

A procedural approach for thermal visualization on buildings

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Abstract

Thermal behaviour analysis on buildings is an important goal for all tasks involving energy flow simulation in urban environments. One of the most widely used simplified thermal models is based on an electrical analogy, where nodes are set to simulate and solve a circuit network. In this paper we propose a procedural approach for automatically locate the nodes of the circuit, according to the building structure. We provide a conceptual technique to efficiently visualize thermal variations over time in buildings. We show that we can simulate and visually represent the variations of the interior temperatures of a building over a period of time. We believe that the technique could be helpful for rapid analysis for changing building parameters, such as materials, dimensions or number of floors.

1. Introduction

The concept of Urban Physics has grown in recent years due to the global increase of the cities. Basically, it provides a way to qualify and quantify the different energy flows, both natural and artificial, that happen in a city. The search for efficient simulation tools is an active research line [Bec12]. In particular, most of the available tools to simulate and analyse thermal behaviour are conceived for the building scale [ESR17] or for the urban or district scale [Cit17]. This work focuses on the thermal behaviour of a building, as its accurate analysis can be a useful tool for urban planning, architecture and other related fields.

Computing a complete thermal simulation at the building scale requires taking into account both the thermal and the geometric models of the building. Most of the simplified thermal methods are based on an electrical analogy. By selecting nodes on a building model, a circuit network can be represented and solved for temperature assessment. The node settings, however, is generally done manually. This is not only error-prone, but also very complicated for a multi-storey building, and almost impossible at the urban scale. In this work, as a first step to simulate thermal behaviour at urban scale, we propose a method for procedurally generating the circuit network for a single building structure.

This paper presents a new technique to simulate the variation of temperatures in a building over time. Given a geometric model and the corresponding physical construction parameters (i.e., conductance and heat capacity) our system parses the model and gen-

erates a set of rules representing the interfaces for heat transfer. From these rules, the technique procedurally generates an electrical circuit that simulates the conductive heat exchange at each step of the simulation. In addition to this convective calculation, a radiative calculation is also performed simulating the heat exchange between the exterior surfaces of the buildings and its surrounding heat flows, using climatic data. As a result, we obtain a thermal calculation system that allows to analyse the temperature changes over time. Our main contribution is the automatic generation of the circuit network, the resolution of the system in short simulation times and the visualization of the temperature changes in the building. The potential benefit of our approach is that it could be easily used for testing optimal node configurations, or analyse building parameter changes. Our tests show promising results, where we show as a conceptual technique that the building model parameters, such as the number of floors or the materials, can be easily changed to analyse temperature variations.

2. Related Work

Methods for computing transient heat transfer in buildings can be classified into two categories: explicit solutions of the heat diffusion equation [Cla01] or model simplification techniques [KR07]. For the first case, finite difference numerical methods can be used. The main drawback is that the computational effort is prohibitive for early stage simulation goal projects. On the other hand, the simplified models offer a good compromise between simplicity, data requirement and computational effort. The electrical analogy simplification is one of the most used methodologies to represent heat transfer [KR07].

The main idea of the electrical analogy is to connect rooms (or floors) of a building by nodes that represent the wall conductivity

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and capacitance [Nah17]. The walls and roofs may be represented by many layers and could also be linked to the outside temperature. By solving the composed circuit network, dynamic temperatures over time could be provided. One of the drawbacks of this approach is that the nodes are mostly set manually [Lev15]. That is, by defining convenient zones, the nodes are established. Furthermore, as a simplification model, it is difficult to know in advance how many nodes may be required and where to put them before testing the system with the corresponding building parameters. To overcome this difficulty, we propose a rule-based methodology that automatically generates the circuit system.

The potential impacts of climate change on buildings were investigated by means of transient building energy simulations and hourly weather data, normally to calculate the demand for heating and cooling [Fra05]. Weather stations allow access to climatic data collected over decades, in order to use that reliable source as input for recreate simulations of the thermal behaviour of buildings [Rem08].

The seminal work by Parish and Müller [PM01] about urban modelling, followed by the key work by Wonka et al. [WWSR03] and Müller et al. [MWH*06] about procedural buildings, produced a blossom in urban modelling research. All these efforts resulted in the origin of commercial packages, like Esri's CityEngine [Esr14], or Epic's UDK [Epi12], focused on, or with modules for, procedural urban design. In this work we also use rule generation to procedurally generate electric circuits that simulate heat exchange.

3. Modelling and Simulation

In this section we describe our technique, how the thermal model is generated and how the simulation over time is performed. We get as input a 3D building model, already simplified into a basic structure, plus all material descriptions. The structure is determined by the building's own geometry, that is, the dimensions of the walls that make it up, and their interconnections. The materials are represented with a database that indicates the number of layers that make up each wall, as well as the physical parameters of the material each layer is composed of. From this structure and these materials, we generate a circuit using an electrical analogy. For the simulation, environmental parameters should be established, such as the initial temperature of building surfaces, external temperatures and dynamic climate-based data for a specific location. As a result, we can calculate the behaviour of the interior and exterior temperatures of the building and their variation over time.

