and feature-based model representation [FG89][FGP*92]. In fact, traditional CAD systems represent only the geometry of a part with no means for coding functional information, while *feature-based modelling* technology has been identified in the engineering community as the solution for associating functional information to geometric data, and consequently for integrating design and downstream applications [SM95][DFG*94]. Instead of designing geometric shapes alone, one of the central goals of modelling with features is to conceptualise shape design with classified elements (features) and add information related to design intent and other specific contexts. While engineering provides the specification of relevant features, formalizing and recognizing features of free-

form, scanned, organic or other kind of shapes is not an easy task. The complexity of the understanding process increases due to the lack of formal definitions of shape features. In this scenario, recent acquisition tools are able to produce huge volumes of precise 3D data, and understanding such complex shapes would provide a significant support to fully exploit the potential of innovative modeling mechanisms such as, for example, modeling by example [FKS04]. Fortunately, the rapid evolution of semantic-based knowledge systems is opening new possibilities for an intelligent exploitation of multi-dimensional media, and metadata and ontologies can be used to support and store the shape understanding process and to produce high-level computational models of 3D shapes [AIM04]. Having this in mind, understanding a shape amounts to shifting from a purely geometric to a semantic-aware level of representation; this process requires tools that, by analyzing the information available (geometry, topology, application context, ...), generate effective high-level computational models that describe shapes and their structures [FS98].

This paper gives an overview of the main techniques developed so far at IMATI-GE / CNR for analyzing and enhancing shape information. The methods discussed work on a geometric model of the shape and produce an enhanced version of the model with respect to its capability of portraying and/or storing knowledge about the shape, be it geometric or structural.

2. Topological Approaches

Mathematics naturally provides a collection of tools that capture and understand the shape of spaces by classifying how they match and differ. For instance, Morse theory [Mil63] sets the foundations for associating the topology of a given manifold to the critical points of a smooth function defined on the manifold. Using Morse theory, it is possible to define a shape description mechanism based on the evolution on the surface of the contour levels of the mapping function.

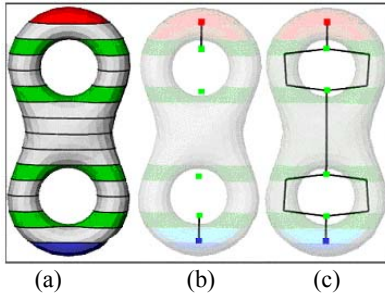


Figure 1: The red region and the blue one respectively correspond to a maximum and a minimum, while the saddles are colored in green (a). Maxima and minima are connected to other critical areas through the region growing process highlighted in light red and light blue (b); finally the graph is completed with the expansion of the saddle areas (c).

The Reeb graphs [Ree46] were firstly introduced in Computer Graphics by Shinagawa et al. [SKK91], and were initially limited to Morse mapping functions. Reeb graphs can be efficiently used for surface analysis and understanding, simplification, similarity evaluation and shape retrieval; for an overview of Reeb graph properties we refer to [BMM*03a]. Morse theory and Reeb graphs for piece-wise linear surfaces are often approached using the paradigm of *simulation of differentiability* [EHZ03], which requires the fulfillment of constraints on the value of the mapping function at the mesh vertices. This approach solves some theoretical problems but may lead to a wrong interpretation of the shape by introducing artifacts, which do not correspond to any shape feature.

Our approach is based on the use of contours for characterizing the surface shape and constructing a topological structure, the **Extended Reeb Graph (ERG)**: We proposed the use of *critical areas* instead of critical points and used the ERG to represent their configuration on the surface. Using contour levels the surface is fully decomposed into sub-regions that are classified as *regular* or *critical* (i.e. maxima, minima or saddles) according to the value of the mapping function and the number of their boundary components [ABS03]. In our representation, a node of the graph is associated to each critical area. Then, arcs between nodes are detected through an expansion process of the critical areas, which tracks the evolution of contour lines. In **Figure 1** the main steps of the ERG extraction process are depicted: first, the arcs between minima (resp. maxima) and

saddles are inserted, expanding all maxima and minima to their nearest (in terms of region expansion) critical area, then the other arcs are detected.

