GUIDED QUALITATIVE AND QUANTITATIVE ANALYSIS OF CARDIAC 4D PC-MRI BLOOD FLOW DATA
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Gutachterinnen / Gutachter:

Prof. Dr.-Ing. Bernhard Preim
Prof. Dr. Gerik Scheuermann
Prof. Dr. Anna Vilanova

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PRELIMINARIES
The genesis and progression of cardiovascular diseases (CVDs) depend on various factors. A better comprehension of patient-specific blood flow hemodynamics has great potential to increase their diagnosis, support treatment decision-making and provide a realistic forecast of such pathologies, facilitating a better implementation of preventative measures. Four-dimensional phase-contrast magnetic resonance imaging (4D PC-MRI) gained increasing importance and clinical attention in recent years. It is a non-invasive imaging modality that allows for time-resolved, three-dimensional measurement of blood flow information. The resulting 4D grid data, which contain vectors that represent the blood flow direction and velocity, are of limited spatio-temporal resolution and suffer from multiple artifacts, making complex image processing methods a prerequisite. Qualitative data analysis aims to depict the course of the blood flow with emphasis on specific flow patterns, such as vortex flow, which can be an indicator for different cardiovascular diseases. For this purpose, flow visualization techniques can be adapted to the cardiac context. Quantitative data analysis facilitates assessment of, e.g., the cardiac function by evaluating stroke volumes, heart valve performances by evaluating percentaged back flows, and fluid-vessel wall interactions by evaluating wall shear stress.

This thesis proposes both qualitative and quantitative data evaluation methods, embedded in a developed software prototype with a guided workflow. A semi-automatic extraction of vortex flow is presented that is based on the line predicates methodology and preserves visually appealing path lines with long and continuous courses. It was tailored towards our targeted user group: Radiologists focused on the cardiovascular system and cardiologists. The extracted path lines were used to establish an overview visualization of aortic vortex flow and to adapt the speed of videos so that the display vortical flow behavior is enhanced. Vortices were grouped into single entities (clustering) and subsequently analyzed according to different criteria that describe properties, such as their rotation direction and elongation. Based on this classification, a simplifying glyph visualization was established.

Moreover, this thesis addresses an improved quantification of the flow rate-based measures, such as stroke volumes, which are prone to errors especially in case of pathologic vortex flow. A robust procedure is presented that analyzes multiple, systematically generated configurations of required measuring planes and evaluates the resulting sample distributions. Additionally, the flow rate calculation is influenced by the dynamic morphology. Therefore, a semi-automatic extraction of corresponding motion information was established and incorporated in an adapted quantification.

Cardiovascular diseases (CVDs) – the number one cause of death in the world [357] – are of great clinical interest. Their initiation and evolution depends on many different variables, such as genetic predispositions, the vessel morphology and the blood hemodynamics. Blood flow in the heart and its surrounding vessels, such as the aorta and pulmonary artery, have been investigated for many decades [403, 466]. Derived information have great potential to improve the diagnosis of CVDs, assess their severity, monitor as well as predict their progression, and support the corresponding treatment decision-making.

2D PC-MRI: Two-dimensional phase-contrast magnetic resonance imaging (2D PC-MRI) [268] was established in the 1980s based on advances in MR velocity mapping [74, 513] using the echo rephasing sequence [323, 376, 402]. It is a non-invasive imaging modality with a decent spatial resolution that provides time-resolved (cine), quantitative 2D data with blood flow velocity information in a plane, which is angulated prior to the scan. The facilitated quantification of different measures related to the cardiovascular function made 2D PC-MRI a useful tool in the clinical routine. The flow rate describes passing blood for each time of the cardiac cycle. Among others, this allows to assess the amount of pumped blood per heart beat – the stroke volume (SV) – and the percentage of blood that flows back into the ventricle – the regurgitation fraction (RF). The latter is increased in case of a malfunctioning aortic (AV) or pulmonary valve (PV). Wall shear stress (WSS) relates to shear forces on the vessel wall induced by nearby blood flow. It is suspected to play an important role in the development of pathologic vessel dilations (ectasia / aneurysm). High pressure gradients can occur in pathologically narrowed (stenotic) valves or vessels, which typically also increases peak flow velocities, since the same amount of blood has to pass a smaller cross-sectional area. The pulse wave velocity denotes the speed of the flow rate curve through a vessel. It may be higher and lower in stiff and elastic vessels, respectively.

4D PC-MRI: Technical progress in the field of MRI nowadays enables four-dimensional (4D) PC-MRI, which was introduced by Wigström et al. [556] in 1996. It is also known as flow-sensitive MRI, MR velocity mapping or 4D flow cardiovascular MR (CMR). This modality is able to provide time-resolved, three-dimensional velocity fields. These data allow for an extensive quantitative analysis, since they contain the full spatio-temporal blood flow information and, e.g., an infinite number of measuring planes can be placed after the scan – in contrast to 2D PC-MRI, where a new scan is required in case of placement errors or the need of information about further locations. Another major advantage is that a qualitative analysis of the three-dimensional, pulsatile blood flow becomes possible. Characteristic flow aspects facilitate a deeper understanding of a patient’s situation, since specific patterns, such as vortex flow, are correlated to different pathologies. There is, e.g., a high probability of emerging systolic vortex flow in the ascending aorta if the aortic valve is bicuspid, i.e., two of the three leaflets are fused [472]. This affects the valve’s opening characteristics [40, 325]. Vortex flow close to the vessel wall may induce high shear forces [197, 520] that increase the risk of aneurysm development [79]. Further

See Section Publications in the appendix for the division of work.
1.1. Structure and Contributions of This Thesis

Understand this mutual influence of hemodynamics and vessel morphology can support treatment decision-making and the corresponding risk assessment. Advances towards higher resolutions and faster acquisitions, as well as studies proving the clinical impact, yielded an increasing interest in 4D PC-MRI in recent years [4].

For the evaluation of 4D PC-MRI data, flow visualization techniques, such as integral lines, can be adapted to the cardiac context. However, the enormous data complexity makes the exploration and analysis a time-consuming, tedious and highly subjective task. There is a need for standardized and fast techniques as well as software with guided workflows in order to make 4D PC-MRI viable for the clinical routine [342]. Standardized methods do not only save time, but also decrease the inter-observer variability, which helps to objectively evaluate larger studies. Fully automatic approaches are often not suitable due to the large variety of anatomical situations. Semi-automation allows to incorporate the expert knowledge of the targeted user group: Radiologists focused on the cardiovascular system and cardiologists.

Overviews about 4D PC-MRI and related clinical studies were provided by: Hope and Herfkens [227], Hope et al. [226], Srichai et al. [483], Ebbers [136], Markl et al. [341, 343], Calkoen et al. [83], Stankovic et al. [488] and Nayak et al. [375]. Sengupta et al. [468] performed an extensive comparison between phase-encoded MRI, echocardiography particle image velocimetry and color Doppler echocardiography for cardiovascular flow visualization. They considered multiple aspects, such as spatio-temporal resolutions, scan time, low- and high-velocity accuracy, and the need for breath-holding. Dyverfeldt et al. [135] – a group of physicists, physicians and biomedical engineers – recently published a 4D PC-MRI consensus paper consisting of shared experiences and ideas. They described the potential clinical and research utility of 4D PC-MRI flow, as well as achieved and open development goals.

The work presented in this thesis aims to support users from the clinical area in evaluating 4D PC-MRI data of the aorta and pulmonary artery. The established methods were integrated in the developed software prototype named Bloodline, which benefits from feedback by our clinical collaborators who have access to the tool for research purposes:

- Matthias Gutberlet, Matthias Grothoff, Franziska Reinhold, Barbara Brenneis, Huong Luu Thi Thanh and Daniel Gräfe from the Department of Diagnostics and Interventional Radiology in the Heart Center in Leipzig, Germany,
- Katharina Fischbach from the Department of Radiology and Nuclear Medicine in the university hospital in Magdeburg, Germany, and
- Uta Preim from the Department of Diagnostic Radiology in the municipal hospital in Magdeburg, Germany.

The background part of the thesis is structured as follows:

- Chapter 2 provides information on the cardiovascular system. This comprises an overview about the heart’s anatomy, the circulatory system as well as selected CVDs that are relevant for the further understanding.
- Chapter 3 explains the basic principle of MRI-based measurements. It proceeds with 4D flow acquisitions and characterizes the obtained image data. A brief comparison to alternative flow imaging modalities is provided.
- Chapter 4 is about artifacts in 4D PC-MRI data and available correction procedures.
- Chapter 5 describes the combined anatomical context and flow visualization and the corresponding qualitative assessment. Emphasis is put on physiological and pathological occurrences of characteristic flow patterns, such as vortex flow.
- Chapter 6 is about the clinical significance and calculation of various quantitative measures.

The following main contributions were made:
1.1. Structure and Contributions of This Thesis

- **Guided Evaluation**: Until now, 4D PC-MRI is primarily used for research purposes. A major reason is the lack of standardized and easy-to-use evaluation software with guided workflows and an automated report generation. Also, data evaluation is challenging for larger studies due to the high manual effort.

The research prototype Bloodline [273] was constantly developed in collaboration with our clinical partners. A detailed description of the workflow is presented in Chapter 7. Bloodline allows to process datasets within 10 min. It integrates a full pre-processing pipeline as well as a quantitative and qualitative data analysis. The use of (semi-)automatic methods enables a fluent workflow. Carefully selected defaults strongly reduce the necessity to adjust parameters. State-of-the-art visualizations can easily be created and saved in order to share results. Bloodline was used to evaluate more than 60 datasets of healthy volunteers as well as patients with various CVDs.

Summarized experiences in the 4D PC-MRI data processing were published as a survey paper [275] to provide other researchers with a good starting point. This can be found in the background Chapters 3–6.

- **Vortex Flow**: One interest of clinicians is vortex flow, which can be an indicator for various pathologic alterations of the cardiovascular system. It is usually assessed manually by evaluating animations of the full flow. Here, due to the high complexity of the 4D flow data, visual clutter is a relevant problem. Especially smaller structures can easily be missed. For example a centrally located helix, which is surrounded by near-wall laminar flow, can be hidden. Vortices are further classified according to criteria that describe their shape, temporal occurrence and behavior, such as the turning direction. The employed criteria are often imperfect, imprecise, oversimplified (binary graduation), and not standardized. Moreover, their assessment is highly subjective. All together, the comparison of different datasets is challenging.

Several methods for the enhanced evaluation of vortex flow are presented in Chapter 8, starting with a GPU-implemented extraction of vortex-representing path lines to reduce visual clutter. This method was excessively used by our clinical collaborators and has been proved as very helpful. Based on this, an overview 2D polar plot visualization of aortic vortex flow was established, which allows a quick comparison of datasets within a study regarding the existence of vortex flow. Rendered videos were modified using time lapse and slow motion to emphasize the temporal visibility of vortex flow. This is a convenient tool for offline evaluations (case discussions) or presentations. A suitable clustering method for the vortex-representing path lines was determined. On top of this, a semi-automatic classification was established, which complements and extends clinical classifiers. This helps to decrease inter-observer variability and to make datasets more objectively comparable.

- **Flow Rate and Stroke Volume Quantification**: The flow rate forms the basis for some essential measures to assess the cardiac function, such as the stroke volume (SV). 4D flow MRI scans suffer from various artifacts, e.g., due to inhomogeneous magnetic fields, critical a priori scan parameters, limited spatio-temporal resolution with related partial volume effects, and respiratory motion. The quantification of flow rates often requires a repeated modification of the employed measuring planes and a result estimation based on experience and plausibility. Furthermore, a 3D approximation of the actually dynamic vessel is usually employed, which introduces additional quantification errors.

Chapter 9 starts with an automation of physicians’ manual approach. Measuring plane configurations were systematically evaluated. Plausible results were suggested, based on an analysis of the obtained samples distribution, and an estimation of the result uncertainty was conveyed. The method provides reliable and reproducible results and is suitable to be integrated in an automated data evaluation. The chapter proceeds with an approach to automatically extract the time-varying vessel morphology solely from the 4D PC-MRI data. Motion information were extracted, processed, incorporated in the SV quantification, and deviations to the approach using static vessels were investigated.
Chapter 10 summarizes this thesis and discusses potential future research topics as well as the potential that lies in 4D PC-MRI.

1.2. Notations

There is an ambiguity regarding the term velocity. It either represents a vectorial flow direction or the scalar length of the flow direction. In this thesis, velocity $\| \vec{v} \|$ consistently denotes a scalar value – the magnitude of the corresponding velocity vector $\vec{v} \in \mathbb{R}^3$. Therefore, velocity is equivalent to the term speed. Vectorial values are explicitly named vector or direction, e.g., flow direction or velocity vector. A vector field is denoted as $V$, spatial positions as $\vec{p} = (x, y, z)$ and spatio-temporal positions as $\vec{p}_t = (\vec{p}, t) = (x, y, z, t)$.

In this thesis, the common terms 2D PC-MRI and 4D PC-MRI are adopted, which actually denote time-resolved (cine) 2D (2D + time) PC-MRI and cine 3D (3D + time) PC-MRI, respectively.

Citations are normally blue [556]. Red citations [275] refer to works where the author of this thesis contributed.
BACKGROUND
This chapter provides an anatomical overview of the heart, the circulatory system, and selected cardiovascular diseases (CVDs) with emphasis on the aorta and pulmonary artery.

2.1. HEART ANATOMY AND BLOOD CIRCULATION

Both heart halves consist of an atrium and ventricle, which are connected through a valve (see Figure 1a). This is the tricuspid and mitral valve in the right (RV) and left ventricle (LV), respectively. The aorta (Ao) is attached to the LV, separated through the tricuspid aortic valve (AV). Tricuspid means that it consists of three leaflets. The crooked aorta is the largest artery in the body with a diameter of about 2.5–3.5 cm and 30–40 cm length [447]. Its vessel sections are commonly divided into the aortic root, located at the AV, the ascending aorta (AAo) behind the AV, the aortic arch and the descending aorta (DAo). The pulmonary artery (PA) is connected to the RV and separated through the pulmonary valve (PV). The main PA (also: pulmonary trunk), directly behind the PV, splits into the left (LPA) and right pulmonary artery (RPA), which reach to the left and right half of lung, respectively.

![Figure 1: (a) Anatomic overview of the heart. (b) Depiction of the systemic (red) and pulmonary circulation (blue). Images a and b were created by the blausen.com staff [44] and made available on Wikimedia Commons under the Creative Commons Attribution 3.0 Unported License.](image)

The main purpose of the cardiovascular system is to control the blood flow to various parts of the body [427]. A heart beat consists of two phases: Systole and diastole. During systole, oxygenated blood is pumped from the left ventricle into the aorta (see Figure 1b). The otherwise closed aortic valve is open at this point. Smaller arteries branch off the aorta and supply the blood to all body regions. The blood is transported back to the right atrium through veins. This part of the circulatory system is called systemic circulation. Also during systole, deoxygenated blood is pumped from the right ventricle into the pulmonary artery through the opened pulmonary valve. The blood is enriched with oxygen in the left and right half of lung and then transported to the left atrium. This process is referred to as pulmonary circulation. The ventricles are refilled during diastole with blood coming from the left and right atrium through the opened tricuspid and mitral valve, respectively. The next heart beat begins.
The heart’s shape facilitates efficient blood supply to the lung [264]. However, the blood flow leads to shear forces on the vessel walls, which causes a continuous remodeling of the heart morphology and vessel tissue [147].

2.2. Cardiovascular Diseases

Cardiovascular diseases (CVDs) are the number one cause of death worldwide [357]. The World Health Organization (WHO) reports:

- "In 2012, an estimated 52 % of all deaths under age 70 was due to noncommunicable diseases (NCDs), and two thirds of those deaths were caused by cardiovascular diseases (CVD), cancer, diabetes and chronic respiratory disease. Premature mortality rates due to NCDs declined globally by 15 % between 2000 and 2012. A major factor is the decrease in CVD mortality, driven by population-level blood pressure improvements, declines in tobacco use and advances in medical treatment. Declines have been greater in high-income countries than in the low- and middle-income countries." (Boerma et al. [49, page 133)

- "The leading risk factor for CVD is high blood pressure." (Boerma et al. [49, page 135])

- "More people die annually from CVDs than from any other cause, with an estimated 17.5 million deaths in 2012 (46 % of all noncommunicable disease deaths) [564]. Of these deaths, an estimated 7.4 million were due to coronary heart disease and 6.7 million were due to stroke. Around one third of these CVD deaths occur in adults age 30–70, which are the focus of the global NCD and sustainable development goals (SDG) targets." (Boerma et al. [49, page 140])

Besides the death toll itself, this represents an enormous cost factor for the health systems. For example, the direct and indirect cost of CVDs and stroke in the United States in 2011 were [373]:

- 215.6 billion USD for heart disease,
- 46.4 billion USD for hypertension,
- 33.6 billion USD for stroke, and
- 24.6 billion USD for other CVDs.

In the European Union, about 196 billion Euro are spent annually. Of this total cost "around 54 % is due to health care costs, 24 % due to productivity losses and 22 % due to the informal care of people with CVDs." (Nichols et al. [382])

In the following, an overview of selected CVDs is given, which are relevant for the further understanding of this thesis. Conventional diagnostic methods and treatments are explained as well.

### Parts of this overview are based on:


See Section Publications in the appendix for the division of work.

### 2.2.1. Alterations of the Vessel Wall

This group of CVDs refers to changes of the vessel wall due to different causes, e.g., inflammatory processes.
ECTASIA AND ANEURYSM: A pathologic vessel dilation up to $1.5 \times$ the original vessel diameter is referred to as ectasia. Above a factor of 1.5 the term aneurysm is applied (see Figure 2a). Aneurysms bear the risk of rupture, which is fatal in most cases for heart vessels.

STENOSIS AND COARCTATION: On the contrary, a pathologic narrowing of the vessel is named stenosis. If the aortic arch is affected, it is referred to as coarctation (see Figure 2a). A stenosis can cause increased flow velocities and raised pressure before the narrowed region. The severity is graded by the percentaged area of the vessel’s cross-section that is blocked. Depending on whether the vessel is blocked equally from all sides or primarily from one side, a stenosis is concentric or eccentric. A potential cause for eccentric stenoses is plaque [240] (see Figure 2b). Moreover the length of the stenotic vessel section is of interest for the classification.

AORTIC DISSECTION: A tear in the inner layer of the aortic wall allows blood to flow between the inner and outer wall layer, which causes their separation (see Figure 2c). The outer layer is widened pathologically and bears a high risk of rupture, which is fatal in most cases. Lansman et al. [298] described different subtypes of aortic dissection.

PULMONARY HYPERTENSION: Pulmonary hypertension [350, 477] (PAH) begins with an inflammation of pulmonary arteries’ (PA) vessel wall cells. Here, small PAs and capillaries in the lung are meant. These PAs become blocked, narrowed or even destroyed, which complicates blood transport. The (right) heart’s workload increases, resulting in raised blood pressure. In the long run, PAH causes a weakening of the heart muscle and eventually its failing.

DIAGNOSIS AND TREATMENT: Shape alterations can be diagnosis with morphological scans from multislice spiral CT or MRI. In combination with contrast agent it is referred to as MR or CT angiography. For example, the latter can be used to assess plaques [186]. Unfortunately, the image resolution of MR or CT is too coarse to obtain information on the vessel wall, e.g., the thickness. Cardiac catheterization in combination with contrast agent and X-rays facilitates the detection of narrowed or blocked vessels. Interventions, such as balloon dilatation, can be performed to widen a narrowed vessel and stents can be placed to provide structural support to the vessel (see Figure 3a). However, catheters are usually inserted in the arm or groin. Navigation to the heart requires in-depth knowledge of the circulatory system and much experience, since the toxic contrast agent in combination with X-ray
imaging can only be used scarcely. Severe stenoses can be treated with a bypass surgery, which is a highly invasive and risky procedure. A possible treatment for pathologic vessel dilations is to insert a vascular prosthesis as an artificial vessel course (see Figure 3b). This detains blood from flowing close to the weakened vessel wall and decreases the chance of further dilation or rupture. An aortic valve-sparing surgery [112] is an advanced procedure where the, e.g., aneurysmatic, aortic root can be replaced while preserving the native aortic valve.

Aortic dissection typically causes a severe pain in the chest, back or between the shoulder blades. Other symptoms comprise clammy skin and an elevated heart rate. Since these could also be caused by other, more common conditions, a dissection might be difficult to diagnose without initial suspicion. However, a quick diagnosis is important to increase the chance of survival in case of rupture [145, 260, 505]. The widened outer layer of the aortic wall can be assessed using X-ray, CT or an MR angiography. As a treatment the aorta can be reconstructed using a stent-graft [139]. If the aortic valve took damage as well, it can be replaced. It has been shown that hemodynamics play an important role in the evolution of aortic dissection [5].

Symptoms like shortness of breath, chest pain and tiredness, which may limit physical activity, are typical for pulmonary hypertension [170]. PAH can be diagnosed by anamnesis or physical examination. Both echocardiography and catheterization are employed to assess right heart pressure and function. Unfortunately, there is no cure for PAH. Corresponding treatments aim at reducing symptoms and increasing the quality of life.

2.2.2. VALVE PATHOLOGIES

Pathological valve alterations comprise morphological changes as well as malfunctions [383].

**STENOSIS:** Similar to vessels, a stenotic valve is abnormally narrowed. This is the most frequent valvular heart disease in Europe and North America [27]. The valve orifice area is used to grade the severity of the stenosis [171]. "The aortic valve area (AVA) must be reduced to roughly one-fourth its normal size before any significant changes occur in hemodynamics [57]." (Garcia and Kadem [171])

**INSUFFICIENCY:** If a valve does not close properly to prevent blood from flowing back, e.g. to the ventricle, it is insufficient, which negatively affects the cardiac function. The percentage of blood that flows back is called regurgitation fraction (RF). It is used as a measure of severity.

![Stent in Coronary Artery](image)

**Figure 3:** (a) A stent was placed via catheterization and provides structural support to the vessel. (b) CT image of a Y-prosthesis as treatment of an aneurysm in the abdominal aorta.

*Image a was created by the blausen.com staff [44] and made available on Wikimedia Commons under the Creative Commons Attribution 3.0 Unported License. Image b was released into the public domain and made available on Wikimedia Commons.*
2.2. Cardiovascular Diseases

Bicuspid Aortic Valve: The most common aortic valve malformation, with a prevalence\(^1\) of 1–2 % \([155]\), is the bicuspid aortic valve (BAV). In this case, two of the normally three valvular leaflets are fused (see Figure 4). In the majority of cases this is inherited, but it also can develop during the lifetime. The constant hemodynamic dysfunction due to the valve’s altered opening characteristics can lead to an aortic valve stenosis and / or insufficiency. “BAV may account for more morbidity and mortality than all other congenital cardiac malformations combined.” (Hope et al. [220])

![Figure 4: Healthy tricuspid aortic valve and different types of bicuspid aortic valves (BAVs) depending on which leaflets are fused. The image is from Martin et al. [344] who published their open access article under the Creative Commons Attribution 4.0 International License.](image)

Diagnosis and Treatment: Cardiac catheterization allows to assess a valve’s function and to determine the blood flow and pressure in the heart chambers (atria, ventricles) as well as the vessels. Besides MRI, BAVs can be diagnosed using an echocardiogram, which is an ultrasound of the heart (see Section 3.2). Valve replacement is a possible treatment of valve pathologies. However, this is highly invasive and should only be performed when a positive benefit-risk ratio is likely. Current medical research investigates less invasive methods, such as transcatheter aortic-valve implantation \([430]\) (TAVI), where a heart valve can be replaced via catheterization.

2.2.3. Complex Congenital Pathologies

This category refers to hereditary diseases that are often characterized by a multitude of conditions.

Tetralogy of Fallot: Tetralogy of Fallot\(^2\) (ToF) is the most common, inherited, cyanotic heart disease. This complex condition consists of four components (see Figure 5):

- A ventricular septal defect (VSD) is a hole in the wall (septum) between the left and right ventricle, which causes a mixture of oxygenated and deoxygenated blood in the left ventricle.
- Due to a pulmonary valve stenosis, the mixed blood from both ventricles is preferably pumped through the aorta. This behavior is called right-to-left shunt.
- The constantly increased resistance due to the PV stenosis promotes the development of a right ventricular hypertrophy – an enlargement of the heart muscle.
- An overriding aorta names a special malpositioning where the aorta is directly positioned over the VSD instead of the left ventricle.

Marfan Syndrome: The Marfan syndrome\(^3\) is a genetic disorder of the connective tissue. Marfan patients show an increased susceptibility to developing aneurysms due to "altered mechanical properties of the aortic wall related to the decreased aortic elasticity." (Geiger et al. [181]) This strongly increases the risk of aortic dissection (see Figures 5b–c). Dormand and Mohiaddin [123] provided a corresponding overview.

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\(^1\)Prevalence is the percentage of a population that has a specific condition.

\(^2\)Named after the French physician Étienne-Louis Arthur Fallot.

\(^3\)Named after the French pediatrician Antoine Marfan.
2.2. Cardiovascular Diseases

![Figure 5: (a) Four components of the tetralogy of Fallot: A ventricular septal defect (VSD), a pulmonary valve (PV) stenosis, an overriding aorta, and a right ventricular (RV) hypertrophy. (b–c) Aortic root dilation with dissection (white arrow in b) in a patient with the Marfan syndrome. Image a was created by F. Gaillard and made available on Wikimedia Commons under the Creative Commons Attribution-Share-Alike 3.0 License. Images b and c are from Dormand and Mohiaddin [123], who published their open access article under the Creative Commons Attribution 2.0 License.](image)

**Diagnosis and Treatment:**

ToF is a cyanotic disease, where the blood oxygen levels are low. Thus, suspicion is raised if a baby has blueish skin. Other symptoms comprise enlarged skin or bones around the fingernails, a poor overall development and occasional dizziness due to the oxygen deficiency. Nowadays ToF is surgically corrected within months after birth [548]. The VSD is closed and the PV stenosis is treated. However, ToF patients are highly vulnerable to developing a post-operative pulmonary valve insufficiency.

Marfan syndrome can be diagnosed using an echocardiogram of the heart, an eye examination to see if the lenses are out of place, and a CT or MRI of the lower back to check if the sac around the spinal column is swollen. The Marfan syndrome itself cannot be cured. Thus, the treatments depend on the developing secondary diseases.

2.2.4. Collective Terms

This section defines umbrella terms for heart conditions with similar symptoms and outcomes.

**Congestive Heart Failure:** Congestive heart failure (CHF) denotes the inability of the heart to supply enough blood to the body. This can be caused by a limited pumping capacity of the heart due to a damaged heart muscle (cardiomyopathy), e.g., from drug abuse or infection. Also, the heart muscle can be damaged during a heart attack, where the coronary arteries, which supply oxygen to the heart muscle, are blocked. This can promote the forming of improperly functioning scar tissue. Other causes are valve-related. Regurgitation fractions up to 5% are considered as physiological and unproblematic [560], whereas higher values can indicate CHF.

**Congenital Heart Disease:** Congenital heart diseases [335] (CHDs) refer to inherited pathologies in general. These can be valve defects, problems with the heart muscle, the vessel walls or conditions of specific blood vessels – for example tetralogy of Fallot or aortic coarctation [530].

**Diagnosis and Treatment:** Besides physical examination, angiographies and blood tests, the individual patient history is important to diagnose CHF or CHD. A gold standard does not exist. Also, treatments depend on the patient-specific symptoms and causes. Modifications of the lifestyle might be appropriate, e.g., in case of a self-induced cardiomyopathy.
2.3. Summary

Circulatory System: The aorta (Ao) is attached to the left ventricle (LV), the pulmonary artery (PA) to the right ventricle (RV). A heart beat consists of systole, where blood is ejected from the LV and RV to the body and lung, respectively, and diastole, where both ventricles are refilled.

Cardiovascular Diseases: CVDs are the number one cause of death in the world and a cost burden to the health systems. Vessel morphology alterations comprise pathologic dilations (ectasia, aneurysm), narrowings (stenosis, coarctation) as well as obstructions (due to plaque or vessel wall inflammation, e.g., of lung capillaries in pulmonary hypertension (PAH)), and tears (aortic dissection, aneurysm rupture). Valve pathologies comprise stenosis (reduced orifice area), insufficiency (no proper closing, blood flows back), and altered morphology (bicuspid aortic valve (BAV)). Congenital diseases (tetralogy of Fallot (ToF), Marfan syndrome) consist of multiple components. Marfan patients are prone to aneurysm development. Congestive heart failure (CHF) and congenital heart disease (CHD) are collective terms for pathologies that affect the sufficient blood supply or that are inherited, respectively.

Conventional diagnostic tools comprise morphological scans (CT, MRI), angiographies (CT and MRI with intravascular contrast agent), patient anamnesis or echocardiography (ECG, ultrasound-based imaging). Cardiac catheterization allows to determine various cardiac function parameters, such as pressure, but is invasive, causes patient stress, and requires highly skilled performing physicians. Vessel dilations can be treated with vessel prostheses (artificial vessel courses, often supported with a stent) or replacement of the corresponding vessel section, e.g., aortic root replacement. A bypass surgery can be performed in case of severe narrowings. Mild cases can be widened via balloon dilatation, where a subsequently placed stent provides structural support to the vessel. Malfunctioning valves can be replaced (highly invasive surgery) or reconstructed (repaired via catheterization, less invasive). ToF is corrected within months after birth, though, later development of pulmonary insufficiency is likely.

Criticism: Blood flow and its interaction with the vessel wall is considered insufficiently with current methods. A fully comprehensive picture of the patient situation might not be obtained.
3. CARDIAC BLOOD FLOW MEASUREMENTS

This chapter explains the ideas behind four-dimensional phase-contrast magnetic resonance imaging (4D PC-MRI) blood flow measurements, characterizes the resulting datasets, and considers selected alternatives based on ultrasound. A basic understanding of the data is essential for the comprehension of existing and development of new analysis methods.

3.1. 4D PHASE-CONTRAST MRI

This section starts with an explanation of general MRI measurements [230], followed by an adaption of this principle that facilitates the measurement of instationary blood.

3.1.1. MRI BASICS

**Introduction:** Magnetic resonance imaging (MRI) [202] is a non-invasive imaging modality that allows the acquisition of slice images. For a historical overview, please consider Rinck [429]. MR scanners are available in most hospitals. Different scan sequences enable a wide variety of possible applications, e.g.:

- $T_1$ and $T_2$ are standard sequences that provide a decent contrast in soft tissue and fluids.
- Functional MRI (fMRI) [163, 321] facilitates the measurement of neuronal activity.
- Diffusion tensor imaging (DTI) [372] allows to obtain directions of fiber bundles in the brain.
- Phase-contrast MRI (PC-MRI) – the central topic of this thesis – enables the measurement of blood flow directions and velocities in, e.g., the heart, the head [20] and the liver [487].

Strong magnetic fields are the basic tool of MRI. Exposure to such fields has no known side-effects. This is a great advantage over other modalities, such as computerized tomography (CT), where ionizing radiation is used that is not always applicable, e.g., in case of pregnancy. Though, MRI has its own restrictions. For instance, it is not applicable for patients with technical gadgets, such as cardiac pacemakers, or if there is metal inside the body. Moreover, it is often more costly in terms of time than other imaging modalities and has a higher consumption of electricity.

The physical principle depicted in this thesis is the classic one, excluding quantum mechanical properties [201]. The goal is to provide only a basic understanding of MRI as well as 4D PC-MRI.

**Spinning Protons:** MRI exploits a physical property of some atomic nuclei called spin [308]. This type of angular momentum can be seen as rotation around an arbitrary, internal axis (see Figure 6). Hydrogen atoms (H), consisting of a proton and an electron, are most relevant for biomedical applications since the human body consists to approximately two third of water (H$_2$O) [544]. This allows to distinguish water from fat tissue, but also to encode fluid movement.

![Figure 6: Protons (green) in their natural state. (a) Each one rotates (blue) around an individual spin axis (red). (b) Countless protons in the body with arbitrary spin axes.](image)
3.1. 4D Phase-Contrast MRI

**Figure 7:** Protons in an active $B_0$ magnetic field.
(a) The proton’s (green) spin axis (red) is not fully aligned with the $B_0$ direction $\vec{B}_0$ (orange). Instead, a precession (orange) like a gyroscope is performed.
(b) A spin axis aligns either parallel or antiparallel to $\vec{B}_0$. Pairs of parallel and antiparallel protons (transparent) erase their contribution to an net (overall) magnetization. However, as the parallel alignment is energetically more favorable, there are slightly more protons in this state, yielding some unmatched protons (dark green). This configuration is called equilibrium.
(c) The sum of their magnetic moments provides a net magnetization $\vec{M}$ (dark red), which solely consists of a longitudinal part $\vec{M}_z$, parallel to $\vec{B}_0$, at this moment.

**Spin Alignment:** Application of an external magnetic field – the $B_0$ main magnetic field – causes a proton’s spin axis to align either parallel or antiparallel with the $B_0$ direction $\vec{B}_0$, where the parallel alignment is energetically more favorable. Consequently, more protons are in this state with a ratio of $1.000.007 : 1.000.000$. This configuration of protons, where solely $B_0$ is active, is referred to as equilibrium.

The spin directions are not fully aligned with $l$ parallel to the $B_0$ direction. Instead, they precess around $\vec{B}_0$ like gyroscopes (see Figure 7a), where the individual precessions are out of phase (unaligned, not synchronized). The precession frequency, named Larmor frequency$^4$, is directly proportional to the magnetic field strength and varies for different nuclei. The Larmor frequency of hydrogen nuclei at 1 T field strength is 42.58 MHz. Typical scanners available in clinics achieve 1.5 or 3 T. Some special productions for research purposes achieve even 7 T. For comparison, the earth’s magnetic field strength is about $3.2 \times 10^{-5} \text{T}$ at 0° latitude and 0° longitude.

The sum of all protons’ magnetic moments yields a net (also: overall) magnetization $\vec{M}$, which increases with higher magnetic field strengths. However, a pair of parallel and antiparallel protons cancel each other out in their effect. Nevertheless, due to the unequal distribution of parallel and antiparallel alignments, there are a few unmatched protons (see Figure 7b). The number of such protons is sufficient for MR imaging due to the vast amount of overall protons in the body, which is approximately $4.7 \cdot 10^{27}$. In the following, the term proton solely refers to unmatched protons. $\vec{M}$ is composed of a longitudinal part $\vec{M}_z$, parallel to $\vec{B}_0$, and a transversal component $\vec{M}_{xy}$, orthogonal to $\vec{M}_z$. The latter is 0 in the equilibrium (see Figure 7c).

**Excitation and Relaxation:** When the protons are excited with a radio frequency (RF) equal to their Larmor frequency from a direction perpendicular to $\vec{B}_0$, about half of the protons spin axes flip by 180°. As a result, the transversal magnetization component $\vec{M}_{xy}$ increases until $\vec{M}$ rotates in a plane perpendicular to $\vec{B}_0$ (see Figure 8). Additionally, the individual precessions synchronize. During this phase coherence, $\vec{M}$ is maximally amplified.

When the RF signal is turned off, the relaxation begins. The protons dephase (spin-spin relaxation) within the constant time $T_2$. It denotes when the transversal magnetization reduced by about 63%. Also, the spin axes recover from the flipping, causing a stepwise restoration of the longitudinal magnetization.

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$^4$Named after the British physicist Joseph Larmor.
3.1. 4D Phase-Contrast MRI

Figure 8: From left to right: Excited with the correct radio frequency (RF) (light blue), the unmatched protons’ (dark green) spin axes (red) synchronize their precession (orange). Additionally, about half of the spin axes are flipped by $180^\circ$ (purple). The overall magnetization vector (dark red) is now rotating in the $\vec{M}_{xy}$ plane (transversal magnetization) perpendicular to $\vec{B}_0$ (orange).

From right to left: When the RF is turned off, the transversal magnetization gradually decreases and the longitudinal magnetization is restored ($T_1$ relaxation). Additionally, the protons’ phases desynchronize ($T_2$ relaxation). During this processes, a measurable RF is released, which is the energy the protons absorbed from the excitation.

Figure 9: One slice in axial orientation of a (a) $T_1$- and (b) $T_2$-weighted MR image of a healthy volunteer’s brain.

$\vec{M}_z$ while the transversal component $\vec{M}_{xy}$ decays. This spin-lattice relaxation happens within the constant time $T_1$, which is longer than $T_2$. It denotes when about 63% of the longitudinal magnetization are restored. Both the $T_1$ and $T_2$ relaxation are independent from one another and happen simultaneously.

**Resonance:** An electromagnetic current is induced in a receiver coil by the rotating transverse magnetization during the relaxation. In other words, the protons release the energy that was absorbed from the RF excitation [353]. This provides a measurable RF signal that depends on the amount of hydrogen nuclei in the corresponding tissue (proton density), the $B_0$ magnetic field strength and the $T_1$ as well as $T_2$ relaxation times.

A repeated excitation and measurement of the magnetic resonance is necessary to acquire an MR image. Thus, the RF excitation is done as a pulse sequence [33], where the signal is rapidly switched on and off again. The time between two excitations (a scan parameter), during which the longitudinal magnetization can recover, is called repetition time (TR). If a low value is chosen, tissue with a short and long $T_1$ time provides a strong and weak signal, respectively. For example, fat appears bright and water dark in a $T_1$-weighted image (see Figure 9a). The time between the excitation and the measurement of the signal is called echo time (TE) and is another scan parameter. Small values focus tissue with short $T_2$ times, such as water, which appears bright in $T_2$-weighted images, whereas fat appears dark [424] (see Figure 9b).
3.1. 4D Phase-Contrast MRI

Spatial Encoding: In order to create a volumetric image, spatial positions within the body must be distinguishable during the acquisition. The idea is to employ three linear magnetic field gradients for this purpose, i.e., spatially varying magnetic fields that are superimposed on the $B_0$ field. The direction of each gradient is perpendicular to the other two:

1. As mentioned before, the Larmor frequency is directly proportional to the magnetic field strength. The $z$-gradient is oriented from head to feet and, therefore, encodes the image slice. Slice distances are a result of the steps in which the RF for excitation is incremented during the scan.

2. The $y$-gradient facilitates the selection of a specific column within a slice. It is turned on and off for a short period of time, directly after the RF, and causes a phase shift of the precessing spin axes. Therefore, it is also called phase-encoding gradient.

3. The frequency-encoding $x$-gradient is active together with the $z$-gradient when the signal is received. It encodes the row by causing a shift in the precession frequencies within the column.

Finally, each image position is uniquely identifiable (see Figure 10). An often employed sequence of RF and gradients pulses is the spin echo sequence, as described by Hahn [198].

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**Figure 10:** Spatial encoding within a slice. (a) The phase of a proton’s precession is illustrated as blue vector. (b) The phase-encoding $y$-gradient causes a different phase shift (blue) for each row ($0^\circ, a^\circ, b^\circ$), whereas the frequency-encoding $x$-gradient alters the speed of the individual precessions (slow, medium, fast). Together they facilitate the selection of a specific position, as indicated by the grayscale background colors. Image b is based on a learning script for physicians about MRI basics by Dr. med. Christoph Pabst from the University Hospital Giessen and Marburg.

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**Figure 11:** The images show a slice in axial orientation of a $T_1$-weighted MR image of a healthy volunteer’s brain. (a) Acquired $k$-space data matrix, which encodes phase and frequency. (b) Image reconstruction via inverse Fourier transform.

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Image Reconstruction: If each position was processed individually, an enormous amount of time would be necessary to obtain a slice image. For example, if a slice’s grid resolution is $256 \times 256$ and each of the 65536 positions takes 10 ms to process, scanning this one slice alone would take nearly 11 min. Therefore, no single points are considered. Instead, row by row is read with a special sequence and composed into a raw data matrix, which is referred to as $k$-space and encodes both the measured phases and frequencies. An inverse Fourier transform allows to convert this data matrix into an image (see Figure 11). In the example above, this reduces the expenditure of time by a factor of 256 to 2.56 s.
3.1.2. Flow Measurement

Besides the vessel morphology, information about the direction and velocity of the intravascular blood flow are of interest in the cardiac context.

**IDEA:** An assumption made in standard MRI acquisitions is that the protons remain stationary. This is not always the case due to various movements within the body, such as the respiratory motion (breathing), the heart contraction, vessel pulsations, swallowing, and flow (moving fluids). This causes phase differences during the acquisitions, resulting in image artifacts like unwanted blurring or ghosting, which denotes displaced image duplications in direction of the phase encoding. However, exactly this motion sensitivity of MRI is exploited for flow measurements. While magnitude images are the ones reconstructed from the k-space, an additional possibility is to generate specific phase images. This is the central idea of blood flow measurements: Encode velocity using phase information.

**BIPOLAR GRADIENTS:** For this purpose, the MRI sequence is slightly modified. After application of the slice-selecting z-gradient and before the phase- as well as frequency-encoding, two bipolar gradients are superimposed on the $B_0$ field. Bipolar means that the gradients have the same strength but opposite directions.

The protons’ spin axes are aligned with the $B_0$ direction and the protons were excited with the RF, which causes phase coherence (see Figure 12a). Now, a new magnetic field gradient (the first of the bipolar gradients) is applied. It causes a position-dependent phase shift in both stationary tissue as well as in...
3.1. 4D Phase-Contrast MRI

3.1.3. Dataset Description

This section is based on:


See Section Publications in the Appendix for the Division of Work.
Three phase (also: gradient or velocity) images $V_{\{x,y,z\}}$ are reconstructed by calculating all phase differences. Each image contains the velocity values in one of the spatial directions $x$, $y$, and $z$ (see Figures 13a–c). From these three components, a 3D velocity vector field $V$ is reconstructed, which forms the basis of all further flow analyses. Another reconstruction process yields undirected flow strengths into three magnitude images $M_{\{x,y,z\}}$. These data are less error-prone to uncorrelated noise (see Figures 13e–g). In an analogous manner, one combined magnitude image $M$ is generated. However, this is not suitable for the calculation of quantitative measures. An anatomy image $A$ is derived from averaging signal intensities (see Figure 13d). There is an ambiguity, as some papers refer to this as magnitude image instead.

A dataset contains a full heart beat, which is the average of multiple cardiac cycles during several minutes. Typical resolutions are $1.5–2.5$ mm between data points in a slice, with slice distances of $2–4$ mm and $20–50$ ms between subsequent time steps, often abbreviated as, e.g., $2 \times 2 \times 3.5$ mm / $40$ ms. This yields a grid with about $150 \times 200$ voxels in each of the $20–50$ slices and $15–40$ temporal positions. The data are usually stored in a 12 bit unsigned integer. Values range from 0 to 4095, where 2048 corresponds to zero velocity, and values below and above 2048 correspond to negative and positive velocities along the current spatial dimension, respectively.

**Figure 13:** Images from a 4D PC-MRI dataset of the aorta (as seen from the side) at a specific time point during the heart cycle.
(a–c) Phase images $V_{\{x,y,z\}}$.
(d) Anatomy image $A$.
(e–g) Magnitude images $M_{\{x,y,z\}}$.
(h) Labeling of the thoracic aorta and heart (red), body (yellow) and air (blue).
Images from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
3.2. Echocardiography

Echocardiography (ECG) is based on ultrasound waves [184] – in the medical context with 1–40 MHz [125]. The sound waves travel through different tissues with different velocities, e.g., fat with 1460 m/s, blood with 1570 m/s, and bone with 3500 m/s. When passing tissue, a part of the waves is reflected (echo). The higher the tissue density is, the stronger is the reflection. An analysis of the transit time allows a calculation of the tissue depth. The echo intensity is mapped to grayscale. The main applications of echocardiography are assessment of the anatomy, ventricular function, and stenotic or insufficient valves (see Figure 14).

3.2.1. Doppler Echocardiography

Doppler ECG (also: Doppler Ultrasound [204]) exploits the Doppler effect\(^5\) (also: Doppler shift), which describes a frequency change of sound waves when the sound source is moving, depending on the direction of this movement. If the sound source comes closer, the frequency increases, and if the sound source moves further away, the frequency decreases (see Figure 15).

Like in common ultrasound examinations, the sound waves are reflected – in this case by blood cells. Due to their movement inside the vessel, the sound waves experience a frequency change directly related to the blood flow velocity \(|\vec{v}|\) [369]:

\[
\Delta F = \frac{2 \cdot F_0 \cdot |\vec{v}| \cdot \cos(\alpha)}{C} \quad \rightarrow \quad |\vec{v}| = \frac{\Delta F \cdot C}{2 \cdot F_0 \cdot \cos(\alpha)} \quad (1)
\]

\(\Delta F\) is the frequency shift due to the Doppler effect, \(F_0\) is the emitted frequency, \(\alpha\) is the angle between the blood flow and sound waves, and \(C\) is the velocity of the sound waves in the corresponding tissue. It can be seen that the angle of the emitted sound waves by the ultrasound probe is critical. Since \(\cos(0) = +1 \equiv 0^\circ\) and \(\cos(\pm \pi) = -1 \equiv \pm 180^\circ\), sound waves aligned parallelly with the flow direction provide the best results. The opposing signs allow to distinguish between blood receding from and approaching the transducer (see Figure 16). No velocities can be obtained for perpendicular

\(^5\)Named after the Austrian mathematician and physicist Christian Andreas Doppler.
alignments, since Equation 1 is not defined for $\cos(\pm \pi/2) = \cos(\pm 3/2 \cdot \pi) = 0$. As a consequence, the resulting image quality strongly depends on the performing physician. Also, this lowers the reproducibility. Attempts have been made to use multiple transducers from different angles in order to obtain true flow vectors. Fox et al. [158] employed two transmitting and one receiving transducer(s) simultaneously. The method was used successfully for phantoms, but has not been established in the clinical routine. Swillens et al. [495] revisited this approach (vector Doppler) in comparison with others. Grothoff et al. [191] investigated the quantification of pulmonary insufficiency in patients with surgically repaired tetralogy of Fallot. They found that acceptable results can be achieved with Doppler echocardiography compared to 4D PC-MRI.

