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Short Papers

Vortex merge graphs in two-dimensional unsteady flow fields

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Abstract

Among the various existing vortex definitions, there is one class that relies on extremal structures of derived scalar fields. These are, e.g., vorticity, λ_2, or the acceleration magnitude. This paper proposes a method to identify and track extremal-based vortex structures in 2D time-dependent flows. It is based on combinatorial scalar field topology. In contrast to previous methods, merge events are explicitly handled and represented in the resulting graph. An abstract representation of this vortex merge graph serves as basis for the comparison of the different scalar identifiers. The method is applied to numerically simulated flows of a mixing layer and a planar jet.

Categories and Subject Descriptors (according to ACM CCS): I.3.8 [Computing Methodologies]: Computer Graphics—Applications J.2 [Computer Applications]: Physical Sciences and Engineering—Physics

1. Introduction

Vortices are a fundamental part of the structural skeleton of time-dependent fluid flows. A lot of research has been devoted to the analysis of their development and interaction. This concerns answers for basic questions as: Which flow configurations cause vortices to arise and dissolve, to merge and split, to grow and shrink? Even two-dimensional flow fields are still an active research area [Tab02, BM10, CKL10].

One approach to investigate the vortex skeleton of a flow field is a feature-based analysis. Here, the explicit geometric embedding of the vortex cores and their regions of influence is extracted. This is typically based on a scalar feature identifier such as the pressure P, the magnitude of Q as defined by Hunt et al. [Hun87], the vorticity ω, the λ_2 measure as defined by Jeong et al. [JH95], or the acceleration magnitude a [KRHH11, FV10]. For a good introduction to these and other vortex criteria, we refer to Fuchs et al. [FKS*10], Post et al. [PVH*03], and Pobitzer et al. [PPF*10].

In the following we focus on vortex cores that are defined by a derived quantity. Several approaches have been published that extract vortex cores based on λ_2 [SRE05, BSER09,SVG*08,SBV*11]. Schneider et al. [SWC*08] apply a contour tree algorithm to quantities such as λ_2 and pressure. They extract vortices as iso-surfaces of these quantities. Sahner et al. [SWTH07] use the Okubo-Weiss criterion to extract vortex and strain skeletons in three dimensional flows. Vorticity was used by Sadlo et al. [SPS06] to extract the vortex dynamics.

Vortex cores based on these quantities are similar but not identical. The differences become more pronounced for strongly unsteady fields considering their development over time. Theisel et al. [TS03] propose "Feature Flow Fields" to track the critical points. Tricoche et al. [TWSH02] and Garth et al. [GTS04] track singularities in vector fields. Bauer and Peikert [BP02] propose a method for vortex core line extraction and tracking in scale-space. Theisel et al. [TSW*05] generalize the use of a the parallel vectors operator to track vortex core lines. Weinkauf et al. [WSTH07] also used the parallel vectors operator to extract cores of swirling particle motion. This extraction approach was also used by Fuchs et al. [FPH*08] for unsteady flow vortices.

In this paper, we focus on the investigation of the temporal evolution of vortex cores. We introduce an algorithm for the extraction of a spatiotemporal vortex skeleton. In contrast to previous approaches, we are able to handle explicit merge events within the framework of combinatorial feature flow fields (CFFF) exploiting its robustness [RKWHar]. This approach enables the noise-resilient extraction and tracking of extremal structures with an importance measure for the individual vortex core lines. The proposed algorithm can be applied to all of the above mentioned vortex identifiers. Exemplary, we test our method for vorticity, acceleration magnitude, and λ_2 and compare the results. To do so, we use two two-dimensional flows: a free shear layer and a data set of a jet. Note that λ_2 was originally defined for three-dimensional flows, but also yields sensible results for two-dimensional flows.

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2. Method

Most vortex core definitions are based on extremal structures of a derived scalar field. For 2D fields these are minima or maxima of the respective scalar field. The temporal evolution of these vortex cores can be revealed by tracking critical points in time. To perform this tracking, we built on the method of combinatorial feature flow fields (CFFF) [RGH*10].

The focus of the original CFFF approach lies on noise resilient extraction of critical lines using homological persistence as spatial importance measure. Since the spatial importance of one of the critical points becomes arbitrarily small as they approach a merge or split point, CFFF does not handle explicit splits and merges of critical lines. However, for vortex core lines, mergers are common events and of special interest to the researchers. Therefore, in the following, a modification of the method is described that enables merge and split detection, while maintaining its ability to handle noise. Since for 2D fields no vortex splits occur, the result is called *vortex merge graph*.

