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Specular Highlight Removal for Real-world Images
Supplementary Material

1. L_0-norm solution

In this section, we privide the proofs of Eqn. (18) in our main paper for completeness. The energy function in Eqn. (18) is:

$$\arg\min_{S} \sum_{j} ||S_j - W_j + \frac{Y_{1,j}}{\rho}||_2^2 + \frac{2\lambda_d}{\rho} C(S_j)\,, \tag{1}$$

where j is the index of the entries of the matrix. The solution of each scalar energy function is updated by

$$S_j = \begin{cases} 0, & \text{if} \quad (W_j - \frac{Y_{1,j}}{\rho})^2 \leq \frac{2\lambda_d}{\rho} \\ W_j - \frac{Y_{1,j}}{\rho}, & \text{otherwise} \end{cases}. \tag{2}$$

Proof Denote by E_j the value of the j-th scalar function in Eqn. (1) as

$$E_j = ||S_j - W_j + \frac{Y_{1,j}}{\rho}||_2^2 + \frac{2\lambda_d}{\rho} C(S_j)\,. \tag{3}$$

(i) When $(W_j - \frac{Y_{1,j}}{\rho})^2 \leq \frac{2\lambda_d}{\rho}$, the function value for non-zero S_j is

$$E_j(S_j \neq 0) = ||S_j - W_j + \frac{Y_{1,j}}{\rho}||_2^2 + \frac{2\lambda_d}{\rho} \geq \frac{2\lambda_d}{\rho} \geq (W_j - \frac{Y_{1,j}}{\rho})^2\,. \tag{4}$$

On the other hand, the function value for the zero-valued S_j is

$$E_j(S_j = 0) = (W_j - \frac{Y_{1,j}}{\rho})^2)^2\,. \tag{5}$$

Since $E_j(S_j \neq 0) \geq (W_j - \frac{Y_{1,j}}{\rho})^2 \geq E_j(S_j = 0)$, the solution is $S_j = 0$ when $(W_j - \frac{Y_{1,j}}{\rho})^2 \leq \frac{2\lambda_d}{\rho}$.
(ii) When $(W_j - \frac{Y_{1,j}}{\rho})^2 > \frac{2\lambda_d}{\rho}$, Eqn. (5) still holds. On the other hand, for non-zero S_j, $E_j(S_j \neq 0)$ has a minimum of $\frac{2\lambda_d}{\rho}$ at $S_j = W_j - \frac{Y_{1,j}}{\rho}$. Since $E_j(S_j = W_j - \frac{Y_{1,j}}{\rho}) = \frac{2\lambda_d}{\rho} \leq (W_j - \frac{Y_{1,j}}{\rho})^2 = E_j(S_j = 0)$, the solution is $S_j = W_j - \frac{Y_{1,j}}{\rho}$ when $\frac{2\lambda_d}{\rho} < (W_j - \frac{Y_{1,j}}{\rho})^2$. □

The proof of the Eqn. (23) is the same as that of Eqn. (18) in our main paper.

2. Convergence curves

In this section, the convergence curves for the five images in Figure 5 in our main paper are illustrated in Figure 1. As can be seen, there is almost no change after the number of iterations is reached to 50. Generally speaking, the 200 $\sim$ 300 iterations is sufficient enough to guarantee that the objective function converges well.

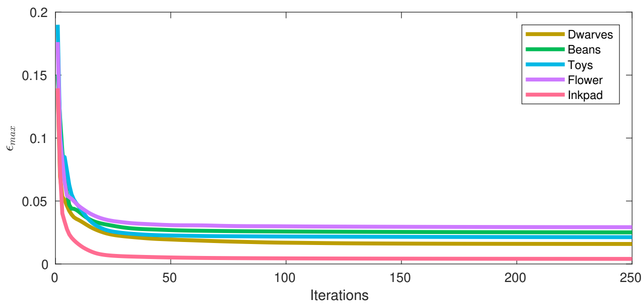

Figure 1: Convergence rate curves of our algorithm for the five images including Dwarves, Beans, Toys, Flower, Inkpad.

References