3D Architectural Modeling: Efficient RANSAC for *n*-gonal primitive fitting

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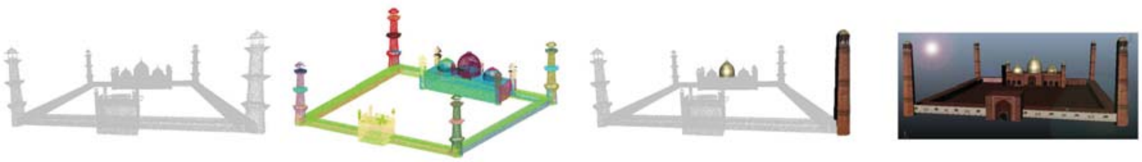

(a) (b) (c) (d)

Figure 1: *(a) Point cloud. (b) Color coded automatically segmented point cloud. (c) Minaret and Dome fitted on segmented point cloud through proposed novel RANSAC based approaches. (d) Final rendered model.*

Abstract

We present a modeling approach to automatically fit 3D primitives to point clouds in order to generate a CAD like model. For detailed modeling we propose a new n-gonal 3D primitive and a novel RANSAC based fitting approach. Non-planar surfaces are modeled through surface of revolution with B-spline profiles. We first reduce the dimension by projecting the 3D data onto a 2D plane. Primitive fitting algorithm is then applied in this 2D space. Our approach compares favorably both with manually and automatically generated models. Not only is it much more time efficient than manual modeling, but it also gives significantly better output than state-of-the-art automatic methods. Since the focal technique of our approach is the fitting of detailed primitives, our results are ideal in the domain of architecture and preservation of heritage.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Modeling—Line and curve generation

1. Introduction

3D reconstruction of architecture has been a popular research area in computer vision and computer graphics. Over the years, significant advances have been made in this domain in the form of semi-automatic and automatic algorithms for 3D reconstruction using input data acquired from photographs, LiDAR scans or aerial imagery (much of which is stored in the form of point clouds).

Many approaches have been proposed that try to recover a set of locally fitted primitives. A seminal work in this direction was a RANSAC based method [SWK07] which fits primitives including planes, cylinders and cones. Existing approaches typically make use of this approach and try to recover global mutual relationships among the fitted primitives. OSnap [ASF*13] and Globfit [LWC*11] are two such recent examples. These approaches are limited in their reconstruction abilities by the primitives supported in [SWK07] and hence most of the existing literature is focused on modeling planar architectural structures only [SSS*08, FP10, ASF*13, NBW14].

An exception to planar fitting is Schematic Surface Reconstruction [WACS12] that describes the architecture with a concise network of horizontal transport curves (floor plan) associated with vertical profile curves to form swept surfaces. However, schematic representation too has its limi-

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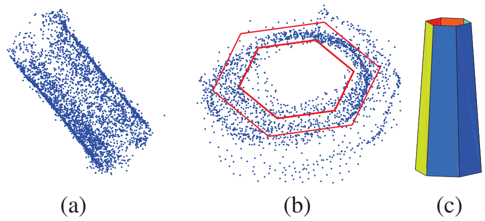


Figure 2: *N-gon fitting pipeline. a) Input 3D point cloud. b) Shape fitting on 2D projection. c) Output 3D n-gon primitive.*

tations as it fails to create a solid, compact 3D CAD-like model, deployable in graphics applications.

Many architectural buildings particularly those in heritage architecture require more detailed modeling of both planar and non-planar elements of architecture. Unlike prior work, we propose a coarse-to-fine modeling approach that automatically recovers the more detailed geometric representation of structural elements. For planar modeling we extend [SWK07] and introduce a novel n-gonal 3D primitive that has a much higher representational ability. We propose a novel RANSAC based shape fitting strategy for fitting of this new primitive. We model non-planar surfaces such as domes by extending swept surface [WACS12] whereby a profile curve, recovered by fitting piecewise Bezier curves, is swept onto a circular transport curve to generate a best fit dome.

