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Direct Limit Volumes: Constant-Time Limit Evaluation for Catmull-Clark Solids
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Supplemental Material

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Introduction

This document contains supplemental material for our short paper "Direct Limit Volumes: Constant-Time Limit Evaluation for Catmull-Clark Solids". All figures and tables that are referenced in the paper can be found in this document.

The Regular Case

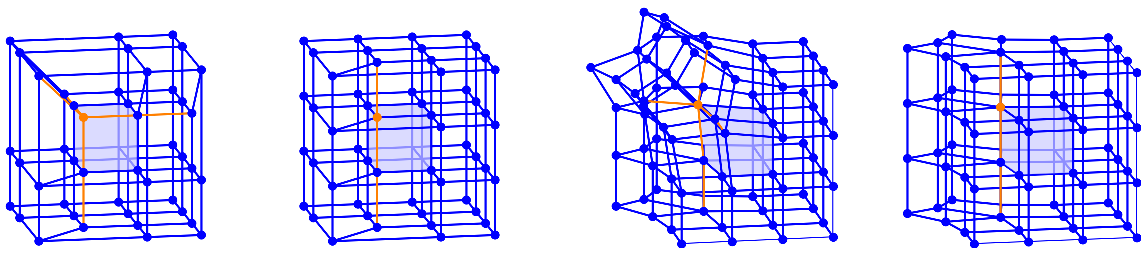
A trivariate element is defined by a regular $4 \times 4 \times 4$-grid of control points and is evaluated with three parameters u, v and $w \in [0,1]$ as

$$e(u,v,w) = \sum_{i=1}^{64} \mathbf{p}_i N_i(u,v,w),$$

where the trivariate basis functions $N_i(u,v,w)$ are defined as a product of three univariate B-spline basis functions

$$N_i(u,v,w) = N_{\lceil i/16 \rceil}(u)\; N_{\lceil i/4 \rceil \% 4}(v)\; N_{i \% 4}(w). \tag{1}$$

Irregular Cases



(a) Irregular volumetric mesh with valence $N = 3$ (b) Layered control mesh for valence $N = 3$ (c) Irregular volumetric mesh with valence $N = 5$ (d) Layered control mesh for valence $N = 5$

Figure 1: *Fully irregular (a), (c) as well as layered (b), (d) local control meshes with EEs of valences $N = 3$ and $N = 5$. The EEs as well as the selected EVs are marked in orange. The element to be evaluated is highlighted in light blue.*

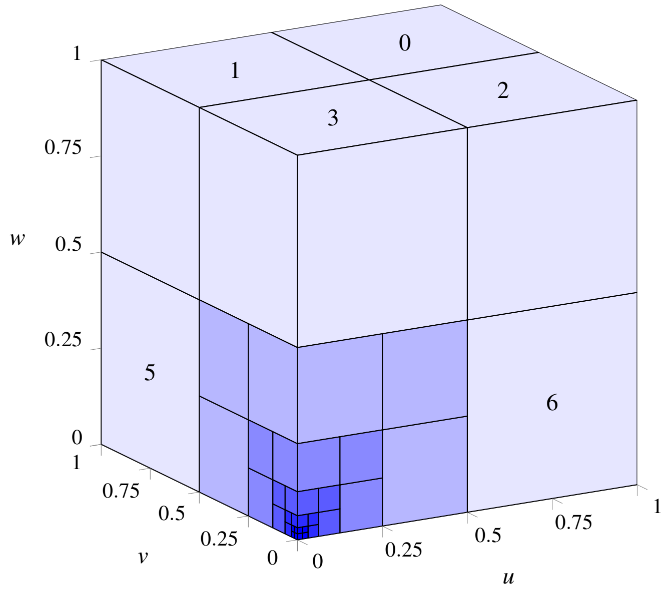
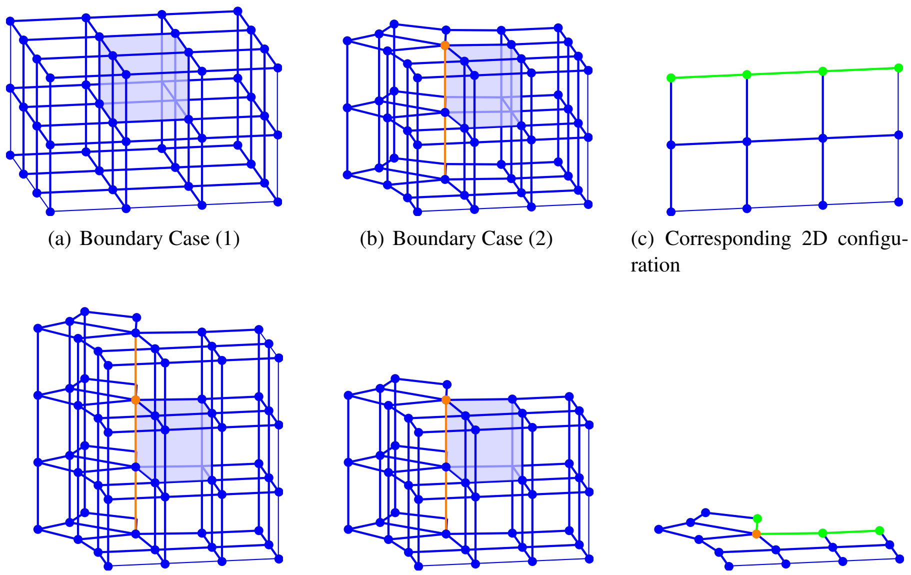