3.1. Thermal modelling

Our simplified thermal model considers both the conduction that flows between the surfaces of the building and the radiation between the exterior surfaces of the building and their surroundings.

To simulate the conductive thermal exchange, a circuit between the outside temperature and the interior temperature of each floor can be created. This is done using an electrical analogy [CFX15]. This analogy replaces the thermal calculation by a conventional electrical circuit that represents the heat flows through the multiple layers of material that compose the walls. Each layer of material

provides resistors and capacitors, making only a portion of the incoming energy reach the interior of the building, thereby altering its interior temperature.

For a single room in a one-floor building, the resultant circuit is shown in Figure 1. Each line of the electrical circuit represents one of the walls that are between the outside temperatures and the interior temperature of the building. The capacitors and resistors represent the resistance of the material and the insulation of the wall. All the figures that show electrical circuits use the electrical symbol notation of the International Electro-technical Commission (IEC) [C*00], used in the software to graphically show the rules.

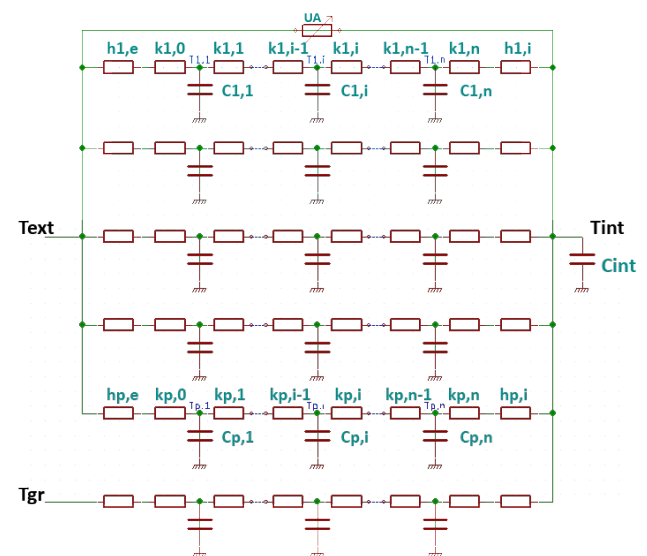


Figure 1: Circuit generated to represent the heat flow between outdoor and indoor temperature in a single-storey building.

However, when growing the building into several floors, the circuit rapidly increases in complexity. Figure 2 shows the circuit for a two-storey building using the electrical analogy simplification. Scaling this problem for many floors, the circuit to represent a building will become too complex to understand and manage.

List of Symbols

This list describes several symbols that are used within circuit figures of the document

C	thermal capacity ($J K^{-1}$)
h_e	exterior convective heat transfer coefficient ($W m^{-2} K^{-1}$)
h_i	interior convective heat transfer coefficient ($W m^{-2} K^{-1}$)
k	thermal resistance ($W m^{-1} K^{-1}$)
T_{ext}	external air temperature (K)
T_{gr}	ground temperature (K)
T_i	wall temperature at material layer i (K)
T_{int_i}	interior temperature at floor i (K)
UA	glazing transmittance by translucent surfaces ($W K^{-1}$)

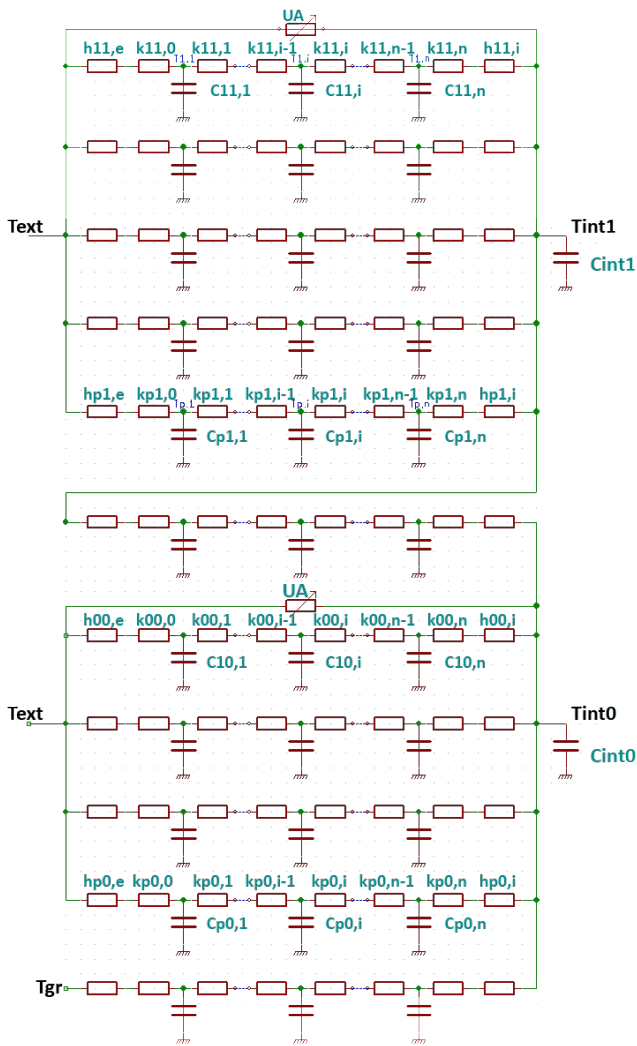


Figure 2: Circuit generated to represent the heat flow between outdoor and indoor temperature in a two-storey building.

