Tutorial: Tensor Approximation in Visualization and Computer Graphics

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Introduction

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Overview

- **Part 1:** Introduction of the TA framework
  - Tucker and CANDECOMP/PARAFAC (CP) tensor decompositions
  - Rank-reduced tensor approximations, ALS methods
  - Useful TA properties and features for data visualization
  - Frequency analysis and DCT equivalence

- **Part 2:** Applications of TA in scientific visualization
  - Implementation details of tensor decomposition and tensor reconstruction algorithms
  - Practical examples (MATLAB, vmmlib)
  - TA-based volume visualization

- **Part 3:** Applications of TA in rendering and graphics
  - Examples for multidimensional datasets in rendering and graphics applications
  - Influence of data organization, parametrization and error metric
  - Clustering and sparsity
  - Processing irregular and sparse input samples
Motivation

• Compact representation of large scale data sets important in many areas of scientific visualization and computer graphics

• Use a mathematical framework for the decomposition of the input data into bases and coefficients

• Key features of a compact data representation:
  ‣ effective decomposition
  ‣ good data reduction
  ‣ fast access and reconstruction

• Tensor approximation methods have shown to be a powerful and promising tool
Decomposition Bases

- Decompositions into bases and weight coefficients can either use a set of pre-defined fixed bases, or computed bases.
- Pre-defined bases are given a priori, often represent some form of frequency analysis, and the decomposition may be fast to compute.
  - e.g. Fourier, Discrete Cosine and Wavelet Transforms.
- Computed bases, learned from the input data, may provide a better data fit, approximation and fast reconstruction.
  - e.g. SVD, PCA and Tensor Decomposition.
Tensor Approximation – TA

- TA: Generalization of low rank SVD matrix approximation to higher order data collections
- Data analysis, bases computation via tensor decomposition followed by rank-reduced reconstruction and approximation
  - data reduction achieved through reduced bases dimensionality

\[ \mathbf{A} = \mathbf{B} \times \mathbf{U}^{(1)} \times \mathbf{U}^{(2)} \times \mathbf{U}^{(3)} \]

Tucker tensor decomposition