Human Activity Modeling on Shape Manifold

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Abstract

In this paper we propose a stochastic modeling of human activity on shape manifold. From a video sequence, human activity are extracted as a sequence of shape. Such sequence is considered as one realization of a random process on shape manifold. Then different activity is modeled by manifold valued random process with different distribution. To solve the stochastic modeling on manifold, we first map the process on the shape manifold to a Euclidean process. Then the process is modeled by linear models such as stationary incremental process and a piecewise stationary incremental process. The mapping from manifold valued process to Euclidean process is known as stochastic development. The idea is to parallelly transport the tangent of curve on manifold to a single tangent space. The advantage of such technique is the one to one correspondence between the process in flat space and the one on manifold. The proposed algorithm is tested on two activity data base \cite{RS01, BGSB05}. The result demonstrate the high accuracy of our modeling in characterizing different activities.

Categories and Subject Descriptors (according to ACM CCS): I.2.10 [Vision and Scene Understanding]: Motion—Shape Modeling and recovery of physical attributes

1. Introduction

Human activity recognition is of interest in a wide range of applications, spanning areas such as security surveillance, person identification and content-based image retrieval. In addition to security applications, the explosively increasing daily usage of video cameras has and continues to motivate a great interest in motion analysis and understanding in video for diverse applications. Recent progress on human activity analysis from video data has been well documented in \cite{PRSV08, AC99}.

As well remarked in \cite{EL04}, the observability of human activity is restricted by the body’s configuration. This implies that for a specific representation, the space of human activity is not necessarily linear. Of the various possible nonlinear representations of human activity \cite{VCC04}, we choose to view any given activity of interest as a shape sequence \cite{BGSB05, VRCC05}.

Different shape representations lead to different shape manifolds. In Kendall’s shape theory \cite{Ken84}, a shape is considered to be a set of landmarks on the boundary of an object. The Kendall pre-shape space is a quotient space (where translation and scaling were quotiented out) and geometrically as a hyper sphere. An AR/ARMA model of human activities was proposed in \cite{VRCC05} by projecting the shape sequences onto the tangent space of Kendall’s pre-shape space. To overcome the problem of systematically picking consistent landmarks of shapes, we consider a shape as a simple and closed planar curve. Such a shape formulation was first proposed in \cite{SL02} and further developed in \cite{KSWM04} with a numerically efficient computation of a geodesic metric between two shapes. With a similar goal of statistical classification of shape sequences as in \cite{VRCC05}, our goal here is to first develop a stochastic model of a curve evolution (representing a shape sequence trajectory on a manifold) on a non-linear space by regressing the problem onto a linear space. In \cite{VRCC05} a shape on a pre-shape sphere is projected onto a tangent space at the mean shape. Adopting such a tangent approximation is, however, valid in only a sufficiently small neighborhood. The invertibility of such a projection on pre-shape sphere only holds when the shape sequence does not cross the “north or south poles” of the hypersphere. Generally on a smooth manifold, the condition of such an orthogonal projection is restricted to a local area of the manifold. In contrast, our proposed regression is intrinsically constructed by a curve development \cite{BC94} as a 1-1 mapping an evolution curve on any smooth manifold to a curve in a flat space. Intuitively, our proposed method,
seeks to exploit all the tangent spaces along the curve on the manifold in order to construct the mapping to flat space. The curve’s tangents at different tangent spaces of the manifold are parallelly transported to a single tangent space. By integration, we proceed to obtain the called curve development in a tangent space.

We exploit the afore-described approach to develop in this paper, an intrinsic stochastic model with a goal to classify activities. Assuming a proper human silhouette segmentation of each frame in a video sequence of interest, a specific activity may be summarized by a sequence of individual closed curves/shapes in a form of an evolution curve on the underlying shape manifold. An reasonable modeling for a activity, for instance like “running”, are expected to describe the different data samples of “running”. In this paper, the set of the different representative curves of “running” are viewed as the realizations of the “running process”, which more precisely is a random process on the shape manifold. As a result, any activity process of interest, may hence be modeled as a manifold valued random process.

Using the development in [Hsu01], any random process on a finite dimensional manifold may be considered as the solution of some stochastic differential equation \( SDE(X_0,V,Z) \) defined on the manifold. In other words, the space of human activities in the same class, is modeled by a solution space of \( SDE(X_0,V,Z) \). It follows that for a certain class of activities, the vector field \( V \) and the driving process \( Z \) characterize the dynamic information on the manifold.