3.2. Rule creation

The growing complexity of the multi-storey building circuit generation motivates the use of rules.

The initial rule created is the "Wall", as is shown in Figure 3. A thermal model based on the electric analogy may vary depending on how many resistors and capacitors are placed and configured in the circuit. For example, a frequently used model is the 4R3C, i.e., four resistors and three capacitors located between the outside temperature and the interior temperature of a building. However, different configurations are also possible, as it is pointed by Fraisse et al. [FVLA02]. In our technique, the type of circuit model is one of the dynamic rules that allow to change the configuration of the case studies to be simulated. It is possible to define if the walls behave like a 4R3C model, a 3R2C model, or any other model that would be required. Taking all of the above into account, the "Wall" rule is parametrizable according to the number of resistances and

capacitors, and also according to the number of material layers and their physical properties.

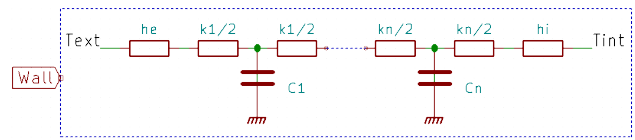


Figure 3: "Wall" design rule.

Another rule designed is the "Room" rule. This rule represents the four walls of the floor of a building, creating the link between the outside temperature of the air and the interior temperature of the floor. See Figure 4. There is also a variable resistance that represents ventilation and windows.

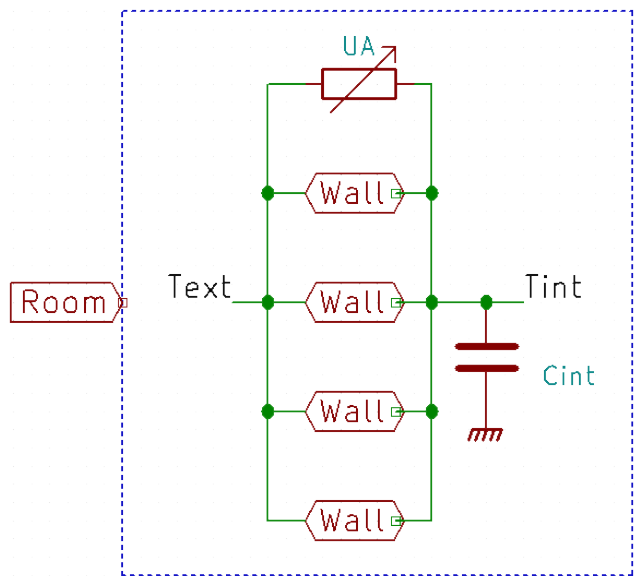


Figure 4: "Room" design rule.

Now, through these two rules, we redesign the two-storey building model again using both "Room" and "Wall" rules, adding a floor and a roof to the building and also the intermediate floors between storeys. See Figure 5. In this type of models, there is an interior temperature for each floor of the building, while there is only a unique outside temperature of the air. In addition, the floor of the building is not connected to the outside temperature of the air, but to the temperature of the ground.

It is important to mention that, in our model, a room represents a single floor of a building, as if a floor had no interior walls. In this way, without independent areas in a same floor, each floor of a building will have a single interior temperature. This is because the simulation assumes that there are no internal heating or cooling systems and, therefore, the temperature variation between walls in the interior is negligible.

We differentiate three kinds of floor: ground-floor, top-floor or

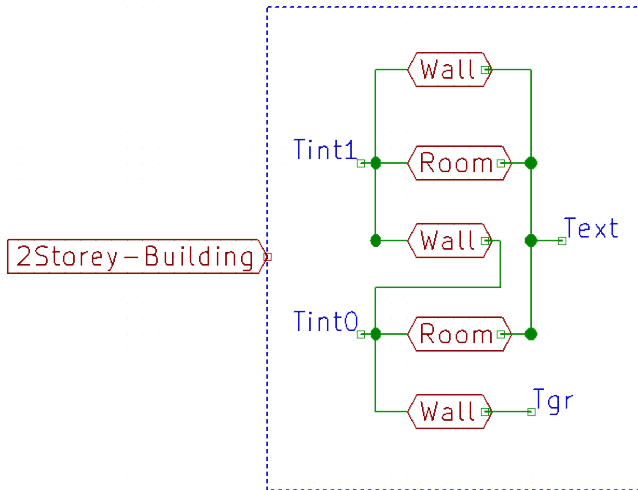


Figure 5: A two-storey building modelled using "Room" and "Wall" rules.

intermediate-floor. The difference is that the ground-floor has its own ground, while the top-floor has the roof directly in contact with the exterior. Intermediate-floors will have their ground floors connected to the roof of the immediately inferior floor, and their ceilings connected to the floor that is immediately above it. In this way, a single plane can be the roof of one floor and floor of a different floor at the same time.

