Abstract
Several methods that use the notion of viewpoint quality have been recently introduced in different areas of computer graphics, such as scene understanding, exploration of virtual worlds, radiosity and global illumination, image-based rendering and modelling. In this paper, we analyze the behavior of three different viewpoint quality measures. The first one is a heuristic measure, the second one is the viewpoint entropy, and the third one is a new measure based on the Kullback-Leibler distance between the projected and actual distributions of the areas of the polygons in the scene. In addition, this paper reviews different applications and introduces a new algorithm using the Kullback-Leibler distance for the selection of a representative set of n views. Our method is based in selecting the view that minimizes the Kullback-Leibler distance between the mixture of the distributions of all selected views and the actual area distribution.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computing Methodologies]: Computer Graphics—Picture/Image Generation

1. Introduction
It seems intuitive to consider that the best viewpoint of a scene is the one that obtains the maximum information about it. A good view must help us to understand the scene or object it represents as much as possible. A notion of the viewpoint quality of a scene is given in Figure 1, where the quality of view $a$ is higher than the quality of view $b$.

![Figure 1: Two views of the same object.](image)

Several methods that use the notion of viewpoint quality have been recently developed to improve computer graphics algorithms \[BDP00, SFR^2002, Vaz03, VFSH03, RFS00, Ple03\]. These methods have been applied to graphics applications such as scene understanding, exploration of virtual worlds, radiosity, global illumination, image-based rendering and modelling. In scene understanding and exploration of virtual worlds, viewpoint quality is used to automatically compute interesting positions and trajectories for a camera exploring a virtual world \[BDP00, Ple03, VS03, AVF04\]. In radiosity, viewpoint quality is used to improve Monte Carlo techniques by allowing a more intelligent shooting of rays from each surface of the scene \[JP98\]. In ray-tracing it can help to decide whether to add more samples to a pixel \[Ple87, RFS02b, RFS02a\]. In image-based modelling, viewpoint quality is used to compute an optimized minimal set of positions of the camera \[VFSH03\].

In this paper, we study three different viewpoint quality measures and review different applications of them. We also present a new algorithm based on the Kullback-Leibler distance for a viewpoint selection of a representative set of $n$ views. The quality of a view will be only discussed from a geometric perspective, although other aspects such as lighting could modify the perception of a scene.

2. Viewpoint Quality Measures
A first measure of the quality of a viewpoint of a scene could be defined as the number of visible details or, more precisely,
the number of surfaces of the scene visible from this point of view. However, this definition of viewpoint quality is not very satisfactory because the size of visible details is also important. We think that a viewpoint quality measure has to take into account the following aspects:

- The number of surfaces visible from the point of view.
- The area of the visible part of each surface of the scene from the point of view.
- The orientation and distance of each visible surface from the point of view.

According to the notion of viewpoint quality presented above, a measure of it has to depend on the number of visible surfaces, the area of the visible part of each surface and the distance and orientation of each visible surface. An adequate combination of these quantities could give a good measure of viewpoint quality. The following three quality measures have these properties.

### 2.1. Heuristic Measure

The quality of a viewpoint of a scene can be computed by the following heuristic measure [BDP99, Ple03]:

\[
C(V) = \frac{\sum_{i=1}^{n} \left( \frac{P_i(V)}{P_i(V) + 1} \right)}{n} + \frac{\sum_{i=1}^{n} P_i(V)}{r},
\]

where \(V\) is the viewpoint, \(C(V)\) is the viewpoint quality of the scene or object, \(P_i(V)\) is the number of pixels corresponding to the polygon \(i\) in the image obtained from the viewpoint \(V\), \(r\) is the total number of pixels of the image (resolution of the image), \(n\) is the total number of polygons of the scene. In this formula, \([x]\) denotes the smallest integer, greater than or equal to \(x\). Observe that the first term in (1) gives the fraction of visible surfaces with respect to the total number of surfaces, while the second term is the ratio between the projected area of the scene (or object) and the screen area (thus, its value is 1 for a closed scene).

