# Efficient Feature-preserving Local Projection Operator for Geometry Reconstruction

Bin Liao[1,2], Chunxia Xiao[1†] and Liqiang Jin[1]

[1]School of Computer, Wuhan University, China
[2]Faculty of Mathematics and Computer Science, Hubei University, China

**Abstract**

*This paper proposes an efficient and feature-preserving locally optimal projection operator (FLOP) for geometry reconstruction. We first develop a bilateral weighted local optimal projection operator. We then present a novel fast FLOP operator based on random sampling of Kernel Density Estimate (KDE), which greatly accelerates FLOP. The experimental results show that the proposed algorithms are efficient and robust for feature-preserving geometry reconstruction.*

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Line and curve generation

## 1. Introduction

With the advances of scanning technologies, reconstructing geometry from raw scanned data is an on-going research topic in computer graphics [HDD*92, LCOLTE07]. Although many methods can produce pleasing reconstruction results, however, due to geometry shape complexity and noise (outliers), in addition, with the high accuracy reconstruction requirement and new arisen applications, many problems remain to be addressed.

Recently, Lipman et al. [LCOLTE07] developed a parameterization-free locally optimal projection operator (LOP) for geometry reconstruction. LOP operates well on raw data without relying on a local parameterization of the points or on their local orientation, and is robust to noise and outlines of raw scanned data. This LOP method is a fixed-point iteration and originated from the multivariate $L_1$ median. However, this method suffers from high computational cost for local optimal minimization and fails to preserve the geometry features well. Furthermore, this method may fail to converge, and oscillate near a solution. Recently, by incorporating an adaptive density weights into LOP, Huang et al. [HLZ*09] modified LOP operator to deal with non-uniform distributions in raw point set data, received more uniform reconstructed point set. They also presented a robust normal estimation method by using a priority-driven normal propagation scheme and an orientation-aware PCA method.

In this paper, we introduce an efficient and feature-preserving locally optimal projection operator (FLOP) for geometry reconstruction. We first develop a bilateral weighted local optimal projection operator for preserving features. Then, inspired by [FK09], we present a fast locally optimal projection operator which is based on random sampling of Kernel Density Estimate (KDE) for the original point-set data. We show that geometry reconstruction results are close to those generated using the complete point set data, to within a given accuracy, while time complexity of the proposed fast FLOP is considerably lower than that of the original FLOP.

## 2. Fast Feature-preserving Local Optimal Projection

### 2.1. Review of Local Optimal Projection

LOP [LCOLTE07] is a parameterization free algorithm for geometry reconstruction. Given the point set data $P = \{p_j\}_{j\in J} \subset \Re^3$, LOP projects an arbitrary point-set $X^{(0)} = \{x_i^{(0)}\}_{i\in I} \subset \Re^3$ onto the set $P$ , where $I, J$ denote the indices sets. The set of projected points $Q = \{q_i\}_{i\in I}$, that minimizes the sum of weighted distances of each $q_i$ to all point set $P$, can be considered as the approximate geometry to the original data set $P$.