An algorithm for representing the ERG of a terrain surface is proposed in [BFS04], where the unique boundary component was virtually closed inserting a global virtual minimum point, so that the outgoing directions from the surface boundary are only descending and the surface is virtually closed. Then, the extension of the method to generic, oriented surfaces with boundary was proposed in [Bia05] (see **Figure 2**). In that case, loops that originate from a boundary component are not distinguishable from those that correspond to a hole through the surface and the cycles in the Reeb graph do not correspond any longer to the genus of the surface. To retrieve the information concerning the topological class of the surface, our idea was to organize the Reeb graph representation into two levels: first, we stored the evolution of the level sets and the boundary components like in the usual ERG representation. Second, the surface characterization is modified assuming that each boundary component is virtually closed and the ERG is simplified according to the new region classification.

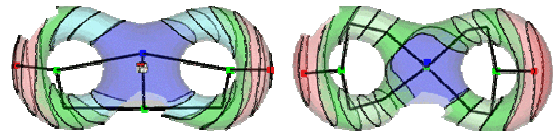


Figure 2: ERG representation of a bi-torus with two boundary wrt the integral geodesic distance defined in [HSK*01] and the distance from the centre of mass.

The proposed framework have been successfully adopted in several application fields like storage, simplification and reconstruction of DTM (Digital Terrain Models) [BFS04]; analysis and synthesis of 3D objects [ABS03]; shape matching [BMM*03b]. In particular, the geometric embedding of the ERG can be efficiently used also for a qualitative quick comparison among discrete objects.

3. Geometric and Structural Approaches

The paradigm of *shape segmentation, analysis, and abstraction* is strongly influenced by the targeted application context. In general, however, we observe that an ideal *shape descriptor* should be able to capture and compute the main features of a given surface and should be based on geometric properties of the shape which are invariant under rotation, translation, and scaling. Furthermore, it should distinguish between global and local features, and organize them into an abstract representation which can be used to automate complex tasks such as matching, retrieval, or comparison of shapes. At IMATI-GE/CNR, recent research is focused on the extraction of *semantic information* based on a priori decomposition of the input shape into building primitives whose meaning is defined by the underlining application context. In section 3.1, we detail a *multi-scale* segmentation method and its applications to the extraction of semantics in the context of virtual humans;

In section 3.2, a hierarchical segmentation method based on known fitting primitives is described.

3.1 Multi-scale Curvature Evaluation and Segmentation: Tailor & Plumber

Tailor [MPS*03] classifies the vertices of a 3D surface represented by a triangle mesh M according to geometric and morphological descriptors evaluated on neighborhoods of increasing size. The set of neighborhoods associated to each vertex $v \in M$ is defined by a set of spheres $\{S(v,r)\}$, centered at v , and whose radii $\{r\}$ represent the scales at which the shape is analyzed. Chosen a scale r , we consider the surface region containing v and delimited by the intersection γ between M and $S(v,r)$; we discard other regions of intersection between the sphere and the mesh that might occur but do not contain v . The number of connected components of γ gives a qualitative characterization of the shape in a neighborhood of each vertex, and the evolution of the length ratio of the boundary components of γ with respect to the radius of the spheres is used to detect specific features such as sharp protrusions or wells, mounts or dips, blends or branching parts. More precisely, if γ has one connected component (see Figure 3(a)), the surface around v is considered equivalent to a disc and its curvature at scale r is approximated by the non-negative ratio $G_r(v) := l_\gamma / r$ [GP74], where l_γ is the length of γ . Then, the value $G_r(v)$ is used to distinguish among *planar*, *sharp*, and *blend* vertices (see Figure 3(b-d)). If $\gamma = \gamma_1 \cup \gamma_2$ has two boundary components (see Figure 3(e)), the vertex v at scale r is classified as *cylindrical* when the ratio between the maximal and minimal length of γ_1 and γ_2 does not exceed a given threshold \mathcal{E} , that is $\gamma_1 = \mathcal{E}\gamma_2$ (see Figure 3(g)). Otherwise, v is classified as *conical* (see Figure 3(f)).