### 2.2. Viewpoint Entropy

The Shannon entropy [CT91] of a discrete random variable \(X\) with values in the set \(\mathcal{X} = \{x_1, x_2, \ldots, x_n\}\) is defined as

\[
H(X) = -\sum_{i=1}^{n} p_i \log p_i,
\]

where \(n = |\mathcal{X}|\) and \(p_i = P[X = x_i]\) for \(i \in \{1, \ldots, n\}\). The entropy gives us the average information or uncertainty of a random variable. If the logarithms are taken in base 2, entropy is expressed in bits. For continuity, we use the convention that \(0 \log 0 = 0\).

To define viewpoint entropy [VFSH01], the relative area
of the projected faces over the sphere $S$ of directions centered in the viewpoint $V$ is used as probability distribution. Thus, the viewpoint entropy is defined by

$$H(V) = - \sum_{i=0}^{N_f} \frac{a_i}{a_t} \log \frac{a_i}{a_t}, \quad (3)$$

where $N_f$ is the number of faces of the scene, $a_i$ is the projected area of face $i$ over the sphere, $a_0$ represents the projected area of the background in open scenes, and $a_t = \sum_{i=0}^{N_f} a_i$ is the total area of the sphere. In a closed scene, or if the viewpoint does not see the background, the whole sphere is covered by the projected faces and consequently $a_0 = 0$. Hence, $\frac{a_i}{a_t}$ represents the visibility of face $i$ with respect to viewpoint $V$. The maximum entropy is obtained when a certain viewpoint can see all the faces with the same projected area $a_i$. So, in an open scene, the maximum viewpoint entropy is $\log(N_f + 1)$ and, in a closed scene, it is equal to $\log N_f$. The best viewpoint is defined as the one that has maximum entropy, i.e., maximum information captured.

The main drawback of viewpoint entropy is that it depends on the polygonal discretization. A high discretized region will heavily attract the attention of the measure. This will be shown in the examples presented in section 1. Also, such as we have defined it, this measure is dependent on the background. Nevertheless, the background can be considered or not into the calculations, depending on whether we want to emphasize or not the relative area covered by the projection.

### 2.3. Kullback-Leibler Distance

We introduce now a new viewpoint quality measure based on the Kullback-Leibler distance \cite{CT91}. The relative entropy or Kullback-Leibler (KL) distance between two probability distributions $p = \{p_i\}$ and $q = \{q_i\}$ over the set $X$ is defined by

$$KL(p \| q) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i}, \quad (4)$$

where, from continuity, we use the convention that $0 \log 0 = 0$, $p_i \log \frac{p_i}{q_i} = \infty$ if $a > 0$ and $0 \log \frac{0}{q_i} = 0$. The KL distance satisfies the information inequality $KL(p \| q) \geq 0$, with equality only if $p = q$. The relative entropy is also called discrimination and it is not strictly a distance, since it is not symmetric and does not satisfy the triangle inequality.

To define a new measure of viewpoint quality we use the Kullback-Leibler distance, where the probability distribution $p$ is given by the relative area of the projected faces over the sphere $S$ of directions centered in the viewpoint $V$ and the probability distribution $q$ is given by the relative area of
faces. Thus, viewpoint quality is defined by

$$KL(V) = \sum_{i=1}^{N_f} \frac{a_i}{n} \log \frac{a_i}{\hat{A}_i},$$

where $a_i$ is the projected area of face $i$, $a_t = \sum_{i=1}^{N_f} a_i$, $A_i$ is the actual area of face $i$ and $\hat{A} = \sum_{i=1}^{N_f} A_i$ is the total area of the scene or object. The viewpoint quality measure can be interpreted as the distance between the normalized distribution of projected areas and the ideal projection, given by the normalized distribution of the actual areas. That is, the minimum value 0 is obtained when the normalized distribution of projected areas is equal to the normalized distribution of actual areas. Otherwise, $KL(V)$ increases when less faces are seen and the proportion of projected areas moves away from the one of actual areas. Thus, to select views of high quality means to minimize $KL(V)$.

While the maximum entropy is very sensitive to both size and number of polygons (see Section 1), the KL measure only takes into account the proportion between the normalized projected area and the normalized actual area, trying to obtain a balanced vision of the object or scene. This is in contrast to the viewpoint entropy, where big polygons are clearly penalized in front of small ones. Note also that the background is not taken into account and that the maximum quality of the KL measure could be obtained with a partial vision of the faces, since only the proportion is considered.