A direct analysis of a manifold valued SDE is, however very challenging. Such a problem is solved in [Hsu01] by first mapping a random process on a manifold to a Euclidean space, where the stochastic analysis is then carried out. Such mapping from manifold to flat space consists of two steps: a horizontal lift is first completed, followed by a curve development (the definition of these differential geometry concepts will be provided in section 2). The advantage of such a framework is in achieving a 1-1 correspondence of the regression from the manifold to the Euclidean space. By constructing a flat connection on the manifold space, we show how the solution space of \( SDE(X_0,V,Z) \) is in 1-1 correspondence to that of \( SDE(X_0,U,W) \) in the linear frame bundle based on the shape manifold (see details below). It is much more convenient to work with \( SDE(X_0,U,W) \), because the vector field \( U \) is a known unique to \( X \). The resulting equation is only characterized by the Euclidean valued driving process \( W \), which is called a stochastic curve development.

In the balance of this paper, we first provide a brief (but sufficient for this development) introduction to manifold geometry and to stochastic analysis on manifolds. The preprocessing on shapes is introduced in Section 2.1, to make the shape manifold finite dimensional. In Section 3, we introduce the stochastic curve development for a human activity process as a mapping from a manifold to a flat space. In Sections 4 and 5, we construct a connection on the shape manifold, and derive the corresponding curve development result for a given human activity.

2. Background

To study human activity as a random process on a shape manifold, we use the shape manifold in [KSWM04]. We numerically compute a connection on shape manifold to map a random process from a manifold to a flat space as in [Hsu01]. In the end, our statistical modeling in the flat space helps achieve activity classification. To proceed, we first provide in Section a brief review of the required background in differential geometry to allow us to define a shape manifold as a working space. We also describe the required tools of parallel transportation and curve development on a shape manifold. For clarity and easy cross-referencing, we summarize the notation adopted throughout the paper in Table 2.

2.1. Shape Manifold

According to [KSWM04], a planar shape is a simple and closed curve in \( \mathbb{R}^2 \)

\[
\alpha(s) : I \rightarrow \mathbb{R}^2,
\]

where an arc-length parameterization is adopted. A shape is represented by a direction index function \( \Theta(t) \). With such a parameterization, \( \Theta(s) \) may be associated to the shape by

\[
\frac{\partial \alpha}{\partial s} = e^{\Theta(s)}.
\]

The ambient space of the manifold of \( \Theta \) is an affine space based on \( \mathbb{L}^2 \). Thus, we have

\[
\Theta \in A(\mathbb{L}^2).
\]

The restriction of a shape to be a closed and simple curve, and invariant over rigid Euclidean transformations. The shape manifold \( M \) is defined by a level function \( \phi \) as

\[
\phi(\Theta) = \left( \int_0^{2\pi} \Theta ds, \int_0^{2\pi} \cos(\Theta) ds, \int_0^{2\pi} \sin(\Theta) ds \right)
\]

\[
M = \phi^{-1}(\pi, 0, 0)
\]

One of the most important properties of \( M \) is that the tangent space \( T\Theta M \) is well defined. Such a property not only simplifies the analysis, but also makes possible the incremental computation,

\[
T\Theta M = \{ f \in \mathbb{L}^2 \mid f \perp \text{span}\{1, \cos(\Theta), \sin(\Theta)\} \}
\]

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Table 1: Notation Table

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>a manifold</td>
</tr>
<tr>
<td>$H$</td>
<td>a horizontal lift in principle fiber bundle</td>
</tr>
<tr>
<td>$f$</td>
<td>a mapping between manifolds</td>
</tr>
<tr>
<td>$f_*$</td>
<td>a push forward between the corresponding tangent spaces</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>a curve on manifold</td>
</tr>
<tr>
<td>$\gamma_*$</td>
<td>the tangent along the curve $\gamma$</td>
</tr>
<tr>
<td>$\tilde{\gamma}$</td>
<td>a horizontal lift of curve $\gamma$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>a curve development of $\gamma$ in Euclidean space</td>
</tr>
<tr>
<td>$V$</td>
<td>a vector field defined on a manifold</td>
</tr>
<tr>
<td>$\tilde{V}$</td>
<td>a horizontal lift of vector field $V$</td>
</tr>
<tr>
<td>$(P,G,M)$</td>
<td>a principal fiber bundle $P$ based on the manifold $M$ with the group actor $G$</td>
</tr>
<tr>
<td>$\mathcal{F}(M)$</td>
<td>linear frame bundle</td>
</tr>
<tr>
<td>$X$</td>
<td>a manifold valued random process</td>
</tr>
<tr>
<td>$W$</td>
<td>a stochastic development of $X$, a Euclidean valued random process</td>
</tr>
<tr>
<td>$U$</td>
<td>a horizontal lift of $X$, a Fiber Bundle valued random process</td>
</tr>
</tbody>
</table>