### 3.2.2. Echo Particle Image Velocimetry

*Particle image velocimetry* (PIV) [2, 508, 558] is an optical method to obtain two-dimensional velocity fields for optically opaque fluids. Tracer particles are seeded, which are then illuminated by a pulsed laser. The now visible particles are recorded by a camera, and the single images are analyzed for the particle patterns (globally, not of individual particles). This yields the flow velocity and direction.

*Ultrasound-based PIV* [108] uses ultrasound beams as imaging source. This principle is nowadays known as *Echo-PIV* and allows the visualization of blood flow within the cardiac chambers. It facilitates the assessment of blood flow directions and, thus, to generate arrow plots (see Figure 17) or to integrate path lines as depiction of flow patterns. Echo-PIV was successfully used in the clinical practice, e.g., by Hong et al. [218] to analyze left ventricular vortex flow patterns.

![Figure 16: Doppler echocardiography of anomalous pulmonary veins. The blood flow directions are color-coded relative to the ultrasound probe in a scale from blue to red, which is a common choice in this context. The image was provided by Nepomuceno et al. [377] on Wikimedia Commons under the Creative Commons Attribution 2.0 Generic License.](image1)

![Figure 17: Intraventricular flow evaluation using echo particle image velocimetry (Echo-PIV). Flow patterns are shown as arrow plot in real-time. The image is a screenshot taken from a video provided by Lampropoulos et al. [294] on Wikimedia Commons under the Creative Commons Attribution 2.0 Generic License.](image2)
3.3. Comparison

The following comparison of 4D PC-MRI, Doppler ECG and Echo-PIV was inspired by Table 1 from Sengupta et al. [468], who recently published a corresponding overview of these imaging modalities. They considered multiple aspects, such as spatio-temporal resolutions, scan time, low- and high-velocity accuracy, and the need for breath-holding.

4D PC-MRI: 4D PC-MRI provides a good spatial, three-dimensional resolution and captures all flow directions accurately. The achieved temporal resolution is typically about 20–50 ms between subsequent phases. Advances in recent years reduced the scan times from more than 30 min to about 8–12 min for the aorta and 10–20 min for the whole heart [488]. Each flow-representing phase image is the average of many cardiac cycles. Thus, small scale features, visible only once every heart beat, might be missing in the data. The data evaluation is performed offline. An infinite number of arbitrarily configured measuring planes can be placed after the scan, since the complete spatio-temporal information are available. This also facilitates a large variety of qualitative and quantitative evaluation methods. The maximum measurable flow velocity (the $V_{ENC}$) is a scan parameter. Exceeding this value leads to image artifacts that can be corrected afterwards – to a certain extent. Breathing control can be used to reduce motion artifacts, but this increases scan times and causes additional patient stress. However, the image quality can also be preserved without it [258]. MRI is not suitable for patients with cardiac pacemakers. Metal stents or implanted artificial valve rings cause local artifacts. 4D PC-MRI is employed to scan large vessels, such as the aorta, pulmonary artery, as well as the cardiac chambers.

Doppler ECG: In 2D, Doppler ECG achieves a high spatial and temporal (4–20 ms) resolution. In 3D, the spatial resolution is still good, but the temporal one is rather low. It has rapid scan times, facilitating an evaluation in real-time. This allows the depiction of small scales features that are only present for short periods of time. However, in clinical settings, only one velocity direction parallel to the transducer can be obtained. High velocities are well-resolved within a certain aliasing limit, but lower velocities are often underestimated or their acquisition suffers from noise. Breathing control is not required. Doppler ECG is mainly used to analyze the heart chambers and aortic flow. The latter may require the use of transesophageal echocardiography, i.e., a small transducer that emits ultrasound waves from the patient’s esophagus [545]. In contrast to 4D PC-MRI, pacemakers and other implanted devices are no limit. Moreover, ultrasound-based examinations are at low cost, which increases the applicability in the clinical routine.

Echo-PIV: Echo-PIV achieves a good spatial 2D resolution – if required even for multiple planes. The temporal resolution of 4–20 ms is comparable to Doppler ECG. An acquisition requires a few heart beats and the evaluation can be performed during the scan or later offline. Echo-PIV provides two in-plane flow directions, but the one through-plane is missing. High velocities may be underestimated, low velocities are captured well. Breathing control is not necessarily needed, since a scan can be performed during one breath-hold. The applications of Echo-PIV are similar to the ones of Doppler ECG: The heart chambers and aortic flow. The costs are similarly low and pacemakers as well as other implants are also no limit.

3.4. Summary

Magnetic Resonance Imaging: MRI builds upon proton excitation with magnetic fields and measuring resonance during relaxation phases. Positions are encoded using magnetic field gradients. There are various scan sequences for MRI that provide decent contrast in soft tissue and fluids or that facilitate the determination of, e.g., neuronal activity and fiber directions in the white brain matter.
4D Phase-Contrast MRI: 4D PC-MRI allows to determine directional blood flow information. It is exploited that moving protons in the blood are surrounded by stationary protons in static tissue. The \( V_{ENC} \) parameter has to be adjusted prior to the scan. It describes the maximum measurable flow velocity, which varies in different vessels. Datasets are 3D + time vector fields in a regular grid. They contain one cyclic heart beat. Spatio-temporal resolutions are about 1.5–2.5 \( \times \) 1.5–2.5 \( \times \) 2–4 mm / 20–50 ms. Phase images \( V_{\{x,y,z\}} \) contain flow strengths and directions. They are used for the subsequent flow analysis and quantification. Magnitude images \( M_{\{x,y,z\}} \) result from another reconstruction. They contain undirected flow strengths and are less noisy. An anatomy image \( A \) depicts averaged signal intensities.

Alternatives: Doppler ECG and Echo-PIV are ultrasound-based modalities. They are cheaper and faster than 4D PC-MRI, but obtained data are less comprehensive. 4D PC-MRI has the highest potential to provide the possibility for a complete qualitative and quantitative flow analysis. Therefore, this modality is the focus of this thesis.
4D PC-MRI acquisitions suffer from various artifacts that alter the measured flow directions. This negatively affects both the qualitative as well as quantitative data evaluation and makes a careful pre-processing a prerequisite. Section 4.1 explains the most important artifacts and corresponding correction steps that can be applied.

Vessel masks are useful in many different ways. They allow to restrict flow integration to the lumen (cavity within the vessel) and establish a morphology visualization. Moreover, they are required for many quantitative analyses. Section 4.2 describes corresponding segmentation approaches for 4D PC-MRI data.

4.1. Correction of Image Artifacts

Chest MRI suffers from motion artifacts due to the beating heart and respiratory motion (breathing). Bakker et al. [19] described general sources of errors particularly for 2D PC-MRI flow measurements that also occur in its 4D counterpart. Among others, this comprises phase wraps (also: aliasing) and velocity offsets due to various causes. Partial volume effects [56] are especially critical for small vessels [13]. Many artifacts can be corrected in a post-processing step. This is necessary to increase the precision of quantitative results and the quality of flow visualizations.

4.1.1. Phase Unwrapping

If the blood flow velocity exceeds the $V_{ENC}$ (recall Section 3.1.2), it appears as a flipped value in the image data, which means that the measured flow seemingly runs in the opposite direction. Assuming that velocities of spatio-temporally adjacent voxels should not differ by more than $V_{ENC}$, such phase wraps can be identified and corrected [46] (see Figure 18). Phase wraps may occur in various MRI-based measurements. Corresponding correction procedures are commonly referred to as phase unwrapping. This thesis follows a classification of phase unwrapping methods by Loecher et al. [320]. A shared weakness of all mentioned methods is that they are not capable of correcting multiple phase wraps, when a pixel is wrapped at least twice.

Methods for MRI: General phase unwrapping methods are not specifically tailored towards the phase-contrast MRI context. Cusack and Papadakis [111] described a robust, iterative, 3D phase unwrapping that estimates the noise per voxel and processes voxels with low noise first. The unwrapping is performed in adjacent voxels of a specified seed point, and a manually initialized noise threshold is gradually increased in each iteration. Jenkinson [245] described PRELUDE – a fully automatic method suitable for $N$-dimensional images that was applied to fMRI data. It is based on a cost-function optimization using a best-pair-first region merging approach. The processing time is about 20 min for larger 3D datasets and might scale unacceptably if applied to 4D PC-MRI data. The employed greedy optimization can get stuck in local minima, but is faster than suggested alternatives like simulated annealing. Langley and Zhao [296] proposed a method based on Chebyshev polynomials, which was tested on phantom as well as human brain datasets, and achieved nearly identical results compared to PRELUDE. Bioucas-Dias and Valadão [39] utilized graph cuts in their proposed PUMA method to
perform an energy minimization based on first-order Markov random fields. PUMA was applied solely to artificial data.

**METHODS FOR PC-MRI:** Other methods were specifically designed for PC-MRI data. They often exploit spatial continuity and phase singularity [12, 34]. Song et al. [480] assumed that phase changes between pixels are less than $V_{ENC}$. A least-squares problem was formulated of which the solution equals the solution of a Poisson equation, allowing the usage of fast Poisson solvers. Though, the success of their algorithm strongly depends on an incorporated sequence of thresholds, which has to be chosen carefully.

**METHODS FOR CINE PC-MRI:** Methods for time-resolved PC-MRI data often consider temporal continuity. Xiang [567] proposed a 1D temporal phase unwrapping where differential velocity maps (DVMs) between adjacent time steps are calculated. Due to lower absolute velocities and temporal continuity of the velocity field, which makes abrupt changes in subsequent time frames impossible, the DVMs are free of aliasing artifacts. A reference velocity map (RVM) is calculated as integration of the DVMs. The DVMs are then used as bridges to connect the RVM with the temporal positions and perform the phase unwrapping. Yang et al. [570] used the anatomy image A to estimate the pixelwise motion between subsequent time steps, which ensures that unwrapped pixels represent the same flow region. Salfity et al. [439] compared the performance of phase unwrapping algorithms that consider one, three and four dimensions. Loecher et al. [318] used a probabilistic measure in a 4D gradient-based approach to decide if a voxel is phase wrapped. Untenberger et al. [514] employed spatial constraints by using a region of interest (ROI) and temporal constraints by demanding temporal continuity in forward and backward direction. The ROI is automatically propagated to all time steps. Schofield and Zhu [459] described a method based on Laplacian operators, which was used for 2D interferometry data and has been shown to be useful for MRI [18, 313, 535]. Loecher et al. [320] combined spatio-temporal continuity in all four dimensions with the Laplacian-based approach and proposed a fully automatic, single-step method.\(^6\)

### 4.1.2. Velocity Offset Correction

Measurement errors can cause a shift (an offset) of the true flow velocities. There are three major causes [326]:

\[^6\]Loecher et al. provide the data and MATLAB code at https://github.com/mloecher/4dflow-lapunwrap (accessed 02/2016)
1. *Eddy currents* [102, 540] in the magnetic field are caused by rapidly switching the velocity encoding gradients.

2. *Concomitant gradient* (also: *Maxwell* terms [32]) are related to terms of the Maxwell equation for the curl and divergence of a magnetic field. A nonlinear, spatially dependent magnetic field, which produces phase errors, is always generated.

3. *Gradient field nonlinearity* [337, 405] denotes that the magnetic field coils are not able to produce a perfectly accurate field. Thus, there is always an inhomogeneous distribution.

Cause 2 and 3 can be corrected during the image reconstruction without user interaction [488]. Bernstein et al. [32] proposed modifications to the phase contrast pulse sequence and an adapted reconstruction method to minimize the effects of Cause 2. Markl et al. [337] showed that measurement errors due to Cause 3 are directly related to model-based predictions, which can be used to correct the gradient field distortion, and presented a corresponding generalized reconstruction.

**Eddy current correction:** However, an *eddy current* (also: *velocity offset, phase offset*) correction is required during the data processing. This systematic, non-constant error, which affects both the stationary tissue and the vessels, can be subtracted from the image. For this purpose, Walker et al. [540] calculated the standard deviation ($\sigma$) for each voxel in the phase image along the temporal dimension (see Figure 19a). The temporal $\sigma$ is highest for air and lowest for static tissue. Vessels are in between. Based on the assumption that obtained flow velocities in static tissue are erroneous, an approximate static tissue mask (see Figure 19b) is created via interactive thresholding on the temporal $\sigma$ image. One plane per phase image slice per temporal position is fitted to the velocity values of the static tissue mask (see Figure 19c) and then subtracted from the corresponding phase image slice. Bock et al. [45, 46] suggested to fit only one plane to slices from the late diastole and use this for the correction of all time steps. This is because here the aorta and pulmonary artery have the least motion, Lankhaar et al. [297] and Chernobelsky et al. [102] showed that such corrections improve quantification results. Fair et al. [154] investigated improvements when using data with a higher signal-to-noise ratio. Lotz et al. [327] pointed out that phase offset corrections can also introduce new errors and have to be applied carefully. They suggested to process only the local surroundings of the target vessel. Lorenz et al. [326] underlined the importance of velocity offset corrections to substantially improve flow visualizations.

4.1.3. *Noise masking*

Bock [45] proposed a masking of noisy flow values in the phase images, mostly present in the lungs or in the surroundings of the patient. Their removal can be important to enhance the quality of anatomical context rendering (see Section 5.1.2) or to improve particle system-based flow visualizations (see Section 5.2.2.1) by diminishing random movements if a particle leaves the vessel. To select air-filled regions, they performed a thresholding on either the anatomical image A or on the temporal $\sigma$ image proposed by Walker et al. [540]. The threshold is either set interactively or using a heuristic, e.g., use...
the top 15 % values in the temporal $\sigma$ image or use the bottom 15 % values in $A$. The resulting binary mask (see Figure 20b) is then multiplied with the flow image $V$ (see Figures 20c–d). For this purpose, the velocity values in each phase image $V_{[x,y,z]}$ should be scaled to $[-V_{\text{ENC}}, +V_{\text{ENC}}]$ so that zero means no flow. However, a careful application is recommended, as parts of the vessel and intravascular flow can inadvertently be removed.

Figure 20: Noise masking.
(a) Anatomical image $A$.
(b) User-adjusted threshold on $A$.
Original (c) and masked (d) flow image $V$. One slice and temporal position of $V_y$ is shown as an example.
Images from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.

Figure 21: Flow profile in the ascending aorta’s cross-section (a) and flow vectors $\vec{v}$ of one slice through the heart (b) before (top) and after (bottom) divergence filtering [389]. Flow velocities are color-coded.
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4.1.4. **DIVERGENCE FILTERING AND REGULARIZATION**

Blood, as an incompressible fluid, should be divergence-free, which might not be the case in the acquired data due to measurement errors. **Divergence filters** suppress these divergent components. Tafti et al. [496, 497] proposed a variational reconstruction method that employs total-variation regularization and incorporates the flow curl and divergence. Loecher et al. [319] described positive effects of divergence-free filters on the integration of stream lines. They evaluated connectivity by counting stream lines, seeded on an emitter plane, that reach another plane inside the vessel before leaving the vessel due to flow field inaccuracies. Bostan et al. [64] additionally incorporated conditions about the flow’s rotational behavior and assumed that flow varies smoothly over time. They introduced a flow field regularization that improved the visualization of swirling patterns in 4D PC-MRI data of the
aorta. However, a quantitative comparison was not performed. Thus, it is not clear if the calculation of quantitative measures remains reliable. For the calculation of pressure differences from measured 4D flow (see Section 6.3.1), divergence filtering did not change the results significantly [355]. Later, Bostan et al. [65] described a regularization based on the nuclear norm, which is the sum of singular values, of the flow field’s Jacobian matrix $J$. A better SNR performance was shown for phantom data and an improved streamline visualization was established for 4D PC-MRI data. Santelli et al. [446] penalized divergence in flow fields, measured with undersampled 4D PC-MRI, using divergence-free wavelets or a finite difference method based on the $\ell_1$-norm of divergence and curl. Ong et al. [389] described a technique based on a wavelet transform that is robust to segmentation errors and improves visualization while preserving quantification results (see Figure 21).

4.2. Vessel Segmentation

For many analysis and visualization tasks, a vessel segmentation or approximation is required. There are various 3D vessel segmentation techniques for angiography data from MRI or CT. Therefore, one approach is to generate an image similar to MR angiography from the 4D PC-MRI data and then use well-established algorithms. Lesage et al. [310] provided a corresponding overview of methods that are not tailored to cardiac vessels. Mirzaee and Hennemuth [365] fused flow images with additional anatomical data to improve the segmentation of, e.g., stenotic vessels. In this section, selected approaches are explained that solely use 4D PC-MRI data.

4.2.1. PC-MRI-based Angiographies

An automatic 4D segmentation is challenging, since image contrast depends on the time-varying blood flow velocities that are typically lower during diastole. Manual 4D segmentation of the whole vessel is not feasible in clinical practice due to the enormous expenditure of time. A common approach is to derive a 3D contrast-enhanced image, which resembles MR angiography [128], but no longer has temporal information.

**TMIP:** A temporal maximum intensity projection (TMIP) obtains the maximum velocity per voxel along the temporal dimension of size $T$. Usually, this technique is applied to the magnitude image $M$ [524]:

$$\text{TMIP}(\vec{p}) = \max_{t} \left( \parallel M(\vec{p}_t) \parallel \right)$$

The TMIP is bright at positions $\vec{p} \in \mathbb{R}^3$, where fast blood flow was present at some time $t = 0 \ldots T - 1$ during the cardiac cycle (see Figure 22a). Inflow jets may appear prominently. Distant vessel sections can lose contrast due to decreasing velocities. Further contrast variations might be caused by the typically parabolic flow profile, which means that the highest velocities are located in the center. This profile can be disturbed in case of vortex flow.

**PCMRA:** A phase-contrast magnetic resonance angiography (PCMRA) image [213] combines the anatomy image $A$ with the phase image $V$. Both have a high vessel contrast, but an opposing high and low contrast for static tissue and noise regions. A PCMRA can be calculated using:

$$\text{PCMRA}(\vec{p}) = \sqrt{\frac{1}{T} \sum_{t=0}^{T-1} A^2(\vec{p}_t) \cdot \parallel V(\vec{p}_t) \parallel^2}$$

or similar formulae [47]. A temporal average instead of the maximum is calculated (see Figure 22b).
4.2. Vessel Segmentation

**LPC:** Chung et al. [103] defined *local phase coherence* (LPC) (see Figure 22c) as the average angle between a normalized velocity vector and its normalized neighboring velocity vectors at $\bar{p}_t^n$:

$$\text{LPC}(\bar{p}_t) = \frac{1}{26} \sum_{\bar{p}_t'} \frac{V(\bar{p}_t) \cdot V(\bar{p}_t^n)}{\|V(\bar{p}_t)\| \cdot \|V(\bar{p}_t^n)\|}$$

The normalization causes insensitivity towards the actual velocities, which might be advantageous in vessels with slower blood flow or if the image contrast is poor due to a $V_{\text{ENC}}$ chosen too high.

**EVC:** Similar to the LPC, Persson et al. [407] and Solem et al. [479] described *eigenvalue coherence* (EVC, also *eigenvalue divergence*) (see Figure 22d), which is based on an eigenvalue analysis of a local velocity tensor:

$$\text{EVC}(\bar{p}_t) = \frac{4 \cdot \lambda_0 \cdot \lambda_1}{(\lambda_0 + \lambda_1)^2}$$

with

$$\{\lambda_0 \geq \lambda_1 \geq \lambda_2\} = \text{eig} \left( \frac{1}{26} \sum_{\bar{p}_t^n} V(\bar{p}_t) \cdot V(\bar{p}_t)^T \right)$$

LPC and EVC both preserve temporal information, which enables the subsequent application of either temporal averaging, as the PCMRA does (see Equation 3), or a TMIP (see Equation 2), where M is substituted accordingly. Temporal averaging produces better results, since the LPC and EVC images tend to be noisy.

4.2.2. Lumen Segmentation - 3D

A 3D vessel mask is an approximation of the dynamic vessel morphology and can be used for the subsequent anatomical context visualization, for quantification purposes, or for the extraction of a centerline. Tagliasacchi [498] provided an overview of centerline extraction methods. The *Vascular Modeling Toolkit* (VMTK) [410] provides corresponding functionality specifically for vascular structures [10].
**Region-based Approaches:** Hennemuth et al. [213] used a *watershed transformation* on a PCMRA image, where the user specifies include and exclude points. Stalder et al. [486] clustered the temporal $\sigma$ image [540] (recall Section 4.1.2) into air, static tissue and vessels using "a maximum-likelihood estimation of the Gaussian mixture model by expectation maximization (EM clustering [196]).“ The method is fully automatic, but does not allow to distinguish between different vessels.

**Graph-based Approaches:** Gülşün and Tek [193] computed a centerline based on a *medialness map* between user-specified seeds on a PCMRA image and extracted the vessel lumen using a graph cut with the centerline as input. *Graph cuts* [68, 244, 315] (see Figure 23) on the TMIP are used in Bloodline (see Chapter 7), where regions inside and outside the vessel are user-provided via drawing.

![Graph cut principle](image)

**Figure 23:** Graph cut principle.

(a) Neighboring pixels (black) are connected as graph with edge weights according to, e.g., inverse image intensity gradients. The weights can be seen as width of pipes in a pipe system. All pixels are connected to two artificial nodes: The source (red) and sink (green). A graph cut determines the maximum flow through this pipe system.

(b) Hereby, the capacity of pipes with a small width is exceeded first.

(c) These bottlenecks cause that the graph falls apart into two regions, which can be interpreted as image foreground and background.

**Model-based Approaches:** Van Pelt et al. [526] used an *active surface model* on the TMIP, where three parameters $\in [0, 1]$ for internal and external forces of the energy minimization can be adjusted. Default values are not given. The computation is done within a few seconds. However, a separation of single vessels is not possible. The method was validated using four datasets of healthy volunteers. Volonghi et al. [536] estimated the vessel via thresholding on a PCMRA image that was filtered with anisotropic diffusion. An initial surface is extracted using marching cubes [324] and a corresponding centerline is approximated. This is used as initialization for an automatic *level set* segmentation [390, 391]. Bergen et al. [31] analyzed each voxel in $V_{(x,y,z)}$ along the temporal dimension. When plotted, one curve should resemble a typical flow curve, whereas outside voxels produce noise curves (see Figure 24). A GPU-based curve fitting is performed to find the mean and standard deviation of the assumed normal distribution. Subsequent interval thresholding generates an approximate segmentation that is used as input for a *geodesic active contour* [95] (GAC) on the temporal position of the anatomy image $A$, where the contrast is highest (typically during peak-systole).

**4.2.3. Lumen Segmentation - 4D**

A 4D vessel segmentation facilitates the depiction of the vessel movement. It can be employed for quantification with increased accuracy and is crucial to calculate measures that interact with the vessel wall, such as wall shear stress.
4.2. Vessel Segmentation

Graph-based Approaches: Section 9.2 describes a procedure to automatically segment the aorta in each temporal position. The user performs an initial 3D graph cut-based segmentation on a TMIP image as basis. The resulting mesh, which is extracted via marching cubes, represents the maximum vessel extent. Based on the assumption that the vessel diameter does not shrink more than 50% during diastole, the 3D graph cut is automatically set up for each time step of the anatomy image. Intravascular regions are initialized using a dilated centerline, regions outside are initialized with a dilated contour of the mesh.

Model-based Approaches: Kainmüller et al. [257] presented a level set approach where multiple sources of information are integrated. A 3D surface is used as a priori shape knowledge, edge information are derived from a Canny edge detector [86], curvature information from mean iso surface curvatures in local 2-neighborhood, and measured flow is incorporated using local phase (LPC) or eigenvalue coherence (EVC). The model was evaluated by segmenting the left ventricle and atrium in each one dataset of a healthy pig and human with 25 and 30 temporal positions, respectively. Segmentation of the first time step took 30 min, whereas each further one required 2 min.

Bustamante et al. [81] created an atlas from the dataset of a healthy volunteer, where multiple vessels were segmented on a PCMRA image. The atlas also contains standardized measuring planes. It was employed to obtain time-resolved segmentations of other datasets using robust affine registrations as initialization, followed by non-rigid registrations as fine-tuning. The net flow volume (see Section 6.1.1) was automatically obtained for each defined measuring plane. Their method requires datasets that were scanned with an equal orientation. Thus, a possible application is the evaluation of studies. However, pathologic cases with vessel morphologies that strongly differ from the atlas are problematic.

4.2.4. Cross-section Segmentation

Quantification methods often require an accurate definition of the lumen in a measuring plane orthogonal to the vessel. Obtaining this from a 3D segmentation without temporal information might introduce errors, since the vessel pulsation is neglected. Also, in contrast to the voxels of the 3D segmentation, measuring planes are not necessarily parallel to the image axes. Consequently, 2D cross-section segmentations that are sampled from a 3D lumen segmentation might have a square-edged boundary. Instead, manual contour drawing can be carried out by the user. However, this might be tedious if multiple evaluations are performed or if time-resolved cross-section segmentations are required. The problem is well-known from the 2D PC-MRI context. There, many tools let the user create one contour that is then automatically propagated over time and readjusted in each temporal position based on the image intensity gradients (edges).
REGION-BASED APPROACHES: Van Pelt et al. [524] detected cross-sections in the TMIP based on an eigendecomposition of a local structure tensor for user-given query positions. The results were used as seeding planes for the subsequent blood flow visualization.

MODEL-BASED APPROACHES: Goel et al. [187] described an automatic method to find vessel cross-sections in the anatomy image. They performed an edge detection on 2D image slices and used a Hough transform [231] to determine the most circular objects in each temporal position. The ascending and descending aorta are highly circular in axial planes. Thus, it is likely that they achieve large values.

4.3. SUMMARY

ARTIFACTS: Phase wraps denote seemingly flipped flow in the phase images. They occur if measured flow exceeds the specified \( V_{ENC} \) parameter. Phase unwrapping is often able to correct these erroneous regions if they are sufficiently enclosed by correct flow values.

Velocity offsets are caused by gradient field inhomogeneities. Assuming that flow should be zero in static tissue, these erroneous flow values can be approximated as gradients and subtracted from the phase images. Carefulness is recommended, since new errors can be introduced.

Air produces extreme noise in the phase images outside the body and in the lungs. These regions can be approximated by analyzing intravoxel standard deviations (IVSD) over time. A rough segmentation, derived via thresholding, can be used to mask the phase images. Caution is advised, since higher thresholds remove parts of the vessels.

Complex divergence filters recreate flow fields with enforced model assumptions. Zero-divergence is demanded, since the incompressible blood should be divergence-free. Such filters can improve the qualitative flow analysis. However, it is not clear to what extent quantification results are preserved.

SEGMENTATION: 3D approximations of the dynamic vessels are commonly employed. For this purpose, special images are calculated that provide high vessel contrast at the cost of temporal information. A temporal maximum intensity projection (TMIP) of the magnitude images is most relevant for this thesis.

Segmentation approaches comprise region-based methods (watershed), graph-based methods (graph cut) and model-based methods (active surface, level set, atlas-based registration). 2D cross-sectional segmentations are performed for quantification purposes. 3D segmentations are performed on a 3D high contrast image, e.g., a TMIP, and adapt methods that were originally made for MR angiography. 3D segmentations can be used to derive arbitrary cross-sectional masks. Moreover, they facilitate the quantification of wall-related measures. 4D segmentations are challenging to obtain due to limited 4D PC-MRI data quality.
Qualitative analysis refers to the visual examination of the acquired cardiac 4D PC-MRI blood flow data. Radiologists are well trained in the slice-based evaluation of medical image data. This allows them to quickly assess vascular morphology changes and create a 3D image in their mind. Thus, visualization of the anatomy mainly serves as context in a comprehensive flow visualization. A requirement, though, is the (semi-)transparent depiction of the vessel front, as seen from the viewer, so that intravascular flow becomes visible.

What is difficult even for medical experts, is to obtain an impression of the time-varying flow using solely the grayscale phase difference images. This issue can be tackled by adapting techniques from the flow visualization community to the cardiovascular context, since 4D PC-MRI data are discrete, time-varying (also: unsteady), 3D vector fields. However, the high data complexity makes additional effort necessary to enhance specific flow characteristics that are interesting for physicians. Also, due to low spatio-temporal data resolutions and a limited signal-to-noise ratio (SNR), a certain robustness of employed methods is also required.

This chapter briefly addresses different approaches to depict the anatomical context and then proceeds with the calculation, rendering and filtering of blood flow representing path lines. Afterwards, the clinical value of visual blood flow evaluation is emphasized by presenting the results of selected medical studies.

5.1. ANATOMICAL CONTEXT VISUALIZATION

When the complete intravascular flow is shown, one can already get a good impression of the vessel shape. In this case, an additional depiction of the vessels would be no critical requirement. However, this might not work when exclusively specific flow features, such as vortices, are shown. Here, the vessel anatomy can be displayed as orientation and contextual information.

There are two basic approaches to visualize the vessels. Vessels are either extracted from 3D lumen segmentations or depicted via direct volume rendering of a 3D high contrast image (recall Section 4.2). Both approaches are outlined in the following. In addition, approaches to extract and visualize time-varying surface meshes are briefly described.

5.1.1. GEOMETRIC SURFACE MESHES

If different vascular structures have separate meshes, single vessels can easily be hidden to focus the evaluation or reduce visual clutter. In addition, the assessment of vascular diameters and cross-sectional areas becomes possible. Furthermore, a segmentation or surface mesh facilitates the extraction of a centerline, which can be used, e.g., for the placement of measuring planes.

5.1.1.1. STATIC VESSELS

To extract triangular surface meshes from 3D segmentations, marching cubes [324] or constrained elastic surface nets [185] can be employed. Mesh-based rendering techniques, such as Phong shading [409],
can be applied to create appealing visualizations. A common way to make intravascular flow visible is to render only the vessel’s back side, as seen from the viewer’s position. Gasteiger et al. [177, 178] used a Fresnel-reflection model [455] to show parts of the culled front faces in order to increase the spatial shape perception: The smaller the angle between a surface normal and the view vector is, the higher the transparency gets. Lawonn et al. [306] additionally emphasized convex and concave regions with an illustrative technique that was inspired by suggestive contours [115] (see Figure 25a). The method is applicable to arbitrary surfaces and thus suitable for the cardiac anatomy. Van Pelt et al. [522, 524] abstracted the surface depiction using a cel shading (see Figure 25b). This denotes a cartoon-like rendering of 3D scenes [116]. Preim and Botha [421] provided an overview of the visualization of vascular structures.

5.1.2. DIRECT VOLUME RENDERING

A direct volume rendering (DVR) can be realized with raycasting, which is a common choice nowadays since it is highly parallelizable and thus suitable for GPU computing [288, 450]. The TMIP turns out very suitable, since it shows the least noise. Unfortunately, viewing the intravascular flow is limited in standard DVR, since it is not simply possible to make solely the back side of the vessel opaque and the front as well as inner regions transparent. Methods that simulate iso surface visualizations by
5.1. **Anatomical Context Visualization**

emphasizing boundaries could use gradients to approximate front face culling. In a work by Behrendt et al. [29], where the author of this thesis contributed, a DVR front face culling was proposed, where the vessel’s back sides are determined during the raycasting (see Figure 26).

Instead, a common approach is to employ a maximum intensity projection (MIP). This avoids unnecessary algorithm complexity and the specification of a transfer function. Due to the 2D nature of MIP, spatial relations get lost. However, when intravascular flow is shown, the user gets a reasonable impression of the vessel shape (see Figure 27a). Venkataraman [531] implemented such an approach as technical demo. A MIP is also suitable for the combination with a geometric mesh, since it can be used as background for the vessel surface rendering (see Figure 27b).

![Figure 26: DVR front face culling by Behrendt et al. [29].](image)
(a) Two Boolean values are stored for each ray during the raycasting procedure. The first defines if a vessel has been entered (at P1) and the second if the vessel has been left again (at P2). Both are determined using an underlying binary segmentation, which was automatically approximated with a threshold on the TMIP. If the vessel is left the ray position is set back by a certain amount, which is a parameter, to P3. Then, only the segment between P2 and P3 is processed by the volume rendering, whereas the rest of the ray does not contribute. This yields a rendering of the vessel’s back side.
(b) Intravascular flow is hidden when standard DVR is used.
(c) Results of the proposed method.

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![Figure 27: Anatomy visualization with direct volume rendering (DVR), more precisely, maximum intensity projection.](image)
(a) Flow gives an impression of the vessel shape. Image kindly provided by B. Behrendt.
(b) Combined geometric surface and DVR.

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5.2. FLOW VISUALIZATION

Analysis of the vessel shape helps to assess morphology-related pathologies, such as dilations or narrowings. However, the investigation of blood flow characteristics facilitates a deeper understanding of a patient’s situation. Segmentations – accurate or approximate – are commonly used to restrict the flow calculation to the inside of the vessel.

Inspired by Post et al. [416], direct as well as geometry- and feature-based flow visualization techniques are explained in the following. Texture-based methods, such as line integral convolution [82], are difficult to adapt to time-varying 3D data. Thus, they are rather uncommon in the cardiovascular context and will not be considered.

5.2.1. DIRECT METHODS

Direct flow visualization techniques are suitable to illustrate basic flow characteristics in a vessel cross-section, whereas 3D and 4D visualizations are rapidly dominated by visual clutter.

5.2.1.1. VELOCITY PROFILES

Blood flow through a cross-section is often color-coded according to the velocities. Rainbow scales are widely used in the medical context, although they might be confusing, obscuring, misleading [59], and should be avoided. Yet, they are used as default in most commercial medical image viewers. Other colormaps, such as red-blue, are often used to depict the flow direction regarding the centerline tangent or the viewing direction. Silva et al. [476] provided an overview of color scales including important guidelines.

HEIGHT FIELDS: The temporal development of the flow profile might be shown in an animation or as a height field (see Figure 28a). This allows to draw conclusions on the distribution of high velocities. Height field visualizations were established early on by Paulsen and Hasenkam [403], who depicted time-varying velocity profiles of canine ascending aortas using data from a hot-film anemometer. The latter “is used to measure the instantaneous velocities of fluid flows [and] is based on the dependence of the sensor (gauge) heat transfer on the fluid velocity, temperature, and composition.” (Shekhter [470])

CROSS-SECTIONAL PATTERNS: Line or arrow glyphs can be helpful to analyze flow patterns in a cross-section (see Figure 28b). Laidlaw et al. [292] performed a comparison of six different 2D vector field visualizations. Yang et al. [568] suggested animated arrow maps to depict pulsatile flow.

5.2.1.2. DIRECT VOLUME RENDERING OF FLOW VELOCITIES

A DVR of flow velocities in one time step illustrates the distribution of fast and slow blood (see Figure 28c). Masking the phase images is recommended to exclude surrounding noise from the visualization (recall Section 4.2.2). However, the visualization includes no directional information, a transfer function has to be specified, and one has to either find a suitable temporal position to show, such as peak systole, or establish a time-varying DVR.

Figure 28: Direct visualization techniques. 
(a) The time-dependent flow through a measuring plane is shown as height fields, where height and color represent the velocity. 
(b) Flow pattern in a cross-section depicted via line glyphs. 
(c) DVR (raycasting) of systolic flow velocities. Image a based on Heiberg et al. [209]. All images from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
5.2. Flow Visualization

5.2.2. Geometry-based Methods

Geometry-based flow visualization techniques depict the course of flow trajectories by means of geometric objects, such as lines, particles, or surfaces. Surfaces can depict enclosures of regions with similar properties. However, they are rather uncommon the blood flow context and therefore only briefly described in the following.

Buonocore [76] described the visualization of blood flow patterns using paths (stream lines, path lines) and linked abnormal flow patterns to atherosclerosis. This concept of flow trajectories has been widely adopted for the visualization of 4D PC-MRI blood flow. Wigström et al. [557] proposed the simultaneous display of particle traces and morphologic slices to increase the comprehension of intracardiac flow patterns.

5.2.2.1. Flow Trajectories

Seeding: A flow trajectory is the solution to an initial value problem of an ordinary differential equation. Each initial value, which is a 4D seed point in the flow field V, will produce one particle path. A seeding strategy is a specific scheme to place seed points within the flow domain (here: the vessel). The goal is to minimize visual clutter, while including salient features in the resulting set of trajectories. Jobard and Lefer [252] and Turk and Banks [511] proposed common techniques for evenly-spaced stream line placement in 2D that resemble appearances from hand-drawn sketches. However, in the 4D cardiac context the situation is more complex.

A uniform distribution places 4D seed points at random spatio-temporal positions within the vessel. To ensure that characteristic flow features, such as vortices, are captured, one seed can be placed in every voxel in each time step, using either the voxel center or another intravoxel random position. A less time- and memory-consuming approach is to guarantee that every voxel is visited at least once per temporal position. This can be done by alternating seeding plus integration and then checking which voxels were visited. The density of the resulting line set depends on the dataset’s grid resolution. Entropy-based methods place seeds in proximity to features, which can be determined by thresholding a corresponding measure like a vortex criterion.

An emitter plane, preferably lying in the vessel’s cross-section, facilitates interactive flow exploration. Again, uniform distribution or a specific scheme on the plane can be used. Van Pelt et al. [522, 525] proposed a probing tool, which is a truncated cone that fits the vessel at a user-specified location. It is as trade-off between planar and volumetric seeding.

Ye et al. [572] described a 3D stream line seeding based on a flow field analysis regarding critical points. This was used to identify important flow patterns that need to be presented. Wu et al. [566] proposed an evenly spaced stream line placement that was also based on a topological feature analysis (singularities, separatrices) of the flow field. Chen et al. [100] presented an adaptive seeding approach for 2D and 3D vector fields based on stream line similarity. Such approaches might be challenging to adapt to the cardiac context due to the amount of noise in the data and the impeded extraction of the flow topology. For a further overview on the seeding problematic, please consider McLoughlin et al. [351].

Calculation: The common approach to calculate blood flow trajectories is to use an integration scheme from the Runge-Kutta family, such as the Dormand-Prince method (DOPR15(4)) [124]. In combination with hardware-accelerated 3D texture look-ups this is suitable for fast GPU computing, as described by Kipfer et al. [266] and Krüger et al. [289]. If only one temporal position is considered, the integration yields a 3D stream line, representing a snapshot of the dynamic flow (see Figure 29). Vortex cores of stream lines and path lines do not necessarily coincide. Thus, only a 4D (3D + time) path line (also: particle path) represents true blood flow trajectories in the cardiac cycle. Path lines can be precalculated in an initialization step, which increases the performance during the visualization (or animation). Another approach is to perform the flow integration in real-time as particle system, where each particle stores a series of recent positions.
5.2. Flow Visualization

**Visualisation:** Particles may be visualized as glyphs, such as arrows, spheres, cones [270], or ellipsoids that are stretched according to the flow velocities [522, 525] (see Figures 30a–b). *Pathlets* (also: *trails*) emphasize the development of a trajectory. Temporal information can be mapped to transparency, so that the opacity is decreased for older positions. In this case, *order-independent transparency* (OIT) [571] is recommended to ensure correct *alpha blending*.

The path lines can be shown all at once without employing the temporal information. Techniques such as *illuminated stream lines* (ISL) [334, 579] and *halos* [11, 151, 152, 241, 347] (see Figures 30d–e) are suitable to enhance the flow visualization. A halo is a contour around the line, as seen from the viewer’s perspective. In case of overlapping lines, the contour of a distant line is interrupted, whereas the nearby contour is continuous. This allows to correctly assess occlusion and enhances depth perception. ISL employ a scheme to consistently select one of the infinite normal vectors of a 3D line segment using the tangent of the line and the direction of the light source. This facilitates the application of Phong illumination [409], which normally would require the use of planar geometric primitives (triangles or quads) that allow the calculation of a needed normal vector. However, if lines are rendered as geometric tubes, the GPU’s workload is vastly increased – especially for dense line sets. A trade-off is to use *view-aligned quads* in combination with the normal vectors provided by ISL. This means to render the lines as quad primitives that always remain orthogonal to the viewer’s direction. A real-time capable implementation can be achieved with the OpenGL geometry shader.

McLoughlin et al. [351] established a corresponding overview of flow visualization techniques. Stoll and Gumhold [490] provided a variety of methods for the enhancement of line primitives, such as mapping information about the flow rotation as texture. If (semi-)quantitative assessment is the focus, a careful use of line visualization techniques is recommended to avoid distractions (see Figure 30c). Illustrative techniques [153, 235] might be adapted to the blood flow context as well, as done by Born [60]. Brambilla et al. [70] provided an overview of illustrative flow visualization techniques.

**Uncertainty:** Numerical approximations in the trajectory calculation cause errors that accumulate for longer lines. Lodha et al. [317] described a corresponding visualization of such uncertainties. However, “*the value of uncertainty visualization is severely limited by the quality and scope of uncertainty data, by the limited confidence in the data itself, and by the perceptual and cognitive confusion that the depiction of this data can generate.*” (Boukhelifa and Duke [66]) Uncertainty glyph visualizations, as established by Hlawatsch et al. [217] and Wittenbrink et al. [559], might be challenging to adapt to the highly complex 4D cardiac context. Otto [394] proposed several uncertainty techniques for general 2D and 3D vector fields.

**Perception:** Ware [542] has shown that long and continuous flow courses can be interpreted efficiently. Forsberg et al. [157] performed a comparison between four 3D integral line renderings: Lines and tubes on monoscopic and stereoscopic monitors. The study participants had to answer five questions regarding the occurrences and characterization of critical points as well as the overall swirling

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**Figure 29:** A space-time diagram depicts the difference between a stream line (red) within one temporal position and a path line (green) in a time-varying vector field. Image from [269].
and flow behavior. They concluded that the best method is task-dependent, but lean towards lines on stereoscopic monitors as an overall favorite.

Günther et al. [194, 195] presented an opacity optimization for 3D stream line sets that was, among others, applied to blood flow in cerebral aneurysms from simulated (CFD) data. An adaption to the cardiac 4D PC-MRI context would require the incorporation of the temporal component and an elaborate handling of noise.

**ANIMATION:** A pathlet visualization can also be achieved with precalculated path lines. Particles (the glyphs) are placed at positions where the current time of the running animation matches the temporal component of the path line. In addition, only a small time frame around the particle position is shown, i.e., all path line points with a temporal distance higher than a threshold are hidden (see Figures 30f–g).

An advantage of precalculated path lines over on-the-fly-integrated particles is that the exact same paths can be evaluated multiple times. *Vortex animations with adaptive speed* (VAAS) [276] are presented in Section 8.3. The technique can be described as view-dependent histogram equalization of feature (vortex) visibility in videos by using time lapse and slow motion. Feature visibility is analyzed for each individual video frame by rendering both the vessel mesh and extracted path lines, which represent the feature of interest, as binary masks, and then forming the ratio of foreground pixels.

**INTERACTION:** Manipulation of the current animation time is possible with a slider or simply via pause, stop and play. Line predicates (see Section 5.2.3.1) allow a threshold-based filtering of particular flow properties of interest, such as high velocities [60, 63]. Van Pelt et al. [522, 525] provided a probing tool suitable for a vessel depiction via DVR. The method facilitates an interactive, qualitative exploration.
of intravascular flow without a segmentation. Created measuring planes for cross-sectional quantification methods (see Section 6.1) are initially placed perpendicularly to the centerline and scaled according to the vessel’s extent. The user can adjust the position by dragging the plane along the centerline. More general flow analysis techniques comprise, among others, the use of focus-context approaches [105], such as magic lenses [36, 169, 347, 533]. Gasteiger et al. [179] presented the FlowLens, which is an adaption of this concept to the enhanced exploration of cerebral aneurysms. Vilanova et al. [534] established an overview of further exploration tools for measured and simulated, cerebral and cardiac data. Alternative focus-and-context approaches, as proposed by Jones and Ma [253], allow the selection of a specific overview of further exploration tools for measured and simulated, cerebral and cardiac data. Alternative of this concept to the enhanced exploration of cerebral aneurysms. Vilanova et al. [534] established an overview of further exploration tools for measured and simulated, cerebral and cardiac data.

5.2.2.2. Flow Surfaces

In contrast to line-based visualization, flow surfaces do not depict a single particle course. Instead, multiple integral lines are calculated, starting from a seed line. Their courses are then triangulated, which yields a surface. Depending on the type of integral lines, this can be, e.g., a stream surface, as introduced by Hultquist [237], or a path surface.

**Usage in Artificial Data:** Research was mainly performed in the field of simulated (CFD) or artificial flow data. Corresponding works investigated the depiction of vortices [174], the adaptation to time-varying vector fields [176, 284, 448], the improvement of performance [448], accuracy [453, 456] and topological correctness [386, 406, 457], and the automatic selection of appropriate stream surfaces [69, 345]. Illustrative techniques were applied to enhance the surface visualization [60, 61, 93, 238].

**Usage in Measured 4D PC-MRI Data:** Stalder et al. [485] extracted unconnected points within vortex regions using the \( \lambda_2 \) criterion (see Section 5.2.3.2) and visualized them as spheres that were scaled according to the vortex strength. Section 8.4.2 describes the generation of a glyph that encloses aortic vortex flow and conveys specific characteristics like the rotational direction.

5.2.2.3. Flow Simplification and Clustering

**Reduction of Visual Clutter:** Visual clutter is a problem for dense line sets. Angelelli and Hauser [8] described a vessel straightening to simplify side-by-side visualizations of integral lines of different temporal positions. In Section 8.2 a polar plot [274] is presented that conveys vortex flow in the aorta as an overview. The temporal component is mapped to the plot’s angle, analogous to a clock, and the course of the centerline is mapped to the radius, starting at the aortic valve location in the center.

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**THE NEXT PARAGRAPH IS PARTLY BASED ON:**


See Section Publications in the Appendix for the Division of Work.

