A Unified Cloth Untangling Framework Through Discrete Collision Detection

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1. Gradient of the V-F Signed Distance

Here we give the derivation of the gradient of vertex-face signed distance function here. If we write \( \mathbf{d} = \mathbf{x}_0 - \sum_{i=1}^{3} \beta_i \mathbf{X}_i \), then vertex-face signed distance is

\[
D(x) = \hat{n} \cdot \mathbf{d},
\]

where \( \mathbf{x}_1, \mathbf{x}_2 \) and \( \mathbf{x}_3 \) are face vertices, and the face normal is defined as \( \hat{n} = \mathbf{n}/|\mathbf{n}| \), and \( \mathbf{n} = (\mathbf{x}_2 - \mathbf{x}_1) \times (\mathbf{x}_3 - \mathbf{x}_1) \). We first give the expression for \( \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} \):

\[
\frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} = \begin{pmatrix}
\frac{\partial n_x}{\partial x_1} & \frac{\partial n_y}{\partial x_1} & \frac{\partial n_z}{\partial x_1} \\
\frac{\partial n_x}{\partial y_1} & \frac{\partial n_y}{\partial y_1} & \frac{\partial n_z}{\partial y_1} \\
\frac{\partial n_x}{\partial z_1} & \frac{\partial n_y}{\partial z_1} & \frac{\partial n_z}{\partial z_1}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
0 & (x_2 - x_3)_z & (x_3 - x_2)_y \\
(x_3 - x_2)_z & 0 & (x_2 - x_3)_x \\
(x_2 - x_3)_y & (x_3 - x_2)_x & 0
\end{pmatrix}
\]

This skew-symmetric matrix is associated with vector \( \mathbf{x}_{23} \) in doing the cross product with any vector \( \mathbf{w} \in \mathbb{R}^3 \),

\[
\frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} \cdot \mathbf{w} = \mathbf{x}_{23} \times \mathbf{w}.
\]

Moreover,

\[
\frac{\partial \hat{n}}{\partial \mathbf{x}_1} = \frac{\partial}{\partial \mathbf{x}_1} \left( \frac{\mathbf{n}}{|\mathbf{n}|} \right) = \frac{1}{|\mathbf{n}|} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} - \mathbf{n} \frac{\partial |\mathbf{n}|}{|\mathbf{n}|^2} \frac{\partial |\mathbf{n}|}{\partial \mathbf{x}_1}
\]

\[
= \frac{1}{|\mathbf{n}|} \frac{\partial \mathbf{n}}{\partial \mathbf{x}_1} - \mathbf{n} \mathbf{n}^T \frac{\partial \mathbf{n}}{|\mathbf{n}|^2} |\mathbf{n}| \frac{\partial |\mathbf{n}|}{\partial \mathbf{x}_1}
\]

\[
= \frac{1}{|\mathbf{n}|} [I - \mathbf{n} \mathbf{n}^T] \frac{\partial \mathbf{n}}{|\mathbf{n}|^2} \frac{\partial |\mathbf{n}|}{\partial \mathbf{x}_1}
\]

Writing the \( 3 \times 3 \) matrix \( \frac{1}{|\mathbf{n}|} \left( I - \frac{\mathbf{n} \mathbf{n}^T}{|\mathbf{n}|^2} \right) \) as \( \mathbf{N} \), together with Equ. 2 and 3 we have

\[
D_{x_1} = \frac{\partial}{\partial \mathbf{x}_1} (\hat{n}^T \mathbf{d})
\]

\[
= \frac{\partial \hat{n}}{\partial \mathbf{x}_1} \mathbf{d} - \beta_1 \hat{n}
\]

\[
= \mathbf{N} (\mathbf{x}_{23} \times \mathbf{d}) - \beta_1 \hat{n}
\]

Similarly there are

\[
D_{x_2} = \mathbf{N} (\mathbf{x}_{13} \times \mathbf{d}) - \beta_2 \hat{n},
\]

\[
D_{x_3} = \mathbf{N} (\mathbf{x}_{12} \times \mathbf{d}) - \beta_3 \hat{n},
\]

\[
D_{x_0} = \hat{n}
\]

Obviously, there is

\[
\sum_{i=0}^{3} D_{x_i} = 0.
\]

