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A Unified Cloth Untangling Framework Through Discrete Collision Detection

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1. Gradient of the V-F Signed Distance

Here we give the derivation of the gradient of vertex-face signed distance function here. If we write $\mathbf{d} = \mathbf{x}_0 - \sum_{i=1}^{3} \beta_i \mathbf{x}_i$, then vertex-face signed distance is

$$D(\mathbf{x}) = \hat{\mathbf{n}} \cdot \mathbf{d}, \tag{1}$$

where $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ are face vertices, and the face normal is defined as $\hat{\mathbf{n}} = \mathbf{n}/|\mathbf{n}|$, and $\mathbf{n} = (\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1)$. We first give the expression for $\frac{\partial \mathbf{n}}{\partial \mathbf{x}_1}$

$$\begin{aligned} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} &= \begin{pmatrix} \frac{\partial n_x}{\partial x_{1x}} & \frac{\partial n_x}{\partial x_{1y}} & \frac{\partial n_x}{\partial x_{1z}} \\ \frac{\partial n_y}{\partial x_{1x}} & \frac{\partial n_y}{\partial x_{1y}} & \frac{\partial n_y}{\partial x_{1z}} \\ \frac{\partial n_z}{\partial x_{1x}} & \frac{\partial n_z}{\partial x_{1y}} & \frac{\partial n_z}{\partial x_{1z}} \end{pmatrix} \\ &= \begin{pmatrix} 0 & (\mathbf{x}_2 - \mathbf{x}_3)_z & (\mathbf{x}_3 - \mathbf{x}_2)_y \\ (\mathbf{x}_3 - \mathbf{x}_2)_z & 0 & (\mathbf{x}_2 - \mathbf{x}_3)_x \\ (\mathbf{x}_2 - \mathbf{x}_3)_y & (\mathbf{x}_3 - \mathbf{x}_2)_x & 0 \end{pmatrix}. \end{aligned}$$

This skew-symmetric matrix is associated with vector $\mathbf{x}_{23}$ in doing the cross product with any vector $\mathbf{w} \in \mathbb{R}^3$,

$$\frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} \mathbf{w} = \mathbf{x}_{23} \times \mathbf{w} \ . \tag{2}$$

Moreover,

$$\begin{aligned} \frac{\partial \hat{\mathbf{n}}}{\partial \mathbf{x}_1} &= \frac{\partial}{\partial \mathbf{x}_1} \left(\frac{\mathbf{n}}{|\mathbf{n}|}\right) \\ &= \frac{1}{|\mathbf{n}|} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} - \frac{\mathbf{n}}{|\mathbf{n}|^2} \frac{\partial |\mathbf{n}|}{\partial \mathbf{x}_1} \\ &= \frac{1}{|\mathbf{n}|} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} - \frac{\mathbf{n}}{|\mathbf{n}|^2} \frac{\mathbf{n}^T}{|\mathbf{n}|} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} \\ &= \frac{1}{|\mathbf{n}|} \left(\mathbf{I} - \frac{\mathbf{n}\mathbf{n}^T}{|\mathbf{n}|^2}\right) \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} \end{aligned} \tag{3}$$

Writing the 3×3 matrix $\frac{1}{|\mathbf{n}|}(\mathbf{I} - \frac{\mathbf{n}\mathbf{n}^T}{|\mathbf{n}|^2})$ as $\mathbf{N}$, together with Equ. 2 and 3 we have

$$\begin{aligned} D_{\mathbf{x}_1} &= \frac{\partial}{\partial \mathbf{x}_1} (\hat{\mathbf{n}}^T \mathbf{d}) \\ &= \frac{\partial \hat{\mathbf{n}}}{\partial \mathbf{x}_1} \mathbf{d} - \beta_1 \hat{\mathbf{n}} \\ &= \mathbf{N}(\mathbf{x}_{23} \times \mathbf{d}) - \beta_1 \hat{\mathbf{n}} \end{aligned} \tag{4}$$

Similarly there are

$$D_{\mathbf{x}_2} = \mathbf{N}(\mathbf{x}_{31} \times \mathbf{d}) - \beta_2 \hat{\mathbf{n}}, \tag{5}$$

$$D_{\mathbf{x}_3} = \mathbf{N}(\mathbf{x}_{12} \times \mathbf{d}) - \beta_3 \hat{\mathbf{n}}, \tag{6}$$

$$D_{\mathbf{x}_0} = \hat{\mathbf{n}} \tag{7}$$

Obviously, there is

$$\sum_{i=0}^{3} D_{\mathbf{x}_i} = \mathbf{0}. \tag{8}$$

2. Gradient of the E-E Signed Distance

Here we give the derivation of the gradient of edge-edge signed distance function here. The collision normal is defined as $\mathbf{n} = (\mathbf{x}_1 - \mathbf{x}_0) \times (\mathbf{x}_3 - \mathbf{x}_2)$, and we write $\mathbf{d} = \beta_0 \mathbf{x}_0 + \beta_1 \mathbf{x}_1 - \beta_2 \mathbf{x}_2 - \beta_3 \mathbf{x}_3$, then

