Optimal spectral descriptors

Generic Q-dimensional spectral descriptor of the form

$$\mathbf{f}_{\boldsymbol{\tau}}(x) \quad = \quad \sum_{k \geq 1} \begin{pmatrix} \tau_1(\lambda_k) \\ \vdots \\ \tau_Q(\lambda_k) \end{pmatrix} \phi_k^2(x)$$

parametrized by frequency responses $\boldsymbol{\tau}(\lambda) = (\tau_1(\lambda), \ldots, \tau_Q(\lambda))^\top$

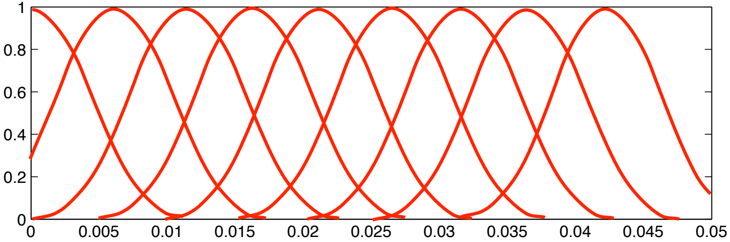
Litman, Bronstein 2014

Optimal spectral descriptors

Generic Q-dimensional spectral descriptor of the form

$$\mathbf{f}_{\mathbf{A}}(x) = \sum_{k \geq 1} \mathbf{A} \begin{pmatrix} \beta_1(\lambda_k) \\ \vdots \\ \beta_M(\lambda_k) \end{pmatrix} \phi_k^2(x)$$

parametrized by frequency responses $\boldsymbol{\tau}(\lambda) = (\tau_1(\lambda), \ldots, \tau_Q(\lambda))^\top$ represented in some fixed basis $\beta_1(\lambda), \ldots, \beta_M(\lambda)$ by an $Q \times M$ matrix $\mathbf{A}$



Litman, Bronstein 2014

Optimal spectral descriptors

Generic Q-dimensional spectral descriptor of the form

$$\mathbf{f}_{\mathbf{A}}(x) = \mathbf{A} \underbrace{\sum_{k \geq 1} \begin{pmatrix} \beta_1(\lambda_k) \\ \vdots \\ \beta_M(\lambda_k) \end{pmatrix} \phi_k^2(x)}_{\mathbf{g}(x)}$$

parametrized by linear combination coefficients $\mathbf{A}$ of geometry vectors $\mathbf{g}(x) = (g_1(x), \ldots, g_M(x))^\top$

Litman, Bronstein 2014

Optimal spectral descriptors

Generic Q-dimensional spectral descriptor of the form

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Litman, Bronstein 2014

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- Optimal $\mathbf{A}$ in the spirit of Wiener filter:
  - attenuate frequencies with large noise content (deformation)
  - pass frequencies with large signal content (discriminative geometric features)

Litman, Bronstein 2014

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Litman, Bronstein 2014

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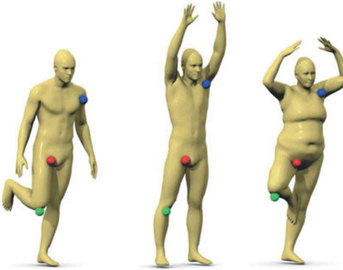
- Optimal $\mathbf{A}$ in the spirit of Wiener filter:
  - attenuate frequencies with large noise content (deformation)
  - pass frequencies with large signal content (discriminative geometric features)
- Hard to model axiomatically...
- ...yet easy to learn from examples!

Litman, Bronstein 2014

Optimal spectral descriptors

- Given a set of points x, knowingly similar points (positive) x_+, and knowingly dissimilar points (negative) x_-, with respective geometry vectors $\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-$



Litman, Bronstein 2014

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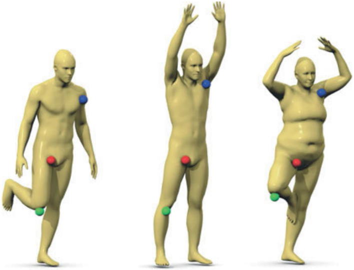

- Find optimal $\mathbf{A}$ by minimizing the loss

$$\min_{\mathbf{A}:\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \mathbb{E}\,\gamma\|\mathbf{f}_\mathbf{A}(x) - \mathbf{f}_\mathbf{A}(x_+)\|^2 - (1-\gamma)\|\mathbf{f}_\mathbf{A}(x) - \mathbf{f}_\mathbf{A}(x_-)\|^2$$

Litman, Bronstein 2014

Optimal spectral descriptors

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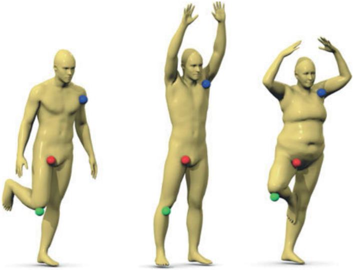


