Spherical Harmonic Lighting of Wavelength-Dependent Phenomena

Clifford Lindsay and Emmanuel Agu

Worcester Polytechnic Institute, Worcester, MA, USA

Abstract

Rendering of objects that exhibit iridescence has previously been limited to simple lighting conditions generated from a few point or directional lights. We extend this idea by modeling light interference as a Bidirectional Reflectance Distribution Function (BRDF) and precomputing the physically based surface response to hemisphere lighting in terms of a low order spherical harmonic basis. Parameterizing by view direction, we can store the light interference effect in a 2D texture map, where each entry contains a vector of spherical harmonic basis function coefficients. Our technique achieves a full spectral representation of interference color by maintaining the spherical harmonic basis in terms of Spectral Power Distribution (SPD) instead of a color-based format such as RGB. In addition, we demonstrate that our approach is amenable to current graphics hardware and can render at real-time frame rates.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Color, shading, shadowing, texture, physically-based modeling

1. Introduction

It is no longer necessary to simulate physical reality at the expense of interactivity. Physically correct models for wavelength-dependent phenomena such as interference, diffraction, dispersive reffraction and scattering are necessary for realistic image generation and require the full range of spectral information in order to render [SFD99, AH03]. There are many different solutions for simulating interference and iridescent effects. The extent to which physical representation of surface structures and optical properties of light contribute to these solutions significantly vary. However, a limitation of these solutions is the use of only simple lighting scenarios [G00, SFC*99, D91, D94]. A more physically accurate model for lighting would be to utilize a hemisphere surrounding a point. As described in this paper, we use this technique to render interference.

Rendering interference effects for real-time applications would benefit video games and virtual reality by adding additional realism to the experience. Little has been done to render interference effects at interactive frame rate except for [IMN04] which focuses on simple lighting models and deformable soap bubbles. Simulations of interference have been implemented primarily via non-realtime renderer such as in a ray tracer [D91, D94, G00, SM90, L96, GMN94].

Our approach differs by rendering surfaces in real-time that exhibit interference effects with complex lighting environments. By, modeling interference as a BRDF, we can accelerate rendering by precomputing the surface response to hemisphere lighting using Spherical Harmonics (SH) basis functions. The interference calculations are done as a preprocess, employing the Composite Model (CM) [SFD99] representation for each of the SH basis color channels, thus allowing for full spectral respresentation.

2. Related Work

Shading using spherical harmonics basis functions has been proposed for offline and interactive rendering with a focus on representing incident environment maps, BRDFs, and Radiance Transfer. Ramamoorthi and Hanrahan [RH01] represent diffuse reflection maps analytically in terms of SH basis. Westin *et* al [WAT92] represent physically based BRDFs using a matrix of SH basis coefficients, where Ramamoorthi and Hanrahan [RH02] represent isotropic BRDFs using a Spherical Harmonic Reflection Map. Kautz *et al* [KSS02] approximate arbitrary BRDFs using a 2D table of SH basis indexed by view direction and combine interreflections and self-shadowing. Sloan *et al* [SK02] represent radiance transfer, glossy surfaces and account for interreflections and self-shadowing using a transfer matrix and per vertex SH basis coefficients. However, none of these methods include wavelength dependent phenomena.

Wavelength dependent BRDF models have been developed for use in simulating phenomena that require spectral representation. Gondek *et al* [GMN94], Agu [AH03] and Stam [Sta99] develop models that strives for accuracy at the expense of interactivity. Where Granier and Heidrich [GH03] use a simple layered BRDF that represents color by sampling the spectrum at 3 locations to ease conversion to the RGB color space. Neither of these models are designed to run at interactive frame rates. In addition our implementation employs the Composite Model [SFD99] to compactly represent spectra for simulating wavelength dependent phenomena instead of uniformly sampling the spectra at many points.

Interference can be seen in many objects like butterfly wings, soap bubbles and oil slicks and produce vivid colors. Early work on interference fringes was done by Smits and Meyer [SM90]. Dias [D91] developed a model for rendering interference based on color bands. Glassner [G00] derived an analytic model interference on clusters of soap bubbles. Interference from multilayer films has been studied by [HKY*01, GMN94, HKY*00]. These models for interference are too computationally intensive for real-time rendering or have been designed to work with an offline render such as a ray tracer. This contrasts with our implementation, which is designed to take advantage of GPU hardware and run at real-time frame rates.