To automate the generation of circuits, we design the procedural code described in Algorithm 1. This algorithm generates the circuits through the rules that have been established with respect to the building. These rules are determined by 3D model itself after parsing the structure, such as the size and the shape of each room type, the number of floors of each building, the number of layers of material per wall and their physical properties such as its area, its thickness, its density, its heat capacity, its thermal conductivity, etc.

This algorithm runs through the structure of the building and, using the design rules, generates the electrical circuit from layer to layer, from wall to wall, and finally from floor to floor, inter-connecting each element in a single final circuit. Temperatures and physical parameters give value to its resistors and capacitors according to the physical properties of its layers of material and the interior, exterior and ground temperatures, all provided as input to the function. Finally, a heat flow is established for each wall of each floor, based on the difference of temperatures between the outside and the inside, as well as the resistors and capacitors of its layers of material.

As the generation of circuits at the structural level is based solely on geometry, there are no limitations with respect to its complexity. The function can generate electrical circuits from buildings with more complex geometries, since the computational cost of the generation will always remain linear. The only limitation detected is the treatment of buildings with different geometries between their floors. When treating the thermal behaviour of the floor/ceiling between two floors, the generator does not take into account its total surface, but the partial surface that is common to the two inter-

Algorithm 1 Procedure to generate the electrical circuit for a building

```

function GENERATECIRCUIT(buildingModel, temperatures)
  circuit ← CREATEEMPTYCIRCUIT(Tair, Tint, Tgrnd)
  floors ← GETNUMBEROFFLOORS(buildingModel)
  layers ← GETLAYERSFORWALL(buildingModel)
  physics ← GETPHYSICALPARAMS(buildingModel)
  Tair ← GETAIRTEMPERATURE(temperatures)
  Tint ← GETINTTEMPERATURE(temperatures)
  Twall ← GETWALLTEMPERATURE(temperatures)
  Tgrd ← GETGRDTEMPERATURE(temperatures)
  for i := 1 to i = floors do
    room ← CREATEROOM(Tair, Tint, Twall)
    for j := 1 to j = 5 do
      wall ← CREATEWALL(Tair, Tint, Twall, Tgrd)
      for k := 1 to k = layers do
        layer ← CREATELAYER(physics)
        wall ← ADDLAYER(layer)
      end for
      room ← ADDWALL(wall)
    end for
    circuit ← CONNECTNEWFLOOR(room)
  end for
  circuit ← INITIALIZECIRCUIT()
  return circuit
end function

```

connected floors. Finally, as for the physical parameters, in the hypothetical case that the source building model is incomplete, it is possible to fill the input missing data manually through some user intervention.

3.3. Radiative heat

Our thermal model takes into account the radiation of short waves (SW) and long waves (LW) that the exterior surfaces of the building receive at each step time of the simulation.

The Stefan-Boltzmann's law allows to calculate the radiative exchange of heat between two black bodies [CG14]. A black body is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency or angle of incidence. Considering the walls of buildings as grey bodies with emissivity ϵ ($0 \leq \epsilon \leq 1$), the heat transfer between a surface i and its surrounding can be expressed as:

$$Q = A\epsilon\sigma(T_i^4 - T_j^4) \quad (1)$$

This calculation must be done in both directions, calculating the heat flow emitted and absorbed by each body, obtaining a total heat flow. The resulting heat flow causes a change in the exterior temperature of the walls of the building, thus affecting also the convective calculation of the heat flow between the outside and the inside.

3.4. Dynamic Simulation

We provide a method for simulating temperatures over time. The simulation process is described with the procedure shown in Algorithm 2. The method takes as input the geometry for the building model, the desired simulation time and the time step.

Algorithm 2 Procedure to simulate the heat exchange on a building

```

procedure SIMULATE(buildingModel, simTime, timeStep)
  local variables
    circuit, circuit of the building
    t, current simulation time
  end local variables
  SETPARAMETERS()
  circuit ← GENERATECIRCUIT(buildingModel, temperatures)
  t ← 0
  while t < simTime do
    RADIATIVEPASS(buildingModel)
    CONDUCTIVEPASS(circuit)
    UPDATETEMPERATURES(circuit, buildingModel)
    UPDATEVISUALIZATION(buildingModel)
    t ← t + timeStep
  end while
end procedure

```

First, the algorithm initializes the parameters. This includes environmental, material and simulation parameters. Three data structures are also used as input parameters, the first one containing the exterior air temperatures, and the other two containing the pre-computed short wave and long wave flows received by each surface of the building throughout the simulation time. SW flows are obtained directly from the exterior temperatures using the Perez model [PSM93], while LW flows are obtained using the Ångström model [Å15]. After that, the algorithm generates the electrical circuit corresponding to the building of the case study, according to the geometry of the model introduced as input.

For the simulation, for each time step, the algorithm performs a radiative pass. For this calculation, the two heat flux files pre-computed representing the SW and LW fluxes, are used. It is important to note that the orientation of the walls of a building will influence the SW and LW flows that they will receive, according to the climatic data, pre-calculated from air temperatures and geometry.

