Specular Highlight Removal for Real-world Images

Supplementary Material

1. $L_0$-norm solution

In this section, we provide the proofs of Eqn. (18) in our main paper for completeness. The energy function in Eqn. (18) is:

$$\arg\min_S \sum_j ||S_j - W_j + \frac{Y_{1,j}}{\rho}||^2_2 + \frac{2\lambda d}{\rho} C(S_j),$$

where $j$ is the index of the entries of the matrix. The solution of each scalar energy function is updated by

$$S_j = \begin{cases} 
0, & \text{if } (W_j - \frac{Y_{1,j}}{\rho})^2 \leq \frac{2\lambda d}{\rho} \\
W_j - \frac{Y_{1,j}}{\rho}, & \text{otherwise}
\end{cases} \quad (2)$$

**Proof** Denote by $E_j$ the value of the $j$-th scalar function in Eqn. (1) as

$$E_j = ||S_j - W_j + \frac{Y_{1,j}}{\rho}||^2_2 + \frac{2\lambda d}{\rho} C(S_j).$$

(i) When $(W_j - \frac{Y_{1,j}}{\rho})^2 \leq \frac{2\lambda d}{\rho}$, the function value for non-zero $S_j$ is

$$E_j(S_j \neq 0) = ||S_j - W_j + \frac{Y_{1,j}}{\rho}||^2_2 + \frac{2\lambda d}{\rho} \geq (W_j - \frac{Y_{1,j}}{\rho})^2 + \frac{2\lambda d}{\rho}.$$ 

On the other hand, the function value for the zero-valued $S_j$ is

$$E_j(S_j = 0) = (W_j - \frac{Y_{1,j}}{\rho})^2.$$ 

Since $E_j(S_j \neq 0) \geq (W_j - \frac{Y_{1,j}}{\rho})^2 \geq E_j(S_j = 0)$, the solution is $S_j = 0$ when $(W_j - \frac{Y_{1,j}}{\rho})^2 \leq \frac{2\lambda d}{\rho}$.

(ii) When $(W_j - \frac{Y_{1,j}}{\rho})^2 > \frac{2\lambda d}{\rho}$, Eqn. (5) still holds. On the other hand, for non-zero $S_j$, $E_j(S_j \neq 0)$ has a minimum of $\frac{2\lambda d}{\rho}$ at $S_j = W_j - \frac{Y_{1,j}}{\rho}$.

Since $E_j(S_j = W_j - \frac{Y_{1,j}}{\rho}) = \frac{2\lambda d}{\rho} \leq (W_j - \frac{Y_{1,j}}{\rho})^2 = E_j(S_j = 0)$, the solution is $S_j = W_j - \frac{Y_{1,j}}{\rho}$ when $\frac{2\lambda d}{\rho} < (W_j - \frac{Y_{1,j}}{\rho})^2$. □

The proof of the Eqn. (23) is the same as that of Eqn. (18) in our main paper.

2. Convergence curves

In this section, the convergence curves for the five images in Figure 5 in our main paper are illustrated in Figure 1. As can be seen, there is almost no change after the number of iterations is reached to 50. Generally speaking, the 200 $\sim$ 300 iterations is sufficient enough to guarantee that the objective function converges well.

References