As for the vortex extraction only minima and maxima are relevant, our approach does not consider saddle points. This simplifies the integration of merge and split events. For the case of minima or maxima, the basic idea of CFFF is to trace stream lines in a combinatorial vector field. Thereby, two minima or maxima are connected, if the stream lines in each others gradient field uniquely connect these points. Note that a stream line ends or starts at a minimum or maximum respectively, if it starts in its basin. Therefore, another interpretation of the CFFF approach is that two minima or maxima of adjacent time slices are connected, if they fall in the topological basin of each other.

Extraction of the vortex merge graph – The first step is the extraction of the critical point for each time slice individually, which are connected in a second step. Let C_t be the set of critical points at time t. On this set two functions are defined: forward tracking $F_t : C_t \to C_{t+1}$ and backward tracking $B_t : C_t \to C_{t-1}$. They assign a critical point $c_1 \in C_t$ to another critical point c_2 in the next or previous time step, respectively, if c_2 falls into the basin of c_1. Two critical points c_1, c_2 of subsequent time steps are called *uniquely* tracked, if $F_t(c_1) = c_2$ and $B_{t+1}(c_2) = c_1$. CFFF considers only such unique tracking; the resulting lines therefore cannot merge or split. To allow for mergers and separations, we drop the uniqueness tracking condition. Instead, we extract both the forward and backward tracking functions F and B for all time steps resulting in the tracking graph.

For tracking of vortices in 2D flow fields, we are interested in minima or maxima of a derived quantity. Sometimes, only a subset of the extrema is relevant. For instance, in the case of the acceleration magnitude, the minima have to be classified by the Jacobian as described in [KHNH11]. From the relevant subset of extremal points, at first the unique tracking graph is generated resulting in the set of core lines M. To compute the mergers, for each end point e_t of a line $e \in M$ the following condition is tested:

$$\exists T > 0 : F_{t+T}(\ldots F_{t+1}(F_t(e_t))) \in e' \in M. \qquad (1)$$

Loosely speaking, it is tested if the repeated evaluation of the forward tracking of the end point hits another tracked core line. If such a T is found, the line generated by $F_{t+T}(\ldots F_{t+1}(F_t(e_t)))$ is added to an additional set of core lines M' that represents the merge lines. The vortex merge graph is then given by the union of M and M'. Note that the search, i.e., T, is only bounded by the finite temporal domain and a repeated evaluation of the forward tracking always leads to another vortex core line or to some extremal point on the domain boundary. To extract the split points, in the above described approach the end points are replaced by start points and the forward by the backward tracking field. This is not needed for 2D flows, since split events do not occur in 2D flows in theory. The tracking graph is a merge tree. In three dimensions, the extraction of mergers and splits would work in the same way as described above.

Note that the importance measure for the individual vortices in M is given by integrated persistence as proposed [RKWHar]. It is a combination of the feature lifetime [KHNH11] and persistence [EHZ01]. This enables a hierarchy of the vortex cores, important for complex flows.

Representation of the vortex merge graph – The forward and backward tracking are discrete functions – they are given for each critical point in each time slice. Thus, the tracking functions for all time steps can be interpreted as a single directed graph containing both backward and forward tracking. This representation eases the implementation, since known efficient graph algorithms can be used for the evaluation.

To store the graph efficiently, the following requirements have been identified: (i) each critical point should be stored only once; (ii) the edges have to be stored including their direction; (iii) the implementation should allow a fast depth search in the graph to trace the connections between the unique vortex core lines; (iv) adding edges should be possible fast, but there is no need for a fast operation to remove them; (v) the coordinates and types of the critical points have to be stored. Note that each critical point can be identified by its time value and a unique identifier in the time slice – we call this a global identifier of a critical point.

Due to these requirements, we assign each critical point a consecutive identifier in the tracking graph – two data structures map the global identifiers to the local ones and vice versa. Each edge is stored as a directed pair of node identifiers. For each node, the associated edges are stored distinguishing between ingoing and outgoing edges. In addition, the coordinates and types of the critical points are stored in two additional data structures.