† Corresponding to Chunxia Xiao, Email: cxxiao@whu.edu.cn

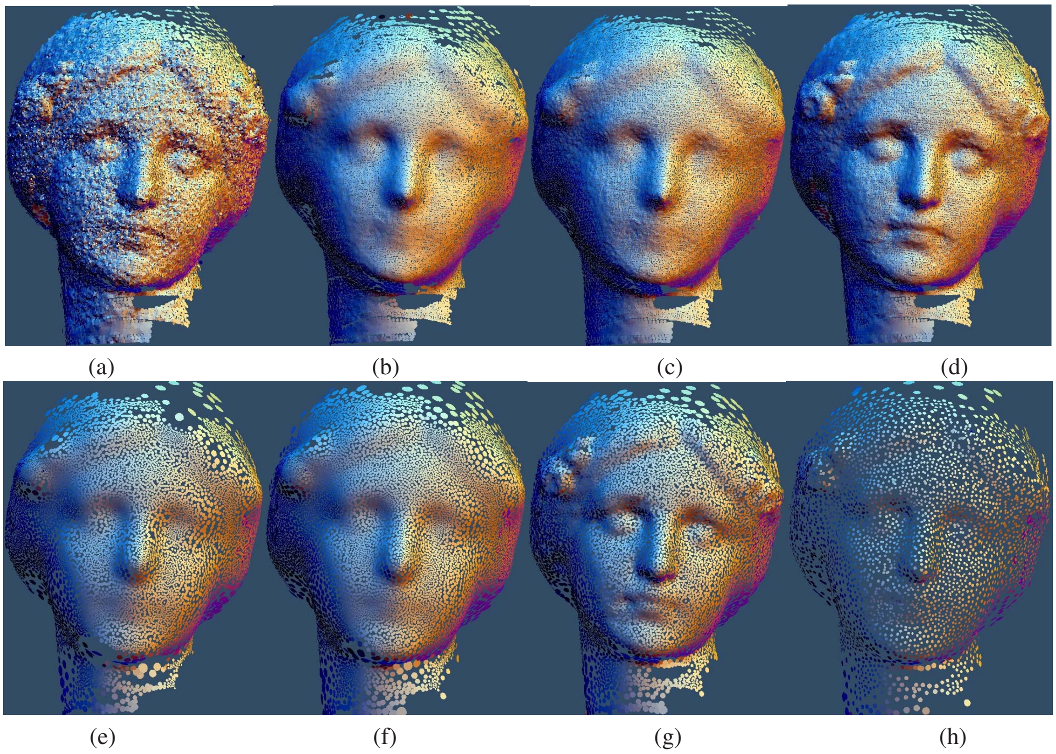



**Figure 1:** *(a) Raw scanned data (65,235 points), (b)(c)(d) reconstruction results using 38,051 points applying methods of [LCOLTE07], [HLZ*09] and our method, respectively. (e)(f)(g) reconstruction results using 16,041 points applying methods of [LCOLTE07], [HLZ*09] and our method, respectively. (h)reconstruction results using our method with 8,143 points.*

More specially, LOP defines the desired points $Q$ as the fixed point solution of the equation

$$Q = G(Q)$$

where

$$G(C) = \arg \min_{X=\{x_i\}_{i\in I}} \{E_1(X,P,C) + E_2(X,C)\}$$

$$E_1(X,P,C) = \sum_{i\in I}\sum_{j\in J} \| x_i - p_j \| \, \theta(\| c_i - p_j \|)$$

$$E_2(X,C) = \sum_{i'\in I} \lambda_{i'} \sum_{i\in I\setminus\{i'\}} \eta(\| x_{i'} - c_i \|) \, \theta(\| c_{i'} - c_i \|)$$

Here $\theta(r)$ is a fast-decreasing smooth weight function with compact support radius $h$ defining the size of the influence radius (for example, $\theta(r) = e^{-r^2/(h/4)^2}$). The term $E_1$ drives the projected points $Q$ to approximate the geometry of $P$, which is also called multivariate $L_1$ median. The term $E_2$ is a repulsion term, and $\eta(r)$ is another decreasing function preventing $x_{i'}$ from getting too close to other points. $\{\lambda_i\}_{i\in I}$ are balancing terms between the two cost functions.

### 2.2. Feature-preserving LOP

We observe that the multivariate $L_1$ median term $E_1(x)$ is defined as the sum of Euclidean distances to the data points, and is located using only a space distance based weight function $\theta(r)$, not considering geometry features existing on the point-set surface. This definition may fail to preserve geometric features, causing sharp features such as edges and corners blurred. Motivated by the geometry bilateral filtering [FDCO03, JDD03], we propose bilateral weighted LOP operator to reconstruct feature-preserving geometry from noisy point set.

We integrate a feature preservation weight $\theta_r$ into the $L_1$ median term $E_1$ and define a feature preserving projection operator. The term $E_1(X,P,C)$ is modified and defined as following:

$$E_1(X,P,C) = \sum_{i\in I}\sum_{j\in J} \| x_i - p_j \| \, \theta_s(\xi_{ij})\theta_r(\zeta_{ij}),$$

where $\xi_{ij} = \|c_i - p_j\|$ and $\zeta_{ij} = \langle n_i, c_i - p_j \rangle$, $n_i$ is the normal of point $c_i$ which can be estimated using the method [HLZ*09]. The weight function $\theta_r$ is feature preservation weight that penalizes large variation in geometry similarity, which is defined as the height difference of point $p_i$ over the tangent plane of the point $c_i$ [FDCO03]. We define function $\theta_r(x)$ as the standard Gaussian filter $\theta_r(x) = e^{-x^2/2\sigma_r^2}$, and define fast-decaying weight function $\theta_s(x) = e^{-x^2/(h/4)^2}$ with the finite support radius $h$.