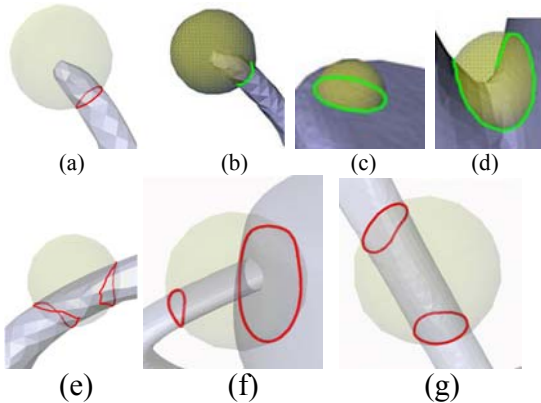


Figure 3: Case of one intersection curve between sphere and mesh (a) and classification of sharp (b), planar (c) and blend (d) vertices. Case of two intersection curves (e) and classification of conical (f) and cylindrical (g) vertices.

If γ has three or more connected components, v is a *branching* and we do not consider other geometric descriptors.

Starting from the above-mentioned vertex classification, *Plumber* [MPS*04] defines a shape decomposition into connected components that are either *tubular features*, identified by regions which can be described as generalized cones or cylinders (e.g., handle-like and protrusion-like features, together with their concave counterparts), and *body regions* identified by patches which connect tubular features. An overview of the main steps of *Plumber* is shown in Figure 4. At the first step, seed limb vertices are located by *Tailor* (see Figure 4(a)), clustered to form candidate seed regions of tubular features, and used to compute a first reliable tube section, called the *medial loop* (see Figure 4(b)).

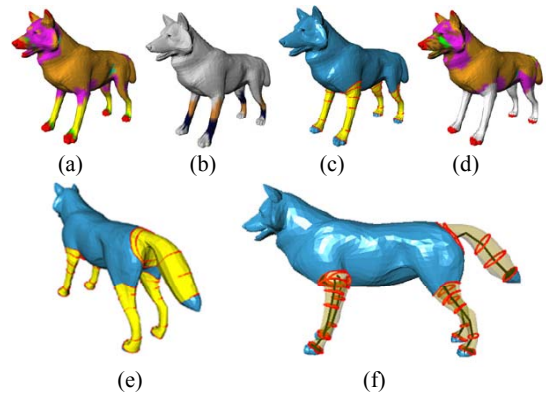


Figure 4: (a) *Tailor* vertex classification where yellow regions represent seed limb patches, (b) seed limb regions with medial loop expansion, (c) iso-contours used to identify cylindrical patches (d) vertex classification repeated at a larger scale. (e, f) Final shape decomposition into cylindrical patches (yellow), body and caps (blue).

This loop is ensured to be around each candidate tube and works as a generator of the feature. Then, the medial loop is moved in both directions on the shape, by using spheres placed not on the surface but at the barycentre of the medial loops iteratively and until the tube is completely swept (see Figure 4(c)). The stop criteria of the iterative procedure are: an abrupt variation of the ratio between the length of the intersection curves and the radius of the current sphere, the end of the tubular feature, or the intersection of a region already classified. The tube detection works in a multi-scale setting (i.e., using a *fine-to-coarse* strategy), starting with the extraction of small tubes first; furthermore, during the iterative procedure triangles are marked as visited and are not taken into account at the following steps (see Figure 4 (d)). The set of radii is automatically set by uniformly sampling the interval between the minimum edge length and the diagonal of the bounding box of M ; however, the user can tune the level of details if he/she has some *a-priori* information or if he/she is searching for features of specific dimension. Furthermore, the multi-scale

approach and the chosen descriptors reduce the influence of noise on the shape evaluation, ensure a greater flexibility to formulate filters for shape analysis, and capture the more representative properties in a detailed description. At the end of the whole process, tubes are labeled with respect to the scale at which they were found (see Figure 4(e, f)). Each conical or cylindrical (tubular) feature T , extracted at scale r , is abstracted by a skeletal line defined by joining the barycentres b_i of the intersection curves γ_i between T and the set of spheres used to sweep the tube. Since the shape and position of T could be arbitrarily complex, the intersection curves are good and simple descriptors for the cross sections of the tube along the centerline. The surface decomposition, the skeletal lines of tubular features, and their spatial arrangement defines a *shape-graph* whose nodes are the extracted primitive shapes, while the arcs code the adjacency relationships among them.

