Spectral Analysis Driven Sparse Matching of 3D Shapes

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Abstract

In this work we present an approach for matching three-dimensional mesh objects related by isometric transformations and scaling. We propose to utilize the Scale invariant Scale-DoG detector and Local Depth SIFT mesh descriptor, to derive a statistical voting-based scheme to robustly estimate the scale ratio between the registered meshes. This paves the way to formulating a novel non-rigid mesh registration scheme, by matching sets of sparse salient feature points using spectral graph matching. The resulting approach is shown to compare favorably with previous state-of-the-art approaches in registering meshes related by partial alignment, while being a few orders of magnitude faster.

1. Introduction

The registration of three-dimensional (3D) objects is a contemporary research challenge that has been given considerable attention in recent years [DK10, ZWW+10, SHM12]. A class of 3D registration schemes relates to point matching in images by assuming that the registered meshes are related by parametric transformations models (similarity, affine, projective), that can be estimated by robust statistical schemes such as RANSAC. Others assume local isometry that can be encoded by graph algorithms [LH05].

Energy minimization schemes formulate the points correspondence via an objective functions, resulting in discrete optimization problems [DK10], while other apply high-order potentials and the corresponding graph matching techniques [ZWW+10]. In dense matching we aim to align all of the common vertices in the registered meshes. Such schemes may operate in the mesh domain by formulating continuous variational problems [BBK06b], or embed the meshes into a space where they can be aligned [SHM12, BBK+10]. Others extend the sparse matching results to compute a dense matching [ZWW+10].

In this work we propose two core contributions:

First, we propose to utilize spectral graph matching to encode and recover local isometries agglomerated over the mesh objects. By applying our approach to isometries with respect to geodesic distances, we derive a non-parametric mesh matching scheme that is able to recover non-rigid alignments. The downside of this approach is its inability to handle transformations consisting of scale changes. For that we propose our Second contribution, that is a straightforward approach for estimating the global scaling between a pair of mesh objects, based on an analysis of the local scales of weakly-corresponding feature points. In particular, the proposed scheme is applicable to meshes related by partial matching.

This paper is organized as follows. In section 2, we survey previous results on mesh registration, and propose our computationally efficient registration approach in Section 3. In Section 4 we show how to estimate global scale differences between 3D shapes using a histogram-based scale estimation approach, and apply it to mesh registration to derive a unified scheme. We verify and exemplify the use of our approach in Section 5, while concluding remarks and future extensions are discussed in Section 6.

2. Related work

The registration of images, 3D models or more generally, sets of points is a fundamental problem in Computer Vision and has been extensively researched [LH05, CFSV04].

The Iterative Closest Point algorithm proposed by Besl and Mckay [BM92], matches each point on one surface to its closest point on the other mesh, and computes a parametric motion model. Its draw back is the lack of global convergence, thus requiring an initial estimate of the relative transformation.

Spectral embedding was also used by Sharma et.
al. [SHM12] that first align the embeddings using histograms related to the eigenvectors of the spectral embedding. Dense matching is computed using an EM based point matching algorithm. Bronstein et. al. [BBK06b] proposed to compute the dense alignment between meshes by extending the MDS algorithm using the Generalized Multi-Dimensional scaling (GMDS). A different approach to surface registration is the minimization of the Gromov-Hausdorff distance between the surfaces, that quantifies the discrepancy in pairwise distances between corresponding vertices in meshes. It was used by Mémpli and Sapiro [MS05] to derive a scale invariant geometry-based mesh similarity measure. Bronstein et. al. [BBK06a] applied the Gromov-Hausdorff within a multi-resolution formulation to compute it efficiently and robustly, and extended it by utilizing the diffusion distances instead of geodesic ones. In this work we follow these methods in making use of the fact that geodesic distances between matching points on a surface are maintained by isometric transforms.

The minimization of the Gromov-Hausdorff distance is the equivalent of the pairwise matching problem that was analyzed by Leordeanu and Hebert [LH05], who formulated it as a discrete optimization problem that is NP-hard. The solution is approximated by spectral relaxation, followed by a greedy discretization scheme. The drawback of this approach is its sensitivity to scale changes and robustly, and extended it by utilizing the diffusion distances instead of geodesic ones. Thus, the mesh alignment is formulated as a pairwise matching problem and is solved by spectral matching [LH05]. An affinity matrix $A \in \mathbb{R}^{N \times L \times L}$ is computed using Eq. 1, such that

$$a_{ij} = \exp\left(-\frac{|d_{ij}^1 - d_{ij}^0|}{\sigma}\right),$$  

where the parameter $\sigma > 0$ sets the tolerance of deviation from perfect isometric matching, and we used $\sigma = 6$ throughout this work. The affinity function adheres to $a_{ij} \approx 1$ for mutually valid point assignments, and $a_{ij} \approx 0$ for invalid ones. Thus, the mesh alignment is formulated as a pairwise matching problem and is solved by spectral matching [LH05]. An affinity matrix $A \in \mathbb{R}^{N \times L \times L}$ is computed using Eq. 1, such that

$$a_{ij} = \exp\left(-\frac{|d_{ij}^1 - d_{ij}^0|}{\sigma}\right),$$  

where $N$ is the number of feature points detected in $M_0$ and $r_{ij}$ is the ranking of the descriptor similarity between $v_i$ and $v_j$. This also implies that this formulation of the assignment problem is asymmetric, such that we explicitly match $M_0$ to $M_1$.