We keep the repulsion term $E_2$ unaltered. Similarly to LOP algorithm [LCOLTE07], the desired projected points $Q$ is the fixed point iteration solution of the equation (2). Given the initial projection point set $X^{(0)}$, we derive our fixed point iterations as follows. We define $X^{(1)} = \{x_i^{(1)}\}_{i\in I}$ by

$$x_{i'}^{(1)} = \frac{\sum_{j\in J} p_j \theta_s\left(\| p_j - x_{i'}^{(0)} \|\right)\theta_r(\langle n_{i'}, p_j - x_{i'}^{(0)}\rangle)}{\sum_{j\in J} \theta_s\left(\| p_j - x_{i'}^{(0)} \|\right)\theta_r(\langle n_{i'}, p_j - x_{i'}^{(0)}\rangle)}.$$

Then, at each iteration $k = 1,2,3,...$, the new projected point $x_{i'}^{(k+1)}$ is computed as:

$$x_{i'}^{(k+1)} = \sum_{j\in J} p_j \frac{\alpha_j^{i'}}{\sum_{j\in J}\alpha_j^{i'}} + \mu \sum_{i\in I\setminus\{i'\}} \left(x_{i'}^{(k)} - x_i^{(k)}\right) \frac{\beta_i^{i'}}{\sum_{i\in I\setminus\{i'\}}\beta_i^{i'}}$$

where

$$\alpha_j^{i'} = \frac{\theta_s(\|x_{i'}^{(k)} - p_j\|)\theta_r(\langle n_{i'}^{(k)}, x_{i'}^{(k)} - p_j\rangle)}{\|x_{i'}^{(k)} - p_j\|}$$

$$\beta_i^{i'} = \frac{\theta_s(\|x_{i'}^{(k)} - x_i^{(k)}\|)}{\|x_{i'}^{(k)} - x_i^{(k)}\|}\left|\frac{\partial\eta}{\partial r}(\|x_{i'}^{(k)} - x_i^{(k)}\|)\right|$$

The $k+1$ iteration result $\{x_{i'}^{(k+1)}\}_{i'\in I}$ is the final projection result. Similar to [HLZ*09], we define $\eta(r) = -r$ which produces locally regular point distribution. Practically, the iteration procedure tends to converge in a very small number of steps, typically around 10. In our experiments, we use the repulsion parameter $\mu$ as $\mu \in [0,1]$.

Although our method incorporates normal information, however, our method does not require accurate normal estimation. In the situations when reconstructing complex models such as thin surface features and close-by surface sheet. we employ the method presented in [MN03] to estimate the normal $n_i$ for point $p_i$. We first clean the data set, then we use a priority-driven normal propagation scheme and an orientation-aware PCA to work complementarily and iteratively for robust normal estimation.

In Fig.1, we give the comparison results with [LCOLTE07] and [HLZ*09]. Clearly, our proposed method preserves the features better and the projected points are uniformly distributed.

### 2.3. Fast Local Optimal Projection

We find a $\hat{E}_1(x)$ that is close to $E_1(x)$ under a user controlled approximation, while using much fewer point set $\hat{P} = \{\hat{p}_j\}_{j\in K} \in R^3$ to generate $\hat{E}_1(x)$, $(|K| \ll |J|)$. In general, we wish to find the point set $\hat{P}$ that minimize the following problem: $\min_{\{\hat{p}_k\}_{k=1}^K} D(E_1(x), \hat{E}_1(x))$ subject to $\hat{E}_1\left(X,\hat{P},C\right) = \sum_{i\in I}\sum_{j\in K} \| x_i - \hat{p}_j \| \, \theta_s\left(\| c_i - \hat{p}_j \|\right)\theta_r(\langle n_i, c_i - \hat{p}_j\rangle)$, where $D$ is a distance measure between these two terms, which can be defined using the $L_p$ type distances.

