Inverse Computational Spectral Geometry

EUROGRAPHICS 2021 Tutorial Proposal

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Short resumes

Prof. Emanuele Rodolà (PhD 2012, University of Venice, Italy) is Associate Professor of Computer Science at Sapienza University of Rome, where he leads the group of Geometric Learning & AI funded by an ERC Starting Grant (2018–) “SPECGEO - Spectral geometric methods in practice” *(directly related to the topics of this proposal)*. Previously, he was a postdoc at USI Lugano (2016–2017), an Alexander von Humboldt Fellow at TU Munich (2013–2016), and a JSPS Research Fellow at The University of Tokyo (2013). He received a number of awards, including Best Papers at 3DPVT 2010, VMV 2015, SGP 2016, 3DV 2019, he has been serving in the program committees of the top rated conferences in computer vision and graphics (CVPR, ICCV, ECCV, SGP, EG, etc.), founded and chaired several successful workshops including the workshop on Geometry Meets Deep Learning (co-located with ECCV/ICCV, 2016–2019), gave tutorials and short courses in multiple occasions at EUROGRAPHICS, ECCV, SGP, SIGGRAPH, SIGGRAPH Asia, and was recognized (9 times) as IEEE Outstanding Reviewer at CVPR/ICCV/ECCV. He spent visiting periods at Stanford, Harvard, Ecole polytechnique and Technion among others. His research interests include geometric deep learning, spectral geometry processing, abstract reasoning and interactions thereof, and has published around 80 papers on these topics.

Dr. Simone Melzi (PhD 2018, University of Verona, Italy) is a post-doctoral researcher at Sapienza University of Rome (Italy). Previously he was a post-doctoral researcher at the École Polytechnique (6 months) and the University of Verona (2018-2019). He graduated in math summa cum laude from “La Statale” University of Milan (2013). He received the Seal of Excellence for the H2020-MSCA-IF-EF-ST-2019 and the EG-Italy PhD thesis award 2018. He has been serving in the program committees and as a reviewer for the top computer graphics conferences and journals. His main research interests are geometry processing, 3D shape analysis and machine learning. He has authored over 20 publications in leading journals and conferences on these topics.

Dr. Luca Cosmo (PhD 2016, University of Venezia, Italy) is currently a post-doc at Sapienza University of Rome (Italy) and has a research fellowship with the University of Lugano (Switzerland) where he also held a one year post-doc position in 2019 under the supervision of Prof. Michael M. Bronstein. From 2016 to 2019 he was a post-doc researcher at the University of Venice and co-founded a start-up working on geometry processing for industrial products inspection. He has been serving in the program committee and as a reviewer for top conferences and journals in computer graphics and computer vision (CVPR, ECCV, SGP, EG, 3DV, etc.). He gave short courses and tutorials in geometry processing related topics at 3DV and SIAM conferences. His research interests span various topics in geometry processing and machine learning, with a particular focus on spectral geometry processing and geometric deep learning. He has authored several high impact publications on these topics in leading journals and conferences.

Prof. Michael Bronstein (PhD 2007, Technion, Israel) is a professor at Imperial College London, where he holds the Chair in Machine Learning and Pattern Recognition. Following the acquisition of his graph deep learning startup Fabula AI in 2019, he joined Twitter as Head of Graph ML Research. He has held visiting appointments at Stanford, Harvard, and MIT, and affiliated with three Institutes for Advanced Study (at TU Munich as Diesel Fellow (2017-2019), at Harvard as Radcliffe Fellow (2017-2018) and at Princeton as a visitor (2020)). He is a Member of Academia Europaea, Fellow of IEEE, BCS, and IAPR, ACM Distinguished Speaker, and the recipient of multiple awards, including five ERC grants, two Google Faculty awards, and two Amazon AWS ML research awards (*all in the fields of geometric computing and ML*). Michael's main research interests are in the interplay between geometry and machine learning. He has coined the term "geometric deep learning" and published extensively on this topic as well as on shape analysis, spectral geometry, and computer vision. He has served on editorial boards and program committees of various journals and venues in the field, and organized multiple meetings on geometry and ML, including recently at IPAM 2018, NeurIPS 2019, and ICML 2020. He is a frequent invited speaker on geometric computing and has given tutorials on these topics at leading conferences such as NeurIPS, CVPR, ICCV, ECCV, SIGGRAPH, Eurographics, and SGP.

Prof. Maks Ovsjanikov (PhD 2011, Stanford University, USA) is a professor at Ecole Polytechnique in France. He works on 3D shape analysis with emphasis on shape matching and correspondence. He has received a Eurographics Young Researcher Award in 2014 "in recognition of his outstanding contributions to theoretical foundations of non-rigid shape matching". He has served on the technical program committees of international conferences including SIGGRAPH and SIGGRAPH Asia, as a member of the editorial board of Computer Graphics Forum and has co-chaired the Symposium on Geometry Processing in 2016. In 2017 he received an ERC Starting Grant from the European Commission and in 2018 he received the Bronze Medal from the French National Center for Scientific Research (CNRS) for research contributions in Computer Science.