**Flow Field and Trajectory Clustering:** Salzbrunn et al. [445] introduced partition-based clustering techniques for general vector fields as a new class that employs the actual vectors, calculated integral lines or topological properties of the flow field. Telea and Van Wijk [501] presented a vector field-based agglomerative hierarchical clustering (AHC) for 2D and 3D data without a temporal component. In AHC every object starts as an individual cluster. Based on similarity, they are merged iteratively until only one cluster remains. The hierarchy, often represented as a tree (dendogram, see Figure 31), conveys the merging process and allows to select an intermediate result with the desired number of clusters. Their (Telea and Van Wijk) proposed elliptical dissimilarity measure is a linear combination of velocity vectorial and positional distances. Ellipses are employed in the process to
derive deviations. Kuhn et al. [290] described a hierarchical method for steady 3D vector fields where a scalar field is calculated that represents the vector fields curvature. Individual cluster surfaces are then generated via marching cubes [324] and color-coded according to an eigenvalue analysis of the Jacobian matrix. The method was applied to simulated (CFD) data of cerebral aneurysms. McLoughlin et al. [352] established an AHC for stream and path lines. Their employed distance matrix describes the pairwise line similarity and was composed of the lines’ curvature, torsion and tortuosity. Rössl and Theisel [432] proposed a spectral clustering for stream lines based on their geometries using pairwise Hausdorff distances\(^7\) [206]. The number of desired clusters is a parameter. Their method was applied to a simulated dataset with three cerebral aneurysms. A clustering that is suitable for vortex-representing path lines in 4D PC-MRI data [362] is determined in Section 8.4.1.

**REPRESENTATIVE LINES:** Yu et al. [573] subdivided time-varying 4D vector fields with an adaptive octree, where each octant stores the average of the contained vectors. The subdivision stops when the resulting octants are too similar. Afterwards, they applied the clustering method by Telea and Van Wijk [501] and calculated representative path lines for each cluster.

Van Pelt et al. [522, 527] performed a hierarchical clustering on the 4D PC-MRI phase image. Average velocities were not considered as a suitable measure to determine clusters, since they do not respect different flow directions. Average velocity vectors were not considered as appropriate, since they do not incorporate the cluster positions. Therefore, two more complex dissimilarity measures were used:

1. The elliptical dissimilarity by Telea and Van Wijk [501], as described in the previous paragraph.
2. The local linear expansion by Carmo et al. [92], which employs linear models to estimate the flow velocities and derives squared errors per cluster. The dissimilarity is the cost to merge two clusters.

Van Pelt et al. discussed performance bottlenecks during hierarchical clustering. They are, to large parts, a result of the repeated search for minimal dissimilarities. To circumvent this problem, they introduced coarse hierarchical clustering, where multiple clusters are merged in each step – all below a certain similarity threshold, for which a reasonable default value was suggested. The number of clusters, i.e., the desired level in the hierarchy, is selected by the user. Finally, an exemplary path line for each cluster is generated as representative. Cluster centers (in 3D or 4D) were chosen as seed points and the line integration was performed for two temporal positions both forward and backward in time.

Instead of clustering, the approach by Born et al. [60, 62] is based on voxelization and skeletonization of line bundles. They first use line predicates (see Section 5.2.3.1) to extract flow lines with a specific property, such as high velocities. The lines are smoothed to decrease the complexity while preserving the main information. Then, the line bundle is voxelized, and a skeleton is extracted using a thinning approach by Palágyi and Kuba [399]. Additionally, they add voxelized vortex core lines to the skeleton that were extracted with a previous approach [63]. A minimal subset of representative lines is determined that visits each skeleton voxel. Typical results contain 1–5 lines. The visualization of the representatives is done in an illustrative way. 3D stream tapes with arrowheads are employed, where similar courses

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\(^7\)Named after the German mathematician Felix Hausdorff.
are fused (see Figure 32). Feedback by two radiologists confirmed the intuitiveness of their method. Neglecting small-scale features was not considered as a disadvantage. The facilitated easier comparison of pre- and post-operative patient data was seen as a clinical benefit.

Oeltze et al. [387] pursued a similar approach of selecting stream line representatives for simulated (CFD) data of cerebral, aneurysmatic vessels. Stream lines are clustered according to different attributes that either describe the flow field (e.g. velocity, vorticity, pressure), the line geometry (e.g. curvature, torsion and adapted mean of closest point distances [106]) or the domain (e.g. vessel wall distance). They compared AHC, k-means [203, 329, 499] as well as spectral clustering (SC) [200, 538] and determined SC as most suitable, closely followed by AHC. One stream line representative was then selected per cluster to establish an overview visualization of the flow behavior. In a subsequent work, Oeltze-Jafra et al. [388] focused on the in-depth analysis of vortex flow in cerebral aneurysms. Besides the depiction of topological properties, such as sources and sinks [212], they map additional properties like the normalized helicity (see Section 5.2.3.2) to the representatives.

5.2.3. FEATURE-BASED METHODS

Feature extraction is used to simplify visualizations or to investigate specific macroscopic blood flow properties. Therefore, the term feature refers to specific flow characteristic, such as high-velocity jets in cardiac vessels. In this section, emphasis is put on salient patterns, since prominent vortical flow behavior is considered as indicator for different cardiovascular pathologies and thus is of great clinical interest. Pobitzer et al. [413] provided an overview of topology-based visualization methods for general, unsteady flow.

5.2.3.1. LINE PREDICATES

Salzbrunn and Scheuermann [442–444] introduced stream and path line predicates as Boolean functions that encode if certain properties of interest are fulfilled or not. A line predicate $\Pi$ maps a point $\vec{p}_t$ of a path line $\mathbb{P}$ to true or false:

$$\Pi : \mathbb{P} \rightarrow \{true, false\}$$

$$\vec{p}_t \mapsto \Pi(\vec{p}_t)$$

(6)

Criteria are based on line geometries or the underlying flow field $V$. These classes are extended in Section 8.1.2. Shi et al. [471] described various attributes especially for path lines. A predicate filters all points that do not lie within a certain value interval. It can be applied to whole lines or to the single points of it. In the latter case, lines can be split into fragments. All points that are mapped to true, are called the characteristic set $C_{\Pi}$:

$$C_{\Pi} = \{ (\vec{p}_t \in D \times T \mid \Pi(\vec{p}_t) = true), \quad \vec{p} \in D, \quad t \in T \}$$

(7)
5.2. FLOW VISUALIZATION

\[ D \] is the \( \mathbb{R}^3 \) domain of the flow field and \( T \) is the set of all temporal positions. Common set operations like union and difference provide the possibility to connect different characteristic sets and combine multiple line predicates. This allows to formulate complex queries. Born et al. [60, 63] used line predicates to extract different features, such as specific flow paths, jets (see Figure 33) or blood with high residence times. Furthermore, they extracted vortices and used predicates to display involved integral lines. Hennemuth et al. [213] filtered by source and target regions, which yielded a connectivity visualization. Gasteiger et al. [177, 180] determined inflow jets and impingement zones in simulated (CFD) blood flow data of cerebral aneurysms, which shows the high flexibility of line predicates. They demanded that stream lines of the inflow jet should contain points where the distance to the vessel wall is minimal, the flow direction changes rapidly and there is a strong deceleration due to the impingement on the wall.

5.2.3.2. VORTEX CORES AND REGIONS

**COMMON METHODS:** Vortex extraction in 2D and 3D vector fields is a well-known problem from the fluid dynamics community. Jiang et al. [250] provided an overview of extraction methods up to 2005, Kolář [280] up to 2007, and Van Gelder [517] up to 2012 plus a discussion of different vortex definitions. Corresponding methods are divided into local and global techniques. Local methods operate mainly on the velocity vectors and their partial derivatives [99], whereas global methods incorporate larger parts of the dataset and, thus, are often more complex as well as computationally expensive. As an example, the analysis of stream and path line geometries is global, since the full set of such integral lines encodes the major flow behavior [60, 63].

Vortex extraction methods either produce core lines (as point sets, line segments or actual lines) or they extract regions with vortex flow behavior. There are various approaches, such as combinatorial methods [248, 249], predictor-corrector schemes [21], the construction of feature flow fields [438, 503], the parallel vectors operator [168, 434], winding angle-based methods [398, 415, 436, 437], and Poincaré-index-based methods [175]. Selected local methods are briefly explained in the following list, as they are relevant for this thesis:
• The **vorticity** \( \vec{\omega} \) (also: curl) of a vector field \( V \) describes the local rotation direction of particles in the flow:

\[
\vec{\omega} = \nabla \times V = \begin{pmatrix}
\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \\
\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \\
\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}
\end{pmatrix}
\tag{8}
\]

The **vorticity magnitude** \( || \vec{\omega} || \) describes the strength of this rotation. Corresponding vortex extraction is based on the assumption that there is a high \( || \vec{\omega} || \) along the vortex core. Depending on the assigned threshold, a smaller or larger vortex region is extracted. Adapted methods were introduced to obtain core lines [491]. A known problem is that the vorticity is high at boundaries, even if there are no vortical structures [127].

• The **normalized helicity** \( H_n \) by Levy et al. [311] is based on the assumption that the direction of the velocity vector \( \vec{v} \) and the vorticity vector \( \vec{\omega} \) are aligned at the vortex core. Thus, it is defined as:

\[
H_n = \frac{\vec{\omega} \cdot \vec{v}}{|| \vec{\omega} || \cdot || \vec{v} ||}
\tag{9}
\]

\( H_n \) is the cosine \( \in [-1, +1] \) of \( \vec{v} \) and \( \vec{\omega} \). The sign provides information on the turning direction of the flow. Vortex core lines are where \( \vec{v} \) and \( \vec{\omega} \) are parallel, i.e., \( H_n = \pm 1 \). Compared to vorticity, the helicity does not produce false-positives at boundaries. However, the results may still contain other false-positives [433].

• The **\( \lambda_2 \) criterion** by Jeong and Hussain [247] and the **Q criterion** by Hunt [239] both are based on the assumption that there is a pressure minimum along the vortex core line. The Jacobian matrix \( J \) is decomposed into its symmetric and anti-symmetric part \( S \) and \( \Omega \), respectively:

\[
S = \frac{1}{2} \cdot (J + J^T) \quad \text{and} \quad \Omega = \frac{1}{2} \cdot (J - J^T)
\tag{10}
\]

The idea of the \( \lambda_2 \) criterion is to find these pressure minima inside the vortex. From the **Navier-Stokes**, **vorticity transport** and **strain-rate transport equation**, a formula is derived that, in addition to \( S \) and \( \Omega \), contains a tensor with information on local pressure extrema. A local minimum is where this tensor has two positive eigenvalues. If the strain-rate transport equation is adapted for incompressible fluids, such as blood, local minima are present where the second eigenvalue \( \lambda_2 \) of the matrix \( S^2 + \Omega^2 \) is negative (with \( \lambda_1 \geq \lambda_2 \geq \lambda_3 \)). However, the existence of local pressure minima is neither a sufficient nor necessary criterion for the existence of a vortex. The mathematical derivation for incompressible fluids including the simplifications is seen as main cause for arising inaccuracies [280, 565]. For example, the \( \lambda_2 \) criterion has problems with differentiating very close vortices [250]. Though, it is commonly considered as effective and reliable [449].

The Q criterion requires the second invariant \( Q \) of the Jacobian matrix to be positive. \( Q \) is defined as \( \frac{1}{2} \cdot (|| \Omega ||^2 - || S ||^2) \). This is the case where the vorticity is greater than the strain-rate. Additionally, it is required that the pressure is smaller than the pressure of the surrounding. Both the \( \lambda_2 \) and Q criterion provide vortex regions and have an implicitly given threshold of 0.

• Sujudi and Haimes [493] proposed a method to extract vortex core lines based on an eigenvalue analysis of the Jacobian matrix. Two complex eigenvalues are a necessary condition, whereas the third one \( \lambda_R \) – with the corresponding eigenvector \( \vec{e}_R \) – is real-valued. \( \vec{e}_R \) describes the direction of the vortex core and is required to be parallel to the velocity vector \( \vec{v} \). Sujudi and Haimes defined the term **reduced velocity** \( \vec{v}_r \) as:

\[
\vec{v}_r = \vec{v} - (\vec{v} \cdot \vec{e}_R) \cdot \vec{e}_R
\tag{11}
\]

Thus, \( \vec{v} \) is projected onto the plane perpendicular to \( \vec{e}_R \). If \( \vec{e}_R \) and \( \vec{v} \) are parallel, the length of \( || \vec{e}_R || \) is 0.
The original method operates on tetrahedral meshes. For each (triangular) side of a tetrahedron it is determined if there is a point where \( \| \vec{e}_R \| = 0 \). If there are exactly two sides with such a point, they are connected to a line segment that represents a part of the vortex core line. Although the method works well [250], the extracted core line is likely to have discontinuities, especially in case of curved vortex cores. This is due to the piecewise linear determination of the Jacobian matrix within the tetrahedrons. Weinkauff et al. [549, 550] introduced cores of swirling particle motion (COSPM) as a direct extension of the reduced velocity to 4D vector fields.

- The torsion \( \tau \) is a measure for 3D lines that describes how sharply a curve \( C(t) \) is twisting out of the plane. It is calculated as:

\[
\tau = \frac{(C'(t) \times C''(t)) \cdot C'''(t)}{\| C'(t) \times C''(t) \|^2}
\]

(Biswas et al. [42] proposed a consensus-based uncertainty model, where the results of four point-based vortex extraction methods are combined in a voting scheme. This facilitates the identification of vortex regions with higher confidence.

**APPLICATION TO 4D PC-MRI:** The majority of methods from the flow analysis community is made for 2D or 3D vector fields and is therefore not directly applicable to 4D PC-MRI data, since flow is inherently time-varying. Evaluating each temporal position independently with a 3D method might introduce errors, since it is not guaranteed that vortices of stream and path lines coincide. Nevertheless, many 3D techniques have been shown to produce reasonable approximations. Stalder et al. [485] used a combination of the \( \lambda_2 \) criterion and the reduced velocity to identify independent points that represent vortex cores mainly in the aorta. Stream lines were seeded in the close surrounding to provide a visual impression of the vortices.

We observed that clinicians are often more interested in the characteristic of a vortex than topological properties such as core lines [270]. Consequently, we aimed at extracting visually appealing path lines with long, continuous, and smooth courses that can be interpreted efficiently [542]. Different local vortex criteria were incorporated in the line predicates technique and the \( \lambda_2 \) criterion was determined as most suitable. Chapter 8 describes this enhanced extraction and assessment of vortex flow.

Elbaz et al. [142, 144] employed the \( \lambda_2 \) criterion to extract vortex core rings, which are a blood transport mechanism in the left and right ventricle. They created a \( \lambda_2 \) scalar field and computed a reference shape signature from a training set using Euclidean distance shape distributions. The best fitting iso value and thus shape of the vortex ring is determined using an iterative search for the best shape distribution match with the reference signature, which is facilitated by a hierarchical clustering. However, vortex core extraction is challenging due to noise in the measured data.

**5.2.3.3. Pattern Matching**

*Vector pattern matching* (VPM) analyzes the similarity of normalized flow vectors in a plane to idealized flow templates via convolution. It was described by Heiberg et al. [208] in order to locate and identify specific 3D flow structures. Each template consists of a pattern, such as right-handedly swirling flow, describing the structure’s cross-section and a second perpendicular pattern that is oriented in direction of the structure (see Figure 34). The largest eigenvalue of a resulting structure tensor per voxel is used as similarity measure. The computational effort is high, since different rotations of the patterns are used to find the maximum similarity. Furthermore, specification of the templates requires a priori knowledge, e.g., about the forward movement along the vortex core (axial velocity).

**Adaptive VPM:** Drexl et al. [127] proposed an adaptive VPM. Here, candidate voxels are identified using a threshold on the vorticity magnitude \( \| \vec{\omega} \| \). The vortex core orientation is then estimated with the vorticity vector \( \vec{\omega} \), and templates are rotated accordingly. Adaptive VPM is 50 % faster than VPM, shows increased robustness, and has a simplified template specification, since it needs only one template for the detection. Among others, they used the dataset of an aneurysmatic aorta in the evaluation and showed the capability of their method to extract vortex cores.
5.3. **Clinical Value of Investigating Flow Characteristics**

The visualization of blood flow courses provides various useful information. For example, Calkoen et al. [85] investigated intraventricular flow. Among others, they visualized the left ventricular inflow using stream lines that were integrated on 4D PC-MRI data, and used this information to re-position measuring planes so that they capture peak velocities better by being perpendicular to the main flow jet [84]. The pursued quantification of flow volumes was more reliable after this correction. Dyverfeldt et al. [131] described challenges in the qualitative evaluation of the right ventricle (RV), such as the minimization of background phase offsets (recall Section 4.1.2) and the simultaneous left and right heart measurement due to greatly differing flow velocities (recall $V_{ENC}$, Section 3.1.2). A corresponding RV flow analysis, as performed, e.g., by Eriksson et al. [147] and Fredriksson et al. [161], provides a better insight regarding the blood transport, which is composed of four main components (see Figure 35). A similar behavior can be observed in the left ventricle (LV). Carlhäll and Bolger [88] described the alteration of this distribution in the failing LV.

Qualitative analysis of such data facilitates a deeper comprehension pathologies by inspecting blood flow courses. Especially vortex flow raised clinical interest, since this is considered as strong indicator for different diseases. This section presents selected clinical studies.

### 5.3.1. Cardiovascular Vortex Flow

Flow in the aorta (Ao) and pulmonary artery (PA) is typically laminar with a parabolic velocity profile. This means that the flow is parallel to the vessel course and the highest velocities are in the center (see Figure 36). In case of vortex flow, this laminar flow behavior can be disturbed. Fundamental research is performed to understand the correlation between flow patterns and various cardiovascular pathologies [53]. In the long term, this could facilitate a better demarcation of physiological [423] and pathological flow characteristics.

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**Figure 34:** Six idealized templates used for vector pattern matching (VPM). The blue pattern lies in the cross-section, whereas the red pattern shows a cut view along the vessel (adumbrated as surrounding circle).

Images based on Heiberg et al. [208] and used in [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
5.3. CLINICAL VALUE OF INVESTIGATING FLOW CHARACTERISTICS

Figure 35: (a) "Schematic depiction of intraventricular flow components based on flow paths through the RV, here illustrated in a long axis view. Direct flow (green): Blood that enters the RV and is ejected during the analyzed heart beat. Retained flow (yellow): Blood that enters the RV but does not leave during the analyzed heart beat. Delayed ejection flow (blue): Blood that leaves but did not enter the RV during the analyzed heart beat. Residual volume (red): Blood that remains in the RV during two or more heart beats." (Fredriksson et al. [161, Figure 1]) (b) "Illustration of the four RV flow components as percentages." (Carlhall et al. [89, Figure 2]) (c–d) Flow visualization of the four flow components in the LV during diastole (c) and systole (d).

Figure 36: Systolic, laminar flow inside the aortic arch of a healthy volunteer has an approximate parabolic velocity profile, which means that the highest velocities are located in the center.

5.3.1.1. PHYSIOLOGICAL OCCURRENCES

There are some natural occurrences of vortices in the great heart vessels. Slight helical flow behavior in the aorta and pulmonary artery is physiological as well as pronounced swirling flow in the ventricles.

**AORTA:** Kilner et al. [263] observed helical flow in the aortic arch or ascending aorta during systole (see Figure 37a) in a study with 10 healthy volunteers. They consider them as a result of the shaped aortic geometry. Bogren et al. [56] described a similar clockwise-rotating flow behavior. However, they stated that it is not perfectly helical or spiral. In another study with 16 healthy volunteers of different age, Bogren and Buonocore [52] confirmed the observations of a right-handed helix in the ascending aorta and aortic arch. Moreover, a slight left-handed helix could be observed in the descending aortas of all participants. Both amplified in older subjects, which was explained with an increase of age-related vessel wall stiffness. According to Jin et al. [251], the wall motion also plays a role in the formation of these physiological patterns. Liu et al. [316] and Morbiducci et al. [371] supposed that helical flow in the aorta could protect from the development of arteriosclerosis.

Frydrychowicz et al. [167] investigated the influence of age and aortic morphology (shape, radius and
angle) on the formation of vortex flow in the thoracic aorta in a study with 64 healthy volunteers. They distinguished between a round, gothic and cubic appearance (see Figures 37b–d). Vortex flow was determined in 75 % of the cases, preferably right-handed helices in roundly shaped aortas. Furthermore, the probability of vortex flow formation was positively correlated with the age of participants and their aortic diameter.

PULMONARY ARTERY: Bogren et al. [56] detected helical flow in the right pulmonary arteries of healthy subjects (see Figure 38a). Bächler et al. [16] observed counter-rotating helices in the main PA. They were not sure about the cause, but suggested specific contraction patterns of the right ventricular outflow tract. This confirms previous findings by Bogren et al. [54] and Sloth et al. [478], who also indicated that the curvature of the PA might play a role in the helix formation. Those assumptions were later confirmed by François et al. [159] and Sundareswaran et al. [494]. However, flow can also be laminar in the whole pulmonary artery (see Figure 38b).

LEFT VENTRICLE: A 3D ring-shaped vortex (vortex ring) forms in the left ventricle during diastole and is an optimized blood transport mechanism (see Figure 39). The corresponding hypothesis about their existence was confirmed by Kim et al. [265]. Schenkel et al. [452] performed a validation using computational fluid dynamics (CFD). Töger et al. [506] and Elbaz et al. [142, 144] confirmed and analyzed this flow behavior using 4D PC-MRI.

RIGHT VENTRICLE: Elbaz et al. [141] also determined vortex rings in the right ventricle, where they are also a physiological flow behavior during diastole. Figure 40 shows results of their comparison with the left ventricle.
5.3. CLINICAL VALUE OF INVESTIGATING FLOW CHARACTERISTICS

5.3.1.2. PATHOLOGICAL OCCURRENCES

AORTA: François et al. [160] investigated blood flow in patients with aortic dissection. The alteration of the flow patterns was in accordance with the disease severity. Future studies might help to "establish prognostic indicators for development of complications or aneurysm growth in patients."

A potential cause for aberrant blood flow patterns in the aorta are morphologic alterations, i.e., narrowings or dilations (see Figure 41). Frydrychowicz et al. [166] performed a study with 28 patients with a coarctation that was surgically corrected in 24 cases. In comparison to 19 healthy volunteers, 25 patients had additional vortex flow, especially in the post-stenotic vessel section. Hope et al. [222] confirmed these findings in a similar study. Prominent vortex flow was detected in 9 of 13 patients with a strongly bent aortic arch after the coarctation was corrected. Frydrychowicz et al. [165] established a link between aneurysm development in the post-stenotic vessel section and the repair of coarctation. Vortex flow was considered as potential cause by Crook and Hope [110] and Hope et al. [225].

Markl et al. [342] considered aneurysms in the thoracic aorta as potential cause for the formation of vortex flow. In a study with 13 aneurysm patients compared to 19 healthy volunteers, Hope et al. [228] found strong, right-handed, helical flow in the ascending aorta in 80% of the patients. Similar studies...
with similar results were performed by Bürk et al. [78], Frydrychowicz et al. [164], Landgraf et al. [295], Markl et al. [338], and Weigang et al. [546].

Examining 26 patients with the Marfan syndrome, Markl et al. [340] found that they have a 3–25% larger aortic diameter. As a result, they had increased helical flow in the descending aorta and in the vessels branching off from the aortic arch. A similar result was determined by Geiger et al. [181].

Kozerke et al. [282] examined flow in patients with bi-leaflet aortic valve prostheses. They found altered flow patterns in close proximity to the valve during early systole as well as adjacent retrograde flow. Kvitting et al. [291] compared six healthy volunteers to two Marfan patients six months after an aortic valve-sparing surgery (recall Section 2.2.1). They described a loss of natural vortex flow after peak systole in the replaced aortic root region. Von Knobelsdorff-Brenkenhoff et al. [537] investigated flow patterns in patients after aortic valve replacement. They determined flow characteristics that are different from healthy volunteers, depending on the concrete valve replacement type.

There are different types of bicuspid aortic valves (BAVs), depending on which cusps are fused. Sievers and Schmidtke [472] provided a corresponding classification as a result of 304 surgeries. Hope et al. [221] found helical flow in the ascending aortas of 75% of their examined 20 BAV patients. These findings were confirmed by Meierhofer et al. [356] in a similar study where 85% of the 18 patients had such flow patterns. Barker et al. [25] and Bissell et al. [40] described a dependency of occurring flow patterns and the BAV fusion type, which allows a prediction of the severity of the flow abnormalities [41]. In a more general study with various aortic valve malformations, Entezari et al. [146] confirmed the influence on the resulting flow in the ascending aorta. BAV patients are prone to aneurysm development, as stated by Barker and Markl [22]. There is increasing evidence that the resulting vortex flow induces altered shear forces on the vessel wall (wall shear stress (WSS), see Section 6.2.1) [24, 223]. The flow eccentricity was linked to aneurysm growth rates by Hope et al. [224], whereas the valve morphology (fusion type) has been shown to produce different WSS distributions by Mahadevia et al. [331]. These might be the cause for these secondary pathologies. A better understanding of the vessel wall-fluid interdependencies may lead to improved clinical guidelines for the treatment of corresponding diseases in the future.

**PULMONARY ARTERY:** In tetralogy of Fallot patients, MRI-based flow parameters are incorporated in the decision for a re-operation or intervention [312, 504].

Reiter et al. [425, 426] used qualitative flow analysis to assess pulmonary hypertension by evaluating vortex flow in the main pulmonary artery. They showed that the period of vortex existence facilitates the measurement of elevated mean arterial pressure. This is a great success, since an extremely dangerous right-heart catheterization, which is normally used to evaluate the pressure [211], is replaced with
non-invasive 4D PC-MRI. Additionally, they observed that larger vessels promote the formation of vortex flow.

**LEFT VENTRICLE:** A disturbance of the left ventricular (LV) vortex ring formation due to various diseases might affect the diastolic function and the overall cardiac health, as stated by Gharib et al. [183] (see Figure 42). Töger et al. [506] determined the ring volume in healthy volunteers and patients with *ischemic cardiomyopathy* – a defect of the heart muscle due to narrowed coronary arteries. The volume was smaller in the patients, implying a loss of efficiency of the blood transport mechanism. In a similar study with 107 subjects, Kheradvar et al. [262] confirmed a different filling behavior in the LV in patients with diastolic dysfunction. Moreover, the vortex formation time might be a useful parameter for the diagnosis and prognosis of heart failure, as suggested by Poh et al. [414] and Belohlavek [30]. Pedrizzetti et al. [404] underlined that LV flow analysis provides new insight in addition to the analysis of conventional function parameters.

![Figure 42](image-url) "Vortex ring core in a control (a) and a patient (b)." (Elbaz et al. [143, Figure 1]) LA = left atrium, RA = right atrium, LV = left ventricle, RV = right ventricle. The images are from Elbaz et al. [143], who published their open access article under the Creative Commons Attribution 4.0 License.

### 5.3.1.3. CLASSIFICATION

Clinical studies focus on more than the pure existence of vortex flow, since some pathologies cause characteristic patterns. A classification aims to describe these characteristics by defining specific properties [55]. So far, this is done completely manually and since there is a smooth transition between some properties, the classification can suffer from a high inter-observer variability. Commonly used criteria are briefly described in the following:

- **The shape** refers to the stretching and forward movement within the vessel. Swirling flow with a rather roundish appearance and a small forward movement is called *vortex* or *vortical flow*. Such flow keeps rotating in the same vessel section. Elongated flow with a strong forward movement is called a *helix* or *helical flow*. In other words by Geiger et al. [181]: "Vortex flow was defined as revolving particles around a point within the vessel with a rotation direction deviating by more than 90° from the physiological flow direction. They resemble recirculating areas within the vessel. Helix flow was considered a movement around an axis with some net forward flow along the flow direction creating some kind of a corkscrew-like motion." They further distinguished between a *global* helix, encompassing the whole vessel, and a *local* helix that occurs only in a specific vessel section.  
- **The size** of a vortex describes how much of the vessel’s cross-section is occupied by swirling flow. *Minor* and *major* describe less and more than 50 %, respectively.  
- A spatial characterization of the vortex occurrence is given by the **vessel section**. For the aorta (Ao) there is the aortic root, ascending aorta (AAo), aortic arch and descending aorta (DAo). For the pulmonary artery (PA) there is the pulmonary trunk (also: main PA) and the left (LPA) as well as right PA (RPA). In the ventricles, the proximity to a specific valve can be used as descriptor.
• The time is the temporal occurrence of the vortex. Usually, it is only distinguished between systole and diastole. Persistent vortices are present during the full cardiac cycle.

• The rotational strength refers to the accumulated angle of swirling flow. Meirhofer et al. [356] used a grading from 0 to 3. Grade 0 was defined as laminar flow. The grades 1, 2 and 3 describe helical flow with maximum a half turn (below 180°), maximum one turn (between 180° and 360°), and more than one turn (above 360°). Geiger et al. [181] had a similar approach with 3 grades: No rotation, below 360°, and above 360°. The term pronounced helical flow was employed for the highest grade.

• The turning direction distinguishes the direction of rotation between left- and right-handed. Since the centerline tangent is used as reference axis, the turning direction describes a clockwise or counterclockwise flow rotation in the vessel’s cross-section [228].

• Nested helical flow is defined as “greater than 180° curvature of the majority of high velocity peak systolic stream lines around slower central helical flow in the ascending thoracic aorta.” (Hope et al. [221])

Section 8.4 describes an approach to cluster vortex entities and automatically obtain an objective classification based on an adaption of the listed criteria.

5.4. Summary

Anatomical Context: Vessels are displayed as contextual information. They are either extracted as a geometric surface from an underlying (3D) segmentation or visualized directly using volume rendering (DVR). Geometric surfaces can be employed for various quantifications, and single vessels can easily be hidden to focus the analysis. Both approaches require front face culling to make the intravascular flow visible.

Flow Visualization: Direct methods are the simplest group of techniques. Arrow plots depict velocity vectors and height fields show time-dependent flow strengths in the vessel’s cross-section. Texture-based methods are uncommon in the blood flow context. Instead, geometry-based methods have been widely adopted, as they are intuitive and easy to interpret. Calculated blood flow trajectories (path lines) are depicted using well-known flow visualization techniques, such as illuminated stream lines (ISL) and halos. Pathlets and particles are employed in the animation of the 4D path lines. The problem of visual clutter was addressed by establishing simplified visualizations. A prerequisite is the clustering of similar line bundles. Selected cluster representatives can be displayed or illustrative arrows as the average course of a cluster. Feature-based methods aim at an extraction of specific flow characteristics. Line predicates are a suitable filtering technique. Among others, they were employed to extract high-velocity inflow jets. A relevant feature is vortex flow. A wide range of global and local vortex core and region extraction methods (for general vector fields) was described in the last decades. An often employed local measure is the $\lambda_2$ criterion. Other feature-based approaches analyze similarities to pre-defined flow patterns via template matching.

Healthy and Pathological Flow Behavior: Physiological flow in the aorta and pulmonary artery is mostly laminar with a parabolic velocity profile, i.e., the highest velocities are in the center. Flow in the left and right ventricle naturally forms vortex rings during diastole, which are an optimized blood transport mechanism. Typical vortex flow patterns are caused by specific pathologies, e.g.:

• Bicuspid aortic valves (BAVs) are likely to produce systolic vortex flow in the ascending aorta. This is associated with increased shear forces (WSS) on the vessel wall and aneurysm development.

• Vortices often occur in post-stenotic vessel sections, which also tend to be dilated.

• The persistence of pulmonary artery vortex flow in patients with pulmonary hypertension (PAH) gradually increases with the mean arterial pressure.
• Ventricular diseases can disturb the vortex ring formation, which decreases the transport efficiency and increases the heart’s workload.

Medical publications classify vortices according to their shape (elongated helix or roundish vortex), rotational strength (e.g. 180° or 360° turn), temporal occurrence (systole, diastole or both), vessel section of occurrence, size (occupancy of the vessel’s cross-section) and turning direction (left- or right-handedly swirling). Employed criteria are mostly binary and would benefit from a revision.
Quantitative measures are essential to assess the severity of pathologies or to support treatment decisions. They allow to derive the cardiac function, vessel wall interaction and intrinsic properties of the blood flow, such as kinetic energy. For example, the stroke volume helps to assess the heart’s pumping capacity. Pathologically low values might indicate heart failure. Stenotic vessels typically show a pressure drop, i.e., there is a high and low pressure before and after the narrowed region, respectively. The re-evaluation of pressure gradients, e.g., after a balloon dilatation (widening of the vessel), may improve the judgment of a patient’s post-treatment situation. Nordmeyer et al. [385] pointed out the benefit of 4D PC-MRI data, where multiple vessels can be evaluated after the scan, since the full spatio-temporal flow information are available. Moreover, quantitative measures facilitate the establishment of comprehensive visualizations of a patient’s situation. Hope et al. [226] provided an overview of such measures with emphasis on the clinical importance.

6.1. CROSS-SECTIONAL METHODS

Measuring planes that are modeled as discrete grids are the basis for many quantifications. An accurate determination of the lumen pixels – the pixels inside the vessel – is required (recall Section 4.2.4). A plane can be aligned orthogonally to the vessel using time-averaged flow vectors, analysis of local structure tensors [524] as estimation of the vessel course, or, if available, the centerline direction.

Measuring Plane Placement: Measuring planes can be evaluated at arbitrary positions, which might impede result comparison between different datasets. Therefore, Schulz-Menger et al. [460] proposed a standardization of the post-processing and data evaluation. Among others, they suggested measuring plane locations in the aorta based on anatomical landmarks (see Figure 43).

Figure 43: (a–b) Standardized measuring plane placements in the thoracic aorta based on anatomical landmarks.
Image a is from the open access article by Schulz-Menger et al. [460]. Image b is from the open access article by Bürk et al. [78]. Both were published under the Creative Commons Attribution 2.0 License.
6.1.1. Flow Rate

The time-dependent flow rate \(fr(t) \text{[ml/s]}\) describes the orthogonally passing blood flow through a plane \(P\) with the normal vector \(\vec{n} \in \mathbb{R}^3\), scale \(\vec{s} = (x_s, y_s) \text{[mm]}\) per cell, and grid size \(\vec{g} = (g_x, g_y)\):

\[
fr(t) = s_x \cdot s_y \cdot \vec{n} \cdot \sum_{x=0}^{g_x-1} \sum_{y=0}^{g_y-1} S(P(x,y),t) \cdot V(P(x,y),t)
\]

with \(S(P(x,y),t) = \begin{cases} 1, & (P(x,y),t) \text{ inside vessel} \\ 0, & \text{else} \end{cases}\)

\(P(x,y) = \vec{p}\) is a position on the plane transformed to 3D world coordinates. \(V(P(x,y),t) \text{[m/s]}\) yields velocity vectors \(\vec{v}\) from the phase image \(V\). If calculated for each temporal position, \(fr(t)\) is periodic, since it represents one full heart beat (see Figure 44).

**Figure 44:** Flow rate curve \(fr(t)\) from a measuring plane in the ascending aorta of a healthy volunteer.

Image from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.

**Flow Volumes:** The forward flow volume (FFV) [ml] and backward flow volume (BFV) [ml] is the area of the \(fr(t)\) curve above and below 0, respectively, scaled with \(10^{-3}\) to obtain [ml]. Thus, the FFV is the blood volume during the whole cardiac cycle through the plane in one direction, which points towards the main blood flow jet, whereas the BFV is the corresponding blood volume through the plane in the opposite direction. The net flow volume (NFV) [ml] is \(FFV - BFV\) or simply the integral of \(fr(t)\). Thus, the NFV is the amount of blood that effectively passes the plane. It can be positive or negative, depending on whether forward or backward flow is predominant [51, 268]. The stroke volume (SV) [ml] describes the pumped blood per heart beat. It is a special case of the NFV, where the measuring plane is located directly above the aortic or pulmonary valve (both values should coincide). The SV can be used as marker for the cardiac function [431], as abnormally low values might indicate the inability of the heart to supply a sufficient amount of blood to the body. The cardiac output (CO) [l/min] is the pumped blood volume per unit time. The cardiac index (CI) [l/min/m²] additionally incorporates the body surface area [m²], which is approximated with a formula that contains a person’s height [cm] and mass [kg].

NFVs can also be used to quantify the severity of shunt flows. For example, a right-to-left shunt allows blood to flow between the right and left ventricle due to a hole in the ventricular septum – the wall between the ventricles. Mauritz et al. [348] quantified the SV above the pulmonary valve in patients with pulmonary hypertension (PAH), which is considered as a standard procedure. However, they showed a limited accuracy of the obtained SVs and suggested to prefer measurements above the aortic valve instead. Eriksson et al. [148] examined the SV in patients with heart failure and in healthy volunteers. They found that the patients’ hearts had a higher workload, although the SVs were equal. Westenberg et al. [554] presented a method for the improved measurement of flow volumes through the mitral valve (MV). They manually defined a MV plane in each temporal position that was used for the corresponding through-flow calculation. Hsiao et al. [234] used SVs to verify an accelerated 4D flow measurement technique that facilitates data acquisition in under 10 min.

The percentage back flow volume (PBFV) [%] is the ratio \(BFV/(FFV + BFV)\). The regurgitation (also: regurgitant) fraction (RF) [%] is the PBFV measured directly above the aortic or pulmonary valve and thus describes the percentage of blood flowing back when the corresponding valve should be closed. RFs below 5 % can also occur in healthy persons and are considered as physiological [560].
INACCURACIES: Hoogeveen et al. [219] pointed out the susceptibility of the flow rate calculation to different imaging artifacts. They suggested a model-based approach that is applicable to small, straight, and cylindrical arteries with a parabolic velocity profile. Therefore, this is not suitable for the cardiac context. Bakker et al. [19] summarized potential sources of errors in 2D PC-MRI acquisitions, such as aliasing (phase wraps).

We determined vortex flow as a main cause for quantification uncertainties [278]. Section 9.1 describes a systematic evaluation of measuring planes with slightly different angulations that yields a distribution of NFVs. A box plot-based graph then illustrates the result variations, which can be seen as uncertainty. Section 9.2 investigates the incorporation of motion information in the SV quantification.

6.1.2. PULSE WAVE VELOCITY

The pulse wave velocity (PWV) [m/s] [71, 349] refers to the "rate of transmission of the systolic pulse through a vessel." (Hope et al. [226]) It is an indicator for arterial stiffness, since it is lower and higher in elastic and stiff vessels, respectively. Stiffness increases with age [134] as well as various physiological, genetic, and cardiovascular risk factors [226]. "The measurement of PWV is generally accepted as the most simple, non-invasive, robust, and reproducible method to determine arterial stiffness." (Laurent et al. [305]) Kröner et al. [287] examined patients with aortic aneurysms and found a strong correlation between the vascular diameter and the PWV. The latter was most reliable in the descending aorta (DAo) and of moderate accuracy in the ascending aorta (AAo). PWV was associated with the mass of left ventricle by Brandts et al. [72]. Amaral et al. [6] found an inverse association of PWV with the total lung capacity. Blacher et al. [43] determined PWV as a marker for the presence of atherosclerosis and a predictor of cardiovascular risk in patients with hypertension. Laurent et al. [304] performed a similar study and showed that PWV is a predictor of cardiovascular mortality in patients with hypertension. Van Elderen et al. [516] examined type 1 diabetes mellitus patients and found PWV plus the related aortic stiffness to be a predictor for white matter brain atrophy.

The description in this section follows Wentland et al. [553] and Dyverfeldt et al. [134], who both provided overviews of MRI-based PWV measurements. They were first discussed by Mohiaddin and Longmore [367]. Markl et al. [339] focused directly on 4D PC-MRI in their overview. Originally, PWV is computed using the Moens-Korteweg equation:

\[
PWV = \sqrt{\frac{E \cdot h}{2 \cdot \pi \cdot \rho}},
\]

where \( E \) [kg/(m\( ^2 \)]) is the vessel’s elastic (also: Young’s) modulus, \( h \) [m] is the wall thickness, \( r \) [m] is the vessel radius and \( \rho \) [kg/m\(^3\)] is the blood density.

PWV FOR MRI: The MRI-based calculation directly relates to the development of the flow curve (also: wave due to its typical appearance) along the vessel’s course:

\[
PWV = \frac{\Delta d}{\Delta t}
\]

\( \Delta d \) [m] describes the intravascular distance (length of the centerline) between two measuring planes and \( \Delta t \) [s] is the temporal offset of the flow rate \( fr(t) \) curves (see Figures 45a–b). The offset between two waves can be obtained using specific landmarks (see Figure 45c) or by comparing whole curves:

- **Time-to-peak (TTP)** uses the curves’ global peak as landmark. This is prone to errors, since the actual peak can easily be missed due to the limited temporal data resolution.
- **Time-to-foot (TTF)** (also: zero crossing) [50, 368, 574] fits a regression line to the upstroke of the waveform and then determines its intersection with either the baseline \( fr(t) = 0 \) or another regression line prior to the upstroke. Often, only data points between 20 % and 80 % of the maximum value are considered [553]. A variation of TTF is to use the time on the fitted line where the velocity corresponds to, e.g., 20 % or 50 % of the peak velocity (threshold method (TH)).
6.1. Cross-sectional Methods

- **Time-to-upstroke** (TTU) (also: derivative method) uses the time of the wave’s first derivative peak value as landmark [15].

- **Sigmoid fitting** approximates a sigmoid function describing the wave’s upstroke [119]. The temporal distance between the sigmoid functions is determined by comparing the positions with the highest curvature.

- **Cross-correlation** (XCorr or just XC) [156] performs incremental temporal shifts of one waveform and uses the temporal offset as result where two curves’ correlation was highest.

- The **Fourier analysis** (FA) method is used to determine two waves’ phase shift by fitting a line to low frequency components of the Fourier transformed waves [303]. The temporal distance is derived from the slope of this regression line.

- The **center of mass** (COM) [134] is calculated as weighted average of points on the curve that are located between 20% of the peak flow rate on the upslope and 20% of the peak flow rate on the downslope of the wave.

**Performance:** Methods that operate on the upslope (TTP, TTF, TH, TTU, sigmoid fitting) tend to produce lower values than methods that incorporate global wave information (XCorr, FA, COM). Thus, Dyverfeldt et al. [134] suggested a careful interpretation of PWV values depending on the employed technique. Comparing TH, TTF and TTU, Mirzaee et al. [366] determined TH as most robust. Wentland et al. [551] reported similarly reliable results of TTU, TTF, and XCorr compared to TTP as well as a good agreement of PWV obtained with 2D and 4D PC-MRI.

**Robustness:** Often, the PWV is obtained using more than two planes. In this case, $\Delta \phi$ and $\Delta t$ are calculated between each plane sample and the first plane. The inverse slope of a fitted regression line yields the PWV. Drexl et al. [126] described a PWV calculation, where the user simply defines a start and end position on the centerline and equidistant planes are generated and evaluated automatically (see Figure 46b). Spottiswoode et al. [482] and Magrath et al. [330] achieved increased robustness by fitting second order polynomial planes to a series of flow curves’ upstrokes (see Figure 46a).

**Figure 45:** (a–b) Temporal offset of the flow curve between measuring planes in the ascending and descending aorta. (c) Depiction of time-to-foot (TTF, blue), time-to-upstroke (TTU, green) and time-to-peak (TTP, red). Images based on Wentland et al. [553] and used in [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
6.1. Cross-sectional Methods

**Drawbacks:** A general problem of the PWV quantification is its strong dependency on typically shaped, noise-free flow rate curves (see Figure 45b). Noise or an ambiguous peak value can lead to implausible results. Especially pathologic cases with vortex flow are likely to produce aberrant flow rate curves depending on the respective measuring plane configurations [278]. Dyverfeldt et al. [134] restrict the PWV quantification to the descending aorta, since the flow rate calculation is more reliable there.

6.1.3. Flow Displacement

Flow in the great arteries typically has an approximate parabolic velocity profile, so that the highest velocities are in the vessel center (see Figure 47a). This pattern can be disturbed by various diseases, causing a displacement of the peak velocities or eccentric flow (see Figure 47b). High velocities close to the vessel wall may induce increased shear forces (wall shear stress), which can initiate a permanent, pathologic dilation.

To quantify eccentric flow jets, Sigovan et al. [473, 475] defined flow displacement \( \in [0, 1] \) in a cross-section as distance between the center position and the center of velocities, which is the velocity-weighted average of all positions in the plane, normalized with the vessel diameter (see Figure 47c). Kari et al. [259] defined the flow compression index \( \in [0, 1] \) as fraction of the area of mid-systolic high velocity flow and the total area of the aorta’s cross-section (see Figures 47d–e).

**Figure 47:** (a) An approximate parabolic velocity profile, which is a typical pattern in great arteries, such as the aorta and pulmonary artery.
(b) Eccentric flow jet in a patient’s ascending aorta.
(c) Flow displacement in a cross-section as difference between the center position (red) to the center of velocities’ (green).
(d–e) More (d) and less compressed (e) distribution of high systolic flow velocities (framed in blue).
Image b is from Sigovan et al. [474], who published their open access article under the Creative Commons Attribution 2.0 License. Images c–e from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
Flow displacement can alternatively be determined as angle between the velocity vector of the main (eccentric) flow jet and the direction of the left ventricular outflow tract (LVOT), as done by den Reijer et al. [117]. Dyverfeldt et al. [133] used the latter to grade the reproducibility of flow quantification in the aorta.

### 6.1. Aortic Valve Area

The aortic valve area (AVA) [171] is an indicator for the severity of an aortic valve stenosis. One has to distinguish between the geometric orifice area (GOA) and the effective orifice area (EOA) (see Figure 48a). GOA is "the anatomical area of the aortic valve orifice", whereas EOA is "the minimal cross-sectional area of the flow jet downstream of the aortic valve." (Garcia and Kadem [171]) The latter is usually the measure of interest. From MRI data it can be obtained via planimetry, i.e., size quantification on a cross-sectional plane. However, Donal et al. [120] showed that such an assessment overestimates the EOA in stenotic BAV patients, which was confirmed by House et al. [232]. In a letter to the editor, Seaman and Sucosky [464] corresponded to this and underlined the mechanical complexity of BAVs, as described in their own article [465]. Especially the degree of valve calcification and pressure recovery are two important factors for the deviation of GOA and EOA. Alternatively, the EOA can be measured using Doppler echocardiography and the continuity equation.