2.2. Connection on Manifold

To study a curve on manifold, we should first overcome the complexity due to the curvature of the manifold. For example, a geodesic in a flat space may simply be described by a straight line. On a shape manifold, however, the "straightness" of a curve is not as straightforward as that in the Euclidean space. In differential geometry, one defines "straightness" by the covariant derivative along a curve. A curve is a geodesic if the covariant derivative of a curve's tangent along the curve is zero everywhere. More generally, the concept of a connection defines the relation between a tangent space along a curve on a manifold. Such a relation may be used to determine the parallelism of two tangent vectors in two different tangent spaces. A connection may be defined in ways to be written in different forms, one of which we exploit to develop our stochastic modeling of a curve on manifold. Specifically, we proceed to describe the parallelism between tangent spaces by specify parallel of basis pair at different tangent space. Such a form of connection is much simpler to calculate and takes place in a so-called principal fiber bundle as a horizontal vector field.

Definition 1 (Principal Fiber Bundle) A principal fiber bundle is a set $(P,G,M)$, where $P,M$ are $C^\infty$ manifolds, and $G$ is a Lie group such that

1. $G$ acts freely on the right of $P$, $P \times G \to P$. For $g \in G$, we shall also write $R_g$ for the map $g : P \to P$.
2. $M$ is the quotient space of $P$ by an equivalence relation under $G$ (any shape subjected to a $g \in G$ is equivalent to itself), and the projection $\pi : P \to M$ is $C^\infty$, so for $m \in M$, $G$ is simply transitive on $\pi^{-1}(m)$.
3. $P$ is locally trivial.

One example of a principal bundle is the linear frame bundle $\mathcal{F}(M)$, which will be extensively used in this paper. For any point $u$ in $\mathcal{F}(M)$ can be written as

$$u = (x, b)$$

where $x \in M$ and $b = e_1, e_2, ..., e_n$ is a basis of the associated tangent space $T_x M$. The group $G$ acting on a fibre is $GL(n)$

Definition 2 (Connection) A connection on the principal bundle $(P,G,M)$ is a d-dimensional distribution $H$ on $P$, where $d = dim(M)$, such that

1. $H \in C^\infty$
2. For every $p \in P$, $H_p + V_p = P_p$
3. For every $p \in P, g \in G$, $(R_g)_*H_p = H_{pg}$.

where $V_p$ is the vertical subspace, $Y \in P_p$ is vertical if and only if $\pi_*Y = 0$

With the definition of a connection on a manifold in hand, we can achieve a horizontal lift from a manifold to a linear frame bundle. More specifically, in this operation we find a "horizontal" curve in a linear frame bundle corresponding to a curve on a manifold. As a result, we discover the appropriate distribution\(^3\) of bases along the curve accomplishing the parallel transport of a shape to its next evolution in the sequence. With the parallel bases along a curve, the parallelism of two tangent bases at different tangent space can be represented by its corresponding coefficients. By preserving the coefficients, we can move all the tangents to a single tangent plane. Such an operation is called a parallel transport along a curve. If we reintegrate all the parallely transported tangents in that single tangent plane, we have a curve in the flat space. This is referred to as a curve development in the tangent plane.

\(^3\) In geometry on manifolds, a distribution is a collection of subspaces of tangent spaces.
In a manifold equipped with a connection, we can construct under certain conditions a 1-1 mapping from a curve on the manifold to a curve in the principal fiber bundle. We subsequently facilitate the curve development onto \( \mathbb{R}^d \). In Figure 1, we illustrate the definition of \( H \) in one particular principal fiber bundle, the frame bundle \( \mathbb{F}(M) \). \( H_p \) could be understood as a subspace of \( T_p\mathbb{F}(M) \) which is smoothly defined for all \( p \in \mathbb{F}(M) \).