The proposed shape segmentation has been used for skeleton extraction from curvature extrema and local parameterization as discussed in [MP02], [PSF04], and has proved particularly suitable for the semantic analysis of human body models. In the following, we discuss how the segmentation method previously described is exploited to extract the semantic content and annotation of human body models from the geometry, structure, and knowledge pertaining to the domain.

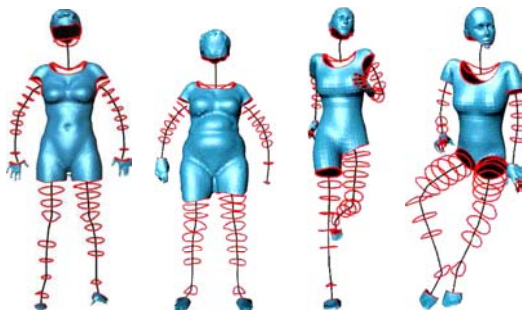


Figure 5: Input virtual humans of different sizes and in different postures; body parts shown in blue; tubular parts are shown as skeletal lines of generalized cylinders and associated cross sections.

Since the input model represents a human body, either virtual or scanned, its relevant tubular features will identify arms, legs, neck, and fingers. The torso and its symmetries are also important data in the anthropometry domain. In general, *Plumber* will not find directly these features, but some of their parts. For instance, a hand will be segmented into five small tubes, possibly with their associated caps, all of them being attached to the same body part. Reasoning on the relative sizes of the features and on the attachment relations among them, makes possible to recognize and automatically measure semantically relevant parts of the human body. Approximating each tube with truncated cones of circular bases, each having the same length of the corresponding tube cross section, enables to calculate an approximation

of the feature volume as the sum of the volumes of each building part (see Figure 5). Then, the ratio volume/length gives an hint on the human limb fatness, in analogy with the Body Mass Index, and the ratio volume/length discriminates between two individuals of same limb thickness but different height. Furthermore, the local variation of the associated shape-graph gives us a precious information about the body posture. In the context of virtual humans, we can exploit the fact that limbs are rigid except at the joints; therefore, the tube axis will be nearly straight, except in a few points, which identify the torsion in the articulation sites. Note that a tube may have at most three articulations: for instance, a leg may comprehend the ankle, knee, and hip joints. Again, the shape graph is used to discriminate each joint, giving an “outward” ordering to each tube, from its attachment to the body towards the tip of the protrusion.

3.2 Hierarchical segmentation

In [GWH01] a hierarchical face clustering approach for meshes was presented. By approximating connected sets of mesh faces with best-fitting planes, the resulting segmentation locates planar regions on the surface, which is useful for several applications.

In [AFS06], this method has been extended to handle fitting primitives belonging to an arbitrary set. A binary tree of clusters is iteratively generated, each of which is fitted by one of the primitives employed. Initially each triangle represents a single cluster; at each iteration, all the pairs of adjacent clusters are considered, and the one that can be better approximated with one of the primitives forms a new single cluster. The approximation error is evaluated using the same metric for all the primitives, so that it makes sense to choose which is the *most suitable* primitive to approximate the set of triangles in a cluster. A prototype has been implemented which uses planes, spheres and cylinders and, for meshes made of 100k faces, the whole binary tree of clusters can be constructed in about 8 seconds on a standard PC (see Figure 6).

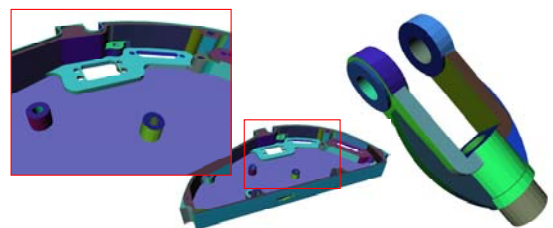


Figure 6: Example of clustering [AFS06] of two models in regions fitted by planes, spheres and cylinders.

