Inverse Computational Spectral Geometry

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Abstract

In the last decades, geometry processing has attracted a growing interest thanks to the wide availability of new devices and software that make 3D digital data available and manipulable to everyone. Typical issues that are faced by geometry processing algorithms include the variety of discrete representations for 3D data (point clouds, polygonal or tet-meshes and voxels) , or the type of deformation this data may undergo. Powerful approaches to address these issues come from looking at the spectral decomposition of canonical differential operators, such as the Laplacian, which provides a rich, informative, robust, and invariant representation of the 3D objects. Reasoning about spectral quantities is at the core of spectral geometry, which has enabled unprecedented performance in many tasks of computer graphics (e.g., shape matching with functional maps, shape retrieval, compression, and texture transfer), as well as contributing in opening new directions of research. The focus of this tutorial is on inverse computational spectral geometry. We will offer a different perspective on spectral geometric techniques, supported by recent successful methods in the graphics and 3D vision communities, as well as older, but notoriously overlooked results. Here, the interest shifts from studying the "forward" path typical of spectral geometry pipelines (e.g., computing Laplacian eigenvalues and eigenvectors of a given shape) to studying the inverse path (e.g., recovering a shape from given Laplacian eigenvalues, like in the classical "hearing the shape of the drum" problem). As is emblematic of inverse problems, the ill-posed nature of the reverse direction requires additional effort, but the benefits can be quite considerable as showcased on several challenging tasks in graphics and geometry processing. The purpose of the tutorial is to overview the foundations and the current state of the art on inverse computational spectral geometry, to highlight the main benefits of inverse spectral pipelines, as well as their current limitations and future developments in the context of computer graphics. The tutorial is aimed at a wide audience with a basic understanding of geometry processing, and will be accessible and interesting to students, researchers and practitioners from both the academia and the industry.

1. Syllabus

- Introduction: Motivation and historical overview
- Problem foundations: Shape-from-spectrum as a classical problem in mathematical physics, inverse eigenvalue problems in matrix calculus, shape-from-metric and shape-from-intrinsic operators
- Background: The forward direction of classical spectral geometry processing
- Inverse spectral geometry in CG: Motivations, applications, and examples in graphics
- Computational techniques: Existing approaches based on formulating an optimization problem, numerical methods and machine learning-based techniques
- Applications: Inverse spectral geometric pipelines addressing practical problems in graphics
- Open problems and future directions: Main limitations of current approaches, next steps and open problems
- Conclusions and Q&A

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