Similar to [FK09], we define the point set $\hat{P}$ using a sampling technique. The sampling procedure is based on random sampling ofKDE $f(x) = \frac{1}{|J|}\sum_{j=1}^J \Theta_H\left(x - p_j\right)$, where function $f(x)$ is defined on the original point set data $P$, and $\Theta_H$ is standard Gaussian Kernel. To sample points $K$ from KDE, for each $k = 1,...,|K|$, we choose $\hat{p}_k$ in following three steps: 1. choose a random integer $r_k \in \{1,...,J\}$; 2. choose a random sample $\delta_k$ from $\Theta_H(\bullet)$; 3. set $\hat{p}_k = p_{r_k} + h\delta_k$. Freedman et al. [FK09] have proved that $\hat{p}_k$ is a proper sample of $f(x)$. When $\hat{E}_1(x)$ is constructed on the samples $\hat{P}$ defined as above, the reduced multivariate $L_1$ median $\hat{E}_1(x)$ is close to $E_1(x)$ defined on complete data under a controlled approximation accuracy.

Using the compact KDE based $L_1$ median term $\hat{E}_1(X,\hat{P},C)$, we perform fast FLOP minimization in the following two steps:

1. Sampling: Take $|K|$ samples of the KDE $f(x)$ to yield $\{\hat{p}_k\}_{k=1}^K$. Construct the new median term $\hat{E}_1\left(X,\hat{P},C\right) = \sum_{i\in I}\sum_{j\in K} \| x_i - \hat{p}_j \| \, \theta_s\left(\| c_i - \hat{p}_j \|\right)\theta_r(\langle n_i, c_i - \hat{p}_j\rangle)$.

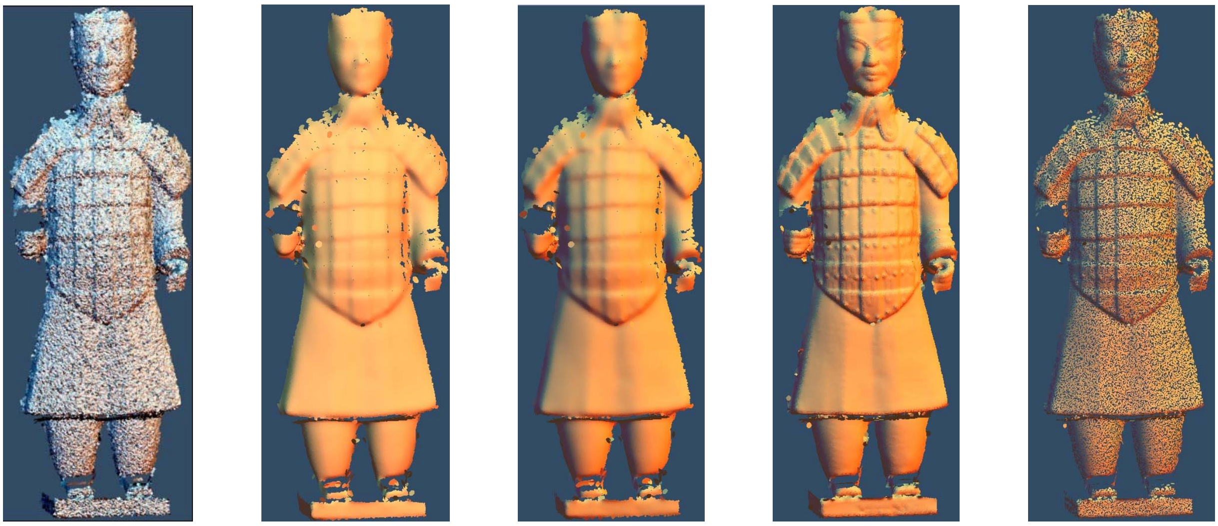
2. Local optimal projection: $G(C) = \arg\min_{X=\{x_i\}_{i\in K}}\{\hat{E}_1(X,\hat{P},C) + E_2(X,C)\}$.

Using the proposed sampling techniques, instead of using all $J$ points to compute the projected points $Q$, we use the reduced set of $K$ samples, which is much smaller than $J$, that is, $|K| \ll |J|$. The computational complexity of $E_1(x)$ have been reduced from $O(J)$ to O(K), and the optimization procedure is greatly accelerated.

