

# Doppler Volume Rendering: A Dynamic, Piecewise Linear Spectral Representation for Visualizing Astrophysics Simulations.

## Supplemental Material

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### 1. Accumulation Cases

The mathematical formulation for all the possible 11 accumulation cases of  $f(x)$  and  $g(x)$  piecewise linear functions is as follows:

Case	Accumulation	Case	Accumulation
Case 1 or 11: $x_2 < y_1 \vee y_2 < x_1$	$f(x) + g(x) =$ $\begin{cases} m_1x + b_1 & x_1 \leq x \leq x_2 \\ m_2x + b_2 & y_1 \leq x \leq y_2 \\ 0, & \text{else} \end{cases}$	Case 2: $x_1 < y_1 < x_2 < y_2$	$f(x) + g(x) =$ $\begin{cases} m_1x + b_1 & x_1 \leq x \leq y_1 \\ (m_1 + m_2)x + (b_1 + b_2) & y_1 < x \leq x_2 \\ m_2x + b_2 & x_2 < x \leq y_2 \\ 0, & \text{else} \end{cases}$
Case 3: $y_1 < x_1 < y_2 < x_2$	$f(x) + g(x) =$ $\begin{cases} m_2x + b_2 & y_1 \leq x \leq x_1 \\ (m_1 + m_2)x + (b_1 + b_2) & x_1 < x \leq y_2 \\ m_1x + b_1 & y_2 < x \leq x_2 \\ 0, & \text{else} \end{cases}$	Case 4: $x_1 < y_1 < x_2 = y_2$	$f(x) + g(x) =$ $\begin{cases} m_1x + b_1 & x_1 \leq x \leq y_1 \\ (m_1 + m_2)x + (b_1 + b_2) & y_1 < x \leq y_2 \\ 0, & \text{else} \end{cases}$
Case 5: $x_1 = y_1 < x_2 = y_2$	$f(x) + g(x) =$ $\begin{cases} (m_1 + m_2)x + (b_1 + b_2), & x_1 \leq x \leq x_2 \\ 0, & \text{else} \end{cases}$	Case 6: $y_1 < x_1 < x_2 = y_2$	$f(x) + g(x) =$ $\begin{cases} m_2x + b_2 & y_1 \leq x \leq x_1 \\ (m_1 + m_2)x + (b_1 + b_2) & x_1 < x \leq y_2 \\ 0, & \text{else} \end{cases}$
Case 7: $x_1 = y_1 < x_2 < y_2$	$f(x) + g(x) =$ $\begin{cases} (m_1 + m_2)x + (b_1 + b_2) & x_1 \leq x \leq x_2 \\ m_2x + b_2 & x_2 < x \leq y_2 \\ 0, & \text{else} \end{cases}$	Case 8: $x_1 = y_1 < x_2 < y_2$	$f(x) + g(x) =$ $\begin{cases} (m_1 + m_2)x + (b_1 + b_2) & x_1 \leq x \leq y_2 \\ m_1x + b_1 & y_2 < x \leq x_2 \\ 0, & \text{else} \end{cases}$
Case 9: $x_1 < y_1 < y_2 < x_2$	$f(x) + g(x) =$ $\begin{cases} m_1x + b_1 & x_1 \leq x \leq y_1 \\ (m_1 + m_2)x + (b_1 + b_2) & y_1 < x \leq y_2 \\ m_1x + b_1 & y_2 < x \leq x_2 \\ 0, & \text{else} \end{cases}$	Case 10: $y_1 < x_1 < x_2 < y_2$	$f(x) + g(x) =$ $\begin{cases} m_2x + b_2 & y_1 \leq x \leq x_1 \\ (m_1 + m_2)x + (b_1 + b_2) & x_1 < x \leq x_2 \\ m_2x + b_2 & x_2 < x \leq y_2 \\ 0, & \text{else} \end{cases}$

## 2. Galaxy Dataset Information

Additional information of the Galaxy dataset, simulating the asymmetries of the center of the Milky Way, by Sormani et al.:

Density range	$\rho \in [7.65 \times 10^{-26} \text{ g/cm}^3, 1.94 \times 10^{-15} \text{ g/cm}^3]$ $\rho_{q=0.05} = 8.1 \times 10^{-23} \text{ g/cm}^3$ , $\rho_{q=0.95} = 3.16 \times 10^{-20} \text{ g/cm}^3$
Gas temperature range	$T \in [130.7 \text{ K}, 481.1 \text{ K}]$ $T_{q=0.05} = 439.0 \text{ K}$ , $T_{q=0.95} = 480.9 \text{ K}$
Velocity ranges	$v_x \in [-342, 328] \text{ km/s}$ , $v_y \in [-238, 242] \text{ km/s}$ , $v_z \in [-140, 117] \text{ km/s}$ $v_{x,q=0.05} = -267 \text{ km/s}$ , $v_{x,q=0.95} = 267 \text{ km/s}$ $v_{y,q=0.05} = -200.0 \text{ km/s}$ , $v_{y,q=0.95} = 200 \text{ km/s}$ $v_{z,q=0.05} = -10.0 \text{ km/s}$ , $v_{z,q=0.95} = 9.39 \text{ km/s}$ (Compare Speed-of-light $c = 299\,792.458 \text{ km/s}$ )
Matter included	neutral hydrogen <i>HI</i> molecular hydrogen <i>H<sub>2</sub></i> carbon monoxide <i>CO</i>

## 3. Synthetic Dataset

With the aim of evaluating our technique in a controlled environment, we created a synthetic dataset, consisting of a radial density field of hydrogen, moved by a vector field with cylindrical dynamics, of thickness around 10% of the side of the volume. The figure shows two sample spectra along respective rays traversing the velocity field.