After the radiative pass, the simulation performs a conductive pass that calculates the heat exchange between the exterior temperature and the interior temperature of the building [CG14]. For this calculation, the procedurally generated circuits for the building are solved at each time step of the simulation, updating the heat flows between the outside temperature and the interior temperature of each floor. To do this, the total resistance of the wall is calculated using the equation 2, where e is the thickness of the wall, A_{wall} is its area and k is its thermal resistance. After that, heat flows are introduced by equation 3, where T_1 is the interior temperature of the floor, T_2 is the exterior temperature and R_{wall} is the total resistance of the wall.

$$R_{wall} = \frac{e}{kA_{wall}} \quad (2)$$

$$Q_{cond} = \frac{T_1 - T_2}{R_{wall}} \quad (3)$$

Finally, the temperatures in the surfaces and interiors of the building are updated according to the solution of the circuit, representing the conductive heat exchange, but also according to the heat absorbed and emitted by the exterior surfaces of the building following Equation 1. In our simulations, the building model is colourized to visually represent the interior temperatures of the building. To calculate the temperature variation resulting from the incidence of heat flow into the interior of the floors of a building, Equation 4 is used, where T_{wall} is the interior wall temperature, e is its thickness, A_{wall} is its area, λ is its thermal conductivity, h is its convection heat transfer coefficient and x is the distance between the centre of the room and the wall.

$$\Delta T = T_{wall} + \frac{Q_{cond}}{A_{wall}} \left(\frac{e+x}{\lambda} + \frac{1}{h} \right) \quad (4)$$

These calculations and temperature updates will continue to be performed at each time step during the simulation time.

4. Implementation

This section describes the implementation details and presents our test results. To implement our technique, the *SideFX Houdini* [Sid17] platform has been chosen as the procedural environment. For the electrical circuit, a Python language library called *ahkab* [Ven17] has been used to create and solve the circuits using the *Netlist* syntax.

Two tests cases has been designed to evaluate the performance of our technique and are presented in the following subsections. All the experiments were performed on a computer with an Intel Core i7-4790 CPU running at 3.60GHz, 16GB of DDRAM3 memory and a GPU NVidia GeForce GTX 770M. The operating system used was Windows 10 Pro 64bits.

4.1. Case Study 1

For this test we used a 3D model consisting in a four-storey building, as shown in Figure 6. The ground floor is 4m high, while the rest of floors are 3m. For the length and the width, all the floors and ceilings are 25x10m. Table 1 shows the environmental parameters. The simulation time used was 7 natural days and the time step was of one hour. For the outside temperature, a meteorological data file containing the air temperature for every hour of a year in Paris, France, was used.

Air density (kg/m ³)	Air heat capacity (J/kgK)	Air infiltration (vol/h)
1.413	1,005	0.25

Table 1: Environmental parameters for the simulation of the case study.

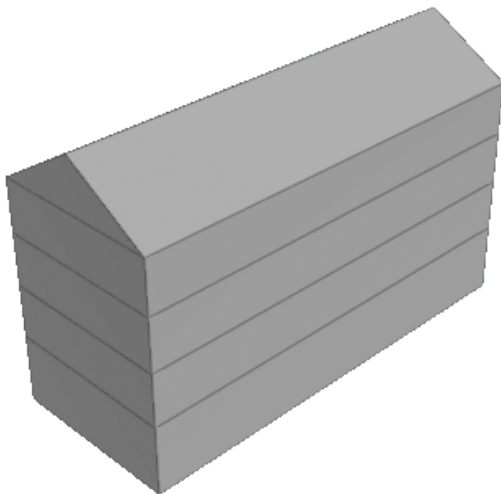


Figure 6: The actual geometry for our first case study, a four-storey building. Its size is 25x10x13m and its floors are 3 meters high, with the exception of the lowest floor, which is 4 meters high.

For this case study, the selected period was the first week after the autumn equinox, from September 22nd to the 29th. All building surfaces and the interior temperature of each floor were initialized to the starting air temperature and the ground temperature was initialized to the mean air temperature of the entire simulation period plus 2°C.

The walls were generated following the model 4R3C, see Figure 7, and the resistances and capacities of the circuits are shown in Table 2, representing the properties of the three material layers that compose the walls.

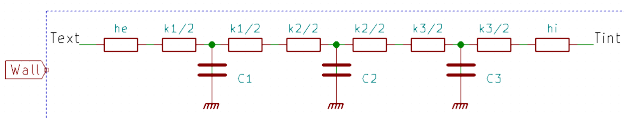


Figure 7: A wall using the 4R3C model, presenting a three-layered isolation.

	Thermal conductivity (W/mK)	Thermal capacity (J/kgK)	Density (kg/m ³)	Thickness (m)
Concrete	1.75	920	2,200	0.2
Polyester	0.04	1,380	30	0.2
Plaster	0.35	800	1000	0.1

Table 2: Material parameters for the simulation of the case study.

The window glazing transmission factor for the glass was set to 75% and, for the radiative pass the surfaces were adjusted with an absorbency of 50% and an emissivity of 90%.