The rest of the paper is as follows. Section 3 gives a brief background on the theory of spherical harmonics (SH). Section 4 outlines an analytical interference function that we shall then approximate using (SH). Section 5 presents our approach including our offline pre-processing and online rendering stages. Section 6 is our conclusion.

3. Background on Spherical Harmonics

Spherical Harmonics is an approach for approximating 2D functions on a sphere and is the analog on the sphere to the Fourier basis on the line or circle. A function parameterized on the sphere may be represented by an infinite sum of scaled spherical harmonics [Gla95]. A finite approximation of the function is achieved by truncating this infinite sum. Below is the general form for Spherical Harmonics:

$$y_l^m(\theta,\phi) = \begin{cases} \sqrt{2}K_l^m\cos(m\phi)P_l^m(\cos\theta) & \text{when } m > 0 \\ \sqrt{2}K_l^m\sin(-m\phi)P_l^{-m}(\cos\theta) & \text{when } m < 0 \\ K_l^0P_l^0(\cos\theta) & \text{when } m = 0 \end{cases} \tag{1}$$

Kautz *et al* [KSS02] has shown that an arbitrary BRDF defined over a sphere S (they also describe the BRDF product function) can be approximated by projecting the BRDF onto SH space. The coefficients from the projection are bidirectional and therefore can be used to reconstruct the original function with minimal loss of information from the original function. This assumes that the original function is a low frequency function and the number of bands used for approximation is sufficient. For low frequency lighting and reflections, this approximation can be accurate with as little as 25 coefficients [KSS02].

$$\tilde{f}(s) = \int_s f(s)y_l^m(s)ds \tag{2}$$

Instead of reconstructing the original equation for a set of operations, the product of the lighting function and BRDF can be approximated with the dot product of their coefficients. In other words, the conversion of incoming irradiance to outgoing radiance is reduced to a product accumulation of the coefficients, where n^2 is the number of coefficients.

$$\int_s \tilde{L}(s)\tilde{f}(s)ds = \sum_{i=0} n^2 L_i f_i \tag{3}$$

4. Analytical Interference BRDF

We extend an analytical solution for reflected interference intensity derived by Glassner [G00] to that of a BRDF. In order to achieve this we need to convert Glassner's closed form solution for thin film to account for hemisphere lighting by discretizing the integral. Summing all the reflected intensities from the hemisphere above a point can account for the total lighting contribution and therefore give an accurate representation of interference. The following equation sums the reflected response over the sphere S.

$$f(s,\lambda) = \sum_s 4R_f(\gamma,\lambda)\sin^2(\frac{2\pi}{\lambda}\omega\eta\cos\theta_t) \tag{4}$$

Where R_f is the Fresnel reflectivity, index of refraction is η, and ω is the thickness of the film. This function neglects transmission from the equation and focuses primarily on reflection.

5. Our Approach

In this section we discuss our approach including the necessary steps for precomputing the interference BRDF, parameterization, storage and rendering considerations.

5.1. Preprocess Steps

The following are the necessary steps for generating a sampling of the interference BRDF for rendering. In general

BRDFs are 5D functions $<\theta_i, \phi_i, \theta_o, \phi_o, \lambda>$, if we consider wavelength as a parameter of the BRDF. The storage considerations, assuming the wavelength varied would be a 5D texture. Current graphics hardware only support textures of 3 dimensions. Thus, we must reduce the size of our BRDF map by 2 degrees through reparameterization. Once we have reduced the size of our map, we have to account for n^2 SH coefficients instead of RGBα values. This essentially adds an additional dimension to the map leaving us with a 3D map. Spherical harmonics is a 2 dimensional orthogonal operator that is defined over on a unit sphere. Low frequency lighting and reflectance functions can be adequately be approximated by Spherical harmonic with relatively few numbers of coefficients (n = 5, 25 coefficients) [KSS02]. Functions that require higher frequency sampling will become band limited by the SH approximation if enough coefficients are not used. Global illumination style lighting tends to be amenable to smoothing with minimal degradation to the visual accuracy. Our pre-processing steps are:

1. Parameterize the reflectance map based on the view vector ω_o. We are storing coefficients, not color values so that we can account for more than RGBα in the map. This can be done by using a 2D/3D map indexed by spherical parameters of the out going view vector $<\theta_o, \phi_o>$.
2. Sample hemisphere at different wavelengths
    a. Sample the hemisphere and determine $<\theta_i, \phi_i>$
    b. For each spherical parameter, $<\theta_i, \phi_i>$, $<\theta_o, \phi_o>$ calculate the composite color values for the Fresnel reflectivity and interference by evaluating equation 4, accumulating the results from all incoming light angles.
3. Project color values to SH basis space (see equation 2) storing them based on view vector $<\theta_o, \phi_o>$.