What sets our algorithm apart from other existing shape fitting approaches is that it can recover the correct parameters of shapes even in the presence of noise, outliers or erroneous shape geometry due to loss of information inherent in point cloud or due to over segmentation of point cloud. This is because our approach performs initial fitting in 2D space. Prior knowledge of primitive shape type makes the algorithm robust to any misleading geometry information that may be present in the point cloud.

2. Our Approach

Given a point cloud, we first perform a coarse primitive fitting using [SWK07]. This approach tends to fit the closest available, often incorrect primitive on many structural elements, particularly n-gonal and non-planar elements (such as gates, minarets and domes) (see Fig. 3). Hence we only use [SWK07] for the point cloud segmentation (Figure 1(b) shows the segmented point cloud). Our shape fitting algorithm then finds the optimal primitive parameters that locally fit this data and produces a CAD like model. Throughout our pipeline, we assume a user marked ground plane which helps us simplify the approach.

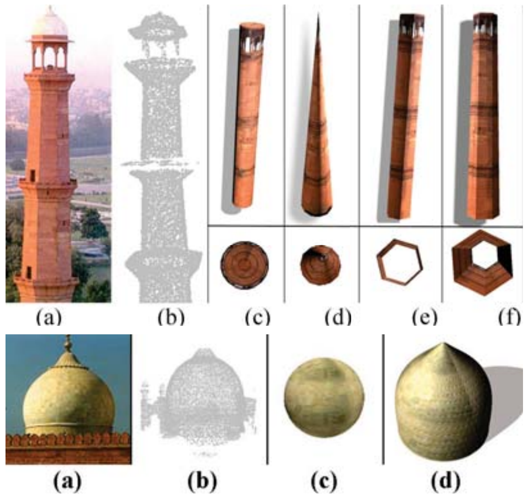


Figure 3: *Primitive fitting comparison between [SWK07] and proposed approach. Row 1: (a) Original minaret. (b) Point cloud (c-d) cylinder and cone fitted by [SWK07] and their cross-section. (e-f) Single radius and multi-radii n-gonal 3D polyhedrons primitive fitted using our approach and their cross-section. Row 2: (a) Original dome. (b) Point cloud (c) Hemisphere fitted by [SWK07]. (d) Bezier curve fitted using our approach.*

2.1. Planar Primitive Fitting

Piecewise planar structures such as hallways and minarets are a major component of heritage architecture. We model such closed structures using regular 3D n-gonal prisms. In this paper, we present a novel shape based RANSAC to fit any regular n-gonal primitive onto an input point cloud. We show that in a two dimensional space, all regular n-gons can be represented unambiguously using only four points. This is also supported by the fact that all regular n-gons have only four degrees of freedom in 2D; namely scale, rotation and two parameters of translation.

Four points must be in a specific formation to constrain a regular two dimensional n-gon (see Fig.4). To model an n-gon where n is even, we need two points on any one of its side, say l_1. l_1 constrains the rotation of n-gon. Similarly, we need the third point on the side l_2 such that $l_2 \parallel l_1$. Knowing l_2, along with l_1 adds the constraint scale and one translation parameter of the shape. To constrain the last free parameter of translation, we now need a point on a side adjacent to $l_1 \vee l_2$. Figure 4 shows the n-gon shape generation from four points as described above. This four point approach can be easily adapted for odd n, by restricting the third point to be on some known side instead of parallel one. While picking four points at random does not guarantee that they will lie on the desired configuration of sides, we try all permutations of these points and, in the spirit of RANSAC, simply discard the hypotheses which are not consistent with observed data.