Thus, the mesh alignment is formulated as a quadratic assignment problem, that can be solved via spectral relaxation [LH05], where we compute $\phi_0$, the leading eigenvector of $A$, and apply the Hungarian algorithm to discretize $\phi_0$ [EKG12], and recover the most probable hard assignments. The proposed scheme is summarized in Algorithm 1:

**Algorithm 1**

1. Compute the sets of local descriptors $\{v_i\} \in M_0$ and $\{v'_j\} \in M_1$.
2. $\forall v_i \in M_0$ find the subset of $M_1$ containing $L$ closest points in terms of descriptor similarity.
3. Apply Eq. 2 to compute the pairwise affinity matrix $A$.
4. Compute $\phi_0$, the leading eigenvector of $A$.
5. Apply the Hungarian Algorithm to $\phi_0$ to compute the discrete assignment result.

\[ \text{For each such pair matching candidate we define a pairwise affinity measure} \]

\[ a_{ij} = \exp\left(-\frac{|d_{ij}^1 - d_{ij}^0|}{\sigma}\right), \]  

\[ \forall i, j \in M_0 \text{, } r_{ij} = \text{the ranking of the descriptor similarity between } v_i \text{ and } v_j. \]

\[ \text{This also implies that this formulation of the assignment problem is asymmetric, such that we explicitly match } M_0 \text{ to } M_1. \]
4. Robust scale estimation

One of the downsides of the mesh alignment scheme presented in the previous section, is its inability to align meshes related by significant scalings. Namely, Eq. 1 assumes an isometric transformation that is scaling-free. To alleviate this limitation, we propose a straightforward approach for estimating the relative global scalings between pairs of meshes. For that we utilize the Scale-DoG interest point detector, and the corresponding LD-SIFT descriptor proposed by Darom and Keller [DK12].

Given a pair of 3D models $M_0$ and $M_1$ we apply the Scale-DoG detector to detect the interest points and corresponding local scales $\{v_i, s_i\} \in M_0$ and $\{v'_j, s'_j\} \in M_1$, respectively. Let $v_i \in M_0$ and $v_j \in M_1$ be a pair of corresponding points, matched by their descriptors similarity

$$i' = \arg \min_k |x_i - x_{k}|^2, \forall v_j \in M_0, \forall v_{k} \in M_1 \quad (3)$$

The ratio of their local scales $\Delta s_{ij} = s_j / s_i$ is an estimate of the global scaling between the two models. Thus, we propose to compute the histogram of $\log(\Delta s_{ij})$ for all pairs of corresponding interest points detected in $M_0$ and $M_1$.

Using Eq. 3 might result in a significant number of outlier matches. But the distribution of their scaling ratios is uniform over the range of scale ratios, while the inlier scaling measurements concentrate over a short interval centered at the true scaling value. This approach does not utilize global attributes of the meshes and can thus be used to recover the relative scaling of partial matches.

Given the scale ratio $\Delta s$ between the two models Eq. 1 is reformulated as

$$\tilde{a}_{i', j'} = \exp\left(-\frac{|d_{ik}^0 - \Delta s \cdot d_{ik}^1|}{\sigma}\right), \quad (4)$$

and as in Section 3.

5. Experimental results

In this section we experimentally verify the proposed mesh matching and scale estimation schemes. We applied both of the proposed methods to models containing arbitrary scale ratios, and considered full and partial matchings†.

5.1. Sparse matching

We tested the sparse matching scheme on models taken from the TOSCA [BBK08] and SHREC’11 [BBB’11] datasets. For computational reasons we used graph distance computed by the Dijkstra algorithm as an approximation for geodesic distances, limited the number of matched vertices in $M_0$ to 1,000, and considered $L = 5$ matching candidates for each such vertex. Figure 2 shows successful sparse matching of hundreds of points between three model pairs related by scaling and an isometric transformation.

5.2. Scale detection

We tested the proposed scale detection scheme using a 100 bins Log-Scale histogram on models taken from the SHREC’11 challenge dataset including one model consisting of 52,565 vertices under different transformations to test our method on a more realistic and difficult scenario, by scaling one of the meshes in each pair. Figure 3 shows partial man models for which scaling has been applied, and the corresponding Log-Scale histogram and the rescaled models.

5.3. Timing results

The proposed scheme was implemented in Matlab with some critical parts coded in C. In Table 1 we report the timing results of the proposed scheme. Our experiments were carried out on a computer running an 2.6GHz Intel i5 processor with 4GBytes of memory. It follows that the proposed methods requires tens of seconds to align hundreds of points
Figure 2: Sparse matching of various scaled models. (a) \(|V| = 27,894, 505 matches, \Delta s = 0.53\). (b) \(|V| = 25,290, 541 matches, \Delta s = 0.53\). (c) \(|V| = 52,565, 176 matches, \Delta s = 1.2\). (d) \(|V| = 4,344, 329 matches, \Delta s = 0.68\).

Table 1: Running times of the proposed methods.

<table>
<thead>
<tr>
<th>Models (Vertices)</th>
<th>Points matched</th>
<th>Features extraction &amp; Groedel distances</th>
<th>Scale detection</th>
<th>Affinity</th>
<th>Spectral matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cat (27,894)</td>
<td>505</td>
<td>68.18 sec</td>
<td>0.33 sec</td>
<td>4.16 sec</td>
<td>1.53 sec</td>
</tr>
<tr>
<td>Dog (25,290)</td>
<td>541</td>
<td>37.67 s</td>
<td>0.11 sec</td>
<td>4.88 sec</td>
<td>1.11 sec</td>
</tr>
<tr>
<td>Wolf (5,844)</td>
<td>329</td>
<td>4.95 s</td>
<td>0.16 sec</td>
<td>0.50 sec</td>
<td>0.16 sec</td>
</tr>
<tr>
<td>Man (52,565)</td>
<td>176</td>
<td>123.98 s</td>
<td>0.32 sec</td>
<td>4.91 sec</td>
<td>1.34 sec</td>
</tr>
</tbody>
</table>

References