The most natural applications of this framework are to be searched in reverse engineering [VM02]. If the model is known to be made of a well defined set of primitives, for example, the algorithm may accept a *plug-in* for each of them in which the computation of the fitting parameters and the error are implemented. Moreover, being a greedy method, the level of accuracy is somehow reflected by the cluster hierarchy which, once computed, may be

interactively navigated by the user through a *slider* which sets the desired number of clusters or a threshold error.

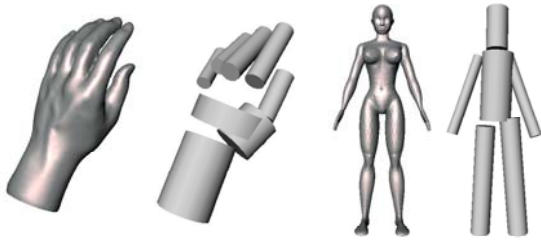


Figure 7: Some models along with the corresponding abstractions represented by the fitting cylinders.

In this context it is worth to consider that natural shapes, and in particular humans and animals, form a class of 3D models whose shape can be effectively abstracted through approximating cylinders. If only this kind of primitive is used, the clustering algorithm described in [AFS06] represents an automatic way to derive a hierarchical view of the shape (see Figure 7) as described by Marr in [Mar82].

4. Application to Local and Global Parameterization

Existing methods of local parameterization are mainly driven by targeted applications and by simplification schemes where patches are generally not related to features of the surface. Usually, triangle or quadrilateral-shaped patches are used to build the base mesh, as they well support subdivision schemes and spline-based representations. The segmentation methods described in the previous sections can be used as the basis of a novel local parameterization framework where the only assumption is that each patch has 0-genus, without constraining the number of boundaries. Independently of the specific segmentation which defines the charts, we classify the patches according to the number of their boundary components into three types: generalized cones, cylinders, and bodies. Then, each patch is reduced to a disk-like primitive through an appropriate cut and unfolded on the parameterization domain, as discussed in [PSF04]. An example is shown in **Figure 8**.

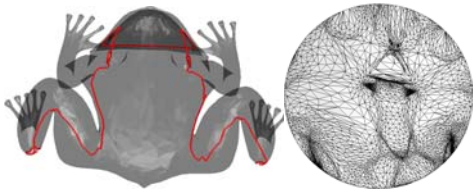


Figure 8: Cut-graph and parameterization on the unit circle of a primitive of 0-genus and four boundary components.

This *general* approach provides a higher degree of flexibility for the choice of the segmentation method and for the identification of patches which represent *features* of the input shape. With respect to methods based on mesh simplification, a feature-based segmentation could

produce better results from the point of view of the parameterization: patch boundaries could be drawn at lines of high curvature, symmetries of the input surface could be exploited, or patches could be defined to identify topological primitives. Using the Reeb Graph, it is also possible to find a family of cut-graphs of closed and bordered surfaces with an arbitrary genus (see Figure 9).

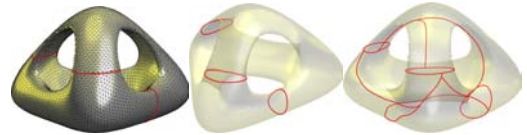


Figure 9: (a-b) Identification and cut of the topological handles, (b) cut-graph.

5. Discussion

Clearly, each method described in the above sections has its advantages and drawbacks; each of them, however, is suitable within a particular application context.

According to theoretical results, the ERG produces a compact iconic representation of the model that can be enriched with geometric information along arcs and nodes (for instance, the critical sections of the object, where topological changes occur or the corresponding surface sub-parts). The features identified by the ERG representation depend on the choice of the mapping function.