![Figure 48: (a) "Effective orifice area (EOA) computed using jet shear layer detection method (JSLD). Panel (A) shows flow stream lines through an orifice (geometric orifice area (GOA) = 1.66 cm² and effective orifice area (EOA) = 1.00 cm². Dashed line represents a plane crossing the vena contracta (∼10 mm from the orifice). Panel (B) shows a normalized velocity map at the vena contracta. Panel (C) shows AST shear layers computed from (B), JSLD identifies the inflexion points from the velocity profile corresponding to noise production, shear layers from vorticity and separation regions at the vena contracta position lead to EOA estimation (blue circle with transparency). Panel (D) shows the correspondence between normalized velocity profiles and normalized AST profiles, gray lines indicate the corresponding voxel on EOA identified shear layer." (Garcia et al. [172, Figure 2])

(b) Left: The JSLD. AST map creation, contour detection and EOA calculation. Right: Single plane velocity truncation (SPVT) as alternative method. A threshold of 0.65 is applied to the normalized velocity map. Counting pixels then yields the EOA.

The images are from Garcia et al. [172, Figures 2 and 3], who published their open access article under the Creative Commons Attribution 2.0 License.

Inspired by vortex sound theory [233, 314, 420], Garcia et al. [172] introduced the jet shear layer detection method (JSLD) (see Figure 48). Flow downstream of an aortic stenosis produces high vorticity, which is responsible for sound generation [58]. JSLD employs the acoustical source term (AST), which
can be computed using the velocity vector $\vec{v}$ and vorticity vector $\vec{\omega}$ of the flow [255, 256]. It is defined as $\nabla(\vec{\omega} \wedge \vec{v})$, where $\wedge$ is the exterior (also: wedge) product. Thus, the AST "is an amplification of [the] vorticity map by local velocity magnitude." (Garcia et al. [172]) Finding a contour on the AST map that connects the local maxima allows to derive the EOA as the enclosed area. The feasibility of JSLD was subsequently shown in a study with aortic valve stenosis patients [173].

6.2. Surface-based Methods

Surface-based measures describe fluid-vessel wall-interactions and the resulting effects on the vascular wall. Research on this mutual influence can increase the understanding of the initiation and progression of related vascular diseases. Corresponding calculations typically depend on an accurate vessel segmentation and geometry.

6.2.1. Wall Shear Stress

Wall shear stress (WSS) [Pa] represents the force tangential to the inner layer of the vessel wall caused by nearby, complex blood flow. A WSS vector $\vec{\tau}_{WSS}$ describes the corresponding direction of the shear force (shear stress). Exposure to increased shear forces over a long period of time promotes pathologic vessel dilations [79]. For example, significantly increased WSS was detected in the ascending aortas of BAV patients by Van Ooij et al. [519] (see Figure 49). Geiger et al. [182] determined increased WSS in patients with the Marfan syndrome, although the aortic dimensions were comparable to healthy volunteers. Truong et al. [509] examined WSS in patients with pulmonary hypertension (PAH) compared to healthy volunteers. They showed that pathologic dilations are more likely in PAH patients, while net flow volumes were similar to healthy volunteers. As consequence of the same amount of flow passing a larger cross-sectional area, the WSS decreases. This might influence the cellular function.

WSS is a time-resolved measure, calculated as the product of the shear rate $\dot{\gamma}$ [1/s] and the dynamic viscosity $\mu$ [Pa·s]. The latter describes the blood’s resistance to gradual deformation by shear stress and is often chosen as $10^{-3} \cdot 3.5$ [400] or $10^{-3} \cdot 3.2$ Pa·s [555] for large arteries. However, $\dot{\gamma}$ has to be derived from the measured flow. In some applications, WSS is time-averaged, calculated at only one specific temporal position, such as peak systole, or the maximum WSS during the cardiac cycle is projected.

Papaioannou and Stefanadis [400] provided an overview of basic principles of and methods for vascular WSS. While WSS can be obtained from CFD blood flow simulations with a very fine grid in near-wall vessel regions, it is more challenging to reliably compute it based on measured flow data. The description in this section follows Potters et al. [418] who specifically described different MRI-based calculations. In addition to reliable flow measurements, especially in proximity to the wall, all WSS methods require an accurate segmentation of the vessel lumen or, more precisely, the vessel wall.

---

**Figure 49:** Average wall shear stress (WSS) atlases of (a) healthy volunteers, (b) patients with a bicuspid aortic valve (BAV), (c) BAV patients with aortic dilation and (d) patients with a stenotic BAV.

The images are from Van Ooij et al. [519], who published their open access article under the Creative Commons Attribution 2.0 License.
WSS APPROXIMATION ON A BOUNDARY CURVE: If a single slice is evaluated, WSS is obtained for the cross-section’s boundary curve that equals a ring on the vessel surface. Simple approximations determine a mean WSS value $WSS_{avg}$ for the whole curve using the Poiseuille formula [333]:

$$WSS_{avg} = \mu \cdot \dot{\gamma} = \mu \cdot \frac{4 \cdot Q}{\pi \cdot r^3}$$

(16)

or similar velocity-based formulae, such as:

$$WSS_{avg} = \mu \cdot \frac{4 \cdot \| \vec{v} \|_{avg}}{r}$$

(17)

$$WSS_{avg} = \mu \cdot \| \vec{v} \|_{max} \cdot \sqrt{\frac{2 \cdot \pi \cdot \| \vec{v} \|_{max}}{Q}},$$

(18)

where $Q$ [m$^3$/s] is the mean volumetric flow rate, $r$ [m] is the radius and $\| \vec{v} \|_{avg}$ [m/s] and $\| \vec{v} \|_{max}$ [m/s] is the average and maximum velocity, respectively. Such methods assume that the flow is steady and laminar and the vessel is straight, cylindrical, and inelastic [400]. This is especially problematic for pathologic cases.

More sophisticated approaches incorporate the actual measured flow profiles in the WSS calculation. Oshinski et al. [392] plotted sampled velocities in the plane against the distance of the corresponding pixels to the vessel wall. The slope of a linear regression line yields the required shear rate $\dot{\gamma}$. The use of a parabolic instead of linear function for the fitting might increase the accuracy [346]. Oyre et al. [395–397] derived the WSS from a 3D paraboloid fitted to large parts of the velocity profile. They excluded pixels close to the wall and in the center. Cheng et al. [101] employed analytical velocity derivatives along the inward pointing normal. They were established by local velocity profile approximations via Lagrangian base functions.

WSS VECTORS $\vec{\tau}_{WSS}$ ON A BOUNDARY CURVE: Newer WSS methods incorporate the additional information provided by 4D PC-MRI data (see Figure 50b). Köhler et al. [279] and Papathanasopoulou et al. [401] fitted a fifth order polynomial to the whole 3D velocity profile of subsequent slices in a bifurcation phantom in order to derive $\vec{\tau}_{WSS}$. Stalder et al. [484] manually defined measuring planes inside the aorta and performed cross-section segmentations. $\vec{\tau}_{WSS}$ were derived from B-splines that were fitted to the velocity values on the segmentation boundary, which represents the vessel wall. Sotelo et al. [481] proposed a method based on finite elements where $\vec{\tau}_{WSS}$ were obtained from a global least-squares stress-projection. Their technique is suitable for arbitrary plane sections of the thoracic aorta.

Figure 50: (a) Velocity vectors (blue) that are sampled along the normal vector (orange) are used to obtain the wall shear stress vector (green) on the vessel surface (red).
(b) $\vec{\tau}_{WSS}$ (green) on the vessel contour (red) on a measuring plane.
(c) Flow impinges on the vessel wall and causes increased WSS.
Images from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
WSS Vectors $\bar{\text{WSS}}$ on the Whole Surface: Bieging et al. [35] introduced a method to obtain $\bar{\text{WSS}}$ on each position of a geometric vessel surface. Consequently, a segmentation of the whole dynamic vessel instead of just a cross-section is required. 3D approximations from a TMIP or similar images with high vessel contrast are often used for convenience (recall Section 4.2.2). Potters et al. [417, 418] used the term vectorial or volumetric WSS. This is defined as:

$$\text{WSS} = \parallel \bar{\text{WSS}} \parallel \quad \text{with} \quad \bar{\text{WSS}} = \mu \cdot \dot{\gamma} = \mu \cdot \frac{\partial \vec{v}}{\partial \bar{n}},$$

where $\bar{n}$ is the normal vector of the corresponding surface mesh position and $\bar{\text{WSS}}$ represents the shear stress [Pa]. Velocity vectors $\vec{V}(\bar{n}) = \vec{v}$ are obtained along the inward pointing normal $-\bar{n}$ with the number of samples as well as the maximum distance from the surface point as parameters [419]. An orthonormal basis $\{\bar{n}, \bar{n}_x, \bar{n}_y\}$ is used to obtain vectors $\vec{v}_\parallel$ that are parallel to the surface’s tangential plane:

$$\vec{v}_\parallel = \vec{v} - (\vec{v} \cdot \bar{n}) \cdot \bar{n}$$

The first derivatives of the $\vec{v}_\parallel$ samples are calculated analytically and evaluated at the vessel wall [518] (see Figure 50a). One-dimensional, interpolating, cubic B-splines with natural boundary conditions can be fitted to the $x$ and $y$ component of $\vec{v}_\parallel$ for this purpose.

Similarity to Cerebral Aneurysms: Extensive research on WSS has been done in the field of cerebral aneurysms [96]. Elevated pressure in combination with increased flow velocities and resulting high WSS have been shown as initiator for local vessel wall dilations by causing a degeneration of elastin as well as smooth muscles of the vessel and an increased production of collagen [97, 359]. However, there are two theories about the role of WSS in the progression and rupture of cerebral aneurysms [67, 358, 469]:

- Theory 1: Exposure to high WSS causes an increased production of nitric oxide. This causes the death (apoptosis) of smooth muscle cells as well as the gradual destruction of endothelial cells, resulting in a lower arterial tone.
- Theory 2: Abnormally low WSS causes inflammation of the vessel wall, which leads to its progressive degeneration.

Guzzardi et al. [197] determined an increased regional WSS in BAV patients as well as a degeneration of elastic fibers and elastin degradation [87]. Clearly, the process of aneurysm development in the aorta has similarities to cerebral vessels. From a clinical standpoint this is interesting, as aortic aneurysms above a certain diameter bear the risk of rupture, which is lethal in most of the cases. Thus, an estimation of the risk level of aneurysm development and rupture by evaluating low and high shear forces has the potential to guide therapeutic decisions in the future.

6.3. Grid-based Methods

Grid-based techniques operate directly on the acquired image data or solve differential equations in finite elements.

6.3.1. Pressure Gradients

Pressure gradients are employed as a measure to assess the severity of obstruction, e.g., due to a stenotic valve or vessel. In such cases, blood has to pass a smaller cross-sectional area or valve orifice. Increased flow velocities and intravascular pressure are the consequence. The current gold standard for pressure measurements is catheterization, which is invasive and stress for patients. However, recent advances facilitate the calculation of pressure information from MRI blood flow data.

Riesenkampff et al. [428] derived pressure fields in patients with aortic coarctation. The promising results suggest that 4D PC-MRI measurements might be a reasonable alternative to invasive catheterizations in
6.3. Grid-based Methods

the future. Thus, the blood’s relative pressure $p$ [mmHg] is an important factor to grade the degree of stenosis (see Figure 51). Nett et al. [380] found a good agreement of pressure gradients obtained from Doppler ultrasound and 4D PC-MRI in patients with aortic stenosis. Pitcher et al. [412] demonstrated the calculation of different pressure components in healthy volunteers and patients (see Figure 52a). Eriksson et al. [150] found a spatially heterogeneous relative pressure in the left ventricle of healthy volunteers (see Figure 52b).

Figure 51: Relative pressure in a patient with aortic coarctation.
(a) Pressure drop derived from 4D PC-MRI data.
(b) Simulated patient situation after treatment.
(c) Pressure along the centerline. The black and red curve show (a) and (b), respectively. The blue enclosure is the aortic arch region, marked with black cross-sections in (a) and (b).

Images kindly provided by M. Neugebauer and used in [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission. Image data are courtesy of Prof. Dr. Titus Kühne, German Heart Center. The pressure visualization technique was established by Schumann and Hennemuth [462].

Figure 52: (a) Aortic pressure in a healthy volunteer (HV2) and three patients with a bicuspid aortic valve (BAV), aortic dissection (AoD), or Marfan syndrome (MFS).
(b) Relative pressure gradient between the left atrium (LA) and ventricle (LV) in a healthy volunteer.

Image a is from the open access article by Pitcher et al. [412]. Image b is from the open access article by Eriksson et al. [149]. Both were published under the Creative Commons Attribution 2.0 License.
BERNOULLI: The simplified Bernoulli equation allows to estimate pressure gradients $\Delta p$ based on the maximum velocity $\parallel \vec{v} \parallel_{\text{max}}$ [205]:

$$\Delta p = 4 \cdot \parallel \vec{v} \parallel_{\text{max}}^2$$

(21)

Due to the quadrature, this is highly sensitive to measurement errors. Oshinski et al. [393] discussed further drawbacks, such as not considering the vessel shape.

NAVIER STOKES / POISSON: For blood, as a viscous, incompressible, laminar, Newtonian fluid, $\Delta p$ can be derived using the pressure Poisson equation (PPE, see Equation 22), which is based on the Navier-Stokes equation [138, 569]:

$$-\Delta p = \nabla \cdot \left( \rho \cdot \frac{\partial \vec{v}}{\partial t} + \rho \cdot (\vec{v} \cdot \nabla) \vec{v} - \mu \cdot \nabla^2 \vec{v} \right)$$

(22)

The divergence-free condition must be met due to the fluid incompressibility, where $\nabla \cdot \vec{v} = 0$. $\rho = 1060 \text{ kg/m}^3$ is the fluid density [48], $\mu = 10^{-3} \cdot 3.2$ or $10^{-3} \cdot 3.5 \text{ Pa} \cdot \text{s}$ is the dynamic viscosity. $\vec{v}$ [m/s] are velocity vectors from the flow field $V$ and $t$ [s] is the time. Gravitational forces can be neglected due to the horizontal patient positioning in the scanner.

Tyszka et al. [512] described an iterative PPE solver that has been shown to produce a good agreement with expected values in phantoms and with values of healthy volunteers from the literature [48]. Ebbers and Farnebäck [137] proposed a multi-grid finite-difference scheme to solve the PPE directly in the segmented vessel. Meier et al. [354, 355] exploited properties of hexahedral voxel grid elements to simplify the incorporation of these boundary conditions and being able to use efficient conjugate solvers due to a symmetric system matrix. Itu et al. [243] solved a 1D Navier-Stokes equation numerically with patient-specific boundary conditions to obtain the pressure distribution along the vessel’s centerline. Mihalef et al. [364] described a method that uses enhanced geometric models, which provide better accuracy than voxel masks of the vessel. Physiologically meaningful boundary conditions are established via tagging of inlets and outlets. Lamata et al. [293] described a separate evaluation of the transient $\rho \cdot \frac{\partial \vec{v}}{\partial t}$, convective $\rho \cdot (\vec{v} \cdot \nabla) \vec{v}$ and viscous component $-\mu \cdot \nabla^2 \vec{v}$ (see Equation 22), which is based on the finite-element method by Krittian et al. [286]. They identified transient effects, originating from the acceleration of the blood, as main cause for relative pressure in the aorta. Donati et al. [121] paid special attention to the viscous component. They automatically fit a tubular mesh in near-wall regions, where the signal-to-noise ratios are lowest, and reconstructed the vector field there. This improved the computation of the required second order derivatives. In other work, Donati et al. [122] derived pressure differences using the work-energy equation for Newtonian fluids.

6.3.2. TURBULENT KINETIC ENERGY

Parts of the blood’s kinetic energy is irreversibly converted into heat due to viscous friction. An elevated level of lost energy increases the heart’s workload and might enhance the risk of ventricular hypertrophy (enlargement). Thus, quantification of the energy loss might be a marker for disease severity [3, 26].

TURBULENT FLOW: Flow turbulences are irregularities and velocity fluctuations of the blood flow. After turbulences were studied in the aorta with a hot-film anemometer [378, 489], they were considered as potential cause for such energy losses [104]. Dyverfeldt et al. [129] described intravoxel velocity standard deviations (IVSD). They are the basis for turbulent kinetic energy (TKE) [J/m$^3$], which is a direction-independent measure of turbulence intensities [130] (see Figure 53). It was exploited that turbulent flow attenuates the signal in the 4D flow measurements. A Reynolds decomposition of the velocity field $V$, which is given by the phase images $V_{(x,y,z)}$, yields a separation into a mean $\overline{V}$ and fluctuating velocity field $V'$, so that $V = \overline{V} + V'$. Assuming a Gaussian distribution, the kinetic energy of the velocity fluctuations (the TKE) corresponds to:

$$\text{TKE} = \frac{\rho}{2} \cdot \left( \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \right),$$

(23)
where $\sigma^2_{[x,y,z]} \left[ m^2/s^2 \right]$ is the variance of velocities in $V_{[x,y,z]}$ and $\rho \left[ kg/m^3 \right]$ is the fluid density [37]. Binter et al. [38] showed that TKE calculation has a relatively low sensitivity to the spatial data resolution.

Loudon et al. [328] found significantly higher peak TKE in patients with bicuspid aortic valve stenosis compared to patients with tricuspid aortic valve stenosis. Zajac et al. [575] determined overall low TKE values in the left ventricles of healthy volunteers. It was increased in patients with cardiomyopathy. Dyverfeldt et al. [132] used TKE to assess the severity of mitral regurgitation. They correlated the degree of flow distortion in the left atrium (LA) to the average LA TKE of one cardiac cycle.

![TKE in a patient with heavy systolic vortex flow in the ascending aorta (a–b) and in a healthy volunteer (c–d).](image)

Figure 53: TKE in a patient with heavy systolic vortex flow in the ascending aorta (a–b) and in a healthy volunteer (c–d). Images from [275] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.

Carlsson et al. [90, 91] quantified kinetic energy (KE) [mJ/ml] in a similar way:

$$KE = \frac{\| \vec{v} \|^2}{2} \cdot m$$

The mass $m$ of blood in the voxel is calculated as volume of the voxel multiplied with the blood density $\rho$, which was set to 1.05 g/ml. They found a higher peak KE during early diastole in the LV compared to the RV and explained this behavior with a possibly higher suction of blood in the LV. Lee et al. [309] examined a patient with surgically repaired congenital heart disease (CHD), which often leads to post-operative pulmonary regurgitation and obstruction. They found that the amount of energy loss was considerably increased compared to a healthy volunteer. In the future, KE could serve as non-invasively assessable biomarker for the disease severity. Jeong et al. [246] performed a similar study in patients with repaired tetralogy of Fallot and found higher KE in both the left and right ventricle compared to healthy volunteers.

### 6.3.3. Vortex Strength

The vorticity $\vec{\omega}$ (also: curl) of a vector field $\nabla \times \vec{V}$ describes local spinning vectors and the vorticity magnitude $\| \vec{\omega} \| [1/s]$ is a measure for rotational strength (recall Section 5.2.3.2). Wong et al. [562] visualized slices through the heart chambers using stream line plots colored by the flow velocities, and vector as well as contour maps depicting the vorticity magnitudes. They analyzed the mean vorticity and / or vorticity for each temporal position and depicted the results in a histogram. In addition, they defined a circulation measure as line integral of the tangential velocity along a circuit enclosing a point of interest [563]. Hess et al. [214] described circulation as fraction of the integral of vorticity magnitudes in an intra-aortic cross-section and the corresponding cross-sectional area. Bächler et al. [16] plotted maximum absolute vorticity magnitudes over time for measuring planes inside a vessel.

2D and 3D metrics to quantify vortex flow in the thoracic aorta were presented by von Spiczak et al. [539]. Six standardized ROIs throughout the aorta were analyzed for minimum, maximum, and average vorticity magnitudes (see Figure 54). An adapted predictor-corrector method [21] in combination with
the $\lambda_2$ vortex criterion was used to extract vortex core lines. The cores were evaluated regarding their strength and elongation over time. Additionally, surrounding vorticity magnitudes were radially sampled for each core line position using eight lines perpendicular to the core. The sampling stops when $\|\vec{\omega}\|$ falls below a given threshold. They defined a vortex’ radial expansion as the circle’s area with a radius equal to the average length of the eight lines.

**Helicity:** Lorenz et al. [325] calculated averaged helicity density $H_d$ and relative helicity $H_r$, which is the helicity before and after normalization (recall Section 5.2.3.2), for equidistant measuring planes inside the aorta as:

$$H_d = \vec{\omega} \cdot \vec{v}$$

and

$$H_r = \frac{H_d}{\|\vec{\omega}\| \cdot \|\vec{v}\|}$$

Morbiducci et al. [370] proposed a helical flow index as the average of local normalized helicity along path lines. This is equal to $H_r$ from Equation 25. Averaging the values of all path lines yields a global assessment of present helicity in the dataset.

### 6.3.4. Lagrangian Coherent Structures

Lagrangian coherent structures (LCS) [199] facilitate the creation of surfaces, e.g., at vortex boundaries, that divide flow into regions with different characteristics. Based on this, Töger et al. [506] established a volume quantification of vortex rings (recall Section 5.2.3), which are a blood transport mechanism in the left and right ventricle.

**FTLE:** LCS are based on finite-time Lyapunov exponents (FTLE) [261, 301] that describe the rate of separation of nearby particles when integrated for a certain time frame $\Delta t$. A flow map [73] contains the end positions of particles that started at the spatio-temporal positions $\vec{p}_t$ and were integrated for $\Delta t$. It usually has a higher resolution than the acquired image data. The concrete upsampling factor is a parameter. The FTLE is defined as:

$$\text{FTLE}(\vec{p}_t) = \frac{1}{\Delta t} \cdot \log(\lambda(\vec{p}_t))$$

with

$$\lambda(\vec{p}_t) = \sqrt{\lambda_{\max}[J(\vec{p}_t)^T \cdot J(\vec{p}_t)]}$$

$J$ is the Jacobian matrix and $\lambda_{\max}$ the maximum eigenvalue.

Krishnan et al. [285] directly employed the FTLE as a stop criterion for particle path calculations (recall Section 5.2.2). If the FTLE is determined close to the vessel boundaries, some of the nearby particles will be seeded inside and some will be placed outside the vessel. Thus, some particles follow the intravascular
flow and some will experience a more or less random movement due to low velocities and / or arbitrary
directions outside the vessel. The resulting high separation allows to estimate the vessel boundaries via
thresholding (see Figure 55). Unfortunately, the boundaries’ FTLE values depend on the flow velocities
and thus may vary for different vessels.

6.3.5. Connectivity
The connectivity is calculated as the number of path lines that pass two specific ROIs [213]. Problems
may arise if the integral lines represent wrong paths due to noise, offset errors or partial volume effects.
There is a strong similarity to diffusion tensor imaging (DTI) [372] and the related fiber tracking, where
probabilistic approaches are employed to calculate paths in uncertain crossings of two or more fiber
bundles.

Uncertainty: The connectivity is also used to establish an uncertainty quantification of the
integration. Friman et al. [162] introduced a probabilistic approach that employs a sequential Monte
Carlo sampling. They showed that each component of a velocity vector $\vec{v}$ can be approximated by a
Gaussian (also: normal) distribution (GD) around the true velocity vector $\vec{u}$:

$$\vec{v} \in GD (\vec{u}, \sigma) \quad \text{with} \quad \sigma = \frac{V_{ENC} \cdot \sqrt{2}}{\pi \cdot SNR}$$

Schwenke et al. [463] incorporated this estimated uncertainty into a fast marching method and calculated
blood flow trajectories as minimal paths.

6.4. Summary
Quantification results can suffer from the low spatio-temporal 4D PC-MRI data resolution and
MRI-related imaging artifacts. Some methods require an accurate segmentation of cross-sections or
the whole vessel.

Cross-Sectional Methods: Cross-sectional methods employ measuring planes that are
perpendicular to the vessel course and thus lie in the cross-section. Standardized placements were
proposed, based on anatomical landmarks. Flow rates are usually shown as time-dependent flow curves.
They form the basis for essential measures to assess the cardiac function:

- The stroke volume (SV) is the amount of pumped blood per heart beat. Too low values can indicate
  an insufficient blood supply to the body.
• **Regurgitation fraction** (RF) denotes the percentaged amount of blood that flows back in case of an improperly closing valve. Increased values (above 5%) can indicate heart insufficiency.

The evaluation of these measures is highly sensitive to the specific plane configuration, i.e., location and angulation. **Pulse wave velocities** (PWV) describe the speed of transmission of flow curves through the vessel. They are higher than the actual flow velocities. PWV values are lower and higher in elastic and stiff vessels, respectively. There are multiple evaluation methods that differ in their results. **Flow displacement** refers to the position of the main flow jet. Eccentric flow jets can be a side effect of various pathologies. The **aortic valve orifice area** (AVA) is used to grade the severity of aortic valve stenosis.

**Surface-Based Methods:** Surface-based methods describe fluid-vessel wall interactions. **Wall shear stress** (WSS) refers to shear forces on the vessel wall caused by nearby, complex blood flow. Exposure to increased WSS over a longer period of time is linked to aneurysm growth. 4D PC-MRI-based calculations often underestimate absolute WSS values. Though, relative information can be obtained, e.g., regions with locally increased WSS.

**Grid-Based Methods:** Grid-based methods operate directly on the grid data structure, e.g., as finite-element techniques. **Pressure gradients** are used to evaluate stenoses. Pressure drop after a stenotic vessel section is a typical behavior. **Turbulent kinetic energy** (TKE) is based on intravoxel velocity standard deviations (IVSD). It describes flow fluctuations that cause energy loss and increase the heart’s workload. A quantification of vortex strengths using vorticity \( \vec{\omega} \) or normalized helicity \( H_n \) is pursued to further assess vortex flow. **Lagrangian coherent structures** (LCS) are based on finite-time Lyapunov exponents (FTLE). They emphasize regions with a high rate of blood flow separation, e.g., vortex boundaries. Among others, LCS were employed to approximate vessel boundaries. **Probabilistic trajectory calculation** can be employed in a connectivity analysis, which also enables an estimation of line integration uncertainties.
GUIDED DATA EVALUATION
4D PC-MRI datasets contain the full spatio-temporal blood flow information and allow a more flexible analysis than data from its 2D counterpart. Recent advances greatly reduced acquisition times to levels that are feasible for the clinical routine. Although it has the potential to replace 2D PC-MRI, 4D flow scans are mainly performed for research purposes at the moment. This points out the need for standardized and guided techniques to analyze these highly complex data. Software solutions that integrate such methods into easy-to-use workflows are of equal importance.

In this chapter, the developed software Bloodline is described in detail. It facilitates data analysis within ten minutes. The analysis comprises an automated data pre-processing, a graph cut-assisted vessel segmentation, semi-automatic vortex flow extraction, and analysis of the stroke volume as well as percentaged backflow. This workflow is described in Section 7.1. Resulting visualizations can easily be saved and shared using the provided one-click solutions for videos of the animated flow and screenshots of the 3D view or GUI. Section 7.2 provides background information on the code design. Section 7.3 and 7.4 characterize Bloodline’s application and alternative software, respectively.

This chapter is based on:


See Section Publications in the appendix for the division of work.

7.1. Workflow

This section summarizes the established processing pipeline, which partially implements methods from the Chapters 4–6. A primary concern during the development of Bloodline was to provide a guided workflow. This was established on the main screen. Processing screens are successively enabled from left to right during the analysis of a dataset. If the software is started for the first time and no datasets were imported yet, there is only one clickable button Import Data (see Figure 56). We received raw 4D PC-MRI data as folder with DICOM files, where every slice in each temporal position is a separate file. They are scanned and converted to 4D images in the Import Data part using information from the DICOM headers. A velocity offset correction (recall Section 4.1.2) is then applied to the flow images using the method by Walker et al. [540] with their suggested default parameters. Back on the start screen, the dataset now appears in the list of available datasets with a short summary on the spatio-temporal resolution and the $V_{ENC}$. The enumeration of already extracted vessels is empty at this time. However, the second part named Vessel Segmentation is now enabled (see Figure 57).
7.1. Workflow

Figure 56: The start screen of Bloodline. The list on the top is empty, since no dataset was imported yet. Therefore, the only clickable button is Import Data on the bottom left. The rest is grayed out.

(a)

(b)

Figure 57: (a) A dataset was successfully imported. (b) It now appears in the list on the upper half of the main screen. No vessel was extracted yet, but the corresponding Vessel Segmentation button is now enabled.

7.1.1. Vessel Segmentation

A temporal maximum intensity projection (TMIP) of the magnitude images and a time-averaged local phase coherence (LPC) are computed. Both yield a high-contrast 3D image and facilitate a static 3D approximation of the dynamic vessel (recall Section 4.2).

The TMIP, which we use by default, emphasizes regions with high velocity magnitudes in at least one time point. Since flow in the aorta is fast, it is clearly visible. However, the normally centrally located main blood flow jet appears prominently, while near-wall regions and vessel sections with greater valve distance may lack contrast. Therefore, the use of a non-automatic segmentation method that allows user corrections seems appropriate. Threshold-based methods, such as region growing, might not handle the varying contrasts, whereas model-based methods, such as active contours, have to compromise between high flexibility in order to capture pathologic vessel shapes and noise robustness. LiveWire [451] would be convenient if one finds a suitable cost function and is willing to process each image slice individually.

We decided to employ a graph cut [68, 244, 315]. For our research purposes, we employ the free (for academic purposes) version of GridCut. Graph cuts require the specification of regions in- and outside the target structure, which is intuitive for medical doctors. The user provides these information by drawing on the slices (see Figure 58). The better the image quality is, the less input is necessary to achieve satisfactory results. Though, detail corrections can be performed if the segmentation includes unwanted or excludes wanted parts.

The graph cut is 3D and operates in a 26–neighborhood per voxel. Thus, input is not required on every
7.1. Workflow

Figure 58: Graph cut segmentation in Bloodline on a TMIP image. The user provides regions inside (green) and outside (red) the vessel (here: the aorta). The resulting segmentation is shown as contour (yellow). The render window on the right shows a preview of the extracted, triangular surface mesh.

Image from [273] © 2015 The Eurographics Association and John Wiley & Sons, reprinted with permission.

slice. Edge weights between nodes in the graph are set to:

\[
\exp \left( -\alpha \cdot \| \nabla I \|^2 \right),
\]

where \( I \) are the \([0,1]\)-scaled image intensities and the tolerance parameter \( \alpha = 1000 \) (our default) was experimentally determined. The higher \( \alpha \) is, the larger the segmented region becomes. Gradients are calculated via finite differences. Noise in the resulting segmentation is reduced with a \(3 \times 3 \times 3\) morphological opening and closing. Phase wraps (recall Section 4.1.1) are corrected within the obtained segmentations [118]. The rest of the flow image is not processed to save time.

7.1.2. Surface Mesh and Centerline Extraction

Marching cubes [324] is employed to automatically extract the vessel surface from the binary segmentation. We apply a low-pass filter by Taubin et al. [500] and reduce the mesh via quadric decimation by Hoppe [229] to minimize discontinuities in of the surface. We currently use implementations from the Visualization Toolkit (VTK):

- \texttt{vtkMarchingCubes} with iso = 0.5,
- \texttt{vtkWindowedSincPolyDataFilter} with 50 iterations and passband = 0.1, and
- \texttt{vtkQuadricDecimation} with topology-preserving enabled and target reduction = 0.8.

The parameters are experimentally determined default values. The low-pass filter was chosen since a comparison of mesh smoothing algorithms [17] revealed its quality for medical surface models.

An aorta is, on average, represented by 2500–3000 equilateral triangles with an edge length of about 5 mm and a mean area of 10–11 mm\(^2\). For comparison, a voxel diagonal is 4.3 mm long. This mesh resolution is sufficient because of the non-complex shape of the aorta and pulmonary artery.

The subsequent centerline extraction (see Figure 59a) by Antiga et al. [10] calculates minimum arc length paths that follow local maxima on a distance map, which is based on the Voronoi diagram of the surface points. A robust implementation is provided by the Vascular Modeling ToolKit [410] (VMTK, \texttt{vmtkcenterlines}). The user specifies a start and one or multiple end points and thus can indirectly
7.1. Workflow

Figure 59: (a) After the centerline was extracted, the vessel is saved. A phase unwrapping, which is restricted to the vessel mask, is performed in the background and the progress is shown. (b) If at least one vessel was extracted, the third part Flow Analysis is enabled for this dataset on the main screen. If another dataset is selected where no vessel was extracted yet it is grayed out again.

exclude undesired vessel branches from the centerline topology. Further parameters are not required. Multiple end points are allowed to create centerlines in branching vessels, such as the pulmonary artery. A branch splitting by Antiga and Steinman [9] is employed to distinguish between the left and right pulmonary artery. We resample centerlines from an interpolating cubic spline (ALGLIB) to guarantee equal distances of 0.5 mm (our default) between successive points. After the mesh(es) and centerline(s) are extracted, the next part Flow Analysis is enabled on the main screen (see Figure 59b).

7.1.3. Qualitative Analysis

Vector Interpolation and Path Line Integration: Path lines are integrated entirely on the GPU using an adaptive step size Runge-Kutta-4 (RK4) method [124]. A velocity vector $\vec{v} \in \mathbb{R}^3$ in the 3D flow field / phase image $V_t$ of one temporal position $t$ is obtained by hardware-accelerated trilinear interpolation. To interpolate a vector at the spatio-temporal position $\vec{p}_t = (x,y,z,t)$, the two temporally adjacent positions $t_0 = \lfloor t \rfloor$ and $t_1 = \lceil t \rceil$ are determined (the cyclic nature of the data has to be considered when ceiling). The vectors $\vec{v}_{t_0}$ and $\vec{v}_{t_1}$ are calculated in the corresponding flow fields $V_{t_0}$ and $V_{t_1}$. Afterwards, $\vec{v}_{t_0}$ and $\vec{v}_{t_1}$ are interpolated linearly, weighted by the inverse temporal distances $1 - \| t - t_0 \|$ and $1 - \| t - t_1 \|$. In other words, we apply quadrilinear interpolation by using the GPU’s fast trilinear interpolation twice and doing the last linear interpolation manually. Because of the rather small dataset resolutions, we are able to store all flow fields of all temporal positions in the GPU’s memory at once. Larger datasets would make an asynchronous data streaming necessary, as shown by, e.g., Venkataraman [531].
We ensure that each voxel of the segmentation is visited at least once in every temporal position. For this purpose, we seed one path line at a random position inside each segmentation voxel at the first temporal position. For each remaining time step, in succession, we determine the voxels that were not visited, create new seeds and integrate the path lines. This approach directly depends on the dataset resolution and can lead to a vast amount of path lines. To alleviate this problem, the binary visited map, which keeps track of the spatio-temporal voxel visits, uses an individual resolution. Our experimentally determined default is $2 \times 2 \times 2$ mm/50 ms. A post-processing to exclude unwanted lines is performed in the following order:

- We assume that flow with velocities less than 0.05 m/s is not interesting.
- We filter lines that are shorter than a half voxel diagonal.
- During the path line integration on the GPU the voxel-based segmentation is used to restrict the calculations to the vessel. Since the vessel mesh runs through voxels, path lines can slightly protrude. These parts are identified and cropped.
- We perform a line reduction so that subsequent points have a minimum distance of 0.25 mm (our experimentally determined default).

All in all, this results in approximately 15000–40000 path lines per vessel.

**Vortex Extraction:** During the full flow integration, we also calculate the $\lambda_2$ vortex criterion for each path line point. To alleviate the impact of the low data resolution and noise, we smooth the values along each path line using a 1D binomial filter with kernel size 3. Contrary to our previous work [270], which is comprehensively described in Section 8.1, we do not crop away parts of the path lines. Instead, we provide the option to flexibly hide all non-vortex parts using a slider that adjusts the $\lambda_2$ threshold (see Figure 60). Filtering flow velocities is possible in the same way. If desired, a 2D polar plot can be generated as overview of present vortices in the aorta [274] (see Section 8.2).

![Figure 60: Qualitative flow analysis in Bloodline allows to filter vortex as well as high-velocity flow and to run an animation. The path lines as well as the vessel surface mesh can be colored according to different properties. The employed color scales can flexibly be adjusted.](image)

Image from [273] © 2015 The Eurographics Association and John Wiley & Sons, reprinted with permission.
7.1. Workflow

**Visualization:** The vessel front is culled and only hinted at with a ghosted viewing, as proposed by Gasteiger et al. [178]. The back faces are rendered with Phong illumination [409]. Path line visualization is enhanced with illuminated stream lines (ISL) and halos to improve spatial and depth perception. Path lines with halos are created in the OpenGL geometry shader as view-aligned quads. Illuminated stream lines (ISL) are implemented in the subsequent fragment shader. In the animation mode, cone-shaped particles with trails are optionally drawn on every position where the current animation time matches a path line’s temporal component. Order-independent transparency (OIT) ensures correct alpha blending. The default line width, particle width, and particle length depend on the dataset’s voxel diagonal. The standard trail length depends on the temporal resolution and number of time steps. Real-time adjustment of all visualization parameters is possible via sliders.

**Media Generation:** Results can easily be shared by taking a high-resolution screenshot of the GUI or the render window. The animated flow can be exported to a 1080p video with a single click. An option to create videos with adaptive speed that enhance extracted vortex flow is provided (VAAS, see Section 8.3). Patient and dataset information from corresponding DICOM tags are automatically added to the top left corner. They are basically the same that are shown on the main screen. The employed color scale for the flow is included on the bottom. If the mesh shows a property, another color bar is rendered on the top. Figure 61 shows one frame of an exported video of a flow animation.

7.1.4. Quantitative Analysis

Measuring planes are automatically oriented orthogonal to the centerline and their size is automatically determined so that they fit the vessel (see Figure 62). The user can drag a plane along the centerline or adjust the angulation, i.e., rotate it. Standardized measuring plane positions, as proposed by, e.g., Schulz-Menger et al. [460], are not yet implemented. A diagram shows the time-dependent flow rate determined for this plane configuration. Additionally, the stroke volume, cardiac output, regurgitation fraction, mean as well as peak velocity, and the vessel diameter are provided. Unfortunately, the calculations are highly sensitive towards the plane’s angulation. Therefore, a robust stroke volume and regurgitation fraction analysis can be performed [278] (see Section 9.1). Another quantifiable measure on the vessel surface is vectorial wall shear stress (recall Section 6.2.1).
7.2. IMPLEMENTATION DETAILS

Bloodline is written in C++ and uses OpenGL for rendering, embedded in a Qt/QML-based GUI. This section gives a rough insight into the code design.

The code is organized in the model-view-controller [283] (MVC) architectural pattern. The model, which is the logical core of the software and contains the data, builds upon BLIB – a library that has been developed for personal use by the author of this thesis. The view employs Qt/QML. Each GUI screen is defined in a separate QML file. The link between QML and C++ is established with each one EventHandler class per screen. The controller is realized as set of controller classes. They are called from the EventHandlers with corresponding parameters that were set on the GUI and invoke corresponding tasks that are executed by the model. The MVC facilitates an easy replacement of the GUI, if necessary, and a general structuring of the code into separated, logical areas.

BLIB:

BLIB is a header-only, heavily templated C++ library: It provides:

- **N-dimensional (ND) vectors** with specialized functionality for specific dimensions, such as cross-product for 3D.
- **ND images** with multivariate entries of an arbitrary arithmetic type. They provide functionality like ND linear interpolation and resampling. Derived types are, e.g., binary segmentations, which enable in-out checks, and flow fields, which facilitate line integration. There are various isotropic and anisotropic image filters. Global filters operate on the whole image (e.g. histogram equalization). Neighborhood filters consider the surrounding data elements (e.g. median). Convolution filters inherit from neighborhood filters and apply a mask (e.g. binomial smoothing). Specialized filters for binary images (e.g. morphological operators) and 4D PC-MRI images (e.g. artifact corrections and TMIP) are available.
- **Meshes** with ND vertices and M indices per cell. Static and dynamic 3D triangular meshes are derived that provide functionality like VTK import and export, extraction from an image (e.g. binary segmentation) via marching cubes, calculation of and access to normal vectors, smoothing, decimation, voxelization, inside / outside checks, and centerline extraction. A kDTree is generated on demand for optimized closest point queries.
7.3. Application

Bloodline is available for research purposes on each one computer in the Heart Center in Leipzig, Germany, where also a Siemens prototype is used, in the University Hospital in Magdeburg, Germany, and on the private computer of Uta Preim who works in the municipal hospital in Magdeburg, Germany. However, the software is not publicly available in a free or commercial manner.

Meanwhile, more than 64 datasets were evaluated in close collaboration with cardiologists and radiologists specialized in the cardiovascular system. Besides 36 healthy volunteers, the following pathologies were present:

- 1 aneurysm in the left subclavian artery,
- 3 aortic insufficiencies,
- 3 ectasias / aneurysms in the ascending aorta,
- 15 bicuspid aortic valves (BAV), some of them with ectatic ascending aortas,
- 1 tetralogy of Fallot with pulmonary insufficiency,
- 3 vascular prostheses, and
- 2 coarctations.

Dataset Acquisition: A 3 T Magnetom Verio (Siemens Healthcare, Erlangen, Germany) with a dedicated 32–channel cardiac coil was used for data acquisition. The imaging parameters were as follows: slice thickness 3.5 mm, flip angle 15°, field of view 340 mm, echo time 3.2 ms, repetition time 6.1 ms, temporal resolution 49 ms, sampling bandwidth 491 Hz/pixel, 10–20 reconstructed phases, one acquisition. The maximum expected velocity ($V_{ENC}$) was set to 1.5 m/s per dimension. A sagittal oblique 3D slab was positioned to include the aorta. Heart and wall motion artifacts were minimized using prospective ECG gating. The spatial resolution is $1.77 \times 1.77 \times 3.5$ mm in a $132 \times 192$ grid with 15–23 slices. Isotropic resolution and respiratory control were not used to achieve shorter acquisition time.
times. The number of time steps varies from 14–21 with a temporal resolution of about 50 ms. Recall Section 3.1.3 for available image data in 4D PC-MRI datasets.

**Usage:** After familiarization, physicians are able to perform a standard evaluation, i.e., vortex flow extraction and stroke volume as well as regurgitation fraction analysis, in less than ten minutes, which was rated as feasible for the clinical routine. The graph cut-assisted segmentation shows high acceptance due to the exploitation of the physicians’ anatomy knowledge. The option to hide vessels in order to reduce occlusions and the independence of specific MRI scanners were appreciated. A suggestion was to let the program perform pending automatic operations, such as path line integrations, for all new datasets at once. This way, the concentrated waiting time could be used for other things.

### 7.4. Alternative Software

Various free or commercial tools are being developed or were already established, such as *FourFlow* by Heiberg et al. [210], *Quantitative Flow Explorer* by Van Pelt et al. [524], *MeVisFlow* by Hennemuth et al. [213], *QFlow ES* by Medis, *CMR 42* by Circle CVI, *Arterys, 4D Flow Demonstrator* by Siemens [23, 467] (see Figure 63a), *EnSight* by CEI (see Figure 63b), *GTFlow* by GyroTools (see Figure 63c), an *OsiriX* plugin by Hüllebrand et al. [236], and *Velomap* by Bock et al. Table 1 shows a rough comparison of selected software.

---

**Figure 63:** Flow visualization with the Siemens 4D Flow Demonstrator (a), EnSight (b), and GTFlow (c). Image a was kindly provided by U. Preim. Image b is a screenshot from a video by Markl et al. [341] that was made available on Wikimedia Commons under the Creative Commons Attribution 2.0 Generic License. Image c is from Valverde et al. [515] who published their open access article under the Creative Commons Attribution 2.0 Generic License.

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<td>- external segmentation via <em>Segment</em> [209] (open source software)</td>
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<td><strong>Quantitative Flow Explorer</strong></td>
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<td><strong>MeVisFlow</strong></td>
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<td>KE, TKE</td>
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<tr>
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<td><strong>VeloMap</strong></td>
<td>- phase wraps</td>
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<td>proprietary, in-house developed MATLAB script for data pre-processing</td>
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<tr>
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<td>- velocity offsets</td>
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<td>- PCMRA export for external segmentation</td>
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### Siemens 4D Flow Demonstrator
- phase wraps
- velocity offsets
- motion
- centerline computation based on medialness map (between 2 specified points), then automatic graph cut; no manual correction
- 3D vessel surface rendering with image slice in the background
- path line integration (seeded on specified cross-sections), animation, color-coding (velocity)
- flow velocities, rates, volumes, regurgitation
- proprietary research prototype

### Ensight
(no details available)
- surface estimation (*no details available*)
- 3D vessel surface rendering with image slice in the background
- path line integration, animation, color-coding (velocity)
- flow velocities, rates, volumes, regurgitation
- WSS on cross-section boundary curves
- pressure
- general flow evaluation (e.g. also CFD data)
- commercial
- multiplanar reformating views of anatomical image

### GTFlow
- phase wraps
- velocity offsets
- noise clipping
- semi-automatic or manual segmentation (*no details available*)
- 3D vessel surface rendering with image slice in the background (cine mode available)
- ROI contours for quantification via freehand / B-spline / polygon / circle / rectangle drawing
- automatic contour propagation
- path line integration, animation, color-coding (velocity), quantification (length, time of arrival, particle count)
- flow velocities, rates, volumes, regurgitation
- WSS
- pressure maps
- TKE
- commercial (for research purposes only)
- session save & restore
- key-frame editor and movie generator
- DICOM anonymization

### Bloodline
- phase wraps
- velocity offsets
- graph cut segmentation on TMIP or LPC (needs in- and outside regions)
- centerline via VMTK (needs 1 start and 1–N end points)
- 3D vessel surface rendering
- prototype: automatic 4D vessel extraction
- path line integration (full coverage guaranteed), animation, color-coding (velocity)
- vortex extraction
- vortex analysis with other, in-house software prototype
- 2D polar plot (aortic vortex flow)
- flow velocities, rates, volumes, regurgitation (robust estimation)
- WSS
- one-click export of screenshots and videos (optionally with adaptive speed)
- proprietary

---

**Table 1:** Comparison of selected 4D PC-MRI evaluation software. Since it is based on publicly available information, completeness cannot be guaranteed and in-depth details might be missing.
7.5. **Conclusion and Future Work**

We presented the cardiac 4D PC-MRI data evaluation software *Bloodline* that allows to process datasets within ten minutes. It integrates a full pre-processing pipeline as well as a quantitative and qualitative data analysis. The workflow was developed in close collaboration with the clinical partners. However, the employed GUI design was made as an initial suggestion and leaves room for future improvements. For instance, the GUI elements take up lots of space, which should be optimized.