5.2. Rendering

Our rendering algorithm is outlined below. This algorithm assumes a tangent frame associated with each fragment. Generating the texture coordinates requires defining the view vector relative to the tangent space coordinate system. Once the texture look up is complete, a dot product of the lighting coefficient and the reflectance coefficients is all that is required per pixel. Most texture hardware can only store values from [-1.0, +1.0] so an appropriate bias and scale of the texture values is required. The rendering algorithm is as follows:

1. The lighting coefficients must be rotated into the proper frame for which rendering is occurring.
2. Then lighting coefficients are uploaded to the hardware or software shader where the dot product will be performed.
3. The appropriate vector to the frame in which the dot product will be calculated is rotated. In our case this will be the view vector in the tangent frame.

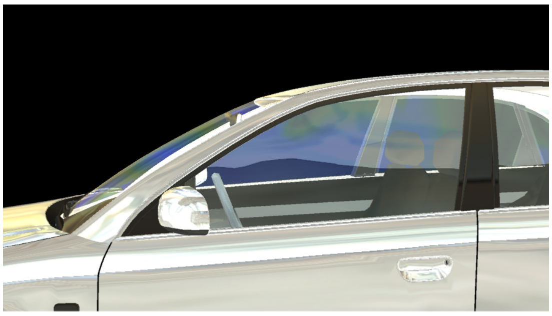

Figure 1: *A car window showing interference rendered using our technique*

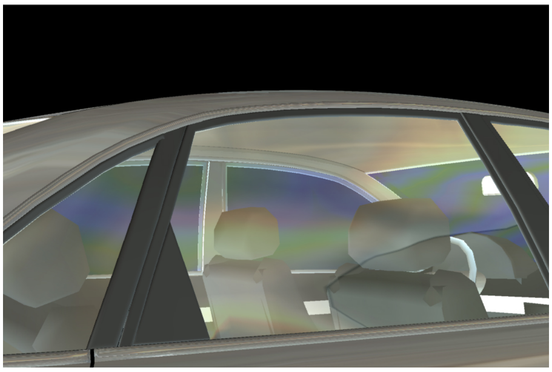

Figure 2: *Another screenshot showing a car window with interference using our technique*

4. The surface reflection coefficients are retrieved from the interference texture map then scaled and a bias is applied.
5. The last step is to apply the dot product of the coefficients. If high dynamic range lighting coefficients are in use, the illumination values may require tone mapping to a displayable range.

Rotation of lighting or reflectance coefficients is a nontrivial task, and is best suited to be computed on the host coefficients uploaded to the GPU as stated in rendering step 1. There are several way to rotate the coefficients involving Euclidian rotations. Refer to Kautz et al. [KSS02] for a more comprehensive explanation of SH rotations. Figures 1 and 2 are sample images of our technique. Our performance measurements show that we are able to render these images at about 65 frames per second.

6. Conclusion and Future Work

We have presented a technique for rendering interference that extends the limitation of few point or area lights. This technique takes into account hemispherical lighting, models interference as a BRDF and precomputes surface response functions in terms of a low order spherical harmonic basis. The physically based surface response is stored in maps that make it amenable to rendering on a GPU. Our approach takes advantage of a full spectral color representation by employing the Composite model. Our system achieves a full spectral representation of interference color by maintaining the spherical harmonic basis in terms of Spectral Power Distribution instead of a color-based format such as RGB. In addition, we demonstrate that our system is amenable to current graphics hardware and can render at real-time frame rates.

Possible extensions to our techniques include using surface microstructures to incorporate pigment materials and multilayer films that would enhance the interference simulation. Additional extensions of this technique would be to apply it to other wavelength based effects such as dispersion, diffraction, polarization, and fluorescence. Our technique could be also be applied to other types of basis functions such as wavelets.