At each time step, the internal temperature of each floor is updated. Figure 8 shows that evolution by colouring the surfaces that conform the model of the building according to a temperature chromatic scale. In this way, it is possible to perceive the changes during the simulation. It can be appreciated how the top floor often differs from the temperature of the rest of the storeys. This is due to its exposure to the sky and its distance from the ground temperature.

The evolution of the interior temperature of each floor of the building during the 7 days of simulation can also be visualized. Plotting the evolution of the outside temperature of the air, it is possible to appreciate its effect on the interior temperatures of the building. See Figure 9. The first thing to notice, is that the interior temperature of the ground floor increases more quickly than the rest. This is due to the contact of its floor with the ground, which has an approximate temperature of 17°C. The simulation of the Case Study 1 took a total of 54.26 seconds.

4.2. Case Study 2

To illustrate another example, the 7-autumn-days simulation has been repeated with the same environmental conditions and the same weather data, but changing some procedural rules, thus generating a different case study. The number of floors of the building has been reduced from 4 to 2. Also the wall model has been changed from 4R3C to 2R1C, see Figure 10, and only one insulating layer of plaster has been left instead of the three original layers. The result can be seen in Figure 11. These results can be compared to Figure 9 to notice how the trends are the same but the temperatures differ. These differences can be better appreciated in Figure 12. Checking those results with the ones in Figure 8, allows to compare the thermal evolution of both case studies at each time step of the simulation. The simulation of the Case Study 2 took a total of 10.54 seconds.

It can be seen how both cases follow a similar trend of temperatures, except that the two-storey building is cooler even at the last steps of the simulation, because there is less isolation on the walls and the first floor does not have an upper floor that makes it conserve its temperature more, despite receiving the heat of the habitually warmest ground floor. In the four-storey building, the ground floor also warms up its superior floors and propagates the effect, but the temperature diminishes as it ascends through the floors of the building.

This example shows a kind of the comparative studies that can be carried out with our technique in order to support decision making regarding the properties that a building should have if it is planned to be built in an environment with a concrete climate, and a certain thermal behaviour is expected. However, we remark that these are preliminary results that are not yet validated with real case studies.

5. Conclusions and Future Work

We have presented a technique to simulate the conductive exchange of heat between the different floors of a building, and between the building and the exterior, by automatically generating and solving the electrical circuits that emulate the conductive heat exchange between the interior temperatures of a building and the outdoor

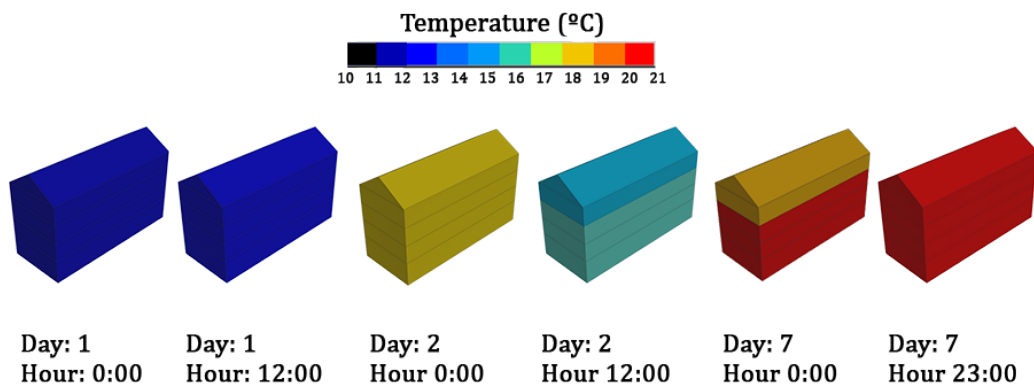


Figure 8: Interior temperatures of the four-storey building, with 4R3C walls, at different instants of the simulation period. The complete evolution of the temperatures can be seen in Figure 9