The use of (semi-)automatic methods enables a fluent workflow. Carefully selected defaults strongly reduce the necessity to adjust parameters. State-of-the-art visualizations can easily be created and saved in order to share results. A unique feature of Bloodline, compared to other free or commercial tools that are being developed or were already established, is the comprehensive analysis of vortex flow.

**Outlook:** In the future, the quantification of further measures shall be facilitated, such as pulse wave velocities (PWV), turbulent kinetic energy (TKE), or flow displacements (recall Chapter 6). Another goal is to automatically generate clinical reports. Hence, larger studies can be better evaluated and gender- and age-specific norm values may be determined. This could benefit from the implementation of standardized measuring planes [460]. Moreover, special functionality for the ventricles shall be provided in the future, such as the analysis of vortex rings. Until now, Bloodline is a proprietary research prototype. A distribution in free or commercial form is planned for the future. A connection to the *Picture Archiving and Communication System* (PACS) would be a useful addition for the use in a clinical environment.
Vortical flow behavior is of great clinical interest, since it can be an indicator for various cardiovascular diseases (CVDs). The qualitative evaluation of vortex flow can be challenging due to the high complexity of the acquired 4D PC-MRI data. This leads to potentially high visual clutter in animations of the path lines as well as in static visualizations where their temporal component is simply ignored. Section 8.1 presents a semi-automatic extraction of vortex flow-representing path lines using the line predicates technique (recall Section 5.2.3.1). This facilitates an enhanced exploration of vortical flow behavior using a single threshold, which is adapted in real-time.

In some cases, medical studies determine the pure existence of vortex flow regardless of further characteristics like the turning direction. This means to evaluate the blood flow of every single case, typically by viewing an animation. In order to support this process, an overview plot visualization for aortic vortex flow is presented in Section 8.2.

A further enhancement of the actual flow animations, preferably of extracted vortex flow, is presented in Section 8.3. The idea was to adaptively employ time lapse and slow motion to emphasize interesting parts of the cardiac cycle.

A more detailed comprehension of cardiovascular vortex flow requires an in-depth analysis of vortex characteristics. Being able to grade single vortex entities requires a clustering as a prerequisite. A corresponding approach that works on the extracted, vortex-representing path lines is explained in Section 8.4.1. Section 8.4.2 proceeds with a mathematical description of employed criteria, which are based on clinicians’ descriptions. They are evaluated for each vortex entity, yielding an objective characterization. An established glyph visualization conveys the main properties.


This section is based on:


See Section Publications in the appendix for the division of work.

This work is a continuation of my Master’s thesis [269]. The following results were already presented there:

- Section 8.1.2: An introduction of new line predicate classes and a formal extension.
- Section 8.1.4: A comparison of local vortex criteria where the $\lambda_2$ criterion was determined as most suitable for the cardiac context.

The following extensions were made as PhD student:

- Sections 8.1.3–8.1.4: The proposed step-by-step vortex extraction procedure was adapted. A GPU implementation increases the clinical feasibility. Instead of seeding $N$ uniformly distributed path lines, it is now ensured that each voxel is visited at least once in every temporal position. This guarantees that no small flow structures are missed. The path line integration, Jacobian matrix estimation, and $\lambda_2$ calculation are performed on the GPU. Cardano’s method [281] was used to solve the arising cubic equation from the eigenvalue analysis. The extraction procedure was adapted so that now a single slider facilitates the vortex filtering.
- Section 8.1.5: A detailed, qualitative case evaluation was performed in collaboration with the clinical partners.
Further comments of this kind are included in the single subsections to provide maximum transparency regarding the reuse of existing results.

Discussions with our collaborating experts – radiologists specialized on the cardiovascular system and cardiologists – revealed that they are often more interested in the characteristic of a vortex than topological properties, such as core lines. Consequently, the aim of this work was to support the blood flow analysis for vortex patterns. This was done by extracting path lines that represent vortex flow (see Figure 64). First, the requirements are analyzed, followed by an extension of the line predicates technique. Then, we describe the generic vortex extraction scheme, where different vortex criteria with particular thresholds are employed. This is followed by comparing the results achieved with different vortex criteria in collaboration with domain experts. Thereby, we define key questions to enable a qualitative comparison.

Figure 64: Physiological vortex in the left ventricle during diastole in this patient with tetralogy of Fallot. Image from [270] © 2013 IEEE Computer Society, reprinted with permission.

8.1. REQUIREMENT ANALYSIS

The majority of vortex extraction methods is designed for 3D steady flow fields, whereas methods for 3D unsteady flow, such as cores of swirling particle motion (COSPM) [550], are rare. Although the vortex cores of stream lines and path lines can be similar when the flow changes only slightly, there is no guarantee that they match.

The identification of the most suitable local vortex criterion in combination with line predicates requires an extensive evaluation. Considered aspects are mainly not precise enough to put them into a mathematical description and, since there is no ground truth, we have to rely on subjective evaluations of domain experts.

To make the characteristics of a vortex easy to interpret, it is necessary to preserve path lines of high quality. Ware [542] described that long lines are well-interpretable at the neuronal level "because a continuous contour will produce stronger mutual excitation [of neurons] than broken but aligned contours." Thus, we have to avoid their fragmentation during the filtering process and instead maintain continuous and smooth courses. In order to achieve the desired high computational performances, the GPU’s potential has to be utilized.

Summarizing, our discussion with domain experts suggests to focus on the following requirements:

- Find the local vortex criterion that provides the best results combined with our extraction procedure.
- Assess the suitability of 3D vortex criteria applied to the 4D blood flow.
- Ensure independent calculations to make our approach easy to implement on the GPU.
- Maintain easy to interpret high-quality path lines during the subsequent filtering.
- Be able to derive clinically relevant information from the results provided by our approach.
8.1.2. Extension of Line Predicates

The content of this section is completely based on:


See section Publications in the appendix for the division of work.

A line predicate \( \Pi \) assigns a value \( \hat{\rho} \) to each point \( \vec{p} \) of an integral line (or \( \vec{p}_i \) of a path line). Born et al. [63] suggested a classification of line predicates according to the necessary data for the calculation: Line-based predicates depend solely on the path lines’ geometry, and derived predicates use the underlying flow field. We renamed these two classes as geometry and flow field predicates, \( \Pi_G \) and \( \Pi_V \), respectively. Moreover, we proposed three further groups in [269]:

- **Mesh predicates** \( \Pi_M \) utilize the explicit vessel representation. An example is the minimal distance of a point to the surface.
- **Stream predicates** \( \Pi_0 \) depend on surrounding path lines. For instance, the number of path lines weighted by their distance to each other can be used as a density value.
- **To modify the values of one integral line in different ways, we created meta predicates** \( \Pi_{\text{meta}} \) as a superclass:
  - **The sum predicate** \( \Pi_\Sigma \) adds up all calculated values \( \hat{\rho}_i \) along an integral line weighted by the corresponding segment lengths \( L(\cdot) \) and assigns this value to each of the \( N \) points:
    \[
    \Pi_\Sigma = \frac{\sum_{i=0}^{N-1} (\hat{\rho}_i \cdot L(p_i))}{\sum_{k=0}^{N-1} L(p_k)}
    \] (29)
    By applying this meta predicate, longer path lines implicitly score higher than shorter ones.
  - **The mean predicate** \( \Pi_\bar{\rho} \) assigns the average value to each point. It is calculated as the sum predicate divided by \( N \).
  - **The smoothing predicate** \( \Pi_{\sigma,n} \) applies a one-dimensional binomial filter with a kernel size of 3 in \( n \) iterations to the values along an integral line.

We use a simplified notation to describe the application of a series of line predicates to all existing path lines. A characteristic set \( C_\Pi \) results in:

\[
C_\Pi = \Pi_1, t_1 \to \Pi_2, t_2 \to \ldots \to \Pi_n, t_n,
\] (30)

where \( \Pi_i \) are the subsequently applied line predicates with the corresponding thresholds \( t_i \). A threshold can be one of the following:

- \( \hat{\rho} \leq v \): All values below or above \( v \) are valid.
- \( v_0 \leq \hat{\rho} \leq v_1 \): All values between \( v_0 \) and \( v_1 \) are valid.
- \( v \% \ = \hat{\rho} \geq \hat{\rho}_{\min} + \frac{v}{100} \cdot (\hat{\rho}_{\max} - \hat{\rho}_{\min}) \): The upper \( v \% \) of the values are valid.
- \( v \% \ = \hat{\rho} \leq \hat{\rho}_{\min} + \frac{v}{100} \cdot (\hat{\rho}_{\max} - \hat{\rho}_{\min}) \): The lower \( v \% \) of the values are valid.

The application of a meta predicate is denoted as \( \Pi_{\text{meta}}(\Pi) \), where \( \Pi_{\text{meta}} \in \{\Pi_\Sigma, \Pi_\bar{\rho}, \Pi_{\sigma,n}\} \) and \( \Pi \in \{\Pi_V, \Pi_G, \Pi_M, \Pi_\Omega\} \). The following characteristic set is an example for a better understanding of the procedure:

\[
C_\Pi = \Pi_{\sigma,n}(\Pi_v^{\text{velocity}}), \quad v_0 \% \to \Pi_G^{\text{length}}, \quad \hat{\rho} \geq v_1 \text{ mm}
\] (31)

After the first step, \( C_\Pi \) contains all points that represent the upper \( v_0 \% \) of the velocity domain after applying a smoothing predicate with \( n \) iterations. From this result, all path lines shorter than \( v_1 \text{ mm} \) are removed in the second step.
8.1.3. Successive Vortex Extraction

The content of this section is primarily based on:


See Section Publications in the appendix for the division of work.

Paragraph Initialization contains a later established extension.

**Initialization:** First, the whole dataset inside the segmentation mask is covered with path lines. 25000–35000 spatially and temporally uniformly distributed path lines were used, depending on the data resolution and the flow field’s changes over time. This produced the desired rather high line density. In a later version, which was established after the appearance of this paper [270], it is ensured that each voxel is visited at least once in every temporal position. This usually yields more path lines, but increases the chance that smaller flow structures are not missed. The computational effort of the path line integration was reduced by implementing a method on the GPU (recall Section 7.1.3).

**Jacobian Matrix Estimation:** Local vortex criteria are in many cases based on an analysis of the Jacobian matrix or a derived matrix based on the Jacobian. So, its robust estimation increases the chance for proper results considerably. The calculation of one single Jacobian matrix for a whole voxel will deliver discontinuities at the voxel boundaries and affect the quality of the results negatively. Instead, we use central differences of the interpolated vectors \( \bar{\mathbf{v}} \in \mathbb{R}^3 \) at the exact query positions \( \bar{\mathbf{p}}_i \in \mathbb{R}^4 \) and the displacements \( \pm \Delta \{x, y, z\} \). The offset in each direction is set to one corresponding voxel dimension and decreased successively if one of the shifted positions is not within the segmentation mask. Based on the assumption that central differences ensure better results than forward and backward differences, they are used even near the segmentation mask boundaries. The multiple vector interpolations enable the smooth capturing of changes of the Jacobian matrix.

**Line Predicate 1 – Finding Vortices:** To extract vortex structures via line predicates, the following procedure is used. First, a local vortex criterion is calculated as flow field predicate \( \Pi_{\text{vortex}} \). Since the result tends to be noisy, a smoothing predicate \( \Pi_{\sigma,25} \) is applied before thresholding: \( \Pi_{\sigma,25}(\Pi_{\text{vortex}}), t_{\text{vortex}} \). On the one hand, line segments with values that indicate swirling behavior are extended. Consequently, longer segments remain after applying the threshold. On the other hand, outliers, i.e., single values indicating non-swirling-regions within a segment of swirling-values, diminish. Hence, thresholding does not interrupt the corresponding segments. As a result, the average filtered path line length increases. The application of a smoothing predicate reduces the fragmentation problem. We recommend to choose a very low threshold, just enough to crop certain non-swirling parts of the path lines. Figure 65a shows a comparison without (top) and with (bottom) the application of a smoothing predicate. The noise reduction and increased average path length is especially noticeable in the right vortex in the distal aortic arch in this example. Due to their local nature, some of the vortex criteria are suitable for parallel calculation on the GPU.

**Line Predicate 2 – Refining the Vortex Shapes:** Sum and mean predicates apply the same value to every point of a path line. As a consequence, path lines can be removed or kept as a whole when thresholding, but further fragmentation is impossible. Therefore, it is continued with the application of a mean predicate of the local vortex criterion: \( \Pi_{\Sigma}(\Pi_{\text{vortex}}), t_{\text{vortex}} \).

**Line Predicate 3 – Post-processing:** Despite the application of the smoothing predicate, it is likely that some short path line fragments will occur, which decrease the visual quality. Thus, an additional post-processing step is applied to remove these fragments properly. A bending energy predicate is employed, which is the sum predicate of curvature values: \( \Pi_{\text{bendingEnergy}}, t_2 = \Pi_{\Sigma}(\Pi_{\text{curvature}}), t_2 \). Based on this measure, a subsequent thresholding preserves long curved path lines. Figure 65b shows the comparison of a result without (top) and with (bottom)
In the following, it is investigated which vortex criterion illustrated in Figure 65c. The application of a bending energy predicate for post-processing. The successive vortex extraction is relevant for this work, it was decided to employ uniformly colored path lines to avoid distractions. Other attributes like directions to color might be misleading for physicians. Since velocities are not ascending aorta. The manual exploration revealed three vortices: One large vortex in the ascending aorta that reliably describe vortex structures in 4D PC-MRI data. The \( \lambda \) criterion by Jeong and Hussain [247] and Q criterion by Hunt [239], reduced velocity by Sujudi and Haimes [493], cores of swirling particle motion (COSPM) by Weinkauf et al. [550], normalized helicity by Levy et al. [311], vorticity, and torsion are considered (recall Section 5.2.3.2). Afterwards, reasonable thresholds for the placeholders \( t_0 \), \( t_1 \), and \( t_2 \) are suggested.

We applied our procedure to ten datasets. Since the results are consistent, for sake of simplicity, the comparison for one representative case is demonstrated (see Figure 66). The patient has an ectasia in the ascending aorta. The manual exploration revealed three vortices: One large vortex in the ascending aorta that is present during the complete heart cycle and two smaller ones in the distal aortic arch that occur only during the systole.
8.1.4.1. Definition of Quality Criteria

A quantitative comparison will be challenging due to the missing ground truth. Thus, we rely on subjective impressions and formulate key questions for the vortex criteria in collaboration with the medical experts. They put great emphasis on the correctness of the results and the capability of our procedure to extract and present different vortex types with expressive path lines. We complemented their expectations with considerations about the vortex criteria’s value domains and the possibility to use universal thresholds. In summary, the evaluation was performed considering the following key aspects:

- **Fixed minimum and maximum:** Is there a fixed minimum and / or maximum of the calculated value domain? This facilitates the identification of a generally applicable threshold for all datasets.

- **Implicit threshold:** Is there an implicit threshold to distinguish between vortex and non-vortex regions? This would make the previous question superfluous.

- **Constant vortex core values:** Are the values of the criterion constant along vortex cores? This aspect influences the filtering behavior. If the values are constant, vortices are filtered from the outside to the inside when the threshold is increased.

- **Correctness:** Is the extraction procedure able to find at least every vortex that was discovered by the physicians’ previously performed manual exploration of the datasets?

- **Helical and vortical flow:** Vortices can be classified according to their appearance: Swirling path lines with a strong movement along the vortex core are helical, whereas path lines that tend to swirl in the plane are vortical. Is the employed vortex criterion able to determine both?

- **Vortex shape:** Is the shape of a vortex pointed out sufficiently? Its swirling characteristic should be comprehensibly presented by the path lines.

- **Path line quality:** The extracted vortices should be presented by path lines with long and continuous courses. False-positive results as well as short path line fragments are unwanted.

All thresholds were manually adjusted at this point to achieve the best possible findings. In addition, it was decided to not use the bending energy line predicate 3 for post-processing in order to emphasize the actual differences between the results.

8.1.4.2. Observations

In the following, the main outcomes of the discussion with the domain experts about the results of each vortex criterion are described. The key questions are also answered in short in the Figures 66a–g. A green check or red cross answers yes / no questions and a green, yellow, or red bar stands for a good, medium, or bad rating of a property. Figure 66h is an anticipation of a result achieved with the final procedure described in Section 8.1.4.3 and provided as a reference. For that reason, this is the only result with applied post-processing.

**Vorticity:** The vorticity (see Figure 66a) preserves helical and vortical path lines, but exhibits a high false-positive rate especially near the vessel boundaries. The quality of the path lines is appropriate. There is a fix minimum value of 0, but the maximum depends directly on the derivatives of the occurring velocities. Thus, there is no fixed peak value among different vortices. The same yields for a general threshold. A small vortex often vanishes before the shape of a larger one is pointed out sufficiently. This can even happen within a single vortex since the criterion is not constant along vortex cores or within vortex regions.

**Torsion:** By using the torsion (see Figure 66b), it is hardly possible to extract satisfactory path lines that meet the expectations. Though the path line density is higher in vortex regions, they are highly fragmented. Not all vortices are captured correctly, and a large amount of false-positive path lines is present. Helical flow is extracted rather than vortical flow, since the torsion describes the twisting of a curve out of the plane. The minimum of 0 and maximum of 1 are fixed values, but a generally applicable threshold could not be determined.
The results of the normalized helicity (see Figure 66c) are very similar to the ones achieved by using the torsion. Especially vortical flow is extracted insufficiently. A unique characteristic of the helicity is its sign that helps to characterize the orientation of a vortex into left- and right-handed. The value domain ranges from $-1$ to $+1$, where 0 is no helicity and $\pm 1$ is the maximum left- or right-handed swirling strength.

**Reduced Velocity:** The reduced velocity (see Figure 66d) is a projection of the velocity vector onto a plane orthogonal to the vortex core direction. If the angle between the velocity vector and the vortex core direction is small, the length of the projected vector, the reduced velocity, is 0. This minimum value is constant along the vortex cores of all vortices. Applying a threshold results in progressively removing off the path lines from the outer regions to the vortex core. This is suitable for cases where the path lines swirl helically along the vortex core direction. Path lines that represent vortical flow tend to swirl in a plane with increasing distance. These path lines are filtered away early when increasing the threshold and, consequently, the extracted vortices may appear thinly. In addition, this problem can lead to the unwanted interruption of path lines. Also, smaller regions of vortical flow are not always captured correctly. An implicit threshold is not given for the reduced velocity, but reasonable values should be close to 0.

**Cores of Swirling Particle Motion:** COSPM (see Figure 66c) is the extension of the reduced velocity from steady to unsteady 3D flow. The results are very similar, which might be an indication that the flow alteration over time is rather small in the cardiac 4D PC-MRI datasets. This fact increases the reliability of steady flow vortex criteria applied to the unsteady blood flow. The minimum of 0 and maximum of 1 are fixed values. As before, a value of 0 means that a position is directly on the vortex core.

**Q Criterion:** The Q criterion (see Figure 66f) was able to extract every single vortex in the datasets. The path line quality meets the requirements and there is an implicit threshold of $Q > 0$. The criterion is neither constant along the vortex core nor has a behavior like the reduced velocity or COSPM regarding the distance to the vortex core. Therefore, increasing the threshold should be avoided, since it cannot be reliably predicted which path lines will be maintained or filtered.

$\lambda_2$ **Criterion:** Like the Q criterion, the $\lambda_2$ criterion (see Figure 66g) was able to detect all vortices. Their results are nearly identical. Using the $\lambda_2$ criterion made it likewise possible to extract all vortices correctly with the same path line quality. The only difference to the Q criterion is that 0 is the maximum value and the implicit threshold is given by $\lambda_2 < 0$.

**8.1.4.3. Summary**

Based on the previous comparison, the $\lambda_2$ and Q criterion have shown the best results. In accordance with the domain experts, a significant difference cannot be seen. Thus, the usage of both criteria is recommended for the extraction of vortices in blood flow. For detailed differences between the $\lambda_2$ and the Q criterion, please consider Sahner et al. [438].

A possible explanation for their robust results is that, in contrast to all other criteria, $\lambda_2$ and Q solely analyze the derivatives of the velocity vectors (the Jacobian matrix). The reduced velocity, for instance, uses the Jacobian matrix and the velocity vectors. This might amplify noise in the data and introduce further errors. It was decided to use the $\lambda_2$ criterion, since it seems to be more established among blood flow-specific works [63, 449, 485].

The implicit threshold of $\lambda_2 < 0$ will be used for the first vortex line predicate. For the second (mean) vortex predicate, the removal of points representing the lower 10% of the value margin is recommended. Post-processing via the bending energy predicate carries the risk of removing short path lines representing very small vortices, especially when other comparatively large vortices exist. Therefore, the user’s judgment is incorporated and this last step is offered as optional, manual post-processing.

Since no vortex was found that was exclusively detected by another vortex criterion and not by $\lambda_2$, it was decided not to combine different vortex criteria. In summary, the final procedure is to cover the whole dataset (inside the vessel segmentation) with path lines and then calculate the characteristic set $C_{\Pi}^{\text{vortex}}$ as:

$$
C_{\Pi}^{\text{vortex}} = \Pi_{\sigma.25}(\Pi_{V^2}^{\lambda_2}), \hat{p} < 0
\rightarrow \Pi_{\sigma}(\Pi_{V^2}^{\lambda_2}), 90\% \\
\rightarrow \Pi_{t_{\text{bendingEnergy}}}, t_{\text{manual}}
$$

(32)

On the average taken over all datasets, the mean path line length, measured after application of the first and last line predicate, becomes three times larger. This supports the claim that path lines of increased quality are obtained.

The only necessary user interaction is to adjust the bending energy threshold for the optional post-processing. Furthermore, if the aforementioned standard parameters are not used, the required input extends to choosing the number of seed points as well as adjusting three successive thresholds.

**Extensions:** In the most recent version of Bloodline, the necessary determination of the Jacobian matrix as well as its eigenvalue analysis is done on the GPU using Cardano's method [281]. Moreover, the extraction procedure was modified. The first step of the procedure remains approximately the same. The $\lambda_2$ calculation is performed on the GPU and the smoothing on the CPU. Contrary to the initial approach, path lines are no longer cropped. Instead, the mean $\lambda_2$ calculation of the second step $(\Pi_{\sigma}(\Pi_{V^2}^{\lambda_2}))$ is only performed on line segments that fulfilled the first predicate. As a result, path lines now consist of zero-valued non-vortex segments or $\lambda_2$-averaged segments. This facilitates the usage of a single value to hide path line segments that are below the user-given mean $\lambda_2$ threshold, which can
be modified interactively with one slider. The originally proposed bending energy predicate is not used anymore, since there is always a risk to exclude smaller vortices with shorter path lines. The slightly increased visual clutter is tolerable, especially during animations.

8.1.5. Results and Informal Evaluation

The vortex extraction method was applied to ten 4D PC-MRI datasets of the aorta and pulmonary artery. In the following, two clinical cases are described in detail, whereas the other ones are presented as an overview. The results of our extraction procedure were shown to one cardiologist as well as two radiologists. They were asked to relate occurring vortex patterns to the present pathologies. The evaluation was performed informally and the domain experts were not required to solve specific tasks. The integration of 30000 path lines and calculation of the $\lambda_2$ criterion is performed in less than 10 ms on a GeForce GTX 680. The data transfer between GPU and CPU takes another 1.7 s. The presented images are directly captured from the animation. In addition to the static path line visualization, freely scalable arrowheads are added at positions where a path line’s time matches the animation time to make the current flow directions easier recognizable.

8.1.5.1. Tetralogy of Fallot with Pulmonary Regurgitation

The first case has tetralogy of Fallot. A VSD surgery has been performed about 20 years ago at the age of one. The patient mentions a fast exhaustion with dyspnea during physical activities, nausea, a pulling pain into the legs, and regularly occurring palpitations. The absence of anginous pain suggests that there is no coronary heart disease. Nocturia, one of many possible symptoms for congestive heart failure, was also not present.

Ultrasonography revealed the existence of distinct diastolic retrograde flow in the pulmonary artery. A closer examination of the heart showed a mild hypertrophy of the right ventricle, although a significant narrowing of the pulmonary artery was not present. There is still a minimal right-to-left shunt, but rather small and therefore clinically irrelevant. According to the admission note, the regurgitation fraction is about 30 % from the pulmonary artery back into the right ventricle. This characterizes a severe pulmonary insufficiency. The hypertrophic right ventricle may be encouraged by the increased blood pressure due to the pulmonary regurgitation.

For clinical research, a 4D PC-MRI measurement was performed. The qualitative analysis of the complete flow in the pulmonary artery shows a clearly visible retrograde flow even without the enhancement of certain data aspects or features. A vortex occurs during diastole when the blood flows back into the ventricles through a small gap, which is caused by an improperly closing valve (see Figure 67).

The quantification results verify the regurgitation fraction of 30 %. Clearly, the situation in the pulmonary artery is pathologic and explains the vortex near the valve. However, the mere existence of such a vortex should not be seen as sufficient evidence, but rather lead to further investigations. All necessary information and explanations of the symptoms for the case are present in this dataset and the domain experts were able to diagnose the same pathology in the pulmonary artery.

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9 A video is available at https://www.youtube.com/watch?v=y3xf5WAFo2M (accessed 03/2016)
8.1.5.2. Aneurysm and BAV with Former Coarctation

The primary disease of the second case was a coarctation, which was widened via balloon dilatation. Based on a suspicion for a re-coarctation nine years later, a new examination was performed combined with the acquisition of a 4D PC-MRI dataset. The patient has no physical limitations, but describes a casually occurring tingling in the left arm, headaches near the temples, dizziness in the morning, and often cold and sweaty hands.

The re-coarctation could be excluded with a cardiac catheterization. An MRI of the heart showed a large aneurysm in the left subclavian artery and an ectatic distal aortic arch. The aneurysm in an early stage was already known from the first examination, but has grown progressively since then. Such an aneurysmatic dilation of the subclavian artery is very rare. Further typical symptoms, such as chest pain, did not occur.

The first look at the overall flow revealed a huge vortex during the full systole and diastole ranging from the dilated left subclavian artery to the descending aorta (see Figure 68). Our vortex extraction procedure reveals no pathological findings in the pulmonary artery, but an abnormal vortex structure near the aortic valve during systole. Hope et al. [221] found swirling flow in the ascending aorta during the systole in 75% of their 20 BAV cases "but in none of the healthy volunteers or patients with a tricuspid aortic valve." Considering these results, it is likely that the BAV causes this vortex in our dataset as well. This assumption, however, could not be confirmed with certainty by the domain experts, but such a flow behavior could indicate a valve defect and lead to further investigations.

Further discussions about the aneurysmatic left subclavian artery revealed another important flow behavior: Blood passes the area of the former coarctation and then, due to the altered vessel shape, keeps flowing straight ahead instead of following the distal aortic arch leading into the descending aorta. The flow impinges on the surface and then starts swirling upwards into the subclavian artery and downwards along the descending aorta. Recent works observed an increased wall shear stress in regions where high velocity flow impinges on the vessel's surface. They agree that abnormally high wall shear stress is one main cause for the initial development of aneurysms [384, 469]. As a consequence, the permanent stress caused by the swirling flow could be one important factor for the patient's progressively dilating vessel.

Again, the 4D PC-MRI data complemented the diagnostic information, helped to explain the symptoms and supported the understanding of the case. The aneurysmatic subclavian artery as well as the ectatic distal aortic arch could be determined easily and the systolic vortex near the aortic valve gave a clear hint towards the BAV.

Figure 67: The pulmonary artery of a tetralogy of Fallot patient. About 30% of the blood flow back into the right ventricle during the diastole due to an improperly closing valve. This percentage is high enough to specify the circumstances as pulmonary insufficiency. Vortices are present in the right pulmonary artery and the pulmonary trunk close to the valve. Image from [270] © 2013 IEEE Computer Society, reprinted with permission.

Figure 68: The pulmonary artery of a tetralogy of Fallot patient. About 30% of the blood flow back into the right ventricle during the diastole due to an improperly closing valve. This percentage is high enough to specify the circumstances as pulmonary insufficiency. Vortices are present in the right pulmonary artery and the pulmonary trunk close to the valve. Image from [270] © 2013 IEEE Computer Society, reprinted with permission.
8.1.5.3. OVERVIEW OF OTHER CASES

Figure 69a shows the patient we presented previously for the vortex criteria comparison. The large vortex in the ascending aorta keeps swirling during the diastole and flows back through the aortic valve into the left ventricle. Further quantification shows a pathologic regurgitation fraction of about 15–20%. According to our experts, this fact can support the decision whether or not to implant an artificial valve.

The patient shown in Figure 69b has a coarctation and a slightly dilated ascending aorta. Analogous to all our other datasets with dilated ascending aortas, a large vortex is present in this part of the vessel.

A similar case is shown in Figures 70a–b. The patient has an ectatic ascending aorta and one large vortex is present during the systole. In contrast to the first example, the vortex disappears in the diastole and no significant retrograde flow is determinable. The pulmonary artery shows a slight vortex in the pulmonary trunk near the valve during the systole. Considering the results regarding aortic valve defects, this could indicate a bicuspid pulmonary valve. Assuming that vortex flow promotes the development of...
vessel dilations and, in turn, bicuspid valves encourage vortex flow, the conclusion could be drawn that bicuspidalities increase the risk of an ectasia development.

Figure 71a shows a patient with a coarctation. The narrowed vessel geometry seems to cause the vortex from the distal aortic arch into the descending aorta. Again, we can observe a vortex in the ascending aorta during the systole that could indicate a BAV.

The case shown in Figure 71b had undergone a bypass operation. There is one large helix present during the systole that is likely promoted by the altered course of the vessel.

The last example in Figure 71c shows a similar case. The patient has a vascular prosthesis implant as treatment of a former, severe coarctation. The occurring vortex patterns are comparable to the ones in Figure 71a and were possibly caused by the same reasons. An additional small vortex is present in the aortic arch. All vortices occur in the systole.

Our two healthy volunteers were, as expected, without pathologic findings. The same yields for the pulmonary arteries of all cases except for the two we showed in Figures 70b and 67.

![Figure 70: The patient has an ectatic ascending aorta.](image)

(a) A large vortex is present during the systole that vanishes in the diastole.
(b) A systolic vortex near the pulmonary valve could indicate a bicuspidality.

![Figure 71:](image)

(a) Patient with a coarctation. Besides a vortex in the ascending aorta, there is one after the narrowed region that is probably evolved by the altered vessel geometry. All vortices occur in the systole.
(b) Patient with a bypass. The altered vessel geometry promotes the formation of a large helix during the systole.
(c) Patient with a former coarctation that was treated with a vascular prosthesis. Three vortices are present during the systole. One is close to the valve and could indicate a defect. A second small vortex is observable in the aortic arch. The third large one occurs in the region of the prosthesis and is likely caused by the anomalous vessel shape.

8.1.5.4. BAV in a Porcine Model

The influence of aortic valve bicuspidalization (BAV, recall Section 2.2.2) was investigated in a study with pigs. Pigs are often used for such purposes, since their cardiac anatomy is similar to human hearts and they are readily available [109]. Three mongrel swines were scanned with 4D flow MRI. Then BAV surgeries were performed:

- The right and left-coronary leaflet (R-L) were fused in swine one.
- The right and the non-coronary leaflet (R-N) were fused in swine two.
- In swine three, it was discovered that it already had an inherited BAV (also R-L type).

Please consider Sievers and Schmidtke [472] or Martin et al. [344] for an explanation of the BAV fusion types. A post-operative scan was performed of swine one and two. Subsequently, vortex flow patterns before and after were compared. Before the surgery both pig one and two had mostly laminar flow during systole. Post-op they had, same as swine three, prominent vortex flow in the ascending aorta or aortic arch (see Figure 72). The study could successfully demonstrate the influence of bicuspidalization of the aortic valve to blood flow patterns.

![Figure 72:](a) Swine one with a tricuspid aortic valve has mainly laminar systolic flow. Path lines are color-coded by velocity. (b) Prominent vortex flow emerges after a surgical bicuspidalization. The images are from our open source article [192], which is available under the Creative Commons 4.0 Attribution License.

8.1.6. Conclusion

It was shown that the approach reliably extracts vortices in the aorta and pulmonary artery based on 4D PC-MRI measured flow information. A result was considered as correct if at least all vortices were extracted that experts were able to discover manually. The extraction achieves real-time capability due to its entire computation on the GPU. A detailed investigation of several vortex criteria was performed in [269] where the $\lambda_2$ criterion was identified as the most appropriate local vortex criterion for cardiac 4D PC-MRI datasets. Yet, the limited diversity of the datasets might alleviate the generality of this result.
As a follow-up to this work, Carnecky et al. [94] further increased the noise robustness of \( \lambda_2 \) calculation in 4D PC-MRI data by suggesting an orthogonal decomposition of the phase images.

Line predicates were incorporated to extract meaningful path lines that describe vortex flow structures. Their quality is high and meets the domain experts’ and our requirements. The experts came to similar conclusions regarding the relation of present pathologies and occurring vortices for the presented cases.

Thus, the straight forward and semi-automatic extraction pipeline in combination with reasonable default parameters ensures the ease of use of our method and minimizes the inter-observer variability.

The results allow a more precise diagnosis, support treatment decisions, and facilitate the understanding of the progression of cardiovascular diseases, such as ectasias and aneurysms. Vortices are a strong indicator for certain pathologies of the cardiovascular system and complement other features like velocities, flow rates, and the amount of retrograde flow. However, the correlation between pathologies and vortex flow patterns is an ongoing research topic. The clinical collaborators indicate that with more confidence in the developed tools, they may replace the standard 2D blood flow measurements in the future. This would be a big step forward, since 4D blood flow measurements would leave the stage of an experimental modality only used for selected cases. An important requirement for future work will be to optimize the workflow and usability of tools.

The line predicate technique is flexible enough to be extended, so that more flow characteristics can be determined (see Born et al. [63]). Possible targets are laminar flow, high velocity flow during the diastole, the separation of antegrade and retrograde flow, flow that impinges on the vessel walls with high velocities, and flow that causes high WSS.

8.2. 2D Plot Visualization of Aortic Vortex Flow

Vortex flow in the great mediastinal vessels, such as the aorta, is presumed to be a strong indication of several pathologies. Therefore, studies with homogeneous patient groups are performed to quantify the probability of vortex occurrence in specific vessel sections during the cardiac cycle. Common path line visualizations of the highly complex 4D PC-MRI datasets allow a qualitative analysis. In the previous section, an adapted line predicates technique to semi-automatically filter vortex flow was established. Such methods are essential to make 4D PC-MRI viable for the clinical routine. Nevertheless, when the path lines are displayed all at once by ignoring their temporal component, heavy visual clutter remains. Particle animations alleviate this problem, but increase the required evaluation time per dataset.

Different works established simplified visualizations of the cardiovascular morphology. Cerqueira et al. [98] proposed the nowadays widely used 2D bull’s eye plot (BEP) of the left ventricle that is based on a standardized segmentation. The plot is simple, unambiguous, and thus a convincing reduction of the 3D information. Based on this, Termeer et al. [502] described a plot visualization of coronary artery disease. Angelelli and Hauser [8] introduced a straightening of tubular structures and applied it to the aorta in order to enhance flow analysis with reference to the main flow direction, which usually is the centerline. Yet, when the whole cardiac cycle is to be analyzed, numerous such visualizations with different temporal positions are required.

In this work, a circular 2D plot is introduced that is adaptable to various flow characteristics. The focus lies on vortex flow due to the strong correlation to cardiovascular pathologies. The plot provides detailed information about the temporal position and approximates the corresponding vessel section. A grid view of different datasets enables the fast assessment of vortex behavior in a user-defined database. A scalar parameter controls the plots’ sensitivity towards vortex flow. The possibility to set already analyzed...
datasets as references enhances the evaluation of new cases. After a brief training, our collaborating cardiologists and radiologists were able to reliably find pathologic cases in the grid view.

### 8.2.1. Method

**Mapping:** Intravascular positions $\vec{p}_t = (x, y, z, t)$ in the 4D PC-MRI dataset consist of a three-dimensional spatial and a one-dimensional temporal component. The circular plot visualization, however, offers merely an angle $\phi$ and a distance $d$ to the center as degrees of freedom.

Medical research papers that correlate the presence of vortex flow patterns to specific pathologies have two central questions: *When* (during the cardiac cycle) does the vortex occur and in *what vessel section?* We decided to map a vortex’ temporal position to the plot’s angle as an analogy to a clock and due to the cyclic nature of the data (see Figure 73).

Now there is only the center distance $d$ left to map the spatial position. The idea is to employ the centerline for this purpose. The projection $q$ of a spatial position in the vessel onto the centerline is determined and used to obtain $d \in [0, 1]$ as:

$$d = \sqrt{\text{length (centerline until } q)} / \text{length (centerline)}$$

Thus, the plot center corresponds to the approximate aortic valve location where the centerline starts. Increasing $d$ encode positions in the aortic arch and descending aorta (see Figure 74). The square root is used to ensure that inner and outer parts of the plot are represented with equally large areas (see Figure 75).

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**Figure 73:** Mapping temporal positions to the plot’s angle – analogous to a clock. The first time step $t_0$ is at 12 o’clock, the direction is clockwise.

**Figure 74:** The distance from the plot center encodes the position on the centerline. Central areas (blue, green) present vessel sections near the aortic valve and in the aortic arch, whereas increasing distances (yellow) show the descending aorta. Image from [274] © 2015 Springer-Verlag Berlin Heidelberg, reprinted with permission.

**Figure 75:** Square roots of distances to the plot center are employed to guarantee equal areas for inner and outer cells.
Vortex Processing: Each centerline point stores $T$ values, where $T$ is the number of time steps in the dataset. These buckets are used to accumulate a quantitative measure, in our case the $\lambda_2$ criterion since it is well suited for vortex extraction in the cardiac 4D PC-MRI context [270]. As a pre-processing, the centerline is equidistantly resampled in 0.5 mm steps via cubic spline interpolation in order to have the same distances in different datasets and a sufficient amount of buckets. Since the employed path line integration uses adaptive step sizes, the vortex-representing path lines are resampled as well in the same manner. Afterwards, every point of each path line is processed. The closest spatio-temporal projection onto the centerline is determined and the $\lambda_2$ value of the path line vertex is split among the two neighboring centerline buckets weighted by their inverse distance to the projection. After evaluating all path lines the mean $\lambda_2$ value is calculated for each bucket (see Figure 76a). A binomial smoothing of the accumulated values is performed as post-processing (see Figure 76b).

Vortices are present where $\lambda_2 < 0$. The smaller $\lambda_2$ is, the stronger is the vortex flow. Unfortunately, the criterion has no fixed minimum, which negatively affects the comparability between different datasets.
As a remedy, we let the user define a minimum $\lambda_2$ value as parameter $\alpha$ within 0 and the minimal occurring $\lambda_2$ value. The closer $\alpha$ is to 0, the more sensitive the visualization is towards vortex flow. The parameter is used to scale as well as clamp the $\lambda_2$ values to $[0, 1]$. The scaled values are then mapped to an arbitrary color scale. We employ the rainbow scale despite their perceptual problems, since this a common choice in the clinical context (see Figure 77).

**Dataset Comparison:** The expressiveness of a single plot might be limited, since an appropriate choice of $\alpha$ is not always clear – especially for pathologic datasets. This problem further increases if there are $\lambda_2$ outlier values. Figure 78 shows two $\alpha$ settings, each one close to 0 and the $\lambda_2$ minimum, for a healthy volunteer. Incorporating a priori knowledge, one would expect a plot that shows almost no critical regions (vortex flow), as only a slight, systolic helix in the aortic arch is expected.

To tackle this problem, a grid view is provided that enables the efficient comparison of multiple datasets. Here, the sensitivity parameter $\alpha$ is used globally for all plots. Additionally, the user has the option to choose known cases as reference plots in addition to the actual cases that are selected for evaluation.

### 8.2.2. Results

We used 14 datasets for the evaluation: Five healthy volunteers, two patients with an ectatic ascending aorta, and seven BAV patients. Each patient has prominent systolic vortex flow in the ascending aorta. The computation time per case is between 2 and 5 s on an Intel i7-3930K depending on the amount of path lines. Figure 77 shows one of the patients with a dilated ascending aorta and depicts the relation between occurring vortex flow and the resulting plot. Figure 79 shows the proposed grid view with all datasets. The two larger plots on the left represent healthy volunteers that were selected as references. The sensitivity parameter $\alpha$ was interactively adjusted so that these two plots merely indicate the slight physiologic helix in the aortic arch during systole. In an informal evaluation, our collaborating radiologists and cardiologists were easily able to spot the remaining three healthy volunteers without pathologic vortex flow.

![Figure 79: Two healthy volunteers are shown as larger reference plots on the left. Three more healthy volunteers (3, 7, 10) are among two patients with an ectatic ascending aorta (1, 11) and seven BAV patients (2, 4–6, 8, 9, 12). Image from [274] © 2015 Springer-Verlag Berlin Heidelberg, reprinted with permission.](image-url)
8.3. Adaptive Animations of Extracted Vortex Flow

8.2.3. Discussion

We presented a two-dimensional circular plot visualization of aortic vortex flow. Domain experts were able to quickly assess whether or not pathologic vortices are present in a dataset and could estimate their positions and temporal extents. Presenting the temporal position based on a clock analogy was considered as intuitive, whereas the mapping of spatial positions using the centerline required a short briefing. A possible application of our method is to get a quick overview of datasets in larger studies. Tasks like counting the BAV patients with systolic vortex flow in the ascending aorta can be performed comfortably. In addition, our method could support the clinical report generation and serve as a summary of a patient’s vortex flow behavior. Our method provides no information about a vortex’ wall closeness, which might be interesting due to the association with high shear forces [96]. However, a visualization or integration of other measures in the plot, derived from arbitrary flow properties or line predicates, is conceivable. The comparability of datasets acquired with different scanners and / or MR sequences has to be analyzed in a future work. Potential problems may arise due to differently scaled λ2 values. Our proposed plot uses exactly one centerline to project the spatial intravascular positions. Another future topic could be the adaption for branching vessels, such as the pulmonary artery.

8.3. Adaptive Animations of Extracted Vortex Flow

While quantitative data analysis allows to assess the cardiac function and monitor disease progression, qualitative methods support the identification of characteristic flow patterns. Flow animations commonly employ pathlets or particles [351]. The extraction of vortex flow facilitates an easier comprehension of the highly complex 4D data [270]. Though, vortices are mostly not present during the full cardiac cycle. An interactive manipulation of the animation time is possible in flow analysis software, but rendered videos typically use a constant speed. As a consequence, many seconds of the video are potentially wasted by displaying an empty vessel or flow outside the focus. There are other works that established flow simplifications or abstractions [275]. However, a great advantage of flow animations is their intuitiveness. Therefore, we enhance this technique by describing view-dependent vortex animations with adaptive speed (VAAS). Here, time-spending is concentrated on flow characteristics of interest, whereas the rest is time-lapsed.

Our collaborating radiologists and cardiologists appreciate the easier vortex analysis from videos and see the principal application in presentations, case discussions, or documentations. We present four diverse datasets in an informal evaluation.

8.3.1. Method

This section describes our work, which can be seen as a histogram equalization for the temporal vortex visibility. 4D PC-MRI data were pre-processed as described in Section 7.1.

Video Settings: A video with the target length $S$ seconds and $FPS$ frames per second consists of $N = S \cdot FPS$ single images. The cyclic 4D PC-MRI dataset has $T$ temporal positions. For standard videos, the temporal offset between successive frames is a constant function:

$$\Delta t(i) = \frac{T}{N - 1} \quad \text{with} \quad i = 0 \ldots N - 1$$

(34)
8.3. Adaptive Animations of Extracted Vortex Flow

**Vessel Mask:** The number of pixels in the current viewing direction that show the vessel mesh is required (see Figure 80a). This can be counted on a binary vessel mask that is rendered with disabled shading and disabled depth testing into the stencil buffer (see Figure 80b). Only front face culling should be enabled, as it normally is when intravascular flow is shown.

**Feature Masks:** Feature images (see Figure 80c) of the extracted vortex flow are created for each frame of the video with constant speed. The pathlets are rendered without illumination or halos, but with depth testing against the front face culled vessel. The animation’s current time \( t \) as well as a user-given parameter \( l \) that controls the pathlets’ length are used to calculate the visibility \( a_{ad}(t, \hat{p}_t, t, l) \) for each path line position \( \hat{p}_t \) with its temporal component \( \hat{p}_t \):

\[
a_{ad}(t, \hat{p}_t, t, l) = \begin{cases} 
1, & \text{if } t - l < \hat{p}_t < t + l \\
0, & \text{else.}
\end{cases} \tag{35}
\]

When checking \( t - l < \hat{p}_t < t + l \), it is important to consider the cyclic nature of the data.

**Feature Visibility Function:** Every value of the discrete, frame-dependent feature visibility function \( f(i) \) (see Figure 80d) represents the ratio of number of foreground (white) pixels in the feature (vortex) mask of that frame and the vessel mask. Each \( f(i) \) is set to \( \max(f(i), h) \) as a correction, where the parameter \( h \) indirectly controls the maximum value in the adaptive speed function \( \Delta_{ad}(i) \). If not restricted, \( \Delta_{ad}(i) \) can skip very large parts of the cardiac cycle if no vortices are visible. We use \( h = 0.05 \cdot \max(f(i)) \) as experimentally determined default. \( f(i) \) is resampled to \( g(k), k = 0 \ldots 100 \cdot N - 1 \) using a periodic spline (Catmull-Rom from ALGLIB). Each value of \( g(k) \) is divided by the spline’s integral as a normalization.