Let \( \tilde{\gamma} \) be the horizontal lift of \( \gamma \) on manifold \( M \). \( \gamma_s^* \) is the tangent along \( \gamma \) such that \( \forall t, \tilde{\gamma}_t \in H_{\gamma(t)} \).

Definition 3 (Horizontal Lift) Let \( \gamma \) be a piecewise \( C^\infty \) curve in \( M \), \( \gamma: [0,1] \rightarrow M \). Let \( p \in \pi^{-1}(\gamma(0)) \). Then there exists a unique lift \( \tilde{\gamma} \) of \( \gamma \) such that \( \tilde{\gamma}_0 = p \).

Definition 4 (Curve Development) Let \( \tilde{\gamma} \) be a piecewise \( C^\infty \) curve in \( M \) starting at \( m \), \( b = \{ e_1, e_2, ..., e_N \} \) a basis at \( m \). Let \( \tilde{\gamma} \) be the horizontal lift of \( \gamma \) in \( \mathbb{F}(M) \). Then by writing,

\[
\beta = \int_0^1 \tilde{\gamma}^{-1} \gamma_s^* ds
\]

where

\[
(\tilde{\gamma}^{-1} \gamma_s^*)_s = \langle e_i(s), \gamma_s^*(s) \rangle
\]

we define a curve in \( \mathbb{R}^N \) which is called a development of \( X \) into \( \mathbb{R}^N \) with respect to \( b \). And \( b \circ \beta \) is a curve in \( T_b M \) called the development of \( X \) in \( T_b M \). \( b \circ Y \) is independent of the choice of basis \( b \) at \( m \). A conceptual distribution is demonstrated in Figure (2).

Definition 5 (Parallel Transport) Let \( \gamma \) be a piecewise \( C^\infty \) curve in \( M \) equipped with a connection \( H \). Let \( \tilde{\gamma} \) be the horizontal lift of \( \gamma \). Let \( V \) be the vector field defined along the curve \( \gamma \). Thus the parallel transport of \( V(\gamma(t)) \) from \( \gamma(t) \) to \( \gamma(t + h) \) can be defined as the mapping between tangent planes \( \tau : T_{\gamma(t)} M \rightarrow T_{\gamma(t + h)} M \),

\[
\tau_h(V(\gamma(t))) = \tilde{\gamma}(t + h) \circ \tilde{\gamma}^{-1}(t) \circ V(\gamma(t))
\]

A more expanded and detailed discussion of connections may be found in [BC94] [KN96].

2.3. Stochastic Horizontal Lift and Development

The horizontal lift and curve development mentioned in the last section may also be extended to a stochastic setting. According to [Hsu01], a stochastic horizontal lift and its development may be defined as follows,

Definition 6 (Stochastic Horizontal Lift and Development) (1) An \( \mathbb{F}(M) \)-valued semimartingale \( U \) is said to be horizontal if there exists an \( \mathbb{R}^d \)-valued semi-functional \( W \) such that

\[
dU_t = \sum_{i=1}^d H_i(U_t) \circ dW_t^i
\]

where \( H_i \) is the fundamental horizontal vector field that span \( H \).

(2) Let \( W \) be an \( \mathbb{R}^d \)-value semimartingale and \( U_0 \) an \( \mathbb{F}(M) \)-valued, \( F_0 \)-measurement random variable. The solution \( U \) of SDE (11) is called the stochastic development of \( W \) in \( \mathbb{F}(M) \).

Its projection \( \pi(U) \) is called a stochastic development of \( W \) in \( M \).

(3) Let \( X \) be an \( M \)-valued semimartingale. An \( \mathbb{F}(M) \)-value horizontal semimartingale \( U \) such that its projection \( \pi(U) = X \) is called a stochastic horizontal lift of \( X \).

With the horizontal lift, a random process \( X_t \) on a manifold may be lifted to a random process \( U_t \) in an associated frame bundle. By the development, the random process in the frame bundle may then be mapped to a Euclidean process \( W \). Under fixed initial conditions, such a two step operation will establish a 1-1 correspondence of a random process on a manifold to one in a flat space. More importantly, the well defined correspondence provides an alternative way to study a random process on a manifold as a more convenient analysis in the Euclidean space. A more detailed discussion of this development will be given in Section 3.