Outline

In the last decades, geometry processing has attracted a growing interest thanks to the wide availability of new devices and software that make 3D digital data available and manipulable to everyone. Typical issues that are faced by geometry processing algorithms include the variety of discrete representations for 3D data (point clouds, polygonal or tet-meshes and voxels), or the type of deformation this data may undergo. Powerful approaches to address these issues come from looking at the *spectral decomposition* of canonical differential operators, such as the Laplacian, which provides a rich, informative, robust, and invariant representation of the 3D objects. Reasoning about spectral quantities is at the core of *spectral geometry*, which has enabled unprecedented performance in many tasks of computer graphics (e.g. shape matching with functional maps, shape retrieval, compression, and texture transfer), as well as contributing in opening new directions of research.

The focus of this tutorial is on *inverse* computational spectral geometry. We will offer a different perspective on spectral geometric techniques, supported by recent successful methods in the graphics and 3D vision communities, as well as older, but notoriously overlooked results. Here, the interest shifts from studying the "forward" path typical of spectral geometry pipelines (e.g. computing Laplacian eigenvalues and eigenvectors of a given shape) to studying the inverse path (e.g. recovering a shape from given Laplacian eigenvalues, like in the classical "hearing the shape of the drum" problem). As is emblematic of inverse problems, the ill-posed nature of the reverse direction requires additional effort, but the benefits can be quite considerable as showcased on several challenging tasks in graphics and geometry processing.

The purpose of the tutorial is to overview the foundations and the current state of the art on inverse computational spectral geometry, to highlight the main benefits of inverse spectral pipelines, as well as their current limitations and future developments in the context of computer graphics. The tutorial is aimed at a wide audience with a basic understanding of geometry processing, and will be accessible and interesting to students, researchers and practitioners from both the academia and the industry.

Syllabus

1. **Introduction:** Motivation and historical overview.
2. **Problem foundations:** Shape-from-spectrum as a classical problem in mathematical physics, inverse eigenvalue problems in matrix calculus, shape-from-metric and shape-from-intrinsic operators.
3. **Background:** The forward direction of classical spectral geometry processing.
4. **Inverse spectral geometry in CG:** Motivations, applications, and examples in graphics.
5. **Computational techniques:** Existing approaches based on formulating an optimization problem, numerical methods and machine learning-based techniques.
6. **Applications:** Inverse spectral geometric pipelines addressing practical problems in graphics.
7. **Open problems and future directions:** Main limitations of current approaches, next steps and open problems.
8. **Conclusions and Q&A.**

Practicalities

Desired course length: Half day (2x90 minutes)

Estimated attendance: 80 people

Target audience: Introductory and Intermediate

Usefulness for the computer graphics community

From spectral quantities, it is possible to generate useful descriptors or signatures for shape retrieval and shape matching, to define energies for optimization problems in mesh registration, simplification and subdivision. All these results were obtained computing the spectrum for an input set of data and then applying some task-specific technique to achieve the desired goal. Our tutorial proposes a change of perspective on this pipeline that gives rise to innovative applications and unexplored directions. Starting from mathematical foundations, we will provide the necessary tools to explore the inverse computational spectral geometry problems suggesting several intriguing and promising questions: how much of the original geometry can we extract from these spectral quantities? How can they describe the real difference between 3D shapes? How can the deformations or relations of these encodings be transferred to the geometry that they are representing? Our tutorial focuses on the applications that one can target from this novel perspective, such as style transfer, shape modelling, geometry exaggeration, shape analogies, among many others. We believe that the contents of the tutorial are new for the majority of the community, and we hope they will foster further research interest in inverse spectral geometry problems.

Background

The tutorial will assume no particular background, beyond some basic working knowledge that is a common denominator for people in 3D computer graphics. We will introduce all the necessary notions and mathematical foundations. We target the tutorial to graduate students, practitioners, and researchers interested in geometry processing, shape analysis, optimization, matching, deformation, retrieval, and transfer.

Relationship to previous short courses/tutorials

To our knowledge, this is the first tutorial to specifically address *inverse* spectral geometric techniques for computer graphics and 3D vision. Several short courses and tutorials have been given on classical ("forward") spectral geometry, even recently, though none of them covers the inverse perspective. Examples of classical courses were given by Levy and Zhang at SIGGA 2009 and SIGGRAPH 2010 and by Bronstein, Ovsjanikov and colleagues at ICCV 2013, ICIP 2015 and CVPR 2017; graduate school lectures held by Patanè at SGP 2019 and by Solomon at SGP 2020; and several dedicated workshops (e.g. Banff 18w5090). Scattered lectures on inverse spectral geometry in graphics have been given by the organizers of this proposal in several occasions at workshop venues such as Dagstuhl, IPAM, Strobl, or as invited lectures in several research institutions, although this has never been done in a unified and widely accessible form before.

Relevant publications

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