Figure 2: *Partitioning of the parameter space for local subdivision. EEs can occur on $u = v = 0$, $u = w = 0$ and $v = w = 0$. The selected EV corresponds to $u = v = w = 0$.*

Boundary Elements



(a) Boundary Case (1)

(b) Boundary Case (2)

(c) Corresponding 2D configuration

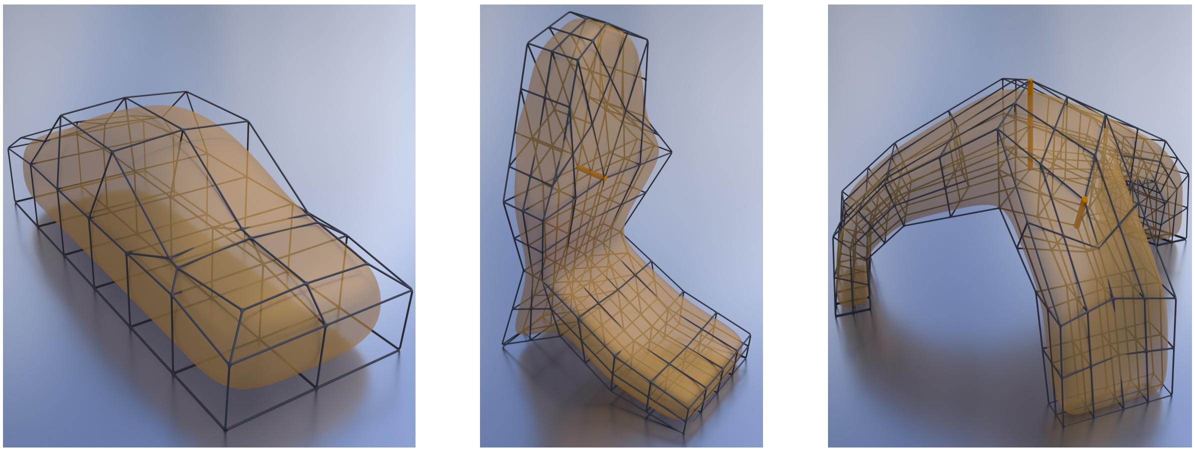
(d) Boundary Case (3)

(e) Boundary Case (4)

(f) Corresponding 2D configuration

Figure 3: *Four different boundary cases, consisting of a regular structure with one boundary layer (a), an irregular structure with one boundary layer on top (b), an irregular structure with the selected EV as part of the boundary (d) and an irregular structure with boundaries orthogonal to, as well as alongside the EE (e). The two-dimensional configuration corresponding to the cases (a) and (b) is depicted in (c). The one for (d) and (e) is depicted in (f). Crease edges are shown in green, EEs and EVs in orange.*

Results



(a) Car model (b) Bookend model (c) Tripod model

Figure 4: *Visualization of the three models that we used for our performance measurements. The Car model (a) consists of 24 regular boundary subdivision cells. The Bookend model (b) is made up of 88 subdivision cells, with two EEs of valence* 5 *(*10 *irregular cells). The Tripod model (c) has a total of 136 subdivision cells, with two EEs of valence* 3 *and two EEs of valence* 5 *(*16 *irregular cells). Extraordinary vertices and edges are marked in orange.*