3. Dynamic of Human Activity on Shape Manifold

In this paper, a human activity as captured in a video clip is represented by a shape/silhouette time sequence. Using the shape representation in Section 2, a shape sequence is hence
viewed as a path on a shape manifold. For a given activity, for example running, there are many different representative paths on the shape manifold. To construct a model which allows such a variability, we view these paths as realizations of an underlying random process on the manifold. The curvature of a manifold, however, generally renders the classical stochastic analysis in a flat space unapplicable.

Hsu in [Hsu01] proposes an efficient analysis framework to construct an invertible mapping from a manifold-valued random process to a Euclidean-valued random process. The essence of the mapping is to compute a Euclidean process that can drive a stochastic differential equation (SDE) to generate a manifold-valued semi-martingale. In a Euclidean space the semi-martingales are much simpler to analyze. In contrast to the orthogonal projection method onto a tangent space around a mean, Hsu’s theory provides a one to one correspondence between a process on a manifold and one on a Euclidean space. This improved accuracy of representation is primarily due to the so-called “rolling without sliding” property of a parallel transport.

As in [Hsu01], our mapping of a random process to a flat space proceeds composite in two steps: firstly, by a horizontal lift, i.e. a manifold-value process \( X_t \) may be mapped to a frame bundle valued random process \( U_t \). Then by development, the frame bundle valued random process is mapped to a Euclidean valued random process. Any random process on the shape manifold, may then be written as a solution to some SDE \( V(X_0, Z) \). Generally we have

\[
X_t = X_0 + \int_0^t \sum_i V_i(X_s) \odot dZ^i_s, \quad (12)
\]

where, \( X_0 \) is the initial condition, \( V_i \) is a smooth vector field defined on \( M \) and \( Z^i_t \) is a Euclidean valued random process driving Equation (12). The stochastic integration here is Stratonovich integration. More intuitively Equation 12 can be understood as \( dX_t = \sum_i V_i(X_s) \odot dZ^i_s \). Thus the dynamic described by equation 12 is characterized by both vector field \( V \) and the driving process \( Z \). However, the form of Euclidean process \( Z \) is variant to different selection of \( V \). In contrast to our goal to construct a \( 1 \to 1 \) mapping from manifold process to Euclidean process, there is no one one correspondence such as \( X \rightarrow Z \) without giving knowledge about \( V \). Previous work in [Hsu01] solve this problem by setting \( V \) equal to the horizontal lift of \( X_t \). Given the uniqueness of horizontal lift, the resulting driving process \( Z_t \) will have the one one correspondence to \( X_t \). Let vector field \( U_t \) be the horizontal lift of \( X_t \) in \( F(M) \), Equation 12 may be rewritten as

\[
X_t = X_0 + \int_0^t \sum_i U^i_t \odot dW^i_t. \quad (13)
\]

According to the definition of frame bundle and stochastic horizontal lift in section 2 we know

\[
U_i = \{e_1, e_2, \ldots, e_i, \ldots, e_n\} \quad (14)
\]

where \( e_i \) is the basis of \( T_NM \). In Equation 13 the differential \( dX_t \) is represented in a selected basis \( U_t \) with corresponding driving process \( W_t \). For an orthogonal basis one can write ,

\[
dW^i_t = \langle e_i, dX_t \rangle \quad (16)
\]

Such rewrite of Equation 12 provides a representation of the random process \( X_t \) on a manifold in terms of the Euclidean random process \( W_t \), which is the stochastic anti-development of \( X_t \) according to the definition in section 2.3. Given a connection \( H, U_t \) is uniquely determined for \( X_t \) up to a group action along the fiber \( \pi^{-1}(X_t) \). So the mapping from \( X_t \) to \( W_t \) is a \( 1:1 \) mapping if the initial condition \( U_{t=0} \) is given.

In the above discussion, we provide a \( 1 \rightarrow 1 \) mapping from \( X_t \in M \) to \( W_t \in \mathbb{R}^{dim(M)} \). The critical point for implementing such a mapping is the specific form of the connection \( H \) which we discuss in Section 4.