**Adaptive Speed Function:** To maintain the target video length \( S \), the integral of the adaptive speed function \( \Delta_{ad}(i) \) has to equal the dataset’s number of temporal positions \( T \), as \( \int_0^T \Delta_{ad}(i) \, di \) does. Each frame of a video with constant speed shows \( p = 1/N \% \) information of the cardiac cycle. In the video with adaptive speed, we equate information with vortex visibility. This is used to transform \( g(k) \) to \( \Delta_{ad}(i) \) via Algorithm 1. If enough information are gathered (\( \text{info} \geq p \)) from \( g(k) \) after \( k_k_{ad} \) steps, \( \Delta_{ad}(i) \) is set to the corresponding portion of the cardiac cycle (see Figure 81). To avoid irritating sudden speed changes, \( \Delta_{ad}(i) \) is smoothed in two iterations with a 1D binomial filter with kernel size 3 (experimentally determined default). A subsequent correction is applied to ensure that:

\[
\int_0^{N-1} \Delta_{ad}(i) \, di \approx \sum_{i=0}^{N-1} \Delta_{ad}(i) = T \tag{36}
\]
8.3. ADAPTIVE ANIMATIONS OF EXTRACTED VORTEX FLOW

Algorithm 1. Construction of the adaptive speed function \( \Delta_{\text{ad}}(i) \) from the resampled feature visibility function \( g(k) \).

\[
p \leftarrow \frac{1}{N} \quad \text{// information threshold} \\
info \leftarrow 0 \quad \text{// gathered information} \\
k_{\text{old}} \leftarrow 0 \\
i \leftarrow 0 \quad \text{// \( \Delta_{\text{ad}} \) index} \\

def \text{for } k = 0 \text{ to } 100 \cdot N - 1 \text{ do} \\
\quad \text{info} \leftarrow \text{info} + g(k) \quad \text{// gather information} \\
\quad \text{if info} \geq p \text{ then} \\
\quad \quad \Delta_{\text{ad}}(i) \leftarrow T \cdot (k - k_{\text{old}})/(100 \cdot N) \\
\quad \quad i \leftarrow i + 1 \\
\quad \quad \text{info} \leftarrow \text{info} - p \\
\quad \quad k_{\text{old}} \leftarrow k \\
\quad \text{end if} \\
\text{end for} 
\]

While this integral is smaller than \( T \), a value \( c \) is added to each \( \Delta_{\text{ad}}(i) \). Afterwards, while the integral is greater than \( T \), \( c \) is subtracted from each \( \Delta_{\text{ad}}(i) \). As default we use:

\[
c = \frac{0.01}{100 \cdot N} \quad (37)
\]

This allows to alter the integral by 0.01 in each iteration. Finally, we set:

\[
\Delta_{\text{ad}}(100 \cdot N - 1) = T - \int_0^{N-1} \Delta_{\text{ad}}(i) \, di \quad (38)
\]

This is done to achieve the exact target video length \( S \). In all diagrams we show the relative video speed \( \Delta_{\text{rel}}(i) \) (see Algorithm 2) instead of \( \Delta_{\text{ad}}(i) \). This is more expressive, since it displays the speed level of the video at each time step.

For the final video rendering, we employ \( \alpha(t, \vec{p}, t, l) \) (see Equation 39) as path line-opacity function. Order-independent transparency (OIT) [571] ensures correct alpha blending of the semi-transparent
pathlets.

\[
\alpha(t, \vec{p}, t, l) = \begin{cases} 
\sqrt{1 - |t - \vec{p}|/(2 \cdot l)}, & \text{if } t - l < p < t + l \\
0, & \text{else.} 
\end{cases}
\]  

(39)

**Extensions:** Our procedure is directly applicable to a region of interest. In case of simultaneously occurring vortices, this allows to emphasize one of them. In addition, another flow property, such as the path lines’ velocity, can be mapped to gray scale. The feature images (previously binary masks) then store color values between 0 and 1 that serve as an implicit weighting. Instead of counting foreground pixels, these color values are accumulated, while the remaining procedure is the same as before. This could also be used to emphasize near-wall vortex flow, which is associated with increased shear forces (wall shear stress). However, disabling line illumination is required to avoid false-positive high feature values.

### 8.3.2. Results

We applied our method to four datasets. The first patient has systolic vortex flow in the dilated ascending aorta (see Figure 82). The adaptive video speed ranged from 19–315 % compared to the constant version. A healthy volunteer with a physiological helix in the aortic arch is shown in Figure 83a. In the adaptive video, time lapse is applied to the diastole, where almost none of the extracted vortex flow is visible. Figure 83b shows a patient with pathologic vessel dilation, heavy systolic as well as diastolic vortex flow in it, and an additional systolic vortex in the aortic arch. A slight speed increase during diastole is achieved when our method is applied to the whole vessel, since vortex flow is visible during the whole cardiac cycle. Figure 83c shows the same patient with a focus (region of interest) on the smaller vortex in the aortic arch. Due to a shorter overall feature visibility, our method has an increased effect in this case. Figure 83d shows an aneurysm patient with a prominent vortex that is present during the full cardiac cycle. The result is similar to Figure 83b.

The employed test system has an Intel Core i7-2600K and a GeForce GTX 680. Our default video setup with 5 s and 50 FPS takes about 50 s to create and analyze the visibility function, and another 45 s to render the actual video. The performance directly scales with the desired FPS and the target video length.

![Figure 82: Patient with systolic vortex flow in the ascending aorta.](image1)

*Diagrams: Corresponding feature (vortex) visibility (blue) with constant (top) and adaptive (bottom) speed (red). The green line in the lower diagram shows the speed level of the video with constant speed as a reference. Image from [276] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.*

10 A video is available at [https://www.youtube.com/watch?v=6k9DTCvYXD](https://www.youtube.com/watch?v=6k9DTCvYXD) (accessed 03/2016)
8.3.3. DISCUSSION

We presented vortex animations with adaptive speed (VAAS) to support the evaluation of cardiac vortex flow patterns. In a figurative sense, the technique is a histogram equalization for vortex visibility. While histogram equalization aims at an optimal use of the available gray values, VAAS aims at an optimal use of given presentation time. The effect, i.e., the use of time lapse and slow motion, is higher the more squeezed the initial feature visibility function is. The results of our method could slightly vary with different line width and pathlet lengths. Also, fast path lines might be favored implicitly, since they produce longer pathlets and thus occupy more pixels in the feature images. The computational effort could be decreased by not analyzing every video frame to create the visibility function. But, features that are visible for only a short time could be missed this way.

Our method processes only the mesh and path line geometry and thus is independent from the underlying image data. An application to simulated CFD data or other vessels is conceivable. Our clinical collaborators appreciate the reduced downtimes, where an empty vessel or uninteresting flow was shown. This was considered as helpful during case discussions, presentations, and for documentation purposes. It was required that the user should explicitly need to enable our method in the software to avoid confusion. Integrating a speedometer was a further idea raised by our clinical collaborators. This would support the recognition of time lapse and slow motion. Additionally, a reference to the current temporal position within the heart cycle should be shown, e.g., as a bar in a flow curve diagram.

8.4. VORTEX CHARACTERIZATION IN THE AORTA

Recent medical works are not only interested in the pure existence of vortex flow, but also in specific characteristics of the vortices. Especially bicuspid aortic valves (BAVs) have been shown to produce different systolic flow patterns [25, 40, 41] in the ascending aorta [221, 356], depending on the valve’s cusp fusion type [472] (recall Sections 2.2 and 5.3.1).

A prerequisite to analyze single vortex entities is their separation by means of clustering. A corresponding approach is described in Section 8.4.1. Afterwards, different criteria for vortex characterization are discussed and evaluated in Section 8.4.2.
8.4. VORTEX CHARACTERIZATION IN THE AORTA

8.4.1. CLUSTERING

This section is based on:

- [360]: M. MEUSCHKE. “COMPUTERGESTÜTZTE ANALYSE VON VERWIRBELUNGEN IN 4D PC-MRI BLUTFLUSSDATEN DER AORTA”. MA THESIS. OTTO-VON-GUERICKE-UNIVERSITY, MAGDEBURG, GERMANY, 2015

See Section Publications in the appendix for the division of work.

There are different types of clustering techniques, depending on what they take into account (recall Section 5.2.2.3). Since we focus on cardiac 4D PC-MRI data, two major classes are relevant:

- Vector field-based methods, e.g., the hierarchical clustering by Van Pelt et al. [527], that operate directly on the voxel grid of the acquired image data, and
- Integral curve-based methods, e.g., the generation of representatives by Born et al. [62], that analyze similarities of calculated flow curves.

Figure 84: The average distance of path lines to their closest projection onto the main centerline (green), which is color-coded in the image, can help to further distinguish nearby vortices. Image from [360], reprinted with permission by M. Meuschke.

8.4.1.1. DISSIMILARITY FUNCTION

This work builds upon the previously described extraction of vortex flow-representing path lines with line predicates (recall Section 8.1). Therefore, we pursued the integral curve-based approach and established a pairwise dissimilarity function for path lines.

Assuming continuous path lines from the extraction, a single vortex is connected in space and time. Thus, the first employed measure analyzes the pairwise spatio-temporal distances of the extracted path lines. We further assume that two path lines of the same cluster start and end at similar spatio-temporal positions. Therefore, instead of computing pairwise distances for the whole path lines, only the start and end points are considered. This simplification was mainly made to speedup calculations.

As an additional measure, the distance of a path line to the centerline was incorporated. It is calculated as average of the distances of each path line point to its closest projection onto the centerline. This can be useful to further distinguish nearby vortices (see Figure 84).

This yields five measures that describe the pairwise path line dissimilarity:

1. The Euclidean distance of the start points,
2. the Euclidean distance of the end points,
3. the temporal distance of the start points,
4. the temporal distance of the end points, and
5. the difference of the average centerline distances.

Each measure is normalized to $[0, 1]$ using corresponding maximum values. For Euclidean distances, this is approximately the length of the centerline. The maximum distance in time is derived from the cyclic dataset’s temporal resolution. The maximum occurring vessel radius, which is determined in the centerline construction based on maximum inscribed spheres [10] and provided by VMTK [410], is used to normalize the average centerline distances. A matrix $M$ is composed with entries $M_{ij}$ that describe the dissimilarity of the path lines $i$ and $j$. Consequently, the main diagonal is composed of zeros and $M$ is symmetric.

More elaborate measures, but with higher computational effort, could be employed as well. For instance, geometry- or flow field-based properties could be incorporated, as done by, e.g., Oeltze et al. [387].

### 8.4.1.2. Clustering Methods

The number of vortices in a dataset often is between 0 and 4. However, the concrete value is unknown. Therefore, we focused on clustering methods that do not require the expected cluster number as a parameter. The following three approaches were tested:

1. Agglomerative hierarchical clustering (AHC).
2. Spectral clustering (SC).
3. Density-based spatial clustering of application with noise (DBSCAN).

AHC initializes each path line as a separate cluster and then recursively merges the two closest clusters until only a single cluster remains. This yields a hierarchy (dendogram, recall Section 5.2.2.3), which facilitates the quick comparison of results with different cluster numbers. Four proximity measures were compared [113]:

1. Single link determines the minimum distance of (single objects from) two clusters.
2. Complete link determines the maximum distance of (single objects from) two clusters.
3. Average link (also: centroid method) determines the distance of two clusters’ (objects’) mean values.
4. Ward’s method [541] finds minimal values of an arbitrary objective function. The sum of squared distances is often employed, which yields Ward’s minimum variance method.

Besides the dissimilarity matrix $M$ and the employed proximity measure, AHC has no further parameters. An automatic suggestion for the optimal number of clusters was derived from the $L$-method by Salvador and Chan [441], which finds the knee in a graph of different cluster numbers (x-axis) and cluster merge distances (y-axis).

SC generates a lower dimensional feature space based on an eigenvalue decomposition of the dissimilarity matrix $M$. In this space, each path line is represented as a single point, facilitating the use of $k$-means clustering. Required parameters are the number of desired clusters and a control for the cluster similarity. Both can be estimated automatically with the self-tuning SC method by Zelnik-Manor and Perona [576].

DBSCAN identifies dense regions and requires two parameters:

1. $\epsilon$ defines the maximum allowed distance for grouping objects into one cluster.
2. $\text{minObjects}$ defines necessary minimum number of objects that form a valid cluster.

Objects with less than $\text{minObjects}$ neighbors in their $\epsilon$-neighborhood are considered as outliers. $\text{minObjects}$ was set to 1 in our tests, so that each path line can be a cluster. $\epsilon$ was set to 10% of the maximum occurring value in the dissimilarity matrix $M$ (experimentally determined). Smaller and larger values led to smaller and larger clusters, respectively, that did not reproduce the ground truth from manual classification.
8.4. Vortex Characterization in the Aorta

<table>
<thead>
<tr>
<th>Name</th>
<th>Num. Vort.</th>
<th>AHC SL</th>
<th>AHC AL</th>
<th>AHC CL</th>
<th>AHC WM</th>
<th>DBSCAN</th>
<th>SC</th>
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<td>all (3)</td>
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<td>all (3)</td>
<td>all (3)</td>
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<td>all (3)</td>
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<tr>
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<td>all (7)</td>
<td>3 (3)</td>
<td>all (3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Clustering results using strongly (first seven) and coarsely (last eight) filtered path line sets. Num. Vort. is the manually determined number of clusters. AHC \{SL, AL, CL, WM\} is AHC in combination with single, average, and complete link as well as Ward’s method. If all vortices could be clustered correctly, the number of applied post-processing steps (interactions) is color-coded as zero (green), up to three (orange) and more than three (red). In brackets are the number of clusters that the methods determined automatically as initial solution. AHC with average link was the only method capable of resembling all manual results.

### 8.4.1.3. Results

In the first part of the evaluation, the most suitable clustering method is determined. We used seven datasets of patients with various diseases (first seven datasets of Table 2). The extracted path lines in these datasets were post-processed manually with a bending energy predicate, as described in Section 8.1. Each patient has at least one pathologic vortex inside the aorta. The maximum number of occurring vortices in a patient is 4. A manual separation (definition of vortex entities) that was employed as a ground truth was performed by the co-author Uta Preim – a radiologist specialized on the cardiovascular system.

The user is allowed to interactively change the number of clusters if the automatic suggestion is not appropriate. Also, clusters can be merged or excluded manually as a post-processing. Up to three interactions were considered as feasible.

AHC with average link was the only method that could reproduce all manual results correctly with up to three interactions each. Therefore, this is considered as most suitable method. The computational effort depends directly on the number of extracted path lines. Between 2 and 9 s were required on an Intel Core i7 with 2 GHz. AHC with single link sometimes connects dissimilar objects through a series of similar objects. Complete link and Ward’s method work better, but still not as good as average link. DBSCAN produces a fixed cluster configuration instead of a hierarchy. It also tends to connect close clusters, and it is difficult to find generally applicable parameters. SC could not separate the vortices correctly in one case and required more than three post-processing interactions in three cases.

In the second part of the evaluation, eight further datasets are examined where the path lines were filtered without the bending energy post-processing (last eight datasets of Table 2). Therefore, the line sets might contain more noise and laminar flow. This test is only performed with AHC – the previously detected winner. Figure 85 exemplary depicts five results. Again, AHC was most reliable in combination with average link, although more than three post-processing interactions were required in four cases.

All path lines are shown in a static view, i.e., the temporal component is ignored so that they are visible all at once with full opacity. Each cluster is visualized in an individual color. Pastel colors are used, as they were appraised as professionally looking by our clinical collaborators.
8.4. VORTEX CHARACTERIZATION IN THE AORTA

8.4.1.4. DISCUSSION

We presented an integral line-based clustering method for vortex flow-representing path lines that were extracted from 4D PC-MRI data of the aorta. The technique facilitates the subsequent analysis of single vortex entities, which is described in Section 8.4.2. A comparison of three different clustering methods was carried out. Agglomerative hierarchical clustering (AHC) in combination with average link was most reliable for both strongly and coarsely filtered path line sets. The amount of necessary interactions for post-processing the determined clusters increased for coarse filtering, but was still below or equal three in many cases.

FUTURE WORK: An adaption of the clustering to branching vessels, such as the pulmonary artery, is a future topic. Here, the measure regarding the average centerline distance is not directly applicable. Furthermore, the straight use of Euclidean distances introduces errors, e.g., if the distance from the aortic valve to the descending aorta is evaluated. Instead, the real intravascular distance (following the centerline) should be considered. Alternatively, distances could be measured in a straightened vessel [8]. Cluster ensembles, as proposed by Strehl and Ghosh [492], could be used to combine and optimize different clustering results. However, this would increase the computational effort significantly.
8.4. VORTEX CHARACTERIZATION IN THE AORTA

8.4.2. CLASSIFICATION

This section is based on:

- [360]: M. MEUSCHEK. “COMPUTERGESTÜTZTE ANALYSE VON VERWIRBELUNGEN IN 4D PC-MRI BLUTFLUSSDATEN DER AORTA”. MA THESIS. OTTO-VON-GUERICKE-UNIVERSITY, MAGDEBURG, GERMANY, 2015

See Section Publications in the appendix for the division of work.

Manual classification approaches for cardiovascular vortex flow, as performed by recent medical works, were described in Section 5.3.1.3. The aim of this work is to provide a semi-automatic method for this task. This does not only speedup the classification process itself, but also increases the result reproducibility.

8.4.2.1. REQUIREMENT ANALYSIS

The classification of single vortex entities, based on extracted path lines, requires a reliable clustering that is able to separate vortices with low spatio-temporal distances correctly and to exclude laminar flow as well as noise. Such an approach, which forms the basis for this work, was described in the previous section.

The employed criteria for the classification should resemble the ideas of already established measures. A drawback of some proposed criteria is their binary nature, even if the described characteristic shows a continuous behavior. For example vortex shapes, which refer to the forward movement, are separated strictly into helical and vortical. The threshold is subjective, leading to potentially high inter-observer variability. However, the underlying property, which might be represented by the average angle between the vortex flow and the centerline direction, provides continuous values. This should be exploited to increase the expressiveness of vortex descriptors. Due to the large diversity of anatomic situations and intravascular flow behaviors, the user should be able to correct results if necessary.

Besides a tabular summary, an overview visualization will be established. For this purpose, the previously introduced vortex plot (recall Section 8.2) will be reused. In addition, a new glyph representation is proposed.

![Figure 86](image_url)

Figure 86: Three basic path lines shapes that are distinguished in the vortex classification. Eigenvalues are derived from a principal component analysis (PCA) of the spatial components of the path line points.

(a) Laminar flow.
(b) Vortical flow.
(c) Helical flow.

Images from [360], reprinted with permission by M. Meuschke.
8.4. VORTEX CHARACTERIZATION IN THE AORTA

8.4.2.2. CRITERIA

VORTEX SHAPE: The path lines of a cluster are analyzed individually for their shape via principal component analysis (PCA). Therefore, the covariance matrix of the spatial positions of one path line’s points is calculated (the temporal component is ignored), and an eigenvalue analysis is performed. The three eigenvalues $\lambda_{1,2,3}$ are sorted descendingly by their absolutes, so that $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$. The eigenvalues are real-valued, since the covariance matrix is symmetric. A path line shape can be (see Figure 86):

- **Laminar**: A straight line. Here, two eigenvalues are 0 and the third is 1.
- **Vortical**: Rotating in the plane with low to no forward movement. Here, two eigenvalues are equal and the third is 0.
- **Helical**: Rotating with stronger forward movement. Here, two eigenvalues are equal and the third one is not 0.

However, path lines will not take one of these idealized shapes. Instead, they will be intermediate shapes that can be described as a compression or stretching of a perfect helix:

$$C(t) = \begin{pmatrix} r \cdot \cos(2 \cdot \pi \cdot t) \\ r \cdot \sin(2 \cdot \pi \cdot t) \\ h \cdot t \end{pmatrix}$$

$t$ is the number of turns of the helix and $r$ is the radius. The influence of $h$, which refers to the elongation, is depicted in Figure 87. Seven different helix shapes, resulting from successively increasing $h$, and the corresponding eigenvalues, which are connected as a parabolic curve, are shown. The parabola is separated into three sections (empirically determined):

1. Vortical flow is until point 3 (beige).
2. Helical flow is from point 3 (beige) to point 7 (last point, blue).
3. Laminar flow is behind point 7 (last point, blue).

The shape analysis identifies the closest point on the parabola and derives the path line classification. However, the mere eigenvalue analysis is not reliable in every case, since the shape behavior of a path line can change along its course. A line that starts vortically and then proceeds laminarly is false-positively classified as helical. As a remedy, the torsion is calculated for all points of path lines that were classified as helical. False-positive helices have a low torsion, whereas true-positive ones have a high torsion. A
threshold is required to separate the two. The torsion $\tau$ of the curve from Equation 40 is given by:

$$\tau = \frac{h}{r^2 + h^2}$$

(41)

Figure 88 shows the relationship of the torsion $\tau$ and the elongation parameter $h$ for $r = 1$. $\tau$ increases for $h < 1$, which represents the transition from a vortical to a helical line. It reaches its maximum of $\tau_{\text{max}} = 0.5$ at $h = 1$ and then decreases monotonously, which is the transition from a helix to a straight line (laminar flow). We set the threshold $\tau_{\text{thresh}}$ to $c \cdot \tau_{\text{max}}$, where $c$ is an empirically determined constant depending on the average vessel radius $r_{\text{avg}}$, which was 7–13 mm in the available datasets:

$$c(r_{\text{avg}}) = \frac{1}{2} \cdot (r_{\text{avg}} - 13) + 3$$

(42)

Equation 42 was derived from a manual categorization of the correction term $c$, as shown in Table 3. Figure 89 depicts an example of the effect of the employed torsion correction.

**Table 3:** Manual specification of the correction term $c$ for the torsion threshold $\tau_{\text{thresh}}$, depending on the mean vessel radius $r$.

<table>
<thead>
<tr>
<th>Mean vessel radius $r$</th>
<th>$c$</th>
</tr>
</thead>
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</tr>
<tr>
<td>9–11 mm</td>
<td>1.5</td>
</tr>
<tr>
<td>11–12 mm</td>
<td>2</td>
</tr>
<tr>
<td>12–13 mm</td>
<td>2.5</td>
</tr>
<tr>
<td>13 mm</td>
<td>3</td>
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</tbody>
</table>

Figure 88: The torsion is determined for helically shaped path lines to perform a correction step. The torsion rises until the elongation parameter $h$ is 1, which describes the transition from a vortical to a helical path line. Afterwards, it decreases monotonously, which describes the transition from a helical to a laminar path line. A torsion threshold $\tau_{\text{max}} = 0.5$ is used for $r = 1$.

Image from [360], reprinted with permission by M. Meuschke.

Figure 89: Classification of the vortex shape without (a) and with (b) torsion correction. Blue, red and green lines represent vortical, helical and laminar flow, respectively. Without torsion correction, many vortical lines are false-positively classified as helical.

Images from [360], reprinted with permission by M. Meuschke.
**8.4. Vortex Characterization in the Aorta**

**Figure 90:** Systole and diastole are distinguished for the temporal occurrence of vortices. Both are separated by the red points. They are automatically determined, but can be corrected by the user if necessary.

(a) Healthy volunteer with no back flow (negative flow rates).

(b) Patient with slightly increased regurgitation fraction.

Images from [360], reprinted with permission by M. Meuschke.

**Figure 91:**

(a) Automatic cusp point (red) detection.

(b–c) Definition of the ascending aorta (red), ascending aortic arch (green), descending aortic arch (purple) and descending aorta (orange).

Images from [360], reprinted with permission by M. Meuschke.

**Temporal Occurrence:** We decided to keep the classification from medical works, so that a vortex can occur during systole, diastole, or both. This requires the definition of two temporal positions: The end of the systole, which is also the beginning of the diastole, and the end of the diastole, which corresponds to the beginning of the systole (see Figure 90). A flow rate curve (recall Section 6.1.1), obtained in the ascending aorta, is used for this purpose. The required measuring plane was placed manually once for each dataset.

If there are no negative flow rates, the first local minimum is used as systolic end point (see Figure 90a). Otherwise, the first zero crossing is used (see Figure 90b). Analogously, the last local minimum or last zero crossing marks the diastolic end point. Automatic suggestions are made, but the user is allowed to perform a manual correction with a simple drag and drop interaction.

**Vessel Section:** To automatically define vessel sections in the aorta, we rely on that there is usually a centerline cusp in the aortic arch (see Figure 91a). In the internal coordinate system, the cusp is simply the centerline point with the minimum y-coordinate. Then, the centerline length $L$ between the first and the cusp point is determined. It is assumed that the centerline starts at the approximate aortic valve location. One third of $L$ is used to separate the ascending aorta from the aortic arch ($cusp - L/3$) and the aortic arch from the descending aorta ($cusp + L/3$). For this work, the aortic arch was separated into ascending and descending aortic arch (see Figures 91b–c).
Again, manual corrections are possible via dragging and dropping the suggested cusp point along the centerline. From discussions with domain experts, we derived that a path line is considered as \textit{in} a specific vessel section when at least 30\% of its points lie in the vessel section.

**Vortex Size:** Medical publications define minor and major vortex flow for vortices that occupy less and more than 50\% of the vessel, respectively. They do not take into account that the size can vary along the course of the vortex.

Vortex core lines can be challenging to compute in 4D PC-MRI data due to low SNR in the image data and other artifacts. Therefore, we calculate a 3D curve $C(u)$ per vortex as approximation of its centerline. This is done via least-squares spline approximation (ALGLIB) of all corresponding path line points of the (clustered) vortex. This yields a piecewise polynomial curve of degree $k$ with $l$ sections. Based on our experiments, $l = k = 4$ was determined as a suitable parameter set. Higher values incorporated outliers too much and lower values did not properly capture the vortex courses. $C(u)$ has three dimensions:

$$C(u) = \begin{pmatrix} f_1(u) \\ f_2(u) \\ f_3(u) \end{pmatrix}$$  

A 1D spline is calculated for each component $f_{(1,2,3)}$, which first requires an ordering of all path line points. For this, the (normalized) eigenvector $\vec{e}$, corresponding to the largest eigenvalue from the PCA shape analysis, is used. Every path line point $\vec{p}_i$ is projected onto $\vec{e}$, yielding a scalar:

$$d_i = \vec{p}_i \cdot \vec{e}$$  

All $d_i$, are sorted ascendingly afterwards. The index $i$ serves as x-component in the spline fitting and the actual point coordinates of $\vec{p}_i$, belonging to $d_i$, are used as y-component. The resulting spline is then sampled in equidistant steps, yielding points $c_j$ with 2 mm distances.

A local coordinate system (LoCoSys) is generated for each $c_j$ where the z-axis corresponds to the centerline tangent. The orientation of subsequent LoCoSys is consistent (see Section 9.1.2.3). The x- and y-axis span a plane in the cross-section. For each path line point, the closest projection onto one of these planes is determined and stored. Then, for each plane with the projected points, a 95\% confidence ellipse is calculated. Thus, 5\% are declared as outliers. This is based on the assumption that the points are normal distributed. The ratio of the ellipse area and the area of the cross-section yields the percentage that is occupied by the vortex in this cross-section. The cross-sectional area is estimated as circle with the vessel radius that was derived from the centerline extraction based on maximum inscribed spheres [10, 410]. To resemble the physicians’ global minor / major classification, the average over all analyzed cross-sections can be calculated.

![Figure 92: Illustration of right- (a) and left-handedly (b) swirling flow in the aorta. Images from [363] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.](image)

**Rotation Direction:** We distinguish right-handed (clockwise) and left-handed (counterclockwise) vortex movements (see Figure 92). Each spatial path line segment (the temporal component is ignored) with its points $\vec{p}_k$ and $\vec{p}_{k+1}$ is projected onto the closest cross-sectional plane belonging to $c_j$ (see previous paragraph). This yields the projected points $q_k$ and $q_{k+1}$ in xy-coordinates of the LoCoSys. The atan2 of the projected points provides an angle $\alpha_k$ and $\alpha_{k+1}$ for each that allows to determine in which quadrant the points are in. We consider three quadrant configurations and propose different solutions (see Algorithm 3 and Figure 93):
1. Both points are on the top or bottom half. Here, a simple comparison of $\alpha_k$ and $\alpha_{k+1}$ allows to determine the rotation direction.

2. Both points are on the right or left half. Here, the order of the first and second points’ quadrants are compared.

3. The points are in diagonal quadrants. Here, it is first determined, which point is in the right and left half. Then, the intersection of the y-axis and a line connecting $q_k$ and $q_{k+1}$ is determined and evaluated.

A path line is classified as right- or left-handedly swirling, depending on the majority of rotation directions of the single segments. In the same manner, a vortex is classified as right- or left-handed according to the majority of path line rotation directions. Additionally, this allows to derive percentages of present rotation directions, which can be used instead of the binary classification.

\[
\begin{align*}
q_k &\leftarrow \text{2D projection of } \vec{p}_k \\
q_{k+1} &\leftarrow \text{2D projection of } \vec{p}_{k+1} \\
\alpha_k &\leftarrow \text{atan2}(q_k.y, q_k.x) \\
\alpha_{k+1} &\leftarrow \text{atan2}(q_{k+1}.y, q_{k+1}.x)
\end{align*}
\]

// both on top or bottom half
if ($\alpha_k > 0$ & $\alpha_{k+1} > 0$) || ($\alpha_k < 0$ & $\alpha_{k+1} < 0$) then
    return ($\alpha_k < \alpha_{k+1}$ ? CCW : CW)
else
    // both on left or right half
    if ($\alpha_k > 0$ & $\alpha_{k+1} < 0$ & $\alpha_k < 0$ & $\alpha_{k+1} > 0$ & $\alpha_k - \pi/2 < \alpha_{k+1} + \pi/2$) then
        return CCW // top right to bottom right quadrant
    else if ($\alpha_k > \pi/2$ & $\alpha_{k+1} > -\pi/2$ & $\alpha_k < -\pi/2$ & $\alpha_{k+1} \leq \pi / 2$) then
        return CCW // bottom right to top right quadrant
    else if ($\alpha_k > \pi/2$ & $\alpha_{k+1} < -\pi/2$ & $\alpha_k < -\pi/2$ & $\alpha_{k+1} > \pi/2$) then
        return CCW // top left to bottom left quadrant
    else if ($\alpha_k < \pi/2$ & $\alpha_{k+1} > \pi/2$ & $\alpha_k > \pi/2$ & $\alpha_{k+1} < \pi/2$) then
        return CCW // bottom left to top left quadrant
    else
        // diagonal quadrants: evaluate y-axis intersection
        d $\leftarrow q_k - q_{k+1}$
        l $\leftarrow -q_{k+1}.x / d.x$
        s $\leftarrow q_{k+1}.y + l \cdot d.y$
        if (($\alpha_k > 0$ & $\alpha_{k+1} \leq \pi/2$) || ($\alpha_k \leq 0$ & $\alpha_{k+1} > -\pi/2$)) then
            return ($s > 0$ ? CCW : CW) // q_k on right half
        else
            return ($s > 0$ ? CCW : CW) // q_k on left half
    end if
end if

Algorithm 3: RotDir.CrossSection(): Determination of the turning direction of a line segment in a cross-section.
8.4. VORTEX CHARACTERIZATION IN THE AORTA

Figure 93: Four examples for the atan2-based determination of the rotation direction in the cross-section. The blue (first) to the red point (second) are a path line segment projected into the plane.
(a) Points are in the same sector.
(b) Points are in neighboring sectors.
(c–d) Points are in diagonal sectors.

8.4.2.3. VISUALIZATION

We provide a simultaneous 2D and 3D visualization to interpret the results.

2D VISUALIZATION: For the 2D visualization, the previously proposed polar overview plot for aortic vortex flow (recall Section 8.2) is adapted. The plot angle still represents the temporal component and the distance from the plot center maps the centerline course. Positions close to the center correspond to the approximate aortic valve location and positions on the outside represent the descending aorta. Instead of continuous distance values, we use four discrete sections for this work, which refer to the four vessel sections:

1. Ascending aorta,
2. ascending aortic arch,
3. descending aortic arch, and
4. descending aorta.

Each plot segment is colored according to the portions of vortex clusters that lie in the corresponding vessel section. We demand that at least 30% of the path line points of a cluster lie in the vessel section. Otherwise, the corresponding plot segment is not colored. The threshold was derived from discussions with domain experts.

3D VISUALIZATION: The 3D visualization is intended to convey the vortex size and turning direction. First, a vortex hull is determined. For this purpose, the confidence ellipses from the previous section are reused (see Figure 94c). Each ellipse is sampled with a fixed number of points that are then transformed back from the local coordinate system (LoCoSys) into world coordinates. Since all LoCoSys have a consistent orientation and the ellipse sampling always starts at the same angle, correspondence of the sampled points of neighboring contours can easily be derived via the points’ indices. This is used to establish a triangulation (see Figure 94d). The vortex size is mapped to the surface color of the glyph (see Figure 94e). A linear blue-to-red color scale was used to emphasize if more than 50% of the vessel are occupied. The rotation direction is depicted as quadstrip around the vortex hull, using an approach by Lawonn et al. [307]. Arrows on the quadstrip are depicted in the original cluster color to establish a link to the 2D visualization.

8.4.2.4. CLASSIFICATION RESULTS

We performed a comparison of 15 datasets against a manually generated ground truth. There were 2 healthy volunteers and 13 patients with various diseases, such as a dilated ascending aorta, coarctation, or bicuspidalitidy of the aortic valve. Each patient has at least one non-physiological vortex somewhere in the aorta.
The ground truth comprises clustering and classification. It was generated manually in collaboration with Uta Preim – one of the coauthors of this work. A standard flow animation with pathlets as well as static visualization of the extracted vortices were used for this purpose. Especially the vortex size was difficult to assess, so that the overall process took 3 h.

The maximum number of vortices in one patient was four. Ten different colors were used to depict the results of the agglomerative hierarchical clustering (recall Section 8.4.1). The six extra colors show laminar flow or noise. Overall, there were 30 vortices in all datasets.

The test computer was an Intel Core i7 with 2 GHz, 12 Gb RAM, and a GeForce GT540M. The implementation is completely CPU-based. The overall computation time for the clustering and the classification was 12–20 s per case, depending on the number of extracted path lines. Figures 95 and 96 exemplary show the results for six cases.
VORTEX SHAPE: All vortex shapes were correctly classified as helix or vortex. Table 4 shows the manual vortex classification and the percentaged amount of helical and vortical path lines that was determined by our approach.

TEMPORAL OCCURRENCE: The systolic and diastolic end points were determined correctly for all datasets. This facilitated the subsequent, accurate classification of each vortex. Systolic vortices were, on average, present during systole by 90.7 % and during diastole by 9.3 %. Vortices that occurred in both cardiac phases were by 68 % systolic flow and by 32 % diastolic flow.

VESSEL SECTION: Five out of ten cases had to be corrected manually by the user. Patients with vessel morphologies that differ strongly from healthy volunteers can be problematic, e.g., if a bypass surgery was performed. The subsequent determination of the vessel sections of each vortex cluster coincided with the ground truth.

VORTEX SIZE: Minor vortex flow was manually classified in eight cases. Here, our approach determined sizes of 16.8–37.1 %, which matches well. Six cases could not be classified manually with certainty. The automatic classification provided sizes slightly below 50 % for three cases (46.9–48.3 %) and sizes slightly above 50 % for the other three cases (50.9–53.6 %). The rest was classified as major vortex flow both manually and automatically.

One vortex (D2 brown) with 37.1 % was overestimated by our method. Here, the assumption that the projected points in the cross-section are normally distributed did not hold. The overestimated size may cause the glyph surface to protrude out of the vessel. In two cases (D2 brown, D3 blue and brown), the employed 95 % confidence ellipses in the cross-section excluded too many path line points from the calculation. The extracted path lines in this dataset had fewer outliers than in other datasets.

ROTATION DIRECTION: The rotation direction was determined correctly 28 times. Two vortices are neither left- nor right-handed. Instead, their core is aligned perpendicularly to the vessel’s centerline. We call this roll over rotation. This is not considered in our current approach and, thus, cannot be classified correctly.
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8.4.2.5. USER STUDY RESULTS

A user study was performed with 12 participants ($P$): 1 physician and 11 researchers with background in medical visualization. Our proposed visualization of clustering and classification results was compared
8.4. Vortex Characterization in the Aorta

to a static visualization of the path lines (temporal component is ignored) and animations.

**Setup:** The participants had to assess:
1. The vessel section (ascending / descending aorta or aortic arch).
2. The temporal occurrence (systole, diastole, or both).
3. The rotation direction (left- or right-handed).
4. The vortex size (fills on average below (minor) or above (major) 50% of the vessel diameter).

The test setup was as follows. Four datasets were used for each task. Two were visualized with both our proposed methods. The 2D plot was used for Task 1 and 2, the 3D glyph for Task 3 and 4. The other two datasets were depicted with the standard path line visualization.

The participants were divided into two groups: A and B. Both groups assessed the same datasets. A always started with the standard visualization, whereas B started with our methods. Thus, each vortex was evaluated with both the standard and our method. The goal of changing the techniques was to reduce learning effects.

The results were compared to the ground truth, and verbal comments from the participants were noted. During evaluation, a questionnaire with two questions was answered with a five-point Likert scale (−−, −, −, +, ++):

1. How difficult was it to solve the task based on the particular visualization method? (In the following tables referred to as **Simplicity**)
2. How certain are you with your assessment? (In the following tables referred to as **Certainty**)

For the interpretation of the Likert scale we provide two values in the following:
1. $m$ is the most frequently given answer.
2. $P$ is the number of participants that answered with + or ++.

**Task 1 – Vessel Sections:** Each four vortices (V1–V4) had to be assessed with the 2D plot and the standard path line visualization by every participant. V1 lies in one vessel section and was correctly assessed by all participants with both techniques. V2–V4 occupy multiple sections. All 36 classifications (12 participants × 3 vortices) of V2–V4 were done correctly using the 2D plot. Additional vessel sections were determined in 8 cases with the standard visualization. The other 28 were assessed correctly. Estimating the percentages of vortices in vessel sections was described as difficult using the standard visualization, which leads to uncertainties. Table 5 shows the answered questionnaire.

<table>
<thead>
<tr>
<th>Simplicity</th>
<th>Certainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$P$</td>
</tr>
<tr>
<td>2D plot</td>
<td>++</td>
</tr>
<tr>
<td>Path line visualization</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 5: Questionnaire results regarding the assessment of the vessel sections. Table from [363] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.

**Task 2 – Temporal Assessment:** The classification of the cardiac phase, systole and / or diastole, was done for three vortices (V1–V3) per technique. V1 is systolic; V2 and V3 occur during both systole and diastole. All vortices were classified correctly with the 2D plot. With the standard path line visualization, two participants classified V2 as systolic only. Table 6 shows the answered questionnaire.

<table>
<thead>
<tr>
<th>Simplicity</th>
<th>Certainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$P$</td>
</tr>
<tr>
<td>2D plot</td>
<td>++</td>
</tr>
<tr>
<td>Path line visualization</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 6: Questionnaire results regarding the temporal assessment. Table from [363] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.
8.4. VORTEX CHARACTERIZATION IN THE AORTA

**TASK 3 – ROTATION DIRECTION:** Three vortices (V1–V3) per technique (3D glyph and standard path line visualization) were assessed in Task 3. About 90 % of V1, located in the ascending aorta, have the same rotation direction. This was classified correctly with both techniques. V2 has a rotation direction distribution of about 70 % : 30 %, which is challenging. V3 lies in the descending aorta. With the standard visualization, V2 and V3 were classified correctly 11 of 18 times. Both were assessed correctly with the 3D glyph. In contrast to the standard visualization, it abstracts the flow, making the analysis less dependent on the local flow complexity. This is reflected in a higher simplicity and certainty in the questionnaire (see Table 7).

<table>
<thead>
<tr>
<th>Simplicity</th>
<th>Certainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D glyph</td>
<td>++ 12/12</td>
</tr>
<tr>
<td>Path line visualization</td>
<td>− 1/12</td>
</tr>
</tbody>
</table>

**Table 7:** Questionnaire results regarding the assessment of the rotation direction.

**TASK 4 – VORTEX SIZE:** Three vortices (V1–V3) per technique were classified according to their size. V1 was minor (below 40 %) or major (above 60 %). This was correctly assessed with the 3D glyph and 3× wrong with the standard path line visualization. V2 and V3 were both rather minor or rather major (50 ± 10 %). One participant wrongly classified a rather major vortex, whereas 18 classifications were wrong using the standard visualization. The problems with the standard visualization coincide well with the answerer questionnaire (see Table 8). Using the standard visualization, a high degree of interaction (scene rotation) was necessary to assess the sizes. This was not required with the color-coded glyph.

<table>
<thead>
<tr>
<th>Simplicity</th>
<th>Certainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D glyph</td>
<td>+ 10/12</td>
</tr>
<tr>
<td>Path line visualization</td>
<td>− 1/12</td>
</tr>
</tbody>
</table>

**Table 8:** Questionnaire results regarding the assessment of the vortex size.

### 8.4.2.6. DISCUSSION AND CONCLUSION

We presented an approach for the semi-automatic classification of extracted, vortex flow-representing path lines. A previously described clustering method was used to group single vortex entities (recall Section 8.4.1). The considered vortex criteria are in accordance with recent medical publications. These are: The vortex size, shape, rotation direction, and its spatial (vessel section) as well as temporal occurrence. Manual classification has a high inter-observer variability. Among others, this is due to a lack of clear and objective definitions of the criteria.

Our approach facilitates a reproducible classification. The analysis is based on the path lines’ geometries as well as their relation to the vessel surface. User corrections can be performed prior to the analysis if necessary. This allows experts to incorporate their specialized knowledge. Since the approach is independent from the underlying 4D PC-MRI data, it is also applicable to other contexts, such as measured or simulated flow in cerebral aneurysms.

The results are presented with a 2D plot that was adapted from a previous work (recall Section 8.2). The plot conveys the temporal occurrence and the occupied vessel sections of the clustered vortices without occlusion. A 3D glyph was designed to present the vortex hull, size in the cross-sections, and the turning direction. The performed user study shows the benefit of our method compared to standard path line visualization and animation.

**FUTURE IMPROVEMENTS:** Eigenvalue analysis of a path line course is error-prone due to the curved vessel shape. For example, a laminar path line that follows the aortic arch is bent instead of straight. The problem could be alleviated if vessel straightening [8] is used prior to the analysis. However, this
increases the algorithm complexity. The size of a vortex is actually a time-dependent measure. A thin vortex with a centerline that is rotating over time like a rope would be classified as major, since it appears as if the whole vessel is occupied in the static visualization. This should be considered in the glyph design and calculation. The current glyph does not directly allow to distinguish between helical and vortical shapes. The forward movement of a vortex could be incorporated in the calculation of the band that shows the rotation direction. Another interesting property, which might be considered in a future work, is the proximity of a vortex to the vessel wall. The assumption of the vessel cusp detection was not always met. A more general approach could perform a PCA of the centerline, find the largest eigenvalue as well as eigenvector, and find the cusp in this local coordinate system. Right now, only vortex flow in the aorta can be assessed. An adaption for, e.g., the pulmonary artery, requires mainly an adaption of the vessel section determination.
9. ENHANCED FLOW RATE QUANTIFICATION

The genesis and evolution of cardiovascular diseases (CVDs) depends on various factors. Recent works have shown that atypical flow patterns, such as vortices, can be found in many pathologies of the cardiovascular system [159, 221]. Yet, when it comes to assessing the severity, quantitative measures are essential. However, quantitative analyses need to be reliable, reproducible, and sufficiently accurate.

It was pointed out that the assessment of flow rates suffers from various inaccuracies in the acquired 4D PC-MRI data (recall Section 6.1.1). Thus, two contributions are presented in this chapter. In Section 9.1 a method is presented that puts emphasis on robustness and physiological plausibility. Net flow volumes (NFVs) are determined automatically, the quantification uncertainty is emphasized, and the development of the NFV along the vessel is reconstructed. Section 9.2 describes an approach to automatically obtain 4D segmentations of the dynamic vessels, which is typically not done due to the high expenditure of time. The segmentations are used to establish a dynamic vessel surface that is visualized on the one hand and incorporated into the quantification of stroke volumes (SVs) on the other hand.

9.1. ROBUST CARDIAC FUNCTION ASSESSMENT

Two of the most important quantitative measures to assess the cardiac function are the stroke volume (SV) and regurgitation fraction (RF). The first describes the volume of pumped blood per heart beat and is the net flow volume (NFV) above the aortic or pulmonary valve. The second characterizes the percentaged backward flow volume (PBFV) into the corresponding ventricle during diastole. Both quantifications require the flow rate in forward and backward direction through a plane that is usually aligned orthogonally to the vessel’s centerline (recall Section 6.1.1). Unfortunately, the calculations are highly dependent on the plane’s configuration. Unlike 2D PC-MRI, where unsatisfying results make a whole new acquisition necessary, 4D PC-MRI datasets contain the patient’s full four-dimensional flow information. This allows to evaluate multiple planes with different positions and angulations after the scan.

In this work, various healthy volunteer and patient datasets were analyzed in close collaboration with radiologists and cardiologists. We carefully observed their manual approach to estimate SVs and RFs while anticipating the high sensitivity towards the plane angulation. From this, we derived an automatic procedure which systematically analyzes multiple angulations for one position on the vessel’s centerline. The obtained statistical information facilitate the robust quantification of the measures and the visualization of uncertainties.