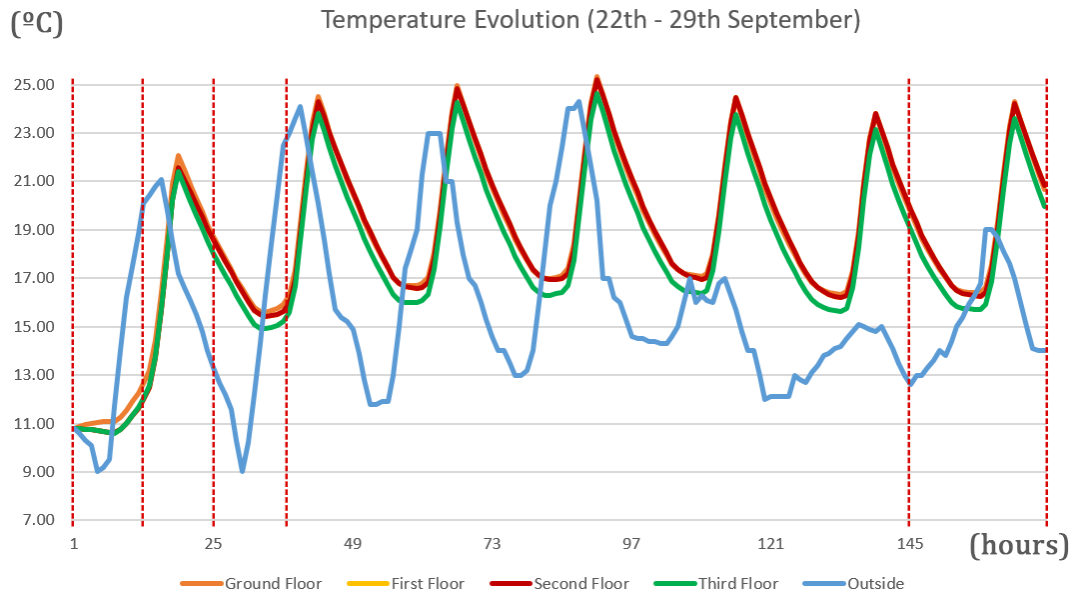


Figure 9: Evolution of the interior temperature of each floor of the four-storey building during a week of autumn, also showing the outside temperature. Red stripes indicate the instants that have been visually represented in Figure 8.

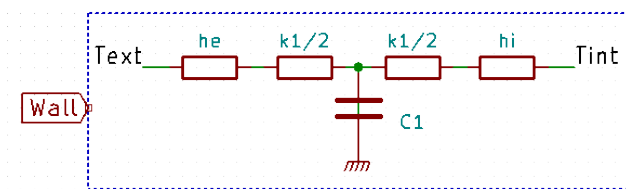


Figure 10: A wall using the 2R1C model, presenting a single-layered isolation.

air temperature. Our technique also presents a radiative calculation

that allows to simulate the thermal evolution of the building surfaces using the pre-calculated short- and long-wave flows that they receive over a period of time.

We provide a way to simulate different case studies by simply changing procedural and simulation parameters. This allows to experiment a variety of situations and to study the changes in thermal behaviour of the model, according to the altered parameters.

Our future work is to extend the usefulness of this technique by improving the radiative component of the thermal calculation, replacing the theoretical pre-calculus of short- and long-wave flows by the simulation of the heat exchange between the exterior surfaces of several buildings in an urban environment, as well as between these surfaces and the sun. That would increase the scope

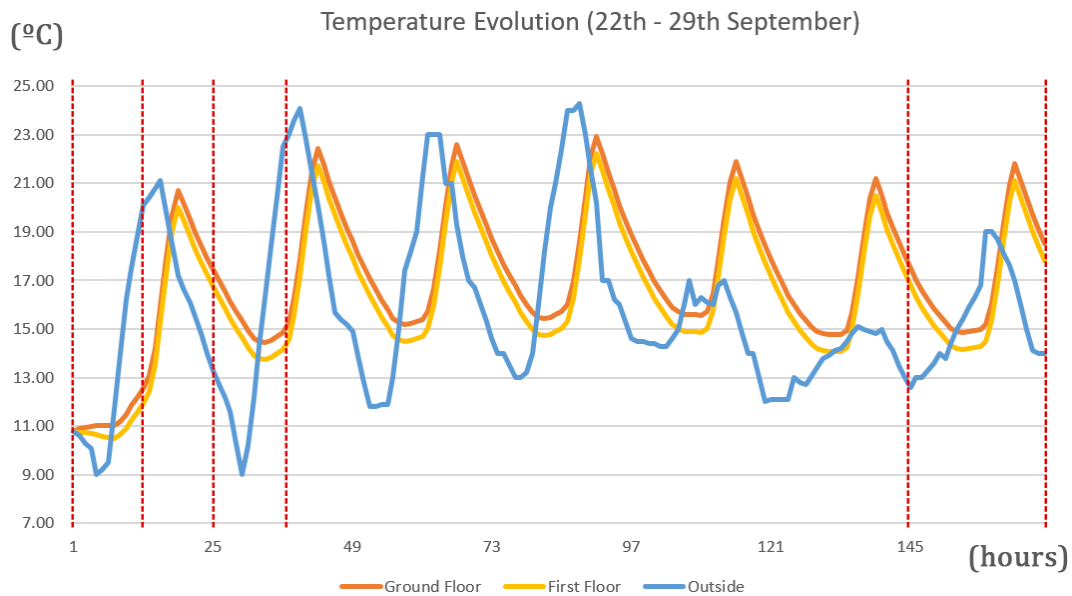


Figure 11: Evolution of the interior temperature of each floor of the two-storey building during a week of autumn, also showing the outside temperature. Red stripes indicate the instants that have been visually represented in Figure 12.