4. Flat Connection on a Shape Manifold

The construction of \( H \) is critical to the implementation of a horizontal lift and a curve development. In theory there may exist many different \( H \) for a given manifold. Once the concrete form of \( H \) is determined, the geometry of a manifold is specified accordingly. Among different kind of connection, we adopt the flat connection for the efficiency of calculate. The implementation of the flat connection \( H \) proceeds by constructing a smooth section \( \sigma : M \rightarrow F(M) \) in the linear frame bundle \( F(M) \). Let \( \pi' : M \times G \rightarrow G \) be the projection from the linear frame bundle to the general linear group. Let \( u = (m \in M, g \in GL(N)) \in F(M) \). We define \( \pi'(u) \) as the following,

\[
\pi'(u) = g\sigma(m)^{-1} \quad (17)
\]

We know given any two point \( u_1, u_2 \) along the fiber there exist a unique \( g \in GL(n) \) such that

\[
u_1 = R_g \circ u_2 \quad (18)
\]

So \( \pi'(u) \) hence implies that the matrix transformation that maps the basis \( \sigma(m) \) to \( g \).
Given \( \pi' \), the flat horizontal vector field can be defined as the kernel of the pushforward of \( \pi' \):

\[
H_p = \text{Ker}(\pi'_s)
\]

(19)

where \( \pi'_s : TF(M) \to TG \) is the push forward of \( \pi' \). To interpret equation 19, we zoom into a single point \( u \in F(M) \). Let \( u = (m_0, \theta_0) \). Let curve \( \tilde{\theta}(t) \in (m, \pi'(m_0)\sigma(m)) \subset F(M) \). Thus we have

\[
\pi'(\tilde{\theta}(t)) = \pi'(m_0)
\]

Consequently,

\[
\pi'_s(\tilde{\theta}(t)) = 0
\]

(21)

Thus \( H_p(u) \) is actually the tangent on the subspace \((m, \sigma(m)\pi'(m_0)) \subset F(M) \).

In the implementation of the smooth section \( \sigma \), we need to assign each point \( \theta \in M \) a basis \( \{e_i\}_{i=1,2,3} \) for the tangent space \( T_0 M \). From Section 2, we know that the tangent space of \( M \) can be written as

\[
T_0 M = \{ v \in S | v \perp \text{span}\{1, \cos(\theta), \sin(\theta)\}\}.
\]

(22)

The essential form of the basis in \( T M \) is a set of partial differential operators. Firstly the basis of \( T M \) is represented by the basis of \( \{B_i\} \) in \( L^2 \), which is orthogonal to subspace \( \text{span}\{1, \cos(\theta), \sin(\theta)\} \). Since \( \tilde{M} \) is the Fourier approximation of \( M \), \( \forall i, \{B_i\} \) is projected onto the Fourier approximation space \( S \) as \( BS_i \). Then the basis of \( TM \) can be extracted as a linearly independent subset of bases functions \( BS_i \). The details of this procedure implementation are as follows.

One can show that the following set of functions is a linearly independent set,

\[
\{1, \cos(\theta), \sin(\theta), \cos(2\theta), \sin(2\theta), \ldots\},
\]

(23)

Let \( B_i \) be the result of a Gram Schmidt orthogonalization of the above basis in ambient space as shown in figure 3

\[
\{v_1, v_2, v_3, B_{i=1,2,3,\ldots}\} = \text{ON}\{1, \cos(\theta), \sin(\theta), \cos(2\theta), \sin(2\theta), \ldots\}
\]

where \( \{v_1, v_2, v_3\} \) are the first three basis vectors from the Gram Schmidt procedure which satisfy \( \text{span}\{v_1, v_2, v_3\} = \text{span}\{1, \cos(\theta), \sin(\theta)\} \). These basis vectors are excluded because they are orthogonal to the tangent space of \( M \). Then \( B_i \) is the ambient representation of the basis of \( T_0 M \).

As shown in figure 3, the orthogonal projection from \( L^2 \) onto \( S \) can be written as a Fourier approximation of \( B_i \) and denoted by \( BS_i \). Letting \( \phi_j \) denote the Fourier basis functions, it follows that

\[
BS_i = \sum_{j=1}^{N_0} \hat{B}_{i,j} \phi_j,
\]

(24)

where

\[
\hat{B}_{i,j} = <B_i, \phi_j>
\]

(25)

for \( 0 < i < \infty \), \( 0 < j < N \). \( \hat{B} \) is the submatrix composed of the first \( N \) columns of the full rank infinite matrix \( <B_i, \phi_j> \). Thus \( \text{rank}(\hat{B}) = N \), then there exists a \( i(k) \) such that \( 0 < k < N \), \( \hat{B}_{i(k), j} \) are linearly independent.