The collaborating experts expect the SV, i.e., the highest NFV, directly above the aortic or pulmonary valve. From there, a continuous decrease along the vessel course is assumed due to smaller vessels that branch off and supply blood to certain body regions. However, noise and a low spatio-temporal data resolution often lead to physiologically implausible results, i.e., the NFV development is not monotonous. We employed these two expectations as heuristics to establish a procedure that fits a function which guarantees the desired behavior. In addition, we adapted this method for the use in branching vessels, such as the pulmonary artery.
9.1. Requirement Analysis

All necessary interactions should fit to the clinicians’ mental model and utilize their in-depth understanding of anatomical shapes including occurring variations, e.g., due to pathologies. Therefore, we have to avoid parameter settings that do not correlate to such medical knowledge and provide reasonable default values for numerical input whenever possible.

The automatic evaluation of different plane angulations should need only a small number of angulation samples in order to limit the computational effort. Thus, an adequate sample distribution is required. The obtained information about the NFV deviation shall be utilized to provide insight into uncertainties.

A proper function has to be determined that represents the expected continuous NFV decrease along the vessel course after a peak value (the SV) above the valve. To ensure that the NFV is properly preserved in case of branching vessels, a comparison and, if necessary, correction of the NFV before and after the junction is required.

To exploit the high computational power of the GPU, algorithm parallelization as well as an appropriate data structure for the measuring planes are required.

9.1.2. Method

In the following, we describe the NFV as well as PBFV evaluation with increased robustness for one measuring plane at a specific position on the centerline. Afterwards, we employ this procedure to determine a physiologically plausible NFV development along the vessel course and derive the SV as well as the RF. The last part is about necessary adjustments to facilitate usage in branching vessels.

9.1.2.1. Observations

Figure 97 shows the typical situation in a patient dataset with pathologic vortex flow where three slightly different measuring planes produce SVs of 35, 83 and 110 ml. Carefully observing our collaborating cardiologists and radiologists in such unsatisfying situations gave insight into their usual approach. They evaluate multiple measuring planes with slightly different positions as well as angulations and estimate a plausible SV based on their experience. We assume that this value is the mean or median of the samples. We also observed that the problem of uncertain quantification is reduced in healthy volunteer datasets.

![Figure 97: Slightly different measuring plane configurations produce 35 (a), 83 (b) and 110 ml (c) NFV in this patient with systolic vortex flow in the ectatic ascending aorta. Images from [278] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.]
9.1.2.2. Evaluation of a Single Position on the Centerline

In this section, we present the robust NFV quantification method which is an automation of the observed manual approach. We start with a detailed description of the employed measuring plane model and provide basic information about the NFV calculation. Then, we proceed with the systematic angulation sample generation, summarize our GPU-supported evaluation procedure, and apply this principle to the PBFV quantification.

Plane Model: Hoogeveen et al. [219] focused on small arteries. They made the simplifying model assumptions that a vessel is cylindrical, straight, and has a parabolic velocity profile. However, the aorta and pulmonary artery are the largest arteries in the body, curved, and the parabolic velocity profile can be disturbed in case of vortex flow. Therefore, their approach might not be suitable for the cardiac context.

We model a measuring plane as grid with the following components:

- $\vec{g} = (g_x, g_y)^T$, $g_{(x,y)} \in \mathbb{N}$ is a parameter that describes the number of rectangles in x- and y-dimension,
- $\vec{s} = (s_x, s_y)^T$, $s_{(x,y)} \in \mathbb{R}$ is the size per rectangle,
- $\vec{c} \in \mathbb{R}^3$ is a center position close to or on the vessel’s centerline,
- $\vec{n} \in \mathbb{R}^3$ is the plane’s normal vector, i.e., angulation, that is commonly set to the corresponding normalized centerline tangent, and
- $\vec{n}_s$ and $\vec{n}_y \in \mathbb{R}^3$ form a local orthonormal system with $\vec{n}$. $\vec{n}_s$ is determined via cross product of $\vec{n}$ with the x-axis $(1, 0, 0)^T$. If $\| \vec{n}_s \| = 0$, the cross product is performed with the y-axis. If the length of $\vec{n}_s$ is still 0, the z-axis is used. $\vec{n}_y$ is obtained as $\vec{n}_s \times \vec{n}$.

These information are sufficient to calculate world coordinates $\vec{w} \in \mathbb{R}^3$ for every grid position $(x, y)$:

$$\vec{w} = \vec{c} + (s_x \cdot \vec{n}_s \cdot (x - \frac{g_x}{2})) + (s_y \cdot \vec{n}_y \cdot (y - \frac{g_y}{2}))$$

(45)

The low memory requirements are advantageous for GPU computation.

Fitting a Plane to the Vessel: The size per rectangle $\vec{s} \in \mathbb{R}^2$ has to be determined by fitting the plane $P$ to the vessel. For this purpose, the plane’s intersection curve with the triangular vessel mesh is determined. Then, the maximum extents in $\vec{n}_s$ and $\vec{n}_y$ dimension are analyzed and $\vec{s}$ is set accordingly so that $P$ is the minimum axis-aligned bounding rectangle (AABR) of the vessel cross-section. $\vec{c}$ is set to the AABR’s center. Figure 98a shows an example of a plane fitted to the ascending aorta.

![Figure 98](image-url) 

(a) A measuring plane was fitted to the ascending aorta of this healthy volunteer. 
(b) The net flow volume (NFV) is obtained as integral of the time-dependent flow rate.

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**Basic Net Flow Volume Quantification:** The time-dependent flow rate \( fr(t) \), \( t \in [0, T-1] \), where \( T \) is the number of temporal positions in the dataset, is required to calculate the NFV for the plane \( P \):

\[
fr(t) = s_x \cdot s_y \cdot \vec{n} \cdot \sum_{x=0}^{g_x-1} \sum_{y=0}^{g_y-1} S_3(P(x,y)) \cdot V(P(x,y),t)
\]

with \( S_3(P(x,y)) = \begin{cases} 1, & \text{if } (P(x,y)) \text{ inside vessel} \\ 0, & \text{else} \end{cases} \) (46)

\( P(x,y) \) is the grid position \((x,y)\) transformed to world coordinates \( \vec{w} \) (see Equation 45). \( V(P(x,y),t) \) are velocity vectors \( \vec{v} \) in the flow field \( V \), which is given by the phase images. The product \( s_x \cdot s_y \) is the area per rectangular grid element. Since a 4D PC-MRI dataset contains one full heart beat, \( fr(t) \) is periodic. The integral of \( fr(t) \) provides the NFV, depicted in Figure 98b. A periodic spline fitting (ALGLIB) is utilized in the implementation. The vessel segmentation is employed to realize \( S_3(P(x,y)) \). This allows to exploit hardware-accelerated 3D texture lookups on the GPU.

**Plane Angulation Sample Generation:** The automatic evaluation of different plane angulation samples \( \vec{a} \), i.e., normal vectors, requires their systematic generation. All possible angulations form a half sphere, where the plane’s original normal vector \( \vec{n} \) points to the top. The maximum angle between \( \vec{n} \) and \( \vec{a} \) is \( \alpha = \pi/2 \approx 90^\circ \). Lower \( \alpha \) values restrict the angulation to a smaller sphere section. Thus, \( \alpha \) describes an angulation tolerance.

The inverse cumulative distribution function (CDF) method allows to generate uniformly distributed samples on a sphere within \([\theta_1, \theta_2]\) and \([\phi_1, \phi_2]\), where \( \theta \) and \( \phi \) are longitude and latitude, respectively:

\[
\begin{align*}
\theta &= \cos(\theta_1) + u \cdot (\cos(\theta_2) - \cos(\theta_1)) \\
\phi &= \phi_1 + v \cdot (\phi_2 - \phi_1)
\end{align*}
\] (47)

\( u \) and \( v \in [0, 1] \) are uniformly distributed random numbers. For \( \vec{n} = (0, 0, 1)^T \), the sphere section is described by \( \phi_1 = 0, \phi_2 = 2\pi \), \( \theta_1 = 0 \), and \( \theta_2 = \alpha \in [0,\pi/2] \). The samples are transformed from spherical coordinates \((1, \theta, \phi)\) to Cartesian coordinates \((x, y, z)\) and used to obtain the angulation sample \( \vec{a} = x \cdot \vec{n}_x + y \cdot \vec{n}_y + z \cdot \vec{n}_z \), where \( \vec{n}_x \) and \( \vec{n}_y \) are part of the plane’s local orthonormal system (LoCoSys).

We generate Poisson-distributed angulations [254] to ensure that the whole sphere section is covered with a small number of samples, which limits computational effort. The parameter \( d_{\text{min}} \) describes the minimum distance of two angulations on the unit sphere surface. A new distribution is calculated for each plane evaluation to avoid bias. Figure 99 depicts the angulation samples.

![Figure 99: Poisson-distributed angulation samples (green) were generated on a \( \alpha = \pi/4 \approx 45^\circ \) sphere section (red) around the plane’s (blue) normal vector (orange).](image)

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9.1. Robust Cardiac Function Assessment

Robust Net Flow Volume Quantification: The NFV determination for one plane is summarized in Algorithms 4 and 5. The evaluation of all angulations provides a range of NFVs with an unknown distribution and possible outliers. Thus, the median of these samples is used as robust result.

Robust Percentaged Backward Flow Volume Quantification: The NFV determination is directly applicable for the PBFV quantification. This requires the forward (FFV) and backward flow volume (BFV). They are calculated as integral of the positive and negative part of the flow rate \( f_r(t) \), i.e., they are the curve’s area above and below zero, respectively. For this purpose, the integral in the ROBUST_NFV procedure (see Algorithm 4) is adapted accordingly. Analogous to the NFV, the FFV/BFV distributions’ median values are used as result. The PBFV for one position on the centerline is then obtained as BFV/(FFV + BFV).

9.1.2.3. Evaluation of a Junction-free Centerline

In the following, we explain how to obtain the SV as well as RF, and a process to fit a physiologically plausible function to the NFV development along the vessel course. The resulting function represents our collaborators’ expectations.

Denoising Plane Sizes: The fitting of planes to the vessel’s cross-section might produce degenerated sizes if the segmentation contains perpendicularly branching vessels. The scales \( s \) per rectangular grid element will be too large which negatively affects the NFV quantification. Therefore, we perform a smoothing of scales along the centerline as a pre-processing step. For this purpose, a consistent alignment of the planes’ local coordinate systems (LoCoSys) is required. Unfortunately, if the LoCoSys of each plane is constructed independently using cross products with the \( x-, y- \), and \( z- \) axis (recall Section 9.1.2.2), there will be a twist, as shown in Figure 100a. Therefore, only the first plane’s LCS \( \vec{n}^0, \vec{n}_y^0, \vec{n}_y^i \) is determined this way. Consistent orientation of \( \vec{n}^1, \vec{n}_y^1, \vec{n}_y^i, i > 0 \) with its predecessor

\[
P \leftarrow \text{measuring plane} \\
A \leftarrow \text{angulation samples} \\
\begin{aligned}
\text{for } i \leftarrow 0 \text{ to size}(A) - 1 \\ /*\text{normal}*/ \vec{P} \vec{n} \leftarrow A_i \\
\vec{P}_x, \vec{P}_y, \vec{P}_z \leftarrow \text{orthonormal system construction} \\
/*\text{scale}*/ \vec{P}_S \leftarrow \text{fit } \vec{P} \text{ to vessel mesh} \\
\text{GPU buffer} \leftarrow \text{add parameters of current } \vec{P} \\
\end{aligned}
\]
\[ \mathbf{n}^{i-1}, \mathbf{n}^i, \mathbf{n}^i \] is achieved by employing the angle \( \theta \) and rotation axis \( \mathbf{r}^i = (r_x^i, r_y^i, r_z^i)^T \) between \( \mathbf{n}^i \) and \( \mathbf{n}^{i-1} \):

\[
\theta = \cos(\mathbf{n}^i \cdot \mathbf{n}^{i-1}), \quad \mathbf{r}^i = \mathbf{n}^i \times \mathbf{n}^{i-1}
\]  

(48)

If \( \| \mathbf{r}^i \| = 0 \), i.e., the normal vectors point in the same direction, then \( \mathbf{n}^i = \mathbf{n}^{i-1} \) and \( \mathbf{n}^i = \mathbf{n}^{i-1} \). Otherwise, \( \mathbf{n}^i = R \cdot \mathbf{n}^{i-1} \) and \( \mathbf{n}^i = R \cdot \mathbf{n}^{i-1} \), where \( R \) is the general rotation matrix around \( \mathbf{r}^i \):

\[
R = \cos \theta \cdot I + \sin \theta \cdot \begin{bmatrix}
0 & -r_z^i & r_y^i \\
- r_z^i & 0 & r_x^i \\
-r_y^i & -r_x^i & 0
\end{bmatrix} + (1 - \cos \theta) \cdot (\mathbf{r}^i, \mathbf{r}^i)^T
\]  

(49)

\( I \) is the \( 3 \times 3 \) identity matrix. Figure 100b shows \( \mathbf{n}_i \) and \( \mathbf{n}_o \) of the aligned LoCoSys after resampling from a cubic spline (ALGLIB), where the parameter \( \rho \in [-15, 15] \) controls a non-linearity penalization. \( \rho = 2 \) is used as default value which, according to the library developers, corresponds to a moderate amount of non-linearity.

**STROKE VOLUME DETERMINATION:** The robust NFV determination (see Algorithm 4) is performed for each of the equidistant centerline points. The x-axis in Figure 101 describes the centerline points in flow direction so that the left part of the plot corresponds to the approximate valve location. The yellow boxes in Figure 101a show the interquartile ranges (IQRs), which are the middle 50% of the sorted NFV samples. Large ranges indicate high sensitivity to the angulation and thus uncertainty in the corresponding vessel section. The thin blue curve interpolates the median NFVs. The SV is obtained as maximum value of this curve. In the following, by NFV and NFV deviations we refer to the median and the IQR of the NFV samples at one centerline position, respectively.

We observed that the IQRs are the most interesting part of the box plots, since outliers, especially behind the valve, may reach unrealistic values, such as 10 or 200 ml, and thus barely provide further insight. Therefore, we modified the visualization by changing the box plots to a continuous area and removing the whiskers for the minimum and maximum. In addition, this visualization is independent from the centerline length, since no number of box plots needs to be specified. Figures 101a–b show a comparison.

The user has the option to limit the evaluation to a part of the centerline in between two specified planes. This can be useful, e.g., if the segmentation contains parts of the ventricles that shall be excluded from the calculation. Additionally, if only the SV and RF are of interest, a restriction to the ascending aorta or pulmonary trunk considerably reduces the computational effort.
**Figure 101:** (a) The net flow volume distributions, obtained with our systematic evaluation of plane angulations for each point of the centerline, are shown as box plots, where the yellow boxes represent the interquartile ranges (IQRs). (b) A modified visualization that is independent from the number of box plots shows the IQR as yellow area graph. (c) The fitted, physiologically plausible functions for the net, forward and backward flow volume are shown in red, green and orange, respectively. The resulting percentaged backward flow volume curve is shown in blue. 84.33 ml SV and 12.95 % RF were determined in this patient with an ectatic ascending aorta. The location is marked by the black vertical line.

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**Plausible Net Flow Volume Development:** Our collaborating cardiologists and radiologists expect a continuous decrease of the NFV after the SV. This is due to smaller vessels that branch off the main vessel and supply blood to certain regions of the body. Yet, even our robust quantification is often not able to produce this behavior. Instead, physiologically implausible local minima and maxima occur. Therefore, we describe a method to enforce this behavior by fitting a monotonous function to the partial NFV function (thin blue curve in Figure 101) right of the SV. In our implementation, this non-trivial task is realized with an iterative cubic, non-linearity penalizing spline fitting (ALGLIB). High and low penalization parameters $\rho$ lead to linear least squares approximation and interpolation, respectively. Our procedure is initialized with $\rho = -5$. After the spline is fitted, its monotonicity is
determined by evaluating the signs of all first derivatives. If it is not monotonous, \( \rho \) is slightly increased by 0.1, otherwise the loop stops.

All NFVs between the valve and SV location cannot be lower than the SV and thus are set to the SV as a correction. Unfortunately, \( C^0 \) continuity of the fitted monotonous function right and corrected function left of the SV is not guaranteed. As a remedy, both functions are concatenated and smoothed in 15 iterations using a one-dimensional binomial filter with kernel size 5. This is solely performed to increase the visual quality of the diagrams. Therefore, no further parameter analysis is necessary at this point. Figure 101c shows the resulting function in red.

The NFV only decreases due to branching vessels and is constant in vessel sections between two junctions. Following this, the question may arise why no step function is employed as model. This approach would require the locations of all branches and, more critical, a guarantee that all branches are present in the segmentation. The latter is problematic because of limited data resolution and low contrast in smaller vessels due to typically lower flow velocities.

**Regurgitation Fraction Determination:** The RF is obtained as PBFV at the SV location. If desired, monotonous functions for the FFV, BFV, and PBFV can be fitted in the same manner as for the NFV. An example is depicted in Figure 101c. The tasks are independent from one another and thus can be performed in parallel threads on the CPU.

### 9.1.2.4. Evaluation of Bifurcations

In the following, the extension of our method for branching vessels, such as the pulmonary artery (PA), is described. The user is required to specify three planes as an initialization. The first one is located at the beginning of the main vessel \( V_{\text{main}} \), the second and third are located at the end of the branches \( B_0 \) and \( B_1 \). Figure 102a shows an example where \( V_{\text{main}} \) is the pulmonary trunk that splits into the left and right PA which is \( B_0 \) and \( B_1 \), respectively. A new branching centerline is calculated that starts at the closest point on the vessel surface to the center of the first plane and ends at the corresponding points from the second and third plane. Afterwards, the centerline is split into \( V_{\text{main}} \), \( B_0 \), and \( B_1 \) and the junction using a branch splitting [9] provided by VMTK. Our method for junction-free centerlines is then applied to each part separately (recall Section 9.1.2.3). However, monotonous NFV functions are only fitted for \( B_0 \) and \( B_1 \). We assume that they are the only branching vessels from \( V_{\text{main}} \). In case of the PA this is a fact. Therefore, we set the NFV in \( V_{\text{main}} \) constant to the SV. Artificial transitions that connect the constant NFV / SV from \( V_{\text{main}} \) with the monotonous NFV curves of \( B_0 \) and \( B_1 \) in the plot are created using cubic Hermite splines (ALGLIB). They represent the junction. The same is performed for the upper and lower boundaries of the IQR area visualizations.

Based on physiologic considerations, we may assume that the sum of the NFV’s at the beginning of the branches (\( NFV(B_0) \) and \( NFV(B_1) \)) equals the NFV (SV in case of the PA) from \( V_{\text{main}} \) (NFV(\( V_{\text{main}} \))). Despite using our proposed quantification with increased robustness, this is not always the case. Thus, a correction step is performed:

\[
\text{NFV}_{\text{diff}} = \frac{\text{NFV}(V_{\text{main}}) - (\text{NFV}(B_0) + \text{NFV}(B_1))}{\text{NFV}(B_0) + \text{NFV}(B_1)}
\]

\[
\text{NFV}(B_0) + = \frac{\text{NFV}_{\text{diff}} \cdot \text{NFV}(B_0)}{\text{NFV}(B_0) + \text{NFV}(B_1)}
\]

\[
\text{NFV}(B_1) + = \frac{\text{NFV}_{\text{diff}} \cdot \text{NFV}(B_1)}{\text{NFV}(B_0) + \text{NFV}(B_1)}
\]

(50)

The plot includes both the fitted monotonous NFV functions before and after correction. Additionally, the IQR, which can be considered as uncertainty, of \( B_0 \) and \( B_1 \) is increased by the values of the corresponding NFV corrections. The upper IQR boundary is increased in case of a positive correction, otherwise the lower IQR boundary is decreased. However, the ratio of NFV(\( B_0 \)) and NFV(\( B_1 \)) before and after correction is the same. Figure 102b shows an example.
9.1. Robust Cardiac Function Assessment

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9.1.3. Results

This section contains an informal evaluation based on selected datasets and a subsequent discussion that includes a justification of employed default parameters.

9.1.3.1. Informal Evaluation

We present six selected, anonymized datasets that were discussed with each two collaborating radiologists and cardiologists. The evaluation was performed informally, i.e., no specific tasks had to be solved. The robust NFV quantification for the whole centerline, SV and RF determination as well as monotonous function fitting took about 30 s on average per dataset on a GeForce GTX 680 and Intel i7-3930K. All images were directly captured from our software (recall Chapter 7) which is used by our clinical collaborators for research purposes.

**Healthy Volunteer:** The first dataset shows a healthy volunteer. Figure 103a illustrates the result of our procedure. The highest NFV deviations and thus sensitivity to different plane angulations were detected in the ascending aorta and at the beginning of the aortic arch. Two likely reasons can be identified from the discussion with the experts. On the one hand, there is a physiological helix during systole. Regions with laminar flow seem to be less susceptible. On the other hand, different angulations of planes close to the valve have, from the outset, higher potential to cause deviations since the highest flow rates are present in this vessel section.

The NFVs show a physiologically implausible local minimum at about one third of the centerline, where the aortic arch ends. A possible explanation is a small segmentation error. The vessel diameter in and after the aortic arch is 1.5 cm but decreases to 1.24 cm at the transition. Figure 103b shows the systolic flow and a measuring plane close to the valve, where our method determined 79.96 ml SV. A RF of 0.08 % indicates a properly functioning aortic valve.
9.1. Robust Cardiac Function Assessment

![Figure 103](image)

Figure 103: (a–b) The plane with the SV is located in the ascending aorta of this healthy volunteer. A local NFV minimum at the end of the aortic arch might be due to a potential segmentation inaccuracy.

(c–e) A large systolic vortex in the dilated ascending aorta of this BAV patient causes high NFV deviations and a SV that is not located directly above the valve. The upper diagram (c) shows the NFV development when only one perpendicular measuring plane per centerline position is evaluated. Our method (d) alleviates the problem of quantified NFVs close to 0 ml. Images from [278] © 2016 The Eurographics Association and John Wiley & Sons, reprinted with permission.

BAV Patient with Dilated Ascending Aorta: This patient has a bicuspid aortic valve (BAV). The upper diagram in Figure 103c shows the NFVs along the vessel course when one orthogonally aligned measuring plane per centerline position is evaluated. A nearly vanishing value at the beginning is conspicuous. Qualitative flow analysis reveals prominent systolic vortex flow in the abnormally widened ascending aorta. This is likely the cause for the enormous deviations in this vessel section. Our robust NFV procedure alleviates the problem, as shown in the lower diagram of Figure 103d. Figure 103e shows the systolic vortex flow and the location of the plane where 136.9 ml SV were determined. Contrary to the employed heuristic, this is not located directly above the valve but rather at the end of the vortex.

The conventionally quantified RF ranges from 3–47 %. The experts did agree on an increased amount of retrograde flow, but struggled to specify a concrete value. Our method determined 4.16 % RF, which lies within the physiological range.

Figure 101 shows a similar patient with vortex flow in the ectatic ascending aorta. The same high sensitivity to different plane angulations was observed. Figure 105 shows the patient’s healthy pulmonary artery. The determined SVs of 80.05 ml and 84.33 ml in the pulmonary artery and aorta, respectively, are physiologically plausible since both values are approximately equal.

Coarctation Patient: The next patient has an aortic coarctation. Qualitative flow analysis shows three systolic vortices: A slight helix above the aortic valve, a small vortex before the narrowing, and one large helix in the post-stenotic vessel section, illustrated in Figure 104c. Figure 104a shows the NFV deviations along the centerline. There is a local NFV minimum in the coarctation at about one
third of the centerline, which again is considered as physiologically implausible. In addition to potential segmentation inaccuracies like in the healthy volunteer, this might be due to the low data resolution. The vessel diameter shrinks about 50% in the narrowed region. Consequently, it is represented by far less voxels, which enhances partial volume effects.

The highest NFV deviations occur in the post-stenotic vessel section, approximately in the middle of the centerline, directly behind the local maximum. This is where the largest vortex is present. The collaborating experts agree that this is the likely cause for the quantification uncertainties. In contrast to the previous BAV patient, the location of the determined 127.7 ml SV is close to the aortic valve despite of the slight helix.

**Bypass Patient:** An extraanatomic bypass surgery was performed to treat this patient’s former coarctation. The altered vessel shape promotes the formation of systolic vortex flow in the ascending aorta, shown in Figure 104d. The obtained diagram is presented in Figure 104b. The SV is located close to the valve despite of the slight helix, like in the previous patient. Moreover, a number of local NFV minima and maxima is noticeable especially at the beginning of the bypass. The numbers 1–3 help to connect the diagram with centerline positions. A possible explanation is that partial volume effects make it difficult to capture the quickly changing flow directions in curved vessel sections. However, our monotonous function fitting produces a physiologically plausible NFV development along the centerline.

**Tetralogy of Fallot Patient:** A surgical correction was performed in this patient with tetralogy of Fallot. The pulmonary insufficiency progressed to a severe state with nearly 25% RF. Figure 102a illustrates the diastolic backward flow in the pulmonary artery. Vortex flow is present in the pulmonary trunk and right pulmonary artery during systole as well as diastole. Our robust quantification for bifurcations, shown in Figure 102b, determined 83.42 ml SV that splits into 52.39 ml and 31.03 ml for the right and left pulmonary artery, respectively. This is physiologically plausible since the right half of the lung consists of three lobes, whereas the left half of the lung with two lobes is smaller due to the neighboring heart.
9.1.3.2. DISCUSSION

The user’s responsibility is to generate a valid segmentation and centerline. For bifurcations, three planes are required which are located in the origin and the two branches. Reasonable default values for other parameters are investigated at the end of this section.

The sensitivity to different plane angulations was visualized as area graphs that represent the middle 50 % (interquartile range, IQR) of box plots. Especially prominent vortex flow seems to cause high uncertainties since corresponding vessel sections show large IQRs. High variations led to careful result interpretations, whereas small deviations created trust.

Contrary to the assumptions, the SVs were not always determined directly above the valve. This is a serious error potential for the common 2D PC-MRI standard in addition to the plane angulation that is required before the acquisition. Though, the automatically obtained SVs show good correspondence with the expected results estimated by our collaborating experts using conventional methods. The SVs are expected between the aortic or pulmonary valve and the location where the first vessel branches off. In the pulmonary artery this is the junction where it splits into left and right. In the aorta, the small coronary arteries close behind the valve are neglected since they receive only a marginal amount of blood (about 5 %). Thus, the location is shortly after the beginning of the aortic arch, where the brachiocephalic artery is located. If solely the SV or RF is of interest, a priori knowledge can speed up our robust quantification method by restricting the calculations to the corresponding vessel sections. The user can specify two planes as start and end position for this purpose.

We described a method to analyze bifurcations. The algorithmic adaption to three or more branches is simple. Yet, the resulting plot visualization may become cluttered if two or more branches receive approximately the same volume of blood. In this case, the subgraphs, especially the IQR areas, would overlap. Fortunately, this problem is of minor relevance for the pulmonary artery since the right pulmonary artery normally receives more blood due to the larger right half of the lung.

**SAMPLE GENERATION PARAMETER EVALUATION:** Reasonable default values increase the practical applicability of our method. We experimentally determined an appropriate standard configuration during the development process, which is a grid size of $\vec{g} = 50 \times 50$, an angulation tolerance $\alpha = \pi/4 \approx 45^\circ$, and Poisson-distributed angulation samples with a minimum distance of $d_{\text{min}} = 0.075$ on the unit sphere surface (recall Section 9.1.2.2). The validation of these parameters is shown exemplary for the SV determination in the BAV patient (recall Section 9.1.3.1), which represents our experiences for all datasets.

New angulation samples are generated for each plane evaluation. Thus, every SV analysis is repeated 100 times per parameter set. The defaults produce $136.9 \pm 0.87$ ml, which is the samples’ mean plus minus the standard deviation ($\sigma$).
Modifications of $\alpha$ produced 146.48 ± 1.27 ml for 15°, 143.24 ± 1.35 ml for 30°, 136.9 ± 0.87 ml for the default 45°, 136.61 ± 0.87 ml for 60°, 150.64 ± 1.18 ml for 75°, and 163.48 ± 1.1 ml for 90°. The $\sigma$ were lowest for 45° and 60° and the best agreement with 140 ml SV estimated by the experts was achieved with $\alpha \in [30^\circ, 60^\circ]$. As a consequence, $\alpha = \pi/4 = 45^\circ$, the middle of this range, seems appropriate as default value.

Altered grid sizes produced 130.08 ± 0.53 ml for 5 × 5, 136.75 ± 1.52 ml for 25 × 25, 136.9 ± 0.87 ml for the default 50 × 50, 136.66 ± 0.36 ml for 75 × 75, and 135.47 ± 0.92 ml for 100 × 100. There were no significant changes above grid sizes of 25 × 25. Though, planes that are fitted to large aneurysms, e.g., with 55 mm vessel diameter, have isotropic rectangle sizes of 55/25 = 2.2 mm in 25 × 25 grids. To ensure that the rectangles are smaller than our data’s finest resolution of 1.77 mm in one dimension, we use 50 × 50 grids as default.

Altered $d_{\text{min}}$ of the Poisson distribution produce 138.73 ± 2.84 ml for 0.165 ($\approx$50 samples), 136.65 ± 1.51 ml for 0.1 ($\approx$125 samples), 136.9 ± 0.87 ml for the default 0.075 ($\approx$200 samples), and 135.76 ± 0.64 ml for 0.05 ($\approx$500 samples). Standard uniform distribution (recall Section 9.1.2.2) yields 140.19 ± 10.81 ml for 50 samples, 137.35 ± 6.38 ml for 125 samples, 137.66 ± 5.32 ml for 200 samples and 136.06 ± 3.18 ml for 500 samples. The $\sigma$ from Poisson distribution are significantly lower than from uniform distribution. It was below 1 ml for $d_{\text{min}} = 0.075$ and reduced only marginally with more samples. The difference between both distributions decreases with an increasing number of samples. However, Poisson distribution requires fewer samples to achieve sufficient results. The higher costs for angulation generation are not substantial. Summarizing, the sample distribution has the highest influence on the results.

9.1.4. Conclusion and Future Work

We presented a method to robustly determine net flow (NFV) and stroke volumes (SV) as well as percentaged backward flow volumes (PBFV) and regurgitation fractions (RF) in cardiac 4D PC-MRI data of the aorta and pulmonary artery. The procedure was developed in close collaboration with cardiologists and radiologists and represents an automation of their manual approach. Statistical information and uncertainties were presented as area graphs which were derived from common box plots. The experts’ expectations were employed as heuristics to establish a procedure that creates physiologically plausible NFV developments along the vessel course. The GPU’s computing power was exploited to achieve average calculation times of 30 s.

In addition to partial volume effects, prominent vortex flow was identified as main cause for uncertainties. This seems plausible for two reasons:

1. 4D flow MRI sequences assume a constant velocity at the time of the measurement. Thus, acquisitions are more and less accurate in laminar and accelerated flows, respectively. The latter can be found, among others, in regions with turbulences.

2. Turbulent flow can cause flow voids if the spatial image resolution is limited. This means that the emitted signal, which is measured (recall Section 3.1.1), is averaged to zero.

Against the assumption, the SV could not be obtained directly above the aortic or pulmonary valve in all cases. This is a source of potential errors for the current 2D PC-MRI standard since the default is to use a plane above the valve that is orthogonal to the vessel’s centerline. However, an advantage of 2D PC-MRI is that the measured slice lies directly in the cross-section and thus has a comparably high resolution, since distances of data points within a MRI slice are usually lower than distances between slices. In contrast, measuring planes in 4D PC-MRI are often aligned with the slice direction. Isotropic resolution is often not used to keep scan times low.

The robust determination of PBFVs and RFs is of particular importance for the discussion of appropriate treatments. The monotonous function fitting for both junction-free and branching vessels was able to generate physiologically plausible NFV developments. Something that needs to be considered in the future is that two digits after the comma suggest a result precision that is simply not given. This might be misleading and thus should be replaced with rounded values.
Due to reasonable default parameters the only required user input is a vessel segmentation and centerline, which both exploit expert knowledge. Hence, our procedure is eligible to be part of an automatic data evaluation procedure in the future that supports clinical report generation. Branching vessels additionally need a specification of the main vessels as well as the branches by placing planes inside each vessel section.

Until now, SV and RF quantification requires knowledge of highly specialized experts on the cardiovascular system and its hemodynamics. Our method reduces this to mainly anatomical knowledge. However, the newly enabled uncertainty assessment still needs experience. More sophisticated visualizations might support the interpretation in the future.

Another future topic might be to investigate the suitability of curved surfaces for the quantification of flow rates and related measures. For this purpose, the technique by Schulze et al. [461] is a good starting point. They proposed the generation of surfaces that are as-perpendicular-as-possible to the flow field. This could be adapted so that cross-sectional planes, which are normally used, become curved cross-sectional surfaces.

### 9.2. Motion-Aware Stroke Volume Quantification

**This section is based on:**

  “Motion-aware stroke volume quantification in 4D PC-MRI data of the human aorta”.  
  pp. 169–79. DOI: 10.1007/s11558-015-1256-4

See Section Publications in the Appendix for the Division of Work.

Stroke volumes (SVs) help to assess the cardiac function and monitor progression of various CVDs, such as BAV and heart insufficiencies. Static 3D approximations of the dynamic vessel wall are widely used to calculate this measure. There is a trade-off between easier, faster pre-processing and accurate information about the heart’s cyclic movement. Techniques with an enhanced level of automation could overcome this issue by speeding up data processing. However, the accurate definition of intravascular hemodynamics requires the determination of the time-dependent wall position, wall orientation, and vessel diameter. Approaches that neglect these components are error-prone. We investigate the reliability of such methods by extracting motion information of the aorta, incorporating them in the SV quantification and examining deviations.

The segmentation of time-varying cardiac image data was aimed at by several research groups from the image processing community (recall Section 4.2.3). Zhao et al. [578] and Abufadel [1] used level sets and deformable models for this purpose, respectively. GPU-accelerated time-varying direct volume rendering was used to display measured anatomical data [531, 577]. Algorithms such as graph cuts [107, 322, 332] only require the specification of areas inside and outside the target structure, which is convenient for physicians since it exploits their anatomical expert knowledge. Based on heuristics that were derived from discussions with collaborating radiologists, we automatically initialize a 3D graph cut for each temporal position of the dataset and obtain a time-dependent binary mask of the aorta.

A mesh model is described that contains motion information as vectorial displacement list per vertex. Displacement vectors are projections from a static vessel surface approximation onto the meshes of each time step. The latter were extracted from the 4D segmentation. This facilitates post-processing of the movement information. We incorporate the vessel dynamics in a motion-aware stroke volume quantification and investigate the reliability of the conventional quantification using static vessels by examining deviations. Various healthy volunteer and patient datasets were used in the evaluation.

#### 9.2.1. Method

In the following, we describe the 4D segmentation of the aorta and the steps to obtain and post-process movement information. After the adjustment of centerlines and measuring planes, the stroke volume
quantification is adapted. We aim at a parameterization that medical doctors can easily adjust based on their expert knowledge. Carefully selected default values are provided for all parameters to ensure feasibility of our method for the clinical routine. Please recall Sections 7.1.1–7.1.2 for the graph cut-based extraction of an initial 3D vessel surface mesh.

### 9.2.1.1. Stroke Volume Quantification in Static (3D) Vessels

The SV calculation in static vessels using a 3D segmentation \( S_3 \) is the same as described previously in Section 9.1.2.2. \( S_3(P(x,y)) \) describes the check whether or not a position is inside the vessel. When applied to each grid element of the measuring plane, it yields a 2D segmentation of the cross-section. The segmentation’s resolution is given by the plane’s grid size \( g \). \( S_3(P(x,y)) \) can be realized using the voxel-based 3D segmentation, which is rather coarse compared to the resolution of the plane’s grid elements. However, it facilitates usage of the GPU, since it can be employed as 3D texture. A smoother result can be obtained by rasterizing the 3D mesh, as depicted in Figure 106. This is what we use for this work.

Figure 106: Mask of an aorta’s cross-section obtained from rasterization of (a) the 3D vessel segmentation and (b) the extracted corresponding triangular surface. Images from [277] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.

### 9.2.1.2. Four-Dimensional Vessel Segmentation

Performing a manual vessel segmentation in every temporal position is too time-consuming for the clinical routine. Therefore, we automatically determine regions inside and outside the aorta for each time step based on the 3D segmentation and employ this to initialize graph cuts. Other segmentation algorithms that require the same kind of input are also suitable. We perform one 3D graph cut for each time step in the 4D anatomical image \( A \).

The 3D segmentation, based on the magnitude images’ TMIP, represents blood-filled vessels during systole and therefore the maximum extent – like an upper boundary. Thus, we assume that voxels outside this segmentation are also outside the segmentation of each temporal position. A safety margin is incorporated to consider inaccuracies. We subtract a kernel size 5 from a kernel size 8 dilated segmentation to obtain a dilated vessel hull. This is about 1–2 cm away from the vessel surface for the data’s spatial resolution of \( 1.77 \times 1.77 \times 3.5 \) mm\(^3\). We observed that a smaller margin does not improve the results of the employed graph cut. Based on discussions with radiologists, a second assumption is made that the vessel diameter does not shrink more than 50% during diastole. Hence, all voxels are specified as inside the vessel that are closer to the centerline than to the wall. The same initialization is used for all time steps and graph cuts are performed separately for each of them.

Image quality, in particular the signal-to-noise ratio, depends on the acquisition time. In patients with severe heart diseases, acquisitions need to be performed in limited time with a quality that is just sufficient for diagnosis. As a consequence, the graph cut tolerance parameter \( \alpha \) (recall Section 7.1.1) is increased by a factor of ten. The resulting segmentations for each temporal position are post-processed in the same manner as described in Section 7.1.1. The same applies to the extraction and post-processing of the polygonal 3D meshes. Figure 107 depicts the process.
Figure 107: Extraction procedure for each temporal position.
(a) Automatic specification of regions inside (green) and outside the aorta (red) as graph cut initialization.
(b) Resulting 3D segmentation.
(c) Post-processed segmentation.
(d) Extracted triangular surface.
(e) Postprocessed vessel mesh.

Figure 108: The projection $\tilde{v}^{\text{dpl}}_i$ of a base mesh vertex $\tilde{v}$ (red) onto the one-ring neighborhood of the closest vertex $\tilde{q}_i$ in the mesh $M_t$ (green) of a temporal position $t$ is determined to obtain the displacement vector $\tilde{d}_i$ (blue) as $\tilde{v}^{\text{dpl}}_i - \tilde{v}$.

Figure 109: (a) Displacement post-processing. The spatial smoothing (orange) is a low-pass filter applied to the displacements $\{\tilde{d}^t_i, \tilde{d}^{\text{dpl}}_i\}$ of the one-ring neighborhood $\tilde{v}$, of the mesh vertex $\tilde{v}$ in each temporal position $t = 0 \ldots T - 1$. The temporal denoising (green) smoothes the displacement list $\tilde{d}^t_i$ of $\tilde{v}$.
(b) Displacement vectors (white lines) of one temporal position without (left) and with (right) noise reduction. The surface is colored according to the displacements.
9.2.1.3. Dynamic (4D) Vessels

**Displacement Vectors:** We aim at extracting motion information that can be post-processed in order to reduce noise. Until now, we obtained independent 3D meshes $M_t$ for each of the $T$ temporal positions $t = 0 \ldots T - 1$ in the dataset. Unfortunately, the meshes may differ in their number of vertices and topology. Thus, a vertex’ time-dependent position $\vec{v}_t^{\text{displ}}$ cannot be derived implicitly. For the association of points on the surfaces with each other, a correspondence problem has to be solved.

A 4D mesh model requires a certain flexibility to capture pathologic vessel morphologies. We employ a 3D triangular surface mesh $M$ with a constant topology as basis. It is the one extracted from the 3D segmentation that was performed on the magnitude images’ TMIP. Every vertex $\vec{v} \in M$ stores a list of $T$ 3D vectors $\vec{d}_t$, in the following referred to as displacement vectors. A vertex’ position $\vec{v}_t^{\text{displ}}$ at time $t$ is $\vec{v} + \vec{d}_t$.

Displacements $\vec{v}_t^{\text{displ}}$ are calculated as closest projection of a vertex $\vec{v} \in M$ onto $M_t$. To do so, the nearest vertex $\vec{q}_t \in M_t$ to $\vec{v}$ is determined. Then, $\vec{v}$ is projected onto each plane spanned by the triangles that $\vec{q}_t$ is part of. If a projection lies outside a triangle, the closest projection onto one of the triangle’s edges is used as result. The displacement vector $\vec{d}_t$ results as $\vec{v}_t^{\text{displ}} - \vec{v}$ (see Figure 108).

**Displacement Postprocessing:** For noise reduction of the displacement vector lists, two post-processing steps are applied, as depicted in Figure 109:

- The *spatial smoothing* aligns displacement vectors in a vertex’ one-ring neighborhood in each temporal position separately. The same low-pass filter [500] as for the mesh smoothing is used for this purpose.
- The *temporal denoising* smoothes the displacement list of each vertex, i.e., the displacements along the temporal dimension. It is realized by fitting a cubic penalized regression spline (ALGLIB).

**Motion Visualization:** A real-time capable visualization of the 4D mesh can be obtained using the GPU, or more precisely the OpenGL shader pipeline. A displaced vertex position $\vec{v}_t^{\text{displ}}$ at $t \in \mathbb{R}$ is calculated as linear interpolation between $\vec{v}_{\lfloor t \rfloor}^{\text{displ}}$ and $\vec{v}_{\lfloor t \rfloor}^{\text{displ}}$. Each vertex is uploaded to the GPU with:

- its position,
- its normal, and an index to access additional information, which are:
  - a list of all triangles,
  - the triangle count per vertex,
  - the corresponding triangle indices, and
  - the displacement list per vertex.

A dataset has about 15 temporal positions. Thus, there are 15 displacement vectors in the list. Linear interpolation of these few samples would not lead to a fluent motion visualization. Therefore, ten times as much displacement vectors are resampled from the spline during the temporal denoising. Figure 110 shows a result of the motion visualization.

**Moving Measuring Planes:** Measuring planes, orthogonal to the centerline and freely movable along it, are a standard to quantify SVs (recall Section 6.1). To ensure that a plane remains perpendicular to the moving centerline $C_t$ and fits the vessel at every temporal position $M_t$, the plane’s center, normal and scale are modified to a list of size $T$ – analogous to the displacement lists. The centers are derived from the centerline positions and the normals from the tangents. The scales are determined so that the plane fits the diameter of the corresponding vessel section in $M_t$. Oversampling for visualization purposes via spline approximation is performed as explained in Section 9.2.1.3. The lists are used in the geometry shader to span the plane at the current time of the heart cycle. Figure 111a shows a measuring plane that fits the vessel at any time and follows the moving centerline, i.e., stays orthogonal.
Aortic Arch

Left Atrium

Mitral Valve

Aortic Valve

Ascending Aorta

Descending Aorta

Figure 10: Cardiac motion visualization. The left ventricle contracts during systole (left) to pump blood into the aorta, resulting in a slightly increased vessel diameter. In the meantime, the left atrium fills and then supplies the left ventricle with new oxygenated blood during diastole (right). Images from [277] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.

Figure 11: (a) A measuring plane inside the ascending aorta follows the vessel movement during the cardiac cycle by changing its size and orientation. An adapted stroke volume quantification incorporates the motion information. (b) Equidistant measuring planes in the ascending aorta between the approximate coronary arteries’ location and the brachiocephalic artery. Images from [277] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.

**Stroke Volumes in Dynamic (4D) Vessels:** The incorporation of motion into the SV quantification requires a slight adaption of the calculation. The check $S_3(\cdot)$, whether or not a position is inside the vessel, becomes a dynamic counterpart $S_4(\cdot)$. In addition, planes have a list of $T$ orthonormal systems $(\vec{n}^t, \vec{n}_x^t, \vec{n}_y^t)$, scales $\vec{s}^t = (s_x^t, s_y^t)$, and center positions $\vec{c}^t$, $t = 0 \ldots T - 1$. The grid size $\vec{g}$ remains constant. Following this, the flow rate calculation $\mathbf{fr}(t)$ and transformation to world coordinates $P(x, y, t)$ have to be adjusted to:

$$\mathbf{fr}(t) = s_x^t \cdot s_y^t \cdot \sum_{x=0}^{g_x-1} \sum_{y=0}^{g_y-1} S_4(P(x, y, t), t) \cdot \mathbf{V}(P(x, y, t), t)$$

with

$$S_4(P(x, y, t), t) = \begin{cases} 1, & (P(x, y, t)) \text{ inside vessel} \\ 0, & \text{else} \end{cases}$$

and

$$P(x, y, t) = \vec{c}^t + s_x^t \cdot \left(x - \frac{g_x}{2}\right) \cdot \vec{n}_x^t + s_y^t \cdot \left(y - \frac{g_y}{2}\right) \cdot \vec{n}_y^t$$

Analogous to the static version, $S_4(P(x, y, t), t)$ is realized with the dynamic triangular surface (see Figure 112).

**9.2.2. Results**

In this section, we present a validation of the 4D segmentations, an overview of the seven datasets, and a discussion. In the implementation, the user is allowed to manually refine the 4D segmentations if desired. However, solely the automatically specified input, as described in Section 9.2.1, was used for this evaluation. The 4D segmentation including the motion extraction were performed in less than 15
seconds per case on an Intel i7-3930K. The stroke volume (SV) quantifications run in interactive speed. All presented images of vessels and flow were directly captured from our software (recall Chapter 7) which is used for research purposes by the clinical collaborators. Plots were created with MATLAB.

The first vessels that branch off the aorta are the coronary arteries (CAs) directly behind the valve. They receive about 5% of the SV. The next branching vessel is the brachiocephalic artery (BA) in the aortic arch. In order to have a larger range of sample positions to compare the static and motion-aware quantification, we use the constant net flow volume in the ascending aorta after the CAs and before the BA as SV. Figure 111b depicts the evaluated planes in 1 mm steps on the centerline.

![Figure 112: Binary mask of the ascending aorta’s cross-section for two of 15 temporal positions of the dataset obtained by rasterization of (a–b) the 4D segmentation, (c–d) the independent 3D meshes for each time step, and (e–f) the 4D mesh with post-processed motion information. Images from [277] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.]