The input point cloud is first projected onto the ground

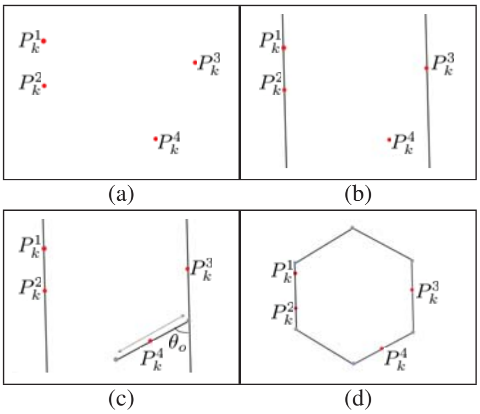


Figure 4: *Visualization of the shape hypothesis generation algorithm as described in Algorithm 1. a) Four initial sample points for n-gon hypothesis generation . b) First two points lie on one side of n-gon and third on a parallel side. c) Fourth point lies on an adjacent line. d) n-gon shape parametrized by the input points.*

plane. Our RANSAC-based shape fitting algorithm 1 that finds the parameters of input n-sided polygon is then run on this 2D data. For each hypothesis, the outer and inner n-gon radii are selected based on the range of input points data. These radii are then adjusted for maximum compactness based on the point density enclosed between the two radii. The fitted n-gon is extruded along the direction of ground plane normal to obtain the 3D coarse model, fitting the input point cloud segment. This pipeline is illustrated in Fig. 2. A similar shape fit algorithm can also be formulated in a three dimensional space using planes and points, but reducing the dimensionality to 2D decreases the complexity of our shape fit algorithm exponentially by imposing fewer degrees of freedom. A visualization of generating shape hypothesis with four sampled points is shown in Fig. 4.

2.2. Non-Planar Primitive Fitting

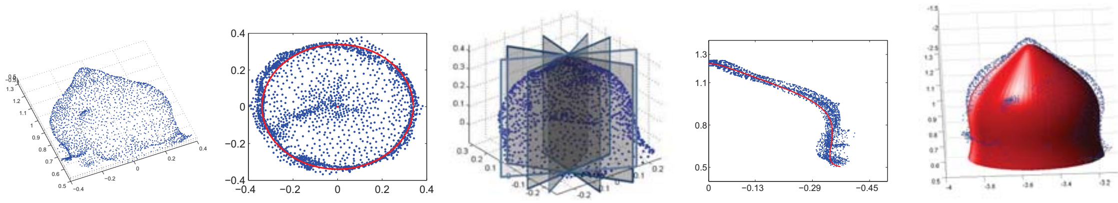
Non-planar structures are usually defined with unique curves. In our work, we focused on non-planar structures which can be described as a surface of revolution around a single axis, such as domes, which are a distinctive feature of heritage architecture. Adapting from [WACS12], we derived a reconstruction pipeline, given in Fig. 5, which detects a circular transport curve from ground plane projection and estimates the vertical profile curve associated with it. The estimated 2D profile curve, which effectively describes the structure curvature, is rotated along the direction of the transport curve to obtain a swept surface.

Given a point cloud segment, with known ground plane, we evaluate its ground plane projection. Using RANSAC, probabilistically optimal center and radius parameters of the circular base are estimated to obtain the axis of rotation for the circular transport curve. Each point on the transport curve t_i has a set of corresponding profile points p_i associated with it. It is possible to evaluate the final profile curve using only p_i for some specific t_i, but a single slice of profile points is insufficient to reconstruct the profile curve accurately because of noise and missing data inherent in point cloud. Hence, all profiles are projected onto a common plane first and then the final profile curve is modeled using Bezier Splines on that plane. In order to achieve this goal, a plane perpendicular to the ground and passing through the axis of rotation is sliced through the point cloud at transport point t_i as shown in Fig. 5 (d). Subsequently, a cluster of profile slices, sampled at an interval of $\delta\theta$, are accumulated by collapsing them onto a common canonical profile plane.