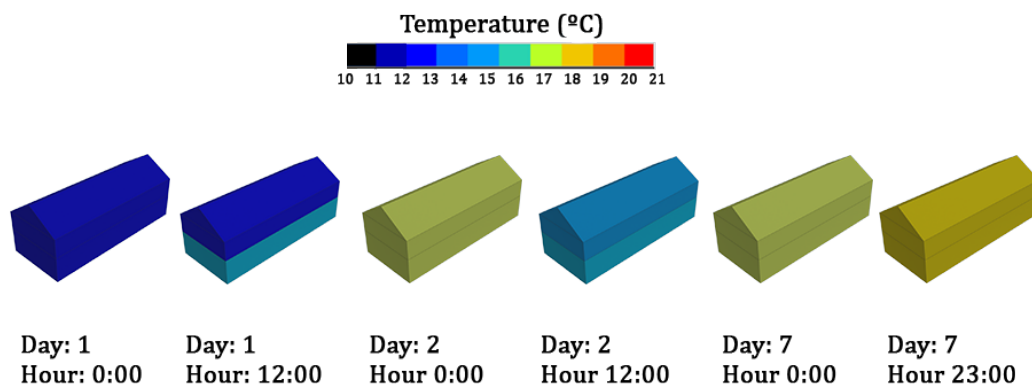


Figure 12: Interior temperatures of the two-storey building, with 2R1C walls, at different instants of the simulation period. The complete evolution of the temperatures can be seen in Figure 11.

of this technique from the building scale to the urban scale. Also, the calculations should be validated by comparing the results with real measurements taken in urban environments, and also with detailed simulated results of such environments modelled in 3D (e.g., with Finite Elements). This last improvement would provide more robustness and potential to our technique, making it capable of performing more complex thermal simulations from a 3D model of an urban environment and a set of parameters and climatic data that determine the conditions to be experienced. We expect that this will make it possible to simulate and visually represent the thermal behaviour of a 3D urban environment, thus allowing the study of more complex physical problems such as heating or cooling demands within certain buildings in an urban setting.

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References

- [Bec12] BECKERS B. (Ed.): *Solar Energy at Urban Scale*. Wiley, 2012. 1
- [C*00] COMMISSION I. E., ET AL.: Letter symbols to be used in electrical technology-part 2: Telecommunications and electronics. *IEC*, (2000), 60027-2. 2
- [CFX15] CHEN Q., FU R. H., XU Y. C.: Electrical circuit analogy for heat transfer analysis and optimization in heat exchanger networks. *Applied Energy* 139 (2015), 81-92. 2

- [CG14] CENGEL Y. A., GHAJAR A. J.: *Heat and mass transfer: fundamentals and applications*. McGraw-Hill Higher Education, 2014. 4, 5
- [Cit17] CITYSIM: Citysim software, 2017. <http://citysim.epfl.ch/>. 1
- [Cla01] CLARKE J.: *Energy Simulation in Building Design (Second Edition)*, second edition ed. Butterworth-Heinemann, Oxford, 2001. 1
- [Epi12] EPICGAMES: Unreal development kit (udk), 2012. <http://udk.com>. 2
- [Esr14] ESRI: Cityengine, 2014. <http://www.esri.com/software/cityengine>. 2
- [ESR17] ESRU: Esru home page, 2017. <http://www.esru.strath.ac.uk>. 1
- [Fra05] FRANK T.: Climate change impacts on building heating and cooling energy demand in switzerland. *Energy and buildings* 37, 11 (2005), 1175–1185. 2
- [FVLA02] FRAISSE G., VIARDOT C., LAFABRIE O., ACHARD G.: Development of a simplified and accurate building model based on electrical analogy. *Energy & Buildings* 34, 10 (2002), 1017–1031. 3
- [KR07] KAMPF J. H., ROBINSON D.: A simplified thermal model to support analysis of urban resource flows. *Energy & Buildings* 39, 4 (2007), 445–453. 1
- [Lev15] LEVINE S.: *Temperature Simulation in Residential Building By Use of Electrical Circuits*. Tech. rep., Washington University in St. Louis, 2015. B.S. in Systems Science and Engineering. 2
- [MWH*06] MÜLLER P., WONKA P., HAEGLER S., ULMER A., VAN GOOL L.: Procedural modeling of buildings. *ACM Trans. Graph.* 25, 3 (2006), 614–623. 2
- [Nah17] NAHON R.: *Modélisation des échanges radiatifs à l'échelle urbaine pour un urbanisme bioclimatique*. PhD thesis, Lille 1, 2017. 2
- [PM01] PARISH Y. I. H., MÜLLER P.: Procedural modeling of cities. In *SIGGRAPH '01: Proceedings of the 28th annual conference on Computer graphics and interactive techniques* (2001), pp. 301–308. 2
- [PSM93] PEREZ R., SEALS R., MICHALSKY J.: All-weather model for sky luminance distribution: Preliminary configuration and validation. *Solar energy* 50, 3 (1993), 235–245. 5
- [Å15] ÅNGSTRÖM A. K.: A study of the radiation of the atmosphere. *Smithsonian Miscellaneous Collections* 65, 3 (1915). 5
- [Rem08] REMUND J.: Quality of meteonorm version 6.0. *Europe* 6, 1.1 (2008), 389. 2
- [Sid17] SIDEFX: Houdini 16, 2017. <http://www.sidefx.com>. 5
- [Ven17] VENTURINI G.: A spice-like electronic circuit simulator written in python, 2017. <https://ahkab.github.io/ahkab/>. 5
- [WWSR03] WONKA P., WIMMER M., SILLION F., RIBARSKY W.: Instant architecture. *ACM Transaction on Graphics* 22, 3 (July 2003), 669–677. Proceedings ACM SIGGRAPH 2003. 2