$$D(\mathbf{x}) = \hat{\mathbf{n}} \cdot \mathbf{d} \tag{9}$$

The expression for $\frac{\partial \mathbf{n}}{\partial \mathbf{x}_0}$ is

$$\begin{aligned} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_0} &= \begin{pmatrix} \frac{\partial n_x}{\partial x_{0x}} & \frac{\partial n_x}{\partial x_{0y}} & \frac{\partial n_x}{\partial x_{0z}} \\ \frac{\partial n_y}{\partial x_{0x}} & \frac{\partial n_y}{\partial x_{0y}} & \frac{\partial n_y}{\partial x_{0z}} \\ \frac{\partial n_z}{\partial x_{0x}} & \frac{\partial n_z}{\partial x_{0y}} & \frac{\partial n_z}{\partial x_{0z}} \end{pmatrix} \\ &= \begin{pmatrix} 0 & (\mathbf{x}_2 - \mathbf{x}_3)_z & (\mathbf{x}_3 - \mathbf{x}_2)_y \\ (\mathbf{x}_3 - \mathbf{x}_2)_z & 0 & (\mathbf{x}_2 - \mathbf{x}_3)_x \\ (\mathbf{x}_2 - \mathbf{x}_3)_y & (\mathbf{x}_3 - \mathbf{x}_2)_x & 0 \end{pmatrix}. \end{aligned}$$

Similarly, there are

$$\begin{aligned}
\frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} &= -\frac{\partial \mathbf{n}}{\partial \mathbf{x}_0}\ . \\
\frac{\partial \mathbf{n}}{\partial \mathbf{x}_2} &= \begin{pmatrix} 0 & (\mathbf{x}_1-\mathbf{x}_0)_z & (\mathbf{x}_0-\mathbf{x}_1)_y \\ (\mathbf{x}_0-\mathbf{x}_1)_z & 0 & (\mathbf{x}_1-\mathbf{x}_0)_x \\ (\mathbf{x}_1-\mathbf{x}_0)_y & (\mathbf{x}_0-\mathbf{x}_1)_x & 0 \end{pmatrix}\ . \\
\frac{\partial \mathbf{n}}{\partial \mathbf{x}_3} &= -\frac{\partial \mathbf{n}}{\partial \mathbf{x}_2}\ .
\end{aligned} \tag{10}$$

Due to Equ. 3 there is

$$\begin{aligned}
D_{\mathbf{x}_0} &= \frac{\partial}{\partial \mathbf{x}_0}(\hat{\mathbf{n}}^T \mathbf{d}), \\
&= \frac{\partial \hat{\mathbf{n}}}{\partial \mathbf{x}_0}\mathbf{d} + \beta_0 \hat{\mathbf{n}}, \\
&= \mathbf{N}\frac{\partial \mathbf{n}}{\partial \mathbf{x}_0}\mathbf{d} + \beta_0 \hat{\mathbf{n}}, \\
&= \mathbf{N}(\mathbf{x}_{23} \times \mathbf{d}) + \beta_0 \hat{\mathbf{n}}
\end{aligned} \tag{11}$$

Similarly, there are

$$D_{\mathbf{x}_1} = -\mathbf{N}(\mathbf{x}_{23} \times \mathbf{d}) + \beta_1 \hat{\mathbf{n}} \tag{12}$$

$$D_{\mathbf{x}_2} = -\mathbf{N}(\mathbf{x}_{01} \times \mathbf{d}) - \beta_2 \hat{\mathbf{n}} \tag{13}$$

$$D_{\mathbf{x}_3} = \mathbf{N}(\mathbf{x}_{01} \times \mathbf{d}) - \beta_3 \hat{\mathbf{n}} \tag{14}$$

Also, there is

$$\sum_{i=0}^{3} D_{\mathbf{x}_i} = \mathbf{0}. \tag{15}$$

3. Conservation of the Momentum

In Equ.(4) of the paper, the diagonal mass matrix $\mathbf{M}$ is meant to maintain the center of mass, so that the angular momentum is least affected when used in a physical simulation. The linear momentum is naturally conserved within each stencil: letting $\Delta\mathbf{x}_i$ denote the position change, there is $\sum(m_i * \Delta\mathbf{x}_i) = \lambda \sum D_{\mathbf{x}_i} = \mathbf{0}$ due to Equ. 8 and Equ. 15 in this supplementary material. Further, scale the above equation by $\frac{1}{\Delta t}$ yields $\sum(m_i * \Delta\mathbf{v}_i) = 0$, which means the momentum of the stencil does not change.