Then we can get a basis for \( T_m M \) by a Gram Schmidt procedure applied to \( BS_{i(k)} \):

\[
e_k = \text{ON}\{BS_{i(1)}, BS_{i(2)} , BS_{i(3)}, \ldots\}.
\]

(26)

5. Stochastic Development of Human Activity

In the shape manifold \( M \) equipped with the flat connection \( H \) defined in Equation (19), the horizontal lift \( U_t \) of \( X_t \) with initial condition \( U_0 \) is computed as follows

\[
U_t = R_{\theta} \sigma(X_t)|_{U_0} \circ \sigma(X_t)
\]

(27)

In the ambient space \( A(S) \), \( \forall t, U_t, \sigma(X_t) \) can be represented by \( N \times N \) invertible matrix. For example \( U_t = [e_1, e_2, \ldots, e_N] \), where \( e_i \in R^N \) span the tangent space \( T_0 M \). In such setting \( U_t \) can be calculated as matrix multiplication in ambient space.

\[
(U_t)_{ij} = \sum_k \sigma(X_t)|_{U_0} \sigma(X_t)_{kj}
\]

(28)

Consequently the development \( W_t \) of \( X_t \) can be written as,

\[
W_t = \int_{0}^{t} (U_t)^{-1} dX_t
\]

(29)

In Figure (4), a few numerical results are demonstrated for \( W_t \) for three activities: walking, running and bending.

As discussed in Section 3, under a specific connection...
The activity data in [BGSB05] includes 10 different activities. Each one has 9 video sequence for 9 different actors. The data base provide the background substraction results of video. We perform level set segmentation to extract the contour of shape. [CSYK09].

We first assume that the incremental process $dW_t$ is stationary and ergodic. Then by the distribution of $dW$ we can characterize different activities. We next proceed to estimate the covariance matrix $K(dW^i, dW^j)$ as the feature of choice for the underlying distribution.

$$K(dW^i, dW^j) = E((dW^i - E(dW^i))(dW^j - E(dW^j)))$$  
(30)

where $dW^i$ is the $i$th element of the vector $dW$. The distance between two different covariance matrices is defined by the Frobenius norm as,

$$D(K1, K2) = \|K1 - K2\|_F.$$  
(31)

The results of $D(K1, K2)$ for the data base in [BGSB05] and CMU-Mobo [RS01] are shown in Figure 5. Using the distance matrix $D$, we may carry the recognition/classification task by using, for example, the leave-one-out algorithm. The nearest neighborhood algorithm is used for classification. If we let $N_B$ be the total number of realizations of B, and $N(B,A)$ the number of realizations of B classified as A activity, we have

$$P(A|B) = \frac{N(B,A)}{N(B)}.$$  
(32)

Table 6 shows the recognition rate of both data bases.

Figure 5: Distance matrix of data base in [BGSB05]: (a) distance matrix of CMU Mobo data base;

To perform a consistent comparison of results published in [BGSB05], we need to change our experiment to the same setting. In [BGSB05] the number of activity observations is
increased by segmenting any video sequence for a given activity into many overlapped chunks. The segments are assumed independent and the classification is carried out. The performance of our proposed method is summarized in the following tables.

7. Conclusion

In this paper, we provide a systematic framework for the stochastic modeling of human activity on shape manifold. In theory, such framework is one one mapping from random process on manifold to the random process in Euclidean space. The implementation developed by adapting the general horizontal lift and curve development with flat connection to the particular shape manifold. The advantage of such mapping is that the problem of modeling on the original shape manifold is now regressed to the traditional modeling in Euclidean environment. In the resulted flat space, the representative random process of activity is modeled as stationary incremental process. The experiment on two different data bases well demonstrate the performance of the proposed modelings of activities.

References


Table 2: table of recognition rate. the number in () is the result in [BGSB05]

<table>
<thead>
<tr>
<th>P activation</th>
<th>bend</th>
<th>jack</th>
<th>jump</th>
<th>pjump</th>
<th>run</th>
<th>side</th>
<th>skip</th>
<th>walk</th>
<th>wave1</th>
<th>wave2</th>
</tr>
</thead>
<tbody>
<tr>
<td>bend</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>jack</td>
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<td>1 (0.98)</td>
<td>0 (0.02)</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>jump</td>
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<td>0</td>
<td>0</td>
<td>1 (0.971)</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>pjump</td>
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<td>0</td>
<td>0</td>
<td>0.944 (1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>0</td>
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<td>0.944 (0.892)</td>
<td>0.0556</td>
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<td>0</td>
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<td>(0.019)</td>
<td>1 (0.972)</td>
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