References

[AH03] Agu E. and Hill F.S. *"A Simple Method for Ray Tracing Diffraction"*, in Proc. 2nd Int'l Workshop on Computer Graphics and Geometric Modeling", CGGM 2003, Montreal Canada.

[RH01] Ramamoorthi R., Hanrahan P., *An Efficient Representation for Irradiance Environment Maps* in Proc. ACM SIGGRAPH 2001, pg 497-500.

[WAT92] Westin S., Arvo J., Torrance K., *Predicting Reflectance Functions from Complex Surfaces* in Proc. ACM SIGGRAPH 1992, pg 255-264.

[RH02] Ramamoorthi R., Hanrahan P., *Frequency Space Environment Map Rendering* in Proc. ACM SIGGRAPH 2002, pg 517-526.

[KSS02] Kautz J., Sloan P., Snyder J. *Fast Arbitrary BRDF Shading for Low-Frequency Lighting Using Spherical Harmonics* in Proc. 13th Eurographics workshop on Rendering, pg 291-296, 2002.

[SK02] Sloan P., Kautz J., Snyder J., *Precomputed radiance transfer for real-time rendering in dynamic, low-frequency lighting environments* ACM Transactions on Graphics, pg 527-536, 2002.

[SFC*99] Sun Y., Fracchia F. D., Calvert T. W., Drew M. S., *Deriving Spectra from Colors and Rendering Light Interference*, IEEE CG&A, 19(4):61–67, July/August 1999.

[GMN94] Gondek J. S., Meyer G. W., NEWMAN, J. G. *Wavelength Dependent Reflectance Functions*, in Computer Graphics Series, 213-220, 1994.

[IMN04] Iwasaki K., Matsuzawa K., Nishita T., Real-time Rendering of Soap Bubbles Taking into Account Light Interference, Computer Graphics International 2004, pp.344-348.

[D91] Dias L.M., *Ray Tracing Interference Color*, IEEE CG&A, Vol.11, No.2, pp.54-60, 1991.

[D94] Dias L.M., *Ray Tracing Interference Color: Visualizing Newton's Rings*, IEEE CG&A, Vol.14, No.3, pp.17-20, 1994.

[L96] Li J., Peng Q., *A New Illumination Model for Scenes Containing Thin Film Interference*, Proc. Pacific Graphics 1996, pp.133-146.

[SM90] Smits B., Meyer G., *Newton's Colors: Simulating Interference Phenomena in Realistic Image Synthesis*, Proc. Eurographics Workshop on Photosimulation, Realism and Physics in Computer Graphics, pp. 185-194, 1990.

[G00] Glassner A., *Soap Bubbles: Part 2*, IEEE C&GA, Vol.20, No.5, pp.76-84, 2000.

[SFD99] Sun Yinlong, Mark S. Drew, and F. David Fracchia, *Representing Spectral Functions by a Composite Model of Smooth and Spiky Components for Efficient Full-Spectrum Photorealism*, IEEE Workshop on Photometric Modeling for Computer Vision and Graphics '99, pp. 4-11.

[GH03] Granier Xavier., Heidrich Wolfgang, *A simple layered RGB BRDF model*, Graphical Models(4): 171-184 (2003)

[HKY*01] Hirayama H., Kaneda K.,Yamashita H., Monden Y., *An Accurate Illumination Model for Objects Coated with Multilayer Films*, Computers & Graphics, Vol.25, pp.391-400, 2001.

[HKY*01] Hirayama H., Kaneda K.,Yamashita H., Monden Y., *Visualization of Optical Phenomena Caused by Multilayer films Based on Wave Optics*, The Visual Computer, Vol.17, No.2, pp.106-120, 2001.

[HKY*00] Hirayama H., Yamaji Y., Kaneda K., Yamashita H., Monden Y., *Rendering iridescent colors appearing on a natural object*. in Proc. Pacific Graphics 2000, pages 15-22, 2000.

[Gla95] Glassner, A. *Principles of Digital Image Synthesis*, Vol 2. Morgan Kaufmann Publishers, San Francisco, 1995.

[Sta99] Stam J, *Diffraction Shaders*, in Proc. ACM SIGGRAPH 1999.