### 2.4. Implementation

To compute the above viewpoint quality measures, we have to estimate the projection of the visible parts of the scene on the screen or on the sphere centered on the point of view. The most accurate estimation of this projection could be obtained by using a hidden surface removal algorithm, working in the user space and explicitly computing the visible part of each surface of the scene. Unfortunately, it is rarely possible in practice to use such an algorithm because of its computational complexity. For this reason, less accurate but also less complex methods, such as z-buffer, have to be used. Before projection, each surface is assigned a different color. The number of pixels with a given color divided by the total number of pixels projected by the object or scene will give the relative area of the surface represented by this given color. With this technique, the three viewpoint quality criteria presented in this section are computed directly by means of an integrated fast display method.

### 2.5. Comparison of Measures

To evaluate the behaviour of the three viewpoint quality measures presented in Section 1, two objects are analyzed:
a cube (Figure 2(a.i)) and an angel (Figure 4(a.i)). In addition, the cube is presented in two different ways: uniformly discretised (Figure 2), with 2 polygons in each face, and non-uniformly discretised (Figure 3), where a face has been discretised in 32 polygons. Figures 2, 3 and 4 have been organized as follows. Rows i, ii and iii show, respectively, the behaviour of the heuristic, entropy and Kullback-Leibler (KL) measures. Columns a and b show, respectively, the views with highest and lowest quality, and columns c and d show two different viewpoint spheres. Blue colors represent the minimum values of the corresponding measures, while red colors represent the maximum values. In the heuristic and entropy cases, the maximum values are interpreted as the best views, and, in the KL case, the maximum values correspond to the worst views. We have not considered the background in the viewpoint in the viewpoint entropy formula to allow for a fair comparison with the other measures, which do not use it.

In Figure 2, we observe that the three compared measures give equivalent views for the best and worst cases. In spite of this, the corresponding spheres of directions show that the entropy and KL measures discriminate better than the heuristic measure. In addition, it can be seen that in this case the entropy and KL measures present a similar behaviour: maximum (or minimum) entropy corresponds to minimum (or maximum) KL value.

In Figure 3, we can clearly observe that heuristic and entropy measures change outstandingly their behaviour with respect to the previous figure, while the KL measure is insensitive to the discretisation. This is an important added value of the KL measure.

In Figure 4 we can see another discriminating feature of KL measure: a ring-like region clustering the lateral bad views of the object (see Figure 4c.iii). Observe also in all three Figures 2, 3 and 4 that the two information-theoretic measures provide us with a better and more informative discretisation of the viewpoint space.

3. Selection of n Views

In this section, we present a new viewpoint selection algorithm based on the Kullback-Leibler distance. Its objective is to find the minimum representative set of views for a given object or scene, in order to well understand it.

The basic idea of the algorithm consists in finding a set of views where the mixed distribution of the projected areas has a minimum KL distance with respect to the actual distribution of the areas. The algorithm proceeds as follows. First, we select the view \( V_1 \) with distribution \( a_1 = \left( \frac{a_{1i}}{\sum_{i=1}^{N} a_{1i}} \right) \) corresponding to the minimum KL distance (maximum quality), where \( a_{1i} \) represents the projected area of face \( i \) for the viewpoint \( V_1 \) and \( d_{1i} = \sum_{i=1}^{N} a_{1i} \). Next, we select \( a_2 = \left( \frac{a_{2i}}{\sum_{i=1}^{N} a_{2i}} \right) \) such that the mixed distribution \( \frac{a_1 + a_2}{2} \) also corresponds to the minimum KL distance, i.e.,

\[
KL(V_1, V_2) = \sum_{i=1}^{N} \frac{a_{1i} + a_{2i}}{2} \log \frac{a_{1i} + a_{2i}}{\frac{a_{1i} + a_{2i}}{2}} \tag{6}
\]

is minimum. At each step, a new mixed distribution \( \frac{a_1 + a_2 + \ldots + a_n}{n} \) is produced until the decrease of the KL distance is lower than a given threshold or a determined number of views is selected.

The set of views obtained by the KL algorithm ensures a representative vision of the object or scene. Note that in some situations it is also possible to obtain an increase of the KL distance (see Figure 8). This happens when a set of good views have been obtained and the next one makes worse the obtained balance with respect to the actual area distribution.