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Dyn. (indep.)</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A − M)/M (%)</td>
<td>−10.64</td>
<td>−4.36</td>
<td>2.44</td>
</tr>
<tr>
<td>(A ∩ M)/(A ∪ M) (%)</td>
<td>86.35</td>
<td>85.23</td>
<td>86.27</td>
</tr>
<tr>
<td>(A Δ M)/(A ∪ M) (%)</td>
<td>13.65</td>
<td>14.77</td>
<td>13.73</td>
</tr>
</tbody>
</table>

Table 9: See Figure 113 for a depiction of the areas.

![Figure 113: Cross-sectional areas that were used to compare the automatic (A) with the manual (M) segmentations. Images from [277] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.]

### 9.2.2.1. Accuracy of 4D Segmentations

For one dataset (see Figures 111b and 115), we generated vessel segmentations for all 43 equidistant planes in the ascending aorta manually by drawing contours. Randomly selected samples were validated by the collaborating experts. On the one hand, we performed this on the TMIP of the magnitude images and compared it to the rasterization of the static (3D) vessel surface (“Static”). On the other hand, this was done in the anatomical images A for each of the 18 temporal positions and compared to the rasterized independent 3D meshes for each temporal position (“Dyn. (indep.)”) as well as the rasterized dynamic vessel with post-processed motion information (“Dynamic”). Planes were always perpendicular to the vessel’s centerline. Each value in Table 9 is an average of all planes. The manual and automatic segmentations overlap by 85.95 ± 0.62%. Cross-sections derived from the static mesh are 10.64% smaller than the manual segmentations. This is the main cause for the 13.65% difference. In the

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11 A video is available at [https://www.youtube.com/watch?v=s0zt3TFwWPE](https://www.youtube.com/watch?v=s0zt3TFwWPE) (accessed 03/2016)
dynamic variant, these sizes differ less. Consequently, large parts of the about 14 % discrepancy are caused by the positioning. The 4D mesh with post-processed motion information directly depends on the independent 3D meshes of each temporal position. Thus, the results of both are similar.

9.2.2.2. Cases

The constant SV between the coronary arteries’ location and the brachiocephalic artery was estimated by the collaborating experts for each case and used as a reference. For this purpose, they were allowed to freely move and rotate a measuring plane, obtain multiple SV samples and then estimate a result. In our experiments, the SV was calculated in three different variants “S”, “D”, and “D2”:

- **S** uses the rasterized 3D vessel surface (see Figure 106b) that was extracted from the segmentation, which is based on the magnitude images’ TMIP.
- **D** employs the rasterized dynamic (4D) surface with post-processed motion information from our proposed method (see Figures 112e–f).
- **D2** is a simulation of the common 2D PC-MRI. Here, planes are fix (constant angulation, size, center) per centerline position, but the segmentation is time-dependent. The 4D segmentation of the cross-sections is derived from rasterization of the 4D mesh (same as in D).

For each variant, the mean of the absolute deviation $\|\mathbf{E}_D(S,D,D2)\|$ per centerline position from the reference was calculated. The following plots in (see Figures 114–120) show the reference in red, $S$ in blue, $D$ in green, and $D2$ in orange. The x-axis shows the centerline positions in 1 mm steps, starting from the approximate coronary arteries’ location on the left. $D$ performed, on average, 1.57 % better than $S$ and $D2$. The improvements ranged from $-1.0 \%$ to $4.12 \%$. $D$ was best in four cases, $S$ in two and $D2$ in one.

**Ectasia 1:** The first patient has a pathologically dilated ascending aorta. Figure 114 shows emerging vortex flow during systole. All SV quantifications are most error-prone in the first half of the examined vessel section, where the vortex is most prominent. The 2D PC-MRI simulation $D2$ performs marginally better than $S$ and $D$ with $\|\mathbf{E}_{D2} = 25.65 \%$.

**Ectasia 2:** In addition to an ectatic ascending aorta, the second patient has an improperly closing aortic valve which causes an abnormal amount of blood swirling back into the left ventricle during diastole. Figure 115 shows the systolic vortex flow. The diagram depicts the 2.96 % improvement of $D$ compared to $S$. Like in the previous patient, the heavy vortex flow causes high quantification uncertainties in the first half of the examined vessel section.

**Bypass:** Extra-anatomic bypass surgery was performed in this patient due to a severe coarctation. The motion extraction shows plausible results: There is a strong movement in the aortic root, no noticeable contraction in the vascular replacement, and then again a pulsating wall motion. The altered vessel shape promotes systolic vortex flow in the ascending aorta, shown in Figure 116. Dynamic quantification ($D$) is, on average, 4.12 % closer to the reference SV than $S$.

**Aneurysm and Coarctation (ANEUCO):** This patient has an aneurysm in the ascending aorta and a coarctation. As illustrated in Figure 111a, there is a heavy movement in the ascending aorta that causes a high variation of the plane angulation. $S$ performs slightly better (0.93 %) than the dynamic counterparts, shown in Figure 117. However, the SV quantification seems to be uncertain due to the present vortex flow.
9.2. Motion-Aware Stroke Volume Quantification

Figure 114: Patient with systolic vortex flow in the ectatic ascending aorta:
- $E_S = 27.19\%$
- $E_D = 26.61\%$
- $E_{D2} = 25.65\%$

Figure 115: The ectatic ascending aorta of this patient causes systolic vortex flow:
- $E_S = 25.79\%$
- $E_D = 22.83\%$
- $E_{D2} = 25.34\%$

Figure 116: A systolic helix in the ascending aorta and aortic arch is promoted by the altered vessel shape of this bypass patient:
- $E_S = 9.33\%$
- $E_D = 5.21\%$
- $E_{D2} = 8.42\%$

Figure 117: Systolic vortex flow in the ascending aorta of this patient with an aneurysm and coarctation:
- $E_S = 18.18\%$
- $E_D = 19.18\%$
- $E_{D2} = 19.11\%$
**Aneurysm and BAV (AneuBAV):** High velocity blood flow passes this patient’s aortic arch and impinges on the wall. A huge vortex emerges that is present during the whole cardiac cycle, shown in Figure 118. Progression for years probably caused the significant dilation of the left subclavian artery. The patient’s bicuspid aortic valve is likely responsible for a smaller systolic vortex in the ascending aorta. The resulting plot illustrates the 0.74% and 1.98% worse performance of D and D2 compared to S, respectively.

**Fallot:** This patient has a pulmonary valve defect as consequence of a surgical tetralogy of Fallot correction. The aorta, however, is free of abnormal flow patterns – only a physiological systolic helix in the aortic arch occurs, as shown in Figure 119. All quantifications achieve similar results. Yet, D with $\varnothing_{E_D} = 11.08\%$ is, on average, 2.7% closer to the reference than the others.

**Healthy Volunteer:** The last dataset is from a healthy volunteer with a slight physiological helix in the aortic arch during systole (see Figure 120). All SV curves match the reference relatively well and show only small average deviations. Yet, with $\varnothing_{E_D} = 4.33\%$, D performs about 2.7% better than the rest.

---

**Figure 118:** Vortex flow after the aortic arch that is present during the whole cardiac cycle. A small systolic vortex in the ascending aorta is a result of the special opening characteristics of the patient’s bicuspid aortic valve:
- $\varnothing_{E_S} = 6.39\%$
- $\varnothing_{E_D} = 7.13\%$
- $\varnothing_{E_{D2}} = 8.37\%$


**Figure 119:** Physiological helix in the aortic arch during systole in this tetralogy of Fallot patient:
- $\varnothing_{E_S} = 13.73\%$
- $\varnothing_{E_D} = 11.08\%$
- $\varnothing_{E_{D2}} = 13.78\%$


**Figure 120:** Physiological helix in the aortic arch during systole in this healthy volunteer:
- $\varnothing_{E_S} = 7.0\%$
- $\varnothing_{E_D} = 4.33\%$
- $\varnothing_{E_{D2}} = 6.9\%$

9.2.2.3. Influences

We analyzed influences on the result deviations by correlating the standard deviation ($\sigma$) of the three obtained SVs from $S$, $D$, and $D2$ per centerline position $i$ to measures that describe differences between the static and dynamic vessels or measuring planes:

- $\Delta Area$ is the standard deviation ($\sigma$) of minimum and maximum plane areas $A_{i}^{S}$, $A_{i}^{D}$, and $A_{i}^{D2}$. High values indicate a strong vessel pulsation.
- $\Delta Normal$ refers to angulation changes of measuring planes in $D$. It is calculated as mean of angles between the average plane normal vector $\bar{n}_{i}^{S}$ and the time-varying plane angulations $n_{i}^{S}$. High values indicate a strong movement of the dynamic centerline.
- $\Delta Center$ is the mean distance of the time-dependent dynamic measuring plane centers $\bar{c}_{i}^{S}$ in $D$ from the average plane center $\bar{c}_{i}^{S}$. Analogous to $\Delta Normal$, high values indicate a strong movement of the dynamic centerline.
- $\Delta Velocity$ is the $\sigma$ of average velocities $|| \bar{u}_{i}^{S} ||$, $|| \bar{u}_{i}^{D} ||$ and $|| \bar{u}_{i}^{D2} ||$ that were sampled on the corresponding planes in $S$, $D$ and $D2$. Different velocity vectors are the result of the deviating sample positions on the measuring planes.

Table 10 shows no ($\rho < 0.25$), low (0.25 $\leq \rho < 0.5$), medium (0.5 $\leq \rho < 0.75$), and high (0.75 $\leq \rho$) Pearson correlation coefficients. The normal ($\Delta Normal$) and plane center variation ($\Delta Center$) are the highest influence on deviations of the resulting SVs. Three high and five medium correlations were found in the datasets for these two measures. This seems plausible because both indicate a strong movement of the vessel or, more precisely, the centerline. The sampled average velocities ($\Delta Velocity$) in the cross-sections are the direct consequence of the resulting different plane configurations and rank third. The differences of the measuring plane sizes ($\Delta Area$) rank fourth. An interpretation could be that there is only a moderate amount of vessel pulsation in many cases. Exclusively no or low correlations were detected in the Ectasia 1 patient. This might indicate that there are other, more complex influences or simply that the differences between the three SVs are not distinctive enough.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Ectasia 1</th>
<th>Ectasia 2</th>
<th>Bypass</th>
<th>AneuCo</th>
<th>AneuBAV</th>
<th>Fallot</th>
<th>Healthy V</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Area$</td>
<td>0.19</td>
<td>0.16</td>
<td>0.24</td>
<td>0.14</td>
<td>0.3</td>
<td>0.69</td>
<td>0.64</td>
<td>0.4</td>
</tr>
<tr>
<td>$\Delta Normal$</td>
<td>0.43</td>
<td>0.02</td>
<td>0.63</td>
<td>0.5</td>
<td>0.84</td>
<td>0.18</td>
<td>0.53</td>
<td>0.45</td>
</tr>
<tr>
<td>$\Delta Center$</td>
<td>0.03</td>
<td>0.74</td>
<td>0.08</td>
<td>0.77</td>
<td>0.56</td>
<td>0.94</td>
<td>0.22</td>
<td>0.48</td>
</tr>
<tr>
<td>$\Delta Velocity$</td>
<td>0.23</td>
<td>0.71</td>
<td>0.03</td>
<td>0.69</td>
<td>0.34</td>
<td>0.96</td>
<td>0.11</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 10: Pearson correlation of different measures to the standard deviation of SVs obtained with $S$, $D$, and $D2$ per centerline position. Table from [277] © 2016 Springer-Verlag Berlin Heidelberg, reprinted with permission.

9.2.3. Discussion

The overall deviations from the reference SV were smallest if there was no prominent vortex flow (Healthy V., AneuBAV, Bypass, Fallot). This coincides with our previous findings [278] (recall Section 9.1).

The differences between the 2D PC-MRI simulation $D2$ and $S$ were, on average, smaller than the differences between $S$ and $D$ – especially if there was just a moderate movement of the dynamic centerline. A possible explanation is as follows: The dynamic aorta has its minimum diameter during diastole. During systole, when the maximum diameter is reached due to the pumped blood, the aorta has approximately the same size as the static vessel approximation. Consequently, the difference between flow rates in $S$ and $D2$ is smallest at this time of the cardiac cycle. The changing diameter causes exclusion of peripheral plane regions from the $D2$ quantification during diastole and during the transition from maximum to minimum vessel diameter. Nevertheless, differences between $S$ and $D2$ remain small because of the low diastolic blood flow velocities with little contribution to the SV. In addition, the aorta...
usually shows a parabolic velocity profile, i.e., the main blood flow jet with highest influence on the SV calculation is located in the center, as depicted in Figure 121.

**LIMITATIONS:** Contrast in the anatomical images strongly varies due to the pulsatile flow, making the automatic segmentation challenging during diastole. Correcting intensity inhomogeneities [440] might increase the robustness. Divergence-free filters [389] may be applied as further phase image pre-processing. The effect of phase dispersion [561] was not considered, but could introduce errors at the vessel boundaries. Scans with retrospective gating could produce different results, since early-systolic values are better captured. Generally, image data with a higher spatio-temporal resolution might generate more significant SV differences.

**9.2.4. CONCLUSION AND FUTURE WORK**

We have presented a cardiac motion extraction that is feasible for the clinical routine due to minimal required interaction. We focused on the aorta, but the application of our method to other vessels, such as the pulmonary artery, is possible. A carefully designed visualization of the results was established. Graph cuts were employed to support segmentation tasks and showed a high level of acceptance among the consulted radiologists because of the intuitive task of specifying regions inside and outside the vessel. Such regions were automatically determined for each temporal position and can be used for other segmentation algorithms as well.

We incorporated vessel dynamics to adjust stroke volume calculation and quantified differences to conventional methods using static vessels and a simulation of 2D PC-MRI. On average, the results differed by 7.82 %. Unfortunately, the incorporation of motion information does not achieve improvements for each position of the centerline and thus is no guarantee for more accurate results. Yet, it performed on average 1.57 % better by having lower differences to the reference stroke volumes estimated by the clinical collaborators. Limitations were pointed out. Variations of the planes’ normal vectors and centers had the highest correlations to differences of the SVs obtained with the different methods. This is plausible since both indicate a strong vessel movement. The limited improvements imply that it is reasonable to employ static 3D vessel approximations to quantify stroke volumes and avoid the additional effort. Yet, the use of robust methods (recall Section 9.1) is recommended due to the calculations’ high sensitivity to the measuring plane angulation [278].

**OUTLOOK:** Motion information open up various opportunities. Wall shear stress is another important measure that strongly depends on the accurate wall position and orientation. Possible improvements by incorporating vessel dynamics should be investigated. Models of the aortic valve facilitate further understanding of vortex formation in, e.g., bicuspid aortic valve patients. The extracted movement could support the determination of the valve’s exact position and location. Finally, the wall movement allows conclusions regarding vessel elasticity, which is important for the risk assessment of aneurysm rupture and thus supports treatment decisions. The derivation of a vascular wall model was a major interest of the consulted experts.

Further research to assess the reliability of the obtained results is necessary. In the future, standardized and (semi-)automatic evaluation methods will enable the fast processing of large studies that are performed for statistical analysis of gender- and age-specific norm values. The derived physiological variations of different flow parameters will support the assessment of disease severity.
10. SUMMARY AND OUTLOOK

This concluding chapter starts with a brief summary of the main contributions of this thesis in Section 10.1. This is followed by a description of potential future topics and research fields in Section 10.2. The last Section 10.3 puts 4D PC-MRI in a bigger picture and points out the potential that the evaluation of such data offers.

10.1. SUMMARY OF CONTRIBUTIONS

This section briefly summarizes the main contributions of this thesis.

OVERVIEW: A comprehensive overview of 4D PC-MRI data measurement, pre-processing, state-of-the-art of quantitative and qualitative data analysis, and visualization was provided. This gives future researchers in the field of 4D flow CMR a strong starting point for further advancements.

GUIDED EVALUATION: The research prototype Bloodline was described. It provides a guided analysis pipeline that facilitates a basic dataset evaluation within ten minutes. This comprises the extraction of a vessel, the corresponding flow calculation as well as vortex extraction, and the quantification of cardiac function parameters, such as the stroke volume (SV) and regurgitation fraction (RF). Reasonable default values limit the amount of parameters that need to be adjusted. Results can comfortably be shared with screenshots or with videos of the flow.

VORTEX FLOW: The assessment of vortex flow is a major interest in the cardiac flow analysis. In this thesis, several methods were presented that support this task, starting with a recap of a semi-automatic extraction of vortex flow-representing path lines of high visual quality. This forms the basis for three subsequent methods:

1. A 2D polar plot was established as overview visualization of aortic vortex flow. A grid view facilitates the quick comparison of datasets within a study regarding the occurrence of pathological vortex flow.
2. Videos with adaptive speed were proposed that optimize the display time of extracted vortex flow. This was made to support offline case discussions and presentations.
3. A suitable clustering method was determined to group single vortex entities. A subsequent classification, which is based on characteristics that were described in recent medical publications, was proposed to minimize inter-observer variabilities and increase the objective comparability of datasets.

FLOW RATE AND STROKE VOLUME QUANTIFICATION: 4D PC-MRI data allow to evaluate arbitrary measuring planes. However, results strongly vary with different plane configurations (position and angulation). A method was presented that automates the manual trial–and–error process to determine reasonable SV results. An initial visualization of the quantification uncertainty was established from the statistical analysis.

Usually, static approximations of the actually dynamic vessel are employed in the quantification of flow rates. This is inherently inaccurate. A method was described to obtain 4D segmentations and dynamic vessels solely from the 4D PC-MRI data. This was incorporated in the SV quantification. Result deviations were analyzed compared to static vessels.

10.2. FUTURE WORK

The field of 4D PC-MRI provides a vast amount of potential future topics. This section presents only selected ideas.
10.2. Future Work

**COUPLING WITH CFD:** Besides MRI-based measurements, flow data can be obtained using *computational fluid dynamics* (CFD). Numerical flow simulations require the specification of various boundary conditions [299], which is often based on assumptions. For example, it is assumed that flow at the vessel wall is at 0 m/s. In addition, information about the inflow and outflow velocity profile are necessary. Among others, CFD simulations are useful to predict the potential outcome of a treatment like valve replacement or stenting [381]. CFD simulations have already been used to verify various measures, e.g., WSS (recall Section 6.2.1) peak locations by Petersson et al. [408], TKE (recall Section 6.3.2) by Arzani et al. [14], systolic pressure drops (recall Section 6.3.1) by Goubergrits et al. [188], and TKE in combination with flow displacements (recall Section 6.1.3) by Andersson et al. [7].

While 4D PC-MRI data suffer from a limited spatio-temporal resolution, noise, and different image artifacts, it is not clear to what extent CFD data truly represent a patient’s situation, since the simulations rely on various model assumptions. However, *data assimilation* allows to link the methods and taking advantages of both. A combined approach can be seen as patient-specific simulation, where the measured data are employed as input, or as 4D PC-MRI data with enforced model assumptions, such as zero-divergence. De Hoon et al. [114] presented a fluid simulation where MRI and CFD are coupled in order to exploit the benefits of both methods and improve the visual analysis of hemodynamics. This is a promising approach that should be further investigated in the future. For example it should be determined to what extent quantification results are influenced.

**UNCERTAINTY:** During this thesis it became clear that the flow data, and consequently the results, are prone to various errors. Spatio-temporal image resolutions are improving as well as the employed MRI sequences in general. In addition, more elaborated post-processing methods, such as divergence filtering (recall Section 4.1.4), are being developed. Though, obtained qualitative and quantitative results should always be interpreted with care.

Uncertainties in the flow trajectory calculation have already been investigated [162, 463] (recall Section 6.3.5). However, such an analysis is usually not performed in a standard data evaluation. This might be due to the high computational effort and the huge amount of resulting information of probabilistic methods. The latter are well-known from *diffusion tensor imaging* and the related *fiber tracking* [372]. Future works could investigate to what extent corresponding methods could be a benefit to the analysis of 4D PC-MRI data, although the situation is slightly different. While fiber tracking has to handle multiple direction within one voxel in a 3D grid, blood flow analysis deals with one direction per voxel in 4D data.

Uncertainty might be even more important in the quantitative data analysis. It has been shown in this thesis how sensitive flow rate calculations are to different measuring plane configurations, and a first uncertainty visualization for stroke volume quantifications has been proposed (recall Section 9.1). With more experience on what are relatively high and low deviations and a better understanding of where the highest uncertainties occur aside from vessel sections with vortex flow, this could be incorporated in a way that the user receives a warning if an analysis is performed in an uncertain region. Other measures, such as WSS, are likely to show a similar uncertainty behavior, since they directly depend on vessel wall position and orientation. The proposed approach of systematically evaluating slightly different configurations could be adapted to the WSS quantification in the future. The same yields for incorporating the vessel’s motion information.

**VENTRICULAR CONTRAST ENHANCEMENT:** 4D PC-MRI image contrast strongly depends on the expected maximum velocities, which are configured prior to the scan with the $V_{\text{ENC}}$ parameter (recall Section 3.1.2). Too low $V_{\text{ENC}}$ estimations produce phase wraps (recall Section 4.1.1), whereas too high estimations result in poor contrast that impedes a subsequent segmentation process.

An appropriate choice is difficult for the ventricles since one usually also wants to measure the left-(LVOT) or right-ventricular outflow tract (RVOT) correctly. This means: Either a low $V_{\text{ENC}}$ is set so that the relatively slow intraventricular flow velocities can be obtained with a decent contrast. In this case, phase wraps in the LVOT / RVOT are likely to occur. Or the $V_{\text{ENC}}$ is adjusted with regard to the LVOT /
RVOT, which yields a limited contrast within the ventricles. Mostly, the latter approach is applied. Here, contrast enhancement would be a beneficial tool. An idea is outlined in the following.

First, the LVOT / RVOT, which should be clearly visible in the data, is segmented by the user. Assuming that blood in a ventricle has to pass the corresponding outflow tract, the flow connectivity could be exploited by performing a backward flow integration starting from the outflow tract. Aforementioned probabilistic approaches might be suitable here to increase the chance that every voxel in the ventricle is reached. Then, image intensities along the path line courses are increased, depending on properties regarding the path lines' appearance (e.g. curvature). The "nicer" a trajectory is, the more contrast enhancement is applied.

**PERCEPTION AND VISUALIZATION:** Other future topics could deal with more perception-based visualization approaches. There is a good understanding of separate visual channels, such as color and motion, that are processed relatively independently from one another [543] as well as the visual perception of flow directions and velocities [542]. This could be exploited to further refine blood flow visualization techniques, especially the visual assessment of quantitative measures color-mapped to the flow trajectories. Existing research on the human visual perception is often focused on simple shapes, such as T and L junctions, and on static images [207]. Further research could put emphasis on accentuating dynamic scenes, e.g., blood flow animations.

Pilar and Ware [411] used special glyphs to convey additional information in 2D meteorological data. Generally, the literature on perception-based visualization techniques in meteorology and oceanology seems to be a good starting point. Such approaches may be customized to the cardiac context. This requires a careful adaptation, evaluation, further refinement, and finally, task-based experiments that compare different visualizations in order to better understand how the design choices affect the accuracy and efficiency of the result interpretation.

Eichelbaum et al. [140] proposed an ambient occlusion method for 3D stream line bundles. Due to its real-time capability, an adaption to path line animations is conceivable. Global illumination has been shown to be effective for the display of dense 3D streamtubes [547]. It should be investigated if ambient occlusion is superior to commonly employed halos regarding the depiction of depth relations during animations. The same questions applies to illustrative techniques for line bundles, as summarized by Isenberg [242]. An overview of further open visualization challenges was given by Van Pelt and Vilanova [523].

**STUDY EVALUATION:** Another goal is to support the evaluation of potentially large studies. This requires the generation of comprehensive reports with standardized, qualitative and quantitative results. The automatic generation of 3D and 4D visualizations (screenshots and videos of the scene) makes finding suitable viewing perspectives necessary that ensures the visibility of all relevant features and structures. Corresponding earlier works, such as by Mühler et al. [374], could be adapted for this purpose.

Dataset comparison is another highly important topic. Verma and Pang [532] proposed tools for general flow fields, Van Pelt et al. [529] specifically for the pairwise comparison of simulated (CFD) datasets of patients with cerebral aneurysms. An investigation should be performed to what extent such methods are suitable for the cardiac context. However, when it comes to evaluating whole studies and finding meaningful correlations, statistical methods are inevitable. Here, the wide field of epidemiology [302, 435] and corresponding analysis methods [267] provide a rich pool of ideas and concepts that might be adapted to blood flow measured with 4D PC-MRI. Moreover, the comparability of datasets acquired with different scanners and / or MR sequences has to be analyzed in a future work.

**CLASSIFICATION:** Vortex classification criteria that are based on recent medical studies were described in this thesis (recall Section 8.4.2). However, there is still room for improvements. For example, proximity to the vessel wall was not considered so far. Also, the size of a vortex can be varying over time. This temporal component was neglected in our initial approach. A corresponding extension also entails the adaption of the proposed glyph visualization.
Moreover, the vessel morphology should be further incorporated to achieve more comprehensive
descriptions of a patient’s situation. For example, the extent and length of a stenosis could improve
the understanding of the potential post-stenotic vortex formation. Both together could be linked to the
often developing post-stenotic vessel dilation.

10.3. 4D PC-MRI Potential

4D PC-MRI enables the measurement of 3D blood flow and its change over the heart cycle. Medical
researchers start using these data to develop an increased understanding of healthy cardiovascular
systems and to find indicators for the genesis and evolution of cardiovascular diseases.

4D flow CMR is expected to significantly improve patient treatment, which was confirmed by recent
medical studies [83, 226, 488, 530]. One goal is, e.g., that it can replace right-heart catheterizations for
pressure estimations. This is not only less stress for the patient, it is also considerably less dangerous.
The clinical feasibility of 4D PC-MRI is increasing constantly. Many research groups are working on
improvements of the employed MR sequences and already achieved a reduction of the acquisition times
to 10–20 min from originally about 45 min. In combination with the development of standardized,
easy-to-use software with guided workflows, report generation, and elaborate dataset comparison as well
as statistical analysis, the evaluation of large studies including follow-up acquisitions becomes possible
and feasible. Then, a long-term goal is to obtain age and gender-specific normative values for different
flow parameters, which could help to refine current treatment guidelines.


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S. Laurent, P. Boutourie, R. Asmar, I. Gautier, B. Laloux, L. Guize, P. Ducimetiere, and A. Benetos. “Aortic stiffness is an independent predictor of all-cause and cardiovascular mortality in hypertensive patients”. In: Hypertension 37.5 (2001), pp. 1236–41. DOI: 10.1161/01.HYP.37.5.1236 (cited on page 67).


<table>
<thead>
<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>AABR</td>
<td>Axis-aligned bounding rectangle</td>
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<td>AAo</td>
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<td>AHC</td>
<td>Agglomerative hierarchical clustering</td>
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<td>Aorta</td>
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<td>Acoustical source term</td>
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<td>BAV</td>
<td>Bicuspid aortic valve</td>
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<td>BEP</td>
<td>Bull’s eye plot</td>
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<td>Backward flow volume</td>
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<td>Broad-use linear speed-up technique</td>
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<td>CDF</td>
<td>Cumulative distribution function</td>
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<td>COM</td>
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<td>Cores of swirling particle motion</td>
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<td>Gigahertz</td>
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<td>Geometric orifice area</td>
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<td>GPU</td>
<td>Graphics processing unit</td>
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<td>GRAPPA</td>
<td>Generalized autocalibrating partially parallel acquisitions</td>
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<td>GUI</td>
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<td>MIP</td>
<td>Maximum intensity projection</td>
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<td>mJ</td>
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<td>mmHg</td>
<td>Millimeters of mercury</td>
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<td>Percentaged back flow volume</td>
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<td>Threshold method</td>
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<td>WSS</td>
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<td>Vortex animations with adaptive speed</td>
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<td>Phase / flow / velocity image</td>
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<td>Spatial dimensions</td>
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### Symbols

- $A$: Anatomy image
- $B$: Main magnetic field
- $B_0$: Direction of the $B_0$ main magnetic field
- $C$: Characteristic set
- $C(r)$: Curve
- $D$: Distance
- $d$: Distance
- $E$: Elastic (Young’s) modulus
- $G$: Shear rate
- $H$: Wall thickness
- $h$: Wall thickness
- $H_d$: Helicity density
- $H_n$: Normalized helicity
- $H_r$: Relative helicity
- $J$: Jacobian matrix
- $L$: $\ell_1$-norm
- $L_1$: Distance / taxicab geometry
- $\lambda$: Eigenvalue
- $\lambda_2$: $\lambda_2$ vortex criterion
- $M$: Magnitude image
- $M$ (without subscript): Overall magnetization
- $M_{xy}$: Transversal magnetization
- $M_z$: Longitudinal magnetization
- $m$: Mass
- $\mu$: Dynamic viscosity
- $N$: Number
- $\nabla \times$: Curl operator
- $\vec{n}$: Normal vector
- $\vec{\omega}$: Vorticity
- $P$: Measuring plane
- $\Delta p$: Pressure gradient
- $p$: Relative pressure
- $\tilde{p}$: 3D position
- $\psi$: Pathline
- $\Pi$: Line predicate
- $\Pi_Y$: Line predicate (flow field)
- $\Pi_{1}$: Line predicate (geometry)
- $\Pi_\ell$: Line predicate (mean)
- $\Pi_M$: Line predicate (mesh)
- $\Pi_{\sigma}$: Line predicate (smooth)
- $\Pi_{meta}$: Line predicate (meta)
- $\Pi_O$: Line predicate (stream)
- $\Pi_{\gamma}$: Line predicate (sum)
- $\tilde{p}_1$: 4D position
- $Q$: Volumetric flow rate
- $\mathbb{R}$: Euclidean space
- $r$: Radius
- $\rho$: Density
- $S$: 3D segmentation
- $S_3$: 4D segmentation
- $S_4$: Segmentation
- $\sigma$: Standard deviation
- $T$: Number of temporal positions
- $T_1$: $T_1$ relaxation time
- $T_2$: $T_2$ relaxation time
- $\tau_{WSS}$: Shear stress
- $\tau$: Torsion
- $\tilde{\tau}$: Time
- $V$: Phase / flow / velocity image
- $\vec{v}$: Velocity vector
- $\| \vec{v} \|$: Velocity magnitude
- $\vec{\omega}$: Vorticity magnitude
- $\| \vec{\omega} \|$: Vorticity magnitude
- $x,y,z$: Spatial dimensions
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JOURNAL PAPERS

– Division of work: I provided the extracted, vortex-representing pathlines, meshes with centerlines, and flow curves for all datasets as well as first classification attempts from my Master’s thesis [269]. I summarized potential classification criteria including rough implementation ideas for two grant proposals which I gave to M. Meuschke as a starting point for her Master’s thesis [360]. The text describes: Vortex sizes as percentaged occupation of cross-sectional planes, the idea to detect the cusp in the aortic arch for a subsequent partitioning, and the basic idea how to determine systole and diastole by separating the flow curve. M. Meuschke designed and implemented a stand-alone software prototype, adapted the 2D polar plot [274], completely established the pathline shape classification, and designed and conducted the user study. I wrote MATLAB code to obtain the turning direction of a line segment in a cross-section, to create consistently oriented cross-sections along the centerline (described in [278]), and provided the basic idea to replace classified vortex clusters with a simplifying glyph, which was designed and realized by M. Meuschke. K. Lawonn provided valuable mathematical support. He and M. Meuschke wrote the paper, while I commented two versions. U. Preim provided feedback from a clinical perspective.

– Division of work: I did the extended literature research and wrote the paper. The coauthors provided, as in the prior version [272], helpful comments. A. Hennemuth provided the suggestion to include divergence filters and Lagrangian coherent structures already in the original version, but joined the list of authors only for the work.

– Division of work: I had fruitful discussions with the clinical partners, especially when the first prototype of a moving vessel was established. The algorithmic design is based on my ideas. I did the implementation and writing of the paper. Each coauthor read the paper at least once and provided comments.

– Division of work: The extension of the original paper [271] to branching vessels was my work. Each coauthor (M. Grothoff joined the team) proofread the paper again and provided feedback.

– Winner of the Karl-Heinz Höhne Prize (MedVis Award), awarded at the Eurographics Workshop on Visual Computing for Biology and Medicine, 2014 (see Figure 122b).
– This work is a continuation of my Master’s thesis [269]. The following extensions were made as PhD student. A GPU implementation increases the clinical feasibility. Instead of seeding N uniformly distributed pathlines, it is now ensured that each voxel is visited at least once in every temporal position. The \( \lambda_2 \) calculation is performed together with the pathline integration on the GPU. The extraction procedure was adapted so that now a single slider facilitates the vortex filtering. A detailed, qualitative case evaluation was performed in collaboration with the clinical partners.
– Division of work: During the Master’s thesis: U. Preim provided feedback from a clinical perspective, and I had discussions with H. Theisel about vortex extraction methods and with R. Gasteiger who supervised me. For the paper: U. Preim provided background information for the patients and verified the text. R. Gasteiger provided detailed comments and tips for writing the paper, and M. Gutberlet, representative for the clinical partners, provided the 4D PC-MRI data and feedback.

CONFERENCE PAPERS (PROCEEDINGS)

Division of work: The idea was mine and discussed with B. Preim. I realized it and wrote the paper. Result feedback was provided by the clinical partners.


Division of work: Initial ideas on distance measures and the results with DBSCAN were provided from my Master’s thesis [269] as starting point for M. Meuschke’s own Master’s thesis [360]. She refined the distance measures, composed a distance matrix, compared different clustering algorithms, and wrote the paper with K. Lawonn. I provided detailed feedback twice and the datasets including the extracted, vortex flow-representing path lines.


Division of work: B. Behrendt received flow data as starting point for his Master’s thesis [28] which is the basis for this paper. He developed the method, wrote the paper, and I gave detailed comments in multiple iterations.


Division of work: A scientific team project [361], which I supervised, was done. Prior to the project, I provided the plot mapping idea, i.e., the mapping of time and vessel section (centerline) to the plot angle and distance from the plot center, respectively, as well as the idea to map individual pathline points to the plot and employ the $\lambda_2$ values for color-coding. I provided the datasets include the vortex-representing path lines and their $\lambda_2$ values. The team established an implementation, tested different visualization styles and conducted a user study. After the project, I reimplemented the mapping, and established a continuous color visualization as well as the grid view to compare datasets. I wrote the paper and had the final version verified by the coauthors.


Division of work: A scientific team project [361], which I supervised, was done. Prior to the project, I provided the plot mapping idea, i.e., the mapping of time and vessel section (centerline) to the plot angle and distance from the plot center, respectively, as well as the idea to map individual pathline points to the plot and employ the $\lambda_2$ values for color-coding. I provided the datasets include the vortex-representing path lines and their $\lambda_2$ values. The team established an implementation, tested different visualization styles and conducted a user study. After the project, I reimplemented the mapping, and established a continuous color visualization as well as the grid view to compare datasets. I wrote the paper and had the final version verified by the coauthors.


Division of work: This short paper summarizes my software prototype Bloodline. I wrote the paper, and B. Preim provided feedback. Including the clinical partners as coauthors points out the collaboration.


Division of work: The idea for this work comes from joint dataset evaluations with U. Preim. I designed and implemented the automatic procedure, which was discussed with all coauthors. I wrote the paper, B. Preim repeatedly gave feedback, and the clinical partners verified the work from a medical perspective.

Posters


Division of work: The data were acquired by the clinical partners from the Heart Center in Leipzig, Germany, represented by M. Grothoff and M. Gutberlet. The subjects were prepared (BAV surgery) by C. D. Etz. The submitted paper (abstract) was written by M. Grothoff. I provided before and after screenshots and videos of the intra-aortic vortex flow.
Awards

(a) VCBM 2014: Honorable mention.

(b) VCBM 2014: Winner of the Karl-Heinz Höhne Prize (MedVis Award).
Figure 122: Research awards.
**PAPER PRESENTATIONS**

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>Location</th>
<th>Reference</th>
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</thead>
<tbody>
<tr>
<td>03/2016</td>
<td>Workshop on Bildverarbeitung für die Medizin (BVM)</td>
<td>Berlin (GER)</td>
<td>[276]</td>
</tr>
<tr>
<td>09/2015</td>
<td>Eurographics Workshop on Visual Computing for Biology and Medicine (EG VCBM). Talk was given by Bernhard Preim</td>
<td>Chester (UK)</td>
<td>[272]</td>
</tr>
<tr>
<td>05/2015</td>
<td>The 36th Annual Conference of the European Association for Computer Graphics (Eurographics). Talk was given via Skype</td>
<td>Zürich (CH)</td>
<td>[273]</td>
</tr>
<tr>
<td>03/2015</td>
<td>Workshop on Bildverarbeitung für die Medizin (BVM)</td>
<td>Lübeck (GER)</td>
<td>[274]</td>
</tr>
<tr>
<td>09/2014</td>
<td>Eurographics Workshop on Visual Computing for Biology and Medicine (EG VCBM)</td>
<td>Vienna (AT)</td>
<td>[271]</td>
</tr>
<tr>
<td>10/2013</td>
<td>IEEE Visualization Conference (VISweek)</td>
<td>Atlanta (USA)</td>
<td>[270]</td>
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**INVITED TALKS**

<table>
<thead>
<tr>
<th>Date</th>
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<th>Title</th>
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<tbody>
<tr>
<td>01/2015</td>
<td>7. Deutsche Kardiodiagnostik-Tage</td>
<td>Leipzig (GER)</td>
<td>4D PC-MRI data evaluation with Bloodline</td>
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**SUPERVISED STUDENT PROJECTS**

<table>
<thead>
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<th>Date</th>
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<tr>
<td>10/2014 – 04/2015</td>
<td>Master’s thesis by Benjamin Behrendt</td>
<td></td>
<td>&quot;Fully automatic visualization of cardiac 4D PC-MRI blood flow data with a smart focus on features&quot;, original title: &quot;Vollautomatische Visualisierung von kardialem 4D PC-MRI Blutflussdaten mit intelligenter Fokussierung auf Features&quot; [28]</td>
</tr>
<tr>
<td>04/2014 – 09/2014</td>
<td>Scientific Project by Julian Hettig</td>
<td></td>
<td>&quot;Contrast-enhancement of the right ventricle in 4D PC-MRI data&quot;, original title: &quot;Kontrastverstärkung des rechten Ventrikels in 4D PC-MRI Daten&quot; [216]</td>
</tr>
<tr>
<td>10/2013 – 04/2014</td>
<td>Scientific Team Project by Monique Meuschke, Julian Hettig and Sarah Pauksch</td>
<td></td>
<td>&quot;Development of a polar plot to depict cardiac blood flow&quot;, original title: &quot;Entwicklung eines Polarplots zur Darstellung kardialer Blutflussdaten&quot; [361]</td>
</tr>
</tbody>
</table>

**TEACHING**

<table>
<thead>
<tr>
<th>Date</th>
<th>Course</th>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>04/2016 – 07/2016</td>
<td>Medical Visualization (ME), Interactive Systems (BE)</td>
<td>ME, BE</td>
<td></td>
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<tr>
<td>10/2015 – 01/2016</td>
<td>Visualization (BE), Computer-aided Diagnosis and Therapy (BS)</td>
<td>ME, BE, BS</td>
<td></td>
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<tr>
<td>05/2015 – 07/2015</td>
<td>Interactive Systems (BE)</td>
<td>ME, BE</td>
<td></td>
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<tr>
<td>04/2015 – 07/2015</td>
<td>Medical Visualization (ME)</td>
<td>ME, BS</td>
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<tr>
<td>10/2015 – 01/2015</td>
<td>Visualization (BE)</td>
<td>ME, BE</td>
<td></td>
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<tr>
<td>04/2014 – 07/2014</td>
<td>Medical Visualization (ME)</td>
<td>ME, BS</td>
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<td>10/2013 – 01/2014</td>
<td>Visualization (BE)</td>
<td>ME, BE</td>
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<tr>
<td>04/2013 – 07/2013</td>
<td>Medical Visualization (ME)</td>
<td>ME, BS</td>
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</table>

**BE: Bachelor exercise  ME: Master exercise  BS: Bachelor seminar**
<table>
<thead>
<tr>
<th>Year</th>
<th>Proposal Status</th>
<th>Funding Body</th>
<th>Title</th>
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<tr>
<td>07/2014</td>
<td>rejected</td>
<td>German Research Foundation (GRF) / Deutsche Forschungsgemeinschaft (DFG)</td>
<td>“Norm value generation and evaluation of pathological flow patterns for different diseases of the semilunar valves using 4D PC-MRI”, original title: ”Normwerterhebung und Evaluation pathologischer Flussparameter bei verschiedenen Erkrankungen der Semilunarklappen mittels 4D-Fluss”</td>
</tr>
<tr>
<td>04/2016</td>
<td>accepted</td>
<td>German Research Foundation (GRF) / Deutsche Forschungsgemeinschaft (DFG)</td>
<td>&quot;Norm value generation for well-established flow parameters in a group of healthy volunteers and 1-year follow-up examinations of patients with selected semilunar valve pathologies using 4D PC-MRI&quot;, original title: &quot;Normwerterhebung etablierter Flussparameter bei einem gesunden Kollektiv und 1-Jahres Verlaufsevaluation ausgewählter Pathologien der Semilunarklappen mittels 4D PC-MRI&quot;</td>
</tr>
</tbody>
</table>
PERSONAL DETAILS

Name: Benjamin Köhler
Date of birth: 04.03.1988
Place of birth: 98617 Meiningen
Civil Status: Unmarried, no children
Nationality: German
Contact: ben.koehler@isg.cs.uni-magdeburg.de

EDUCATION

11/2012 – 06/2016: Otto-von-Guericke University Magdeburg, research assistant / PhD student
  – Teaching: Visualization, Interactive Systems, Computer-aided Diagnosis and Therapy (Bachelor courses), Medical Visualization (Master course)
  – Thesis topic: “Guided qualitative and quantitative analysis of cardiac 4D PC-MRI blood flow data”
  – Degree: Doktoringenieur (Dr.-Ing.), grade: summa cum laude

04/2011 – 10/2012: Otto-von-Guericke University Magdeburg, studies of Computational Visualistics (Master):
  – Thesis title: "Vortex extraction in 4D PC-MRI blood flow data of the aorta”, original title: "Wirbelextraktion in 4D PC-MRI Blutflusssdaten der Aorta”, grade: 1.0
  – Degree: Master of Science (M. Sc.), grade: 1.0

  – Application subject: Medicine
  – Thesis title: "Reconstruction of neuronal fiber bundles using global fiber tracking on an ODF-field derived from HARDI data”, original title: "Rekonstruktion neuronaler Faserbündel mittels globalem Fiber-Tracking ausgehend von einem aus HARDI-Daten erzeugten ODF-Feld”, grade: 1.0
  – Degree: Bachelor of Science (B. Sc.), grade: 1.6

  – Degree: "Abitur”, grade: 1.9
WORK EXPERIENCE

since 07/2016: Otto-von-Guericke University Magdeburg, research assistant
  – GRF grant proposal title: "Norm value generation for well-established flow parameters in a group of healthy volunteers and 1-year follow-up examinations of patients with selected semilunar valve pathologies using 4D PC-MRI", original title: "Normwert-erhebung etablierter Flussparameter bei einem gesunden Kollektiv und 1-Jahres Verlaufsevaluation ausgewählter Pathologien der Semilunarklappen mittels 4D PC-MRI"

  – Topic: Visualization of cardiac 4D PC-MRI blood flow data

03/2011 – 09/2011: Fraunhofer MeVis, Bremen, external student assistant
  – Topic: Integration of the developed method during the Bachelor thesis as a module for MeVisLab

10/2010 – 02/2011: Fraunhofer MeVis, Bremen, combined internship and Bachelor thesis
  – Topic: Global fiber tracking using HARDI data


2002 – 2006: Meiningen, deliverer of advertising leaflets

EXPERTISE

Programming: C/C++, Qt, VTK, OpenGL, GLSL, MATLAB

Applications: Microsoft Office, LibreOffice, MeVisLab, Visual Studio, CLion, QtCreator, QtDesigner, Adobe Photoshop, Gimp, ParaView, Meshlab, Camtasia Studio

Typesetting: LATEX, Microsoft Word, LibreOffice Writer

ADDITIONAL SKILLS

Languages: English (fluent in speech and writing)

Driving License: B

10.10.2016

Ich habe insbesondere nicht wissentlich:

- Ergebnisse erfunden oder widersprüchliche Ergebnisse verschwiegen,
- statistische Verfahren absichtlich missbraucht, um Daten in ungerechtfertigter Weise zu interpretieren,
- fremde Ergebnisse oder Veröffentlichungen plagiiert,
- fremde Forschungsergebnisse verzerrt wiedergegeben.

Mir ist bekannt, dass Verstöße gegen das Urheberrecht Unterlassungs- und Schadensersatzansprüche des Urhebers sowie eine strafrechtliche Ahndung durch die Strafverfolgungsbehörden begründen kann. Die Arbeit wurde bisher weder im Inland noch im Ausland in gleicher oder ähnlicher Form als Dissertation eingereicht und ist als Ganzes auch noch nicht veröffentlicht.”

Magdeburg, den 10.10.2016

______________________________
Benjamin Köhler
Zunächst einmal möchte ich meinem Doktorvater Bernhard Preim für die sehr gute Betreuung danken. In meinen Augen zeichnest du dich insbesondere dadurch aus, dass du dich flexibel auf deine Mitarbeiter einstellst, viele Freiheiten läst, aber gleichzeitig immer intensiv mitdenkst und sehr kompetente Ratschläge gibst. Man hat bei dir immer das Gefühl, dass alles, was deine Mitarbeiter angeht, absolut oberste Priorität hat. Es ist alles andere als selbstverständliche, dass du dich mit so viel Engagement für uns einsetzt. Außerdem weiß ich dein stets offenes Ohr für Berufliches sowie Privates und deine lockere Art sehr zu schätzen.


