Appendix A: Derivation of Equation 8

To simplify the exposition, let us introduce an interpolation parameter \( t = (s - s_k)/(s_{k+1} - s_k) \in [0, 1] \). By substituting Equations 6 and 7 into Equation 5, we can approximate the original integration as follows:

\[
\begin{align*}
I &= \int_{s_k}^{s_{k+1}} L(s) e^{-\int_{s_k}^s \sigma_k(u) du} ds \\
&\approx \int_0^1 ((1-t)L(s_k) + tL(s_{k+1})) e^{-\sigma_k \delta k} ds \\
&= (L(s_{k+1}) - L(s_k)) \int_0^1 te^{-\sigma_k \delta k} dt \\
&\quad + L(s_k) \int_0^1 e^{-\sigma_k \delta k} dt. 
\end{align*}
\] (A.1)

From the transformation above, the problem is simplified to solving the integral in each term of the above equation. The first integral can be solved as follows:

\[
\int_0^1 te^{-\sigma_k \delta k} dt = \int_0^1 \left[ e^{-\sigma_k \delta k} \right]_0^t dt = \frac{1}{\sigma_k \delta k} \left( e^{-\sigma_k \delta k} - 1 \right).
\]

In the same manner, the second integral can be solved as follows:

\[
\int_0^1 e^{-\sigma_k \delta k} dt = -\frac{1}{\sigma_k \delta k} \left[ e^{-\sigma_k \delta k} \right]_0^1 = -\frac{1}{\sigma_k \delta k} (e^{-\sigma_k \delta k} - 1).
\]

Thus, Equation 8 can be obtained by substituting these calculation results into Equation A.1.

Appendix B: Additional results for self-occlusion and obstacles

There are two figures provided in this section; Figure B.1 shows the rendering results for volumes with different magnitudes of extinction within smokes, and Figure B.2 shows the results for volumes partially occluded by an opaque obstacle.