2. Gradient of the E-E Signed Distance

Here we give the derivation of the gradient of edge-edge signed distance function here. The collision normal is defined as \( \mathbf{n} = (\mathbf{x}_1 - \mathbf{x}_0) \times (\mathbf{x}_3 - \mathbf{x}_2) \), and we write \( \mathbf{d} = \beta_0 \mathbf{x}_0 + \beta_1 \mathbf{x}_1 - \beta_2 \mathbf{x}_2 - \beta_3 \mathbf{x}_3 \), then

\[
D(x) = \hat{n} \cdot \mathbf{d}
\]

The expression for \( \frac{\partial \mathbf{n}}{\partial \mathbf{x}_0} \) is

\[
\frac{\partial \mathbf{n}}{\partial \mathbf{x}_0} = \begin{pmatrix}
\frac{\partial n_x}{\partial x_0} & \frac{\partial n_y}{\partial x_0} & \frac{\partial n_z}{\partial x_0} \\
\frac{\partial n_x}{\partial y_0} & \frac{\partial n_y}{\partial y_0} & \frac{\partial n_z}{\partial y_0} \\
\frac{\partial n_x}{\partial z_0} & \frac{\partial n_y}{\partial z_0} & \frac{\partial n_z}{\partial z_0}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
0 & (x_2 - x_3)_z & (x_3 - x_2)_y \\
(x_3 - x_2)_z & 0 & (x_2 - x_3)_x \\
(x_2 - x_3)_y & (x_3 - x_2)_x & 0
\end{pmatrix}
\]

\[
= \frac{1}{|\mathbf{n}|} [I - \mathbf{n} \mathbf{n}^T] \frac{\partial \mathbf{n}}{|\mathbf{n}|^2} \frac{\partial |\mathbf{n}|}{\partial \mathbf{x}_0}
\]
Similarly, there are

\[
\frac{\partial \mathbf{n}}{\partial x_1} = - \frac{\partial \mathbf{n}}{\partial x_0}.
\]

\[
\frac{\partial \mathbf{n}}{\partial x_2} = \begin{pmatrix}
(x_0 - x_1)_z & (x_0 - x_1)_y \\
0 & (x_1 - x_0)_x
\end{pmatrix}.
\]

\[
\frac{\partial \mathbf{n}}{\partial x_3} = - \frac{\partial \mathbf{n}}{\partial x_2}.
\]

(10)

Due to Equ. 3 there is

\[
D_{x_0} = \frac{\partial}{\partial x_0} (\hat{n}^T \mathbf{d}),
\]

\[
= \frac{\partial \hat{n}}{\partial x_0} \mathbf{d} + \beta_0 \hat{n},
\]

\[
= \mathbf{N} \frac{\partial \mathbf{n}}{\partial x_0} \mathbf{d} + \beta_0 \hat{n}
\]

\[
= \mathbf{N}(x_{23} \times \mathbf{d}) + \beta_0 \hat{n}
\]

(11)

Similarly, there are

\[
D_{x_1} = -\mathbf{N}(x_{23} \times \mathbf{d}) + \beta_1 \hat{n}
\]

(12)

\[
D_{x_2} = -\mathbf{N}(x_{01} \times \mathbf{d}) - \beta_2 \hat{n}
\]

(13)

\[
D_{x_3} = \mathbf{N}(x_{01} \times \mathbf{d}) - \beta_3 \hat{n}
\]

(14)

Also, there is

\[
\sum_{i=0}^{3} D_{x_i} = 0.
\]

(15)

3. Conservation of the Momentum

In Eq.(4) of the paper, the diagonal mass matrix \( \mathbf{M} \) is meant to maintain the center of mass, so that the angular momentum is least affected when used in a physical simulation. The linear momentum is naturally conserved within each stencil: letting \( \Delta \mathbf{x}_i \) denote the position change, there is \( \sum (m_i \Delta \mathbf{x}_i) = \lambda \sum D_{x_i} = 0 \) due to Eq. 8 and Eq. 15 in this supplementary material. Further, scale the above equation by \( \frac{1}{\Delta t} \) yields \( \sum (m_i \Delta \mathbf{v}_i) = 0 \), which means the momentum of the stencil does not change.