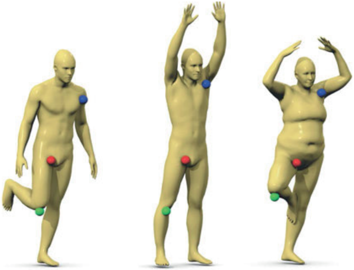
- Find optimal $\mathbf{A}$ by minimizing the loss

$$\min_{\mathbf{A}:\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \mathbb{E}\,\gamma\|\mathbf{A}\mathbf{g} - \mathbf{A}\mathbf{g}_+\|^2 - (1-\gamma)\|\mathbf{A}\mathbf{g} - \mathbf{A}\mathbf{g}_-\|^2$$

Litman, Bronstein 2014

Optimal spectral descriptors

- Given a set of points x, knowingly similar points (positive) x_+, and knowingly dissimilar points (negative) x_-, with respective geometry vectors $\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-$



- Find optimal $\mathbf{A}$ by minimizing the loss

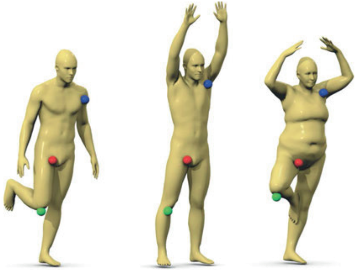
$$\min_{\mathbf{A}:\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \quad \mathrm{trace}\left(\mathbf{A}(\gamma\mathbf{C}_+ - (1-\gamma)\mathbf{C}_-)\mathbf{A}^\top\right)$$

where $\mathbf{C}_\pm = \mathbb{E}(\mathbf{g}-\mathbf{g}_\pm)(\mathbf{g}-\mathbf{g}_\pm)^\top$ is covariance matrix

Litman, Bronstein 2014

Optimal spectral descriptors

- Given a set of points x, knowingly similar points (positive) x_+, and knowingly dissimilar points (negative) x_-, with respective geometry vectors $\mathbf{g}, \mathbf{g}_+, \mathbf{g}_-$



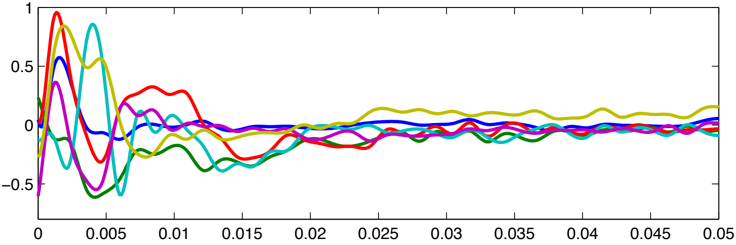
- Find optimal $\mathbf{A}$ by Mahalanobis metric learning

$$\min_{\mathbf{A}:\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \text{trace}\left(\mathbf{A}(\gamma\mathbf{C}_+ - (1-\gamma)\mathbf{C}_-)\mathbf{A}^\top\right)$$

where $\mathbf{C}_\pm = \mathbb{E}(\mathbf{g}-\mathbf{g}_\pm)(\mathbf{g}-\mathbf{g}_\pm)^\top$ is covariance matrix

Litman, Bronstein 2014

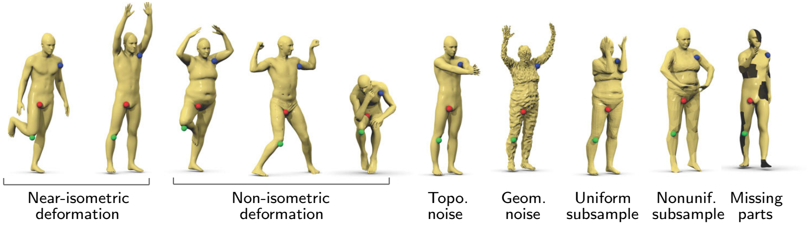
Optimal spectral descriptor



Optimal transfer functions learned from positive and negative examples

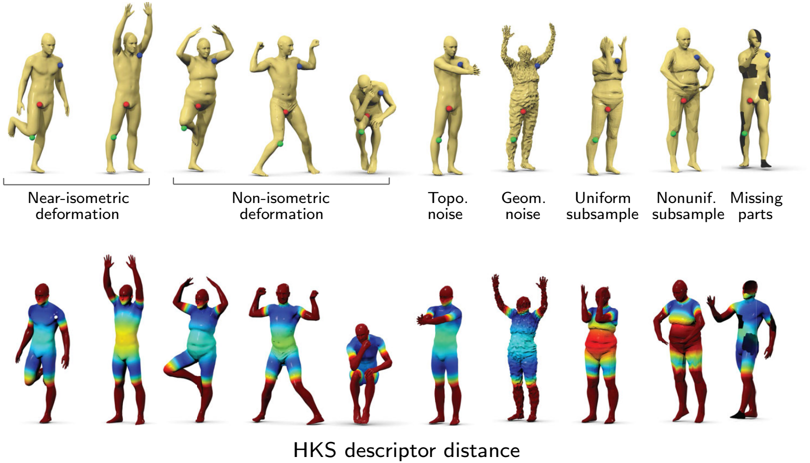
Litman, Bronstein 2014

Descriptor robustness



Descriptors: Sun, Ovsjanikov, Guibas 2009; Aubry, Schlickewei, Cremers 2011; Litman Bronstein 2014; Evaluation: Masci, Boscaini, Bronstein, Vandergheynst 2015; data: Bronstein et al. 2008 (TOSCA); Anguelov et al. 2005 (SCAPE); Bogo et al. 2014 (FAUST)