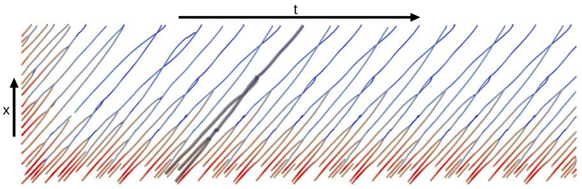

Figure 1: *Vortex merge graph of the mixing layer based on the acceleration magnitude. The time increases from left to right and the flow direction is from bottom to top. Radius and color represent the vorticity magnitude: low and high vorticity correspond to blue/thin and red/thick, respectively.*

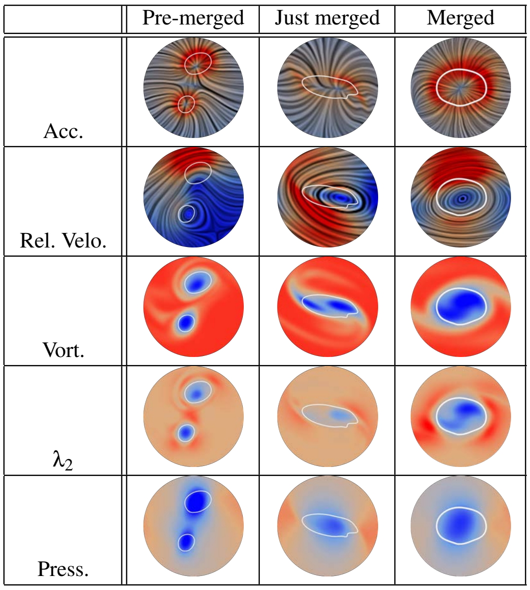

Figure 2: *Comparison of different quantities at different stages of a merge event.*

3. Results

Mixing Layer – The first data set is a 2D mixing layer. It represents a simulated shear flow, sampled on an uniform grid (960×384) and consists of 11001 time steps. The velocity ratio between the upper and the lower stream is 3 : 1. Fig. 1 shows the vortex merge graph of this data set using the acceleration magnitude as feature identifier. The image shows the tracked minima with non-zero vortex strength. It can be observed that the vortices are generated at the front (low x-values) and move with the flow in the x-direction. Note that it is possible that a vortex core ends inside the domain without merging to another vortex core. To investigate the expected merge behavior for this graph, we analyze related quantities on a circular disk around the core before, during and after a merge event. We added the vortex regions identified by the acceleration [KRHH11] as white lines to ease the comparizon. The results are shown in Fig. 2. In the first column each vortex identifier exhibits a similar structure indicating two extrema referring to two vortices. This changes in the second column shortly after the merge event detected by the acceleration magnitude. Then considerable differences can be seen. While vorticity and λ_2 still reveal two clearly distinguishable extremal values, the pressure and the acceleration magnitude show just a single vortex. A couple of time steps later the structures are again very similar. This suggests that the specific time of the merge events of the vortices expressed by these quantities is different. Similar behavior can be observed for other merge events and also for other data set. It seems to be a typical difference of these feature identifiers. Since Q and the vortex strength measure correlate with λ_2 in two-dimensional flows, we omitted these quantities. Inspecting the relative velocity shown in the second row, it becomes clear that there is no distinguished frame of reference that would allow to represent the two merging vortices as vector field singularities.

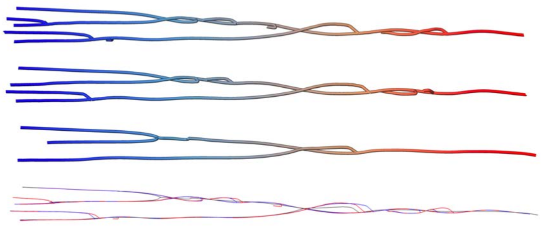

Figure 3: *Vortex cores extracted from different feature identifiers: λ_2 (first, bottom red), vorticity (second, bottom blue) and acceleration magnitude (third, bottom grey). The color of the top vortex graphs refers to time. The merge points in time differ significantly between acceleration and the other two quantities. Vorticity and λ_2 coincide most of the time.*

To underpin the above observations we now compare vortex cores resulting from different scalar measures. Therefore, we focus on one connected component picked from the merge tree. The corresponding results based on acceleration minima, λ_2, and vorticity are shown in Fig. 3. Three interesting observations can be made: First, it is confirmed that for the acceleration magnitude the vortex core lines merge earlier than for the other two quantities. Close to the merge event the core lines differ significantly. Second, outside the merge windows, all three vortex core lines coincide well. While this behavior is expected for vorticity and λ_2, we did not expect this for the acceleration magnitude. Third, even inside the merge windows, the vortex core lines of vorticity and λ_2 coincide until the actual merge event. This could be an effect of the persistence simplification. Note that it is not possible to use the same persistence threshold for different quantities. Thus, the extremal points that depict the vortex core line may be cancelled at slightly different times.