## 3. Experimental results and discussion

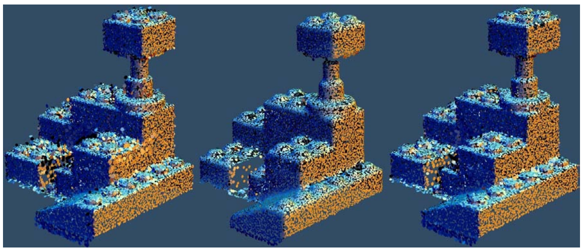
We provide experimental results using the proposed method, and compare our algorithm to related work [LCOLTE07, HLZ*09] on both performance and quality. Our approach is implemented using C++ on a machine equipped with Pentium (R) Dual-Core CPU E5200@2.50GHz with 2GB RAM.

In Fig. 1 and Fig. 2 , we give reconstruction results from the noisy raw scanned data. We have given comparison results with [LCOLTE07] and [HLZ*09]. Compared with [LCOLTE07], our method preserves the features better, and the projected points are distributed fairly. Although the weighted locally optimal projection operator [HLZ*09] improves the points distribution of method [LCOLTE07], however, this method still can not preserve the features well. We also present comparison results with [LCOLTE07] and [HLZ*09] on less projected points in Fig. 1.



(a) (b) (c) (d) (e)

**Figure 2:** *(a) the Terra Catta Warriors point set with slight noise, (b) reconstruction result using existed method [LCOLTE07], (c) reconstruction result of [HLZ*09], (d) result of our algorithm, (e) result of our algorithm with less projected points.*



(a) (b) (c)

**Figure 3:** *(a) The scanned point set, (b) reconstruction result using LOP [LCOLTE07], (c) result of our algorithm.*

In Fig.3, we presented reconstruction results on scanned point set with outliers. At each iteration, we estimate the normal using the method presented in [HLZ*09]. To produce clean point set from the raw scanned data with outlines, as well as preserving the features, at first several iterations, we set a large value for $\sigma_r$ in the feature preservation weight $\theta_r$, which make the FLOP work like LOP. For last several iterations, the value for $\sigma_r$ is small to preserve the geometric features.

## 4. Conclusion and future work

In this paper, we present an efficient FLOP for geometry reconstruction. We first develop a bilateral weighted LOP operator, which takes both spatial and geometric feature information into consideration for feature-preserving geometry reconstruction. We also present sampling technique that is based on the random sampling of KDE to accelerate the FLOP computing.


## Acknowledgment

This work was partly supported by NSFC (No. 60803081, No. 61070081), the Fundamental Research Funds for the Central Universities (6081005).


## References


[FDCO03] FLEISHMAN S., DRORI I., COHEN-OR D.: Bilateral mesh denoising. *ACM Transactions on Graphics (TOG) 22*, 3 (2003), 950–953.

[FK09] FREEDMAN D., KISILEV P.: Fast Mean Shift by compact density representation. In *IEEE Conference on Computer Vision and Pattern Recognition* (2009), IEEE, pp. 1818–1825.

[HDD*92] HOPPE H., DEROSE T., DUCHAMP T., MCDONALD J., STUETZLE W.: Surface reconstruction from unorganized points. *SIGGRAPH 26* (1992), 71–71.

[HLZ*09] HUANG H., LI D., ZHANG H., ASCHER U., COHEN-OR D.: Consolidation of unorganized point clouds for surface reconstruction. *ACM Transactions on Graphics (TOG) 28*, 5 (2009), 1–7.

[JDD03] JONES T., DURAND F., DESBRUN M.: Non-iterative, feature-preserving mesh smoothing. *ACM Transactions on Graphics 22*, 3 (2003), 943–949.

[LCOLTE07] LIPMAN Y., COHEN-OR D., LEVIN D., TAL-EZER H.: Parameterization-free projection for geometry reconstruction. *ACM Transactions on Graphics 26*, 3 (2007), 22.

[MN03] MITRA N., NGUYEN A.: Estimating surface normals in noisy point cloud data. In *Proceedings of the nineteenth annual symposium on Computational geometry* (2003), ACM, p. 328.