Topology	Subdivision Matrix	Eigenstructure	Evaluation	Total	Average per Subdiv. Step	Break-Even Point
Interior cell, $N = 5$	0.02 ms (0.13%)	15.37 ms (99.09%)	0.12 ms (0.78%)	15.51 ms	0.083 ms	1.45
Boundary case (4), $N = 5$	0.35 ms (5.66%)	5.81 ms (92.88%)	0.09 ms (1.45%)	6.28 ms	0.043 ms	2.12
Corner boundary case	0.25 ms (14.57%)	1.42 ms (83.06%)	0.04 ms (2.37%)	1.71 ms	0.018 ms	2.25

Table 1: *Time in milliseconds required for each part of our direct evaluation algorithm for different topologies. For comparison, the last two columns show the average time required for one subdivision step with the non-direct method as well as the break-even point.*

u	v	w	k	n	Direct Evaluation	Required Subdivision Steps	Required Rearrangements	Subdivision-Based
0.51	0.51	0.51	0	1	0.077 ms	1	0	0.024 ms
0.01	0.26	0.26	2	2	0.077 ms	2	0	0.046 ms
0.51	0.51	0.15	4	1	0.063 ms	3	0	0.069 ms
0.01	0.01	0.01	0	7	0.080 ms	7	0	0.150 ms
0.01	0.01	0.33	3	2	0.072 ms	7	5	0.196 ms

Table 2: *Time in milliseconds required for evaluating different parameter values u, v, w with our approach as well as with a non-direct subdivision-based approach, on a cell with boundary case (4) and an EE with $N = 5$ as shown in Figure 3(e). The table also includes the values for k and n, as well as the number of subdivision steps and coordinate system rearrangements for the non-direct approach.*

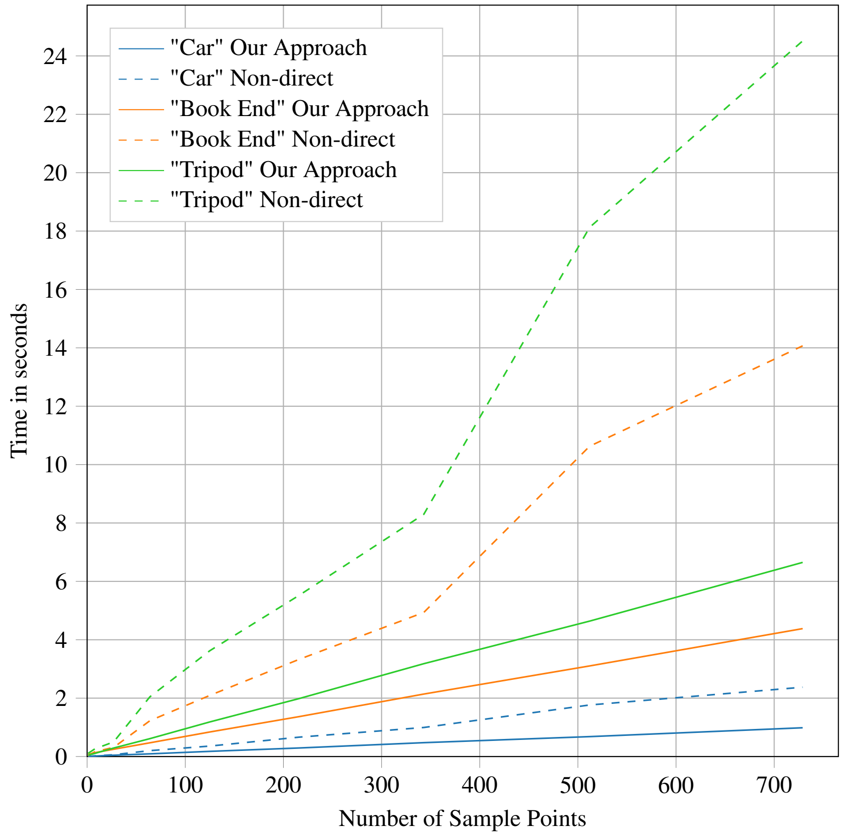


Figure 5: *Timing measurements for evaluating the limit at regular sampling points using three different volumetric subdivision models (Car, Bookend, Tripod). We compare our direct evaluation approach (visualized with solid lines) with the subdivision-based approach presented by Burkhart et al. [BHU10] (dashed lines).*

References

[BHU10] BURKHART D., HAMANN B., UMLAUF G.: Iso-geometric finite element analysis based on Catmull-Clark subdivision solids. *Computer Graphics Forum 29 (5)* (2010), 1575–1584. 4