Since the height function is directly correlated to the definition of a terrain surface and corresponds to the natural direction to observe the model it is the most natural choice for these models. However, the dependence of the graph on the direction of the height function makes it ineffective for matching applications: the same object may have completely different graphs depending on its position in space. On the contrary, the distance from the centre of mass of the input data set induces an ERG which is invariant to the object position in space, resulting suitable for the similarity analysis; in this description, nodes correspond to protrusions and hollows with respect to the center of mass. Other possible choices of mapping function useful for similarity evaluation are the average of geodesic distances proposed in [HSK*01] and the geodesic distance from curvature extrema generated starting from the object protrusions, which are first determined through a robust multi-scale curvature evaluation on the surface [MPS*03]. In the analysis of virtual humans, for example, the pose may be considered an interesting characteristic or not: in Figure 10 the skeletal representation of a human model in two different poses is shown with respect to the height direction Figure 10(a,d), the distance from the center of mass Figure 10(b,e), and the geodesic distance from curvature extrema Figure 10(c,f).

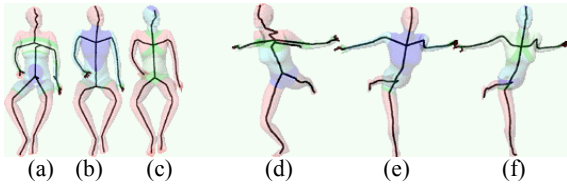


Figure 10: Skeletal representation of a human model in two different positions wrt different choices of the mapping function.

When the shape analysis is performed on regular objects, whose shape is a composition of well-defined primitives, the hierarchical segmentation described in [AFS06] provides an extremely reliable abstraction. Also, this method can be constrained to fit only cylindrical primitives and thus may be used for the analysis of natural shapes as well. Clearly, since the method can fit only straight cylinders, it cannot replace the *Plumber* algorithm. On the other hand, however, it might be used as a support for *Plumber* when the model has tiny unwanted topological connections. For example, a real object may have a tubular feature very close to another part of the surface (i.e., a human's arm close to an ank) and the model resulting from a scanning is likely to connect such close points. In such a case, a prior hierarchical segmentation may be run to extract the regions fitted by the primitives; in the segmented model, unwanted topological connections may be easily identified at junctions of adjacent regions and properly removed by cutting the mesh and filling the resulting holes. After such a topological fix, the *Plumber* algorithm can detect more and/or more complete tubular features even if, in the original model, these were spoiled by unwanted topological connections.

6. Open issues and future research

Skeletons have been studied independently in image analysis by using a discrete geometry approach and in geometric modelling by using *continuous* computational geometry techniques. Our future research will probably include the study and analysis of the two approaches in a common framework, in order to develop more powerful techniques that could be also combined with a multi-scale view. A first effort will be devoted to the description of a shape by means of a collection of different skeletons (a multi-skeleton), either derived by different geometric representations of the shape or related to different sets of feature types. A further potential goal is to decompose a shape into significant parts, based on a prior computation of a skeleton and on extraction of features. Another important objective is studying representations of scientific data sets based on critical field characteristics, which provide a structural description of a field and its iso-surfaces. The focus of this research would be on 3D and 4D scalar fields, the mathematical basis is provided by Morse theory and is intended as a first step for a multi-scale representation. The aim should be supporting visualisation, inspection and retrieval (visual data mining) over the web on large scientific data sets.

In the more general context of our current research, however, the process of analyzing and structuring the raw geometry of a digital shape is seen as a first step towards the difficult task of semantic annotation. Clearly, the semantics of a digital shape is strongly related to the context in which the shape is used, hence a fully automatic annotation is not the scope of our research. On the other hand, particular application contexts are worth to be targeted; it has been shown, for example, how the *Plumber* algorithm can be effectively used to associate a *meaning* to the various parts of a digital shape when it is known to represent a human body. Further research is necessary to make such a virtual human interact with objects within an environment. The definition of the behavior of a digital shape under certain circumstances has been targeted in [AMP*05], where the semantic annotation consists on determining the parts of the shape that can be *grasped* by a virtual hand. The interaction of a virtual human with its environment, however, might be much more complex than simply grasping objects, and significant further research is required in this field.

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