Jet – The second data set used in this paper results from a direct numerical simulation of a planar time-dependent jet. It is sampled on a rectilinear grid with 2449×598 resolution and consists of 4560 time slices.

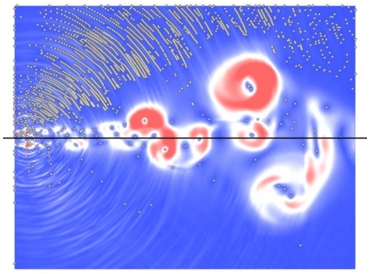

Figure 4: *Minima of the acceleration magnitude of the jet dataset. The top shows all minima contained in that slice and the bottom a filtering by homological persistence with a threshold of 2 % of the range of the field.*

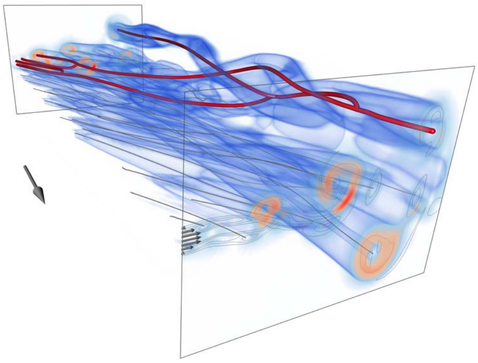

Figure 5: *Vortex merge graph of the jet flow. The direction of time is from back to front. The vortex cores are extracted as tracked minima of the acceleration magnitude.*

While the acceleration of the mixing layer is relatively smooth, the acceleration magnitude of the jet data set shows a superposition of pressure waves originating from the jet. These structures add a lot of spurious critical points to the data which challenge the analysis and tracking. Due to the use of homological persistence our tracking tool is still able to extract a stable merge graph. Fig. 4 depicts the critical points of one time slice of the acceleration magnitude for two different filtering levels. The top half of the image shows all critical points, while in the bottom half the critical points are filtered with a persistence threshold of two percent of the range in that slice. The vortex-related minima of the acceleration magnitude stay, most minima induced by the shock waves are removed.

The result of the vortex core extraction algorithm is shown in Fig. 5 as gray lines. The velocity profile is depicted by the gray arrows in the front plane. The time axis points from back to front and the flow moves from left to right. The acceleration magnitude is depicted as color coding at the front and back slice and as volume rendering (blue) in the space-time volume. Vorticity isolines are added to the front plane. The cores indicated by the vorticity correspond to a subset of the minima of the acceleration magnitude. The red lines highlight one selected vortex merge graph, which reveals the typical merge structure as expected by the flow experts. There is a single vortex at the end of the vortex street that is fed by other vortices merging onto its core.

4. Discussion

This paper proposes a new method to track the evolution of vortices in complex 2D flow data sets including vortex mergers. The detection of merge events is not based on spatial vicinity, but on the *flow* of the vortex cores in a time-dependent feature flow field. Furthermore, in the presented framework, different feature identifiers such as the acceleration magnitude, λ_2, and vorticity can be used. The resulting merge graph complements the analysis of Lagrangian coherent structures (LCS) based on the Finite time Lyapunov exponent (FTLE), e.g., the work of Sadlo et al. [SW10].

The method is embedded in a robust combinatorial framework. The tools work directly on the given grid and do not rely on any interpolation. The filtering capabilities of the combinatorial framework, based on homological persistence, carry over thoroughly to the proposed merge graph extraction. Thus, it becomes possible to handle complex flows, which is very challenging otherwise. With our approach, the applicability of analytic vortex concepts, e.g., Lagrangian equilibrium points (LEPs) [KHNH11], expands from simple textbook examples to real world data sets such as the jet flow.

The generality of the approach lends itself to be used for the comparison of different vortex indicators. There is no commonly accepted mathematical definition of a vortex and not even a set of axioms such a definition should obey. Hence, which of the vortex indicators is *correct* cannot be said. Due to the large variety of applications and research questions in engineering and fluid flow research, a single definition might even be unwanted.

Exemplarily, we considered four different scalar measures. Outside the vortex merge windows, the vortex cores of these quantities show a similar behavior. But, a closer inspection of the merge events reveals some differences. It can be observed that among the four considered identifiers – depending on when mergers happen – two types can be distinguished: first λ_2 and vorticity, and second pressure and the acceleration magnitude. There are also vortex cores that merge for the one type but not for the other. Differences among measures of one type seem not to be conceptual but rather due to algorithmic specifics, which we will further explore in future work.

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