Figures 5, 6 and 7 present the views obtained with our algorithm. Its behaviour is shown in Figure 8. In Figures 5 and 6 the minimum representative set of views is shown, i.e., we stopped when the KL difference between two successive views is lower than a given threshold (see Figure 8). In Figure 7 we show the twelve first views selected by our algorithm, although the minimum representative set of views is given by the four first views.

Observe in Figure 8 how the KL values obtained for the successive mixed distributions for Figures 5, 6 and Figure 7 converge asymptotically to a value that depends on the complexity of the object.

4. Other Applications

Viewpoint quality measures have many potential applications. In this section, we will review briefly some of them, referring the reader to the appropriate bibliography.

- **Scene Exploration.** Automatic exploration of the scene by a virtual camera will be based on incremental evaluation of the viewpoint quality of the scene from the next possible point of view. However, the viewpoint quality of the scene from the next candidate point of view is not enough to ensure intelligent computation of the camera path. In addition, the movement of the camera must obey to the following rules: The camera must avoid fast returns to the starting point or to already visited points. The camera path must be as smooth as possible in order to allow the user to well understand the explored world. A movement with brusque changes of direction is confusing for the user and must be avoided [BDP99, BDP00, Ple03, Vaz03, VFSH03, VS03, AVF04].

- **Molecular Visualization.** Visualization of molecules is relevant for molecular science, a discipline which falls in several areas such as Crystalography, Chemistry and Biology. Two kinds of views are important: for a set of molecules, low entropy views and, for a single molecule, views with high entropy [VFSL02]. In the first case, the views allow to see how the molecules arrange in space and thus infer physical properties. The second case shows
Figure 5: The five most representative views of the mug object selected by the KL algorithm.

Figure 6: The six most representative views of the chair object selected by the KL algorithm.

Figure 7: The twelve first views of the athena object selected by the KL algorithm. The minimum representative set of views is given by the four first views.
how the atoms are arranged in a molecule and allows to infer its chemical properties.

- **Radiosity.** Viewpoint quality can also be used in radiosity, in order to improve Monte Carlo-based computation. Monte Carlo sampling, using rays to distribute the energy of each polygon, is not entirely satisfactory because, on average, the same number of rays is shot to all parts of the scene from a given polygon. This sampling problem may produce noisy images, especially in the case where the scene contains both simple and complex parts. We can improve this distribution by sending more rays to the regions of the scene that contain more information. These regions can be identified using a viewpoint quality measure [JP98].

- **Ray Tracing.** Obtaining a good quality image with ray-tracing demands to cast a lot of rays through each pixel of the screen plane. However, not all pixels need this amount of supersampling. An homogeneous region will need less rays than a region with geometrical discontinuities and/or high illumination gradients. Viewpoint quality can give a measure of the additional sampling needed [Ple87, RFS02b].

- **Image-Based Modelling and Rendering.** Image based rendering allows to compute realistic images at low cost using precomputed ones. However, we can not allow to store an infinite number of images. A selection of the images that best represent or model the scene has to be done. Thus, viewpoint quality has a role to play in image-based modelling. Viewpoint entropy has been used to select this minimum set of points of view to compute the images that best model the scene [VFSH03].

![Figure 8: KL values obtained for the successive mixed distributions for Figures 5, 6 and Figure 7.](image)

5. Conclusions and Future Research

In this paper, we analyzed the behaviour of three different viewpoint quality measures (heuristic measure, viewpoint entropy, and a new Kullback-Leibler distance-based measure) and reviewed different applications of them. A new algorithm using the KL distance for a viewpoint selection of a representative set of \( n \) views has been also presented.

Viewpoint selection using viewpoint quality can also play an important role in data visualization. When complex data need to be shown and/or interpreted, the automatic selection of views can make the process easier. In this sense, molecular visualization shown in Section 4 can be seen as a first step in this direction. Another application area which is worth investigating is protein docking [SMG*98, Vak95]. A protein could move in order to see the other one from the most appropriate viewpoint for docking. Finally we will also try our measures for model simplification: a simplification would be chosen without losing viewpoint quality.

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