Descriptor robustness



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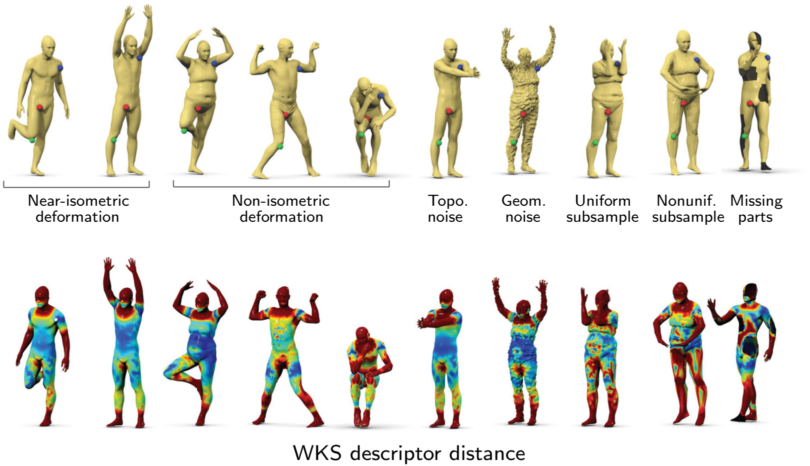

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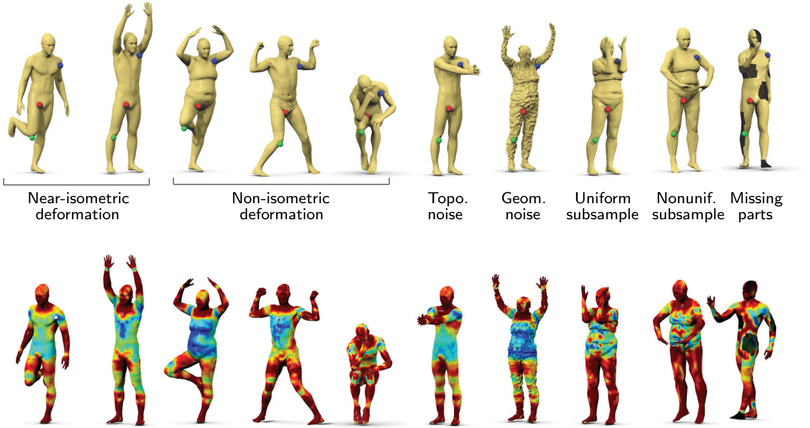
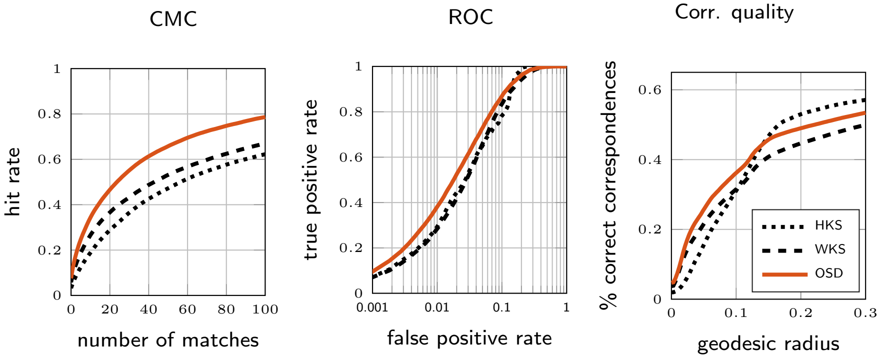


Optimal Spectral descriptor distance

Descriptors: Sun, Ovsjanikov, Guibas 2009; Aubry, Schlickewei, Cremers 2011; Litman Bronstein 2014; Evaluation: Masci, Boscaini, Bronstein, Vandergheynst 2015; data: Bronstein et al. 2008 (TOSCA); Anguelov et al. 2005 (SCAPE); Bogo et al. 2014 (FAUST)

Descriptor performance



Descriptor performance (training and testing: FAUST)

Descriptors: Sun, Ovsjanikov, Guibas 2009; Aubry, Schlickewei, Cremers 2011; Litman Bronstein 2014; Evaluation: Masci, Boscaini, Bronstein, Vandergheynst 2015