Algorithm 1 Generate Shape Hypothesis

```
procedure GENERATESHAPEHYPOTHESIS(p, n)
▷ p is vector of non-collinear and randomly picked four points
▷ n is user provided number of sides of the regular n-gon
  θ0 ← (n − 2) ∗ 180/n                          ▷ interior angle
  C ← φ                              ▷ Shape Hypothesis Candidates
  for each permutation P_k of p do
    compute the line l1 from points P_k^1 and P_k^2
    compute the line l2 passing through P_k^3 such that l2 ∥ l1
    d ← distance⊥(l1, l2)
    compute length of each n-gon side |S| using d and θ0
    if P_k^4 does not lie on a line adjacent to l1 ∨ l2 then
      continue;                              ▷ Invalid permutation
    end if
    choose line l_m from l1 ∨ l2 with minimum distance to P_k^4
    compute l3 passing through P_k^4 and l_m at (180 - θ0)°
    C_k^1 ← l3            ▷ 1^st side of k^th candidate hypothesis C_k
    for i ← 2, n do
      compute gradient of side s_i
      compute intersection point of s_i with C_k^{i−1}
      C_k^i ← s_i
    end for
    if all points in p do not lie on C_k then
      C ← C \ C_k                            ▷ Remove C_k from C
    end if
  end for
end procedure
```

Once all profile points have been accumulated, the profile curve is estimated from plane projected data by estimating a smooth concatenation of cubic Bezier Curves i.e. a B-spline [WPL06], on profile points as shown in Fig. 5 (d). A single Cubic Bezier curve is generally not expressive enough to model most of the curves perfectly. There are two ways of solving the issue: i) to use a higher order Bezier and ii) to concatenate multiple Cubic Bezier curves continuously, which leads to B-splines. Since the former method is computationally expensive, the latter is generally used in



(a) (b) (c) (d) (e)

Figure 5: *Dome fitting pipeline. a) Input 3D point cloud. b) Finding circle parameters of dome transport curve. c) Cluster of profile slices. d) B-spline modeling profile curve. e) Output 3D dome model.*

literature. We used B-splines to model the profile curve as in Fig. 5 (d). Rotating the B-spline along the transport curve yields a 3D model as shown in Fig. 5 (e).

3. Experimental Set-up and Results

We used an SfM point cloud generated by state-of-the-art Autodesk 123D Catch, using **12** self-taken aerial photographs. The point cloud and corresponding mesh generated were very noisy and had many artifacts as shown in Fig. 5(a). This point cloud was then segmented into coarse primitives using [SWK07]. Planes detected in the segmentation phase were grouped together to form geometrical primitives, such as cubes, cuboids, etc. (beyond the scope of [SWK07]) through an interactive GUI. The dome shown in Fig. 5(e) was generated in 8.3 seconds with an input noisy segment of $2,089$ points only. Similarly, the minaret in Fig. 2(c) was generated in 128 seconds, and consisted of a hexagonal base with 2,163 points and a dome on top having 302 points. Compared to the automatic model generated by Autodesk 123D Catch, our approach provides a regularized hexagonal minaret of much better quality.

We also used a synthetic point cloud from an existing CAD model to reconstruct the complete Badshah-i-Masjid, as shown in Fig. 1(a). The synthetic point cloud had $6,27,326$ points. Automatic segmentation resulted in a total of 29 primitive shapes out of which 11 were grouped together using the interactive GUI. The complete model in Fig. 1 (d) was generated in around 70 minutes and used only three basic primitive shapes; namely 11 domes, 8 hexagonal prisms and 8 cuboids. While the manual model has much more detail, our model is a good enough approximation, completed in a fraction of time needed to create a model manually.

During experimentation, we used points on the courtyard to mark the ground plane. We restricted the number of RANSAC iterations to be 10% the size of the input data and the minimum size of inlier set for a successful hypothesis was 5% of the input. All these results were generated using Matlab 2012b, running on an intel core i7, 2.67 GHz with 8GB RAM. After generating the best fit primitives, the results were manually texture mapped in Autodesk Maya 2015.

4. Conclusion

In this work we proposed a new n-gonal primitive and a novel RANSAC approach for its fitting. Unlike plane fitting approaches, our multi-radii n-gonal 3D polyhedronal primitive can preserve various details within the architecture while preserving global mutual relationships within an n-gon. We also proposed an extension of swept surfaces for modeling non-planar structures through B-spline fitting and surface of revolution. Our work can be considered as a new parametric model for heritage architecture.

In future we aim to incorporate clustering of normals to automatically identify the value of parameter n for further automating n-gonal primitive fitting.

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