Making Gabor Noise Fast and Normalized

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Abstract

Gabor Noise is a powerful procedural texture synthesis technique, but it has two major drawbacks: It is costly due to the high required splat density and not always predictable because properties of instances can differ from those of the process. We bench performance and quality using alternatives for each Gabor Noise ingredient: point distribution, kernel weighting and kernel shape. For this, we introduce 3 objective criteria to measure process convergence, process stationarity, and instance stationarity. We show that minor implementation changes allow for $17-24\times$ *speed-up with same or better quality.*

CCS Concepts

•***Computing methodologies*** → ***Texturing;***

1. Introduction

Procedural noise texturing is an important element of the CG toolbox. Gabor noise [LLDD09] and its numerous follow-ups [BLV*10, LLD11, GLLD12, GSV*14, GLM17] allow to procedurally generate stochastic noise instances with controlled spectral properties. It belongs to *sparse convolution* methods [Lew89, vW91] therefore combining 3 ingredients: one or several splat kernels, a set of random splat positions, and a random weighting of splats. The result is an instance of a *Gaussian texture* (*i.e.* having random uncorrelated Fourier phases), which process reproduces a Power Spectrum Distribution (*PSD*) given by the kernel and is normalized to unitary variance.

This procedural method is particularly powerful: the look can be controlled by interactive [LLDD09] or by example [GLLD12] PSD design; it applies to curved surfaces [LLDD09] as well as to solid texturing [LLD11]; and, as a procedural noise, is able to generate infinite spans of zoomable textures.

However, this approach is too costly for being used in interactive applications as it requires 30 to 100 splats per pixel depending on the target quality. Moreover, resulting texture properties are known and controlled for the *process* itself, but the quality and normalization of the resulting *instances* are not ensured: Indeed, even subtle statistic fluctuations may lead to drastic changes in the final appearance, as noises are usually non-linearly post-treated in real-world shaders.

In this paper, we revisit each ingredient of the Gabor noise method: kernel, point distribution, and weighting. For each, we envision the alternatives and discuss their impact on performance and quality based on objective criteria, measured on the seminal case (single frequency lobe).

2. Gabor noise ingredients

Gabor noise is defined as the convolution of a Poisson point distribution with the real part of a Gabor kernel multiplied by signed uniform random weights. In practice, the seminal Gabor noise [LLDD09] is defined for one base kernel g at location $\boldsymbol{x}$ as:

$$n(\boldsymbol{x}) = \sum_{\{\boldsymbol{x}_i\}} w_i \, g(\boldsymbol{x} - \boldsymbol{x}_i)$$

$$g(\boldsymbol{d}) = K e^{-\frac{\pi}{r^2}\|\boldsymbol{d}\|^2} \cos(2\pi F_0 \boldsymbol{d} \cdot \boldsymbol{\omega}_0),$$

where $\{\boldsymbol{x}_i\}$ are Poisson point process impulses and $\{w_i\}$ are i.i.d. uniform weights in $[-1,1]$. User parameters control the magnitude $K \in \mathbb{R}^+$, the spatial frequency $F_0 \in \mathbb{R}^+$, the main orientation $\boldsymbol{\omega}_0 \in \mathbb{R}^2$, and the kernel radius $r \in \mathbb{R}^+$. In practice, complex textures are obtained by combining several $n_j(\boldsymbol{x})$ functions (with their own set of parameters). For efficiency, implementations rely on a virtual grid of cells of size r. Impulses are generated cell-wise, and kernels are truncated to $\|\boldsymbol{d}\| \leq r$, so that only the 3×3 cells that may contribute to the current pixel are evaluated.

The number of splats per pixel λ is proportional to the number N of impulses per cell: $\lambda = \pi N$. The minimal N to be used for the splat process to converge to a given target quality is let to user trial-and-error, and the alternatives for the different components (weights, kernels, and point distribution) that could decrease the cost for a given target quality have not been studied. We propose a study of alternative Gabor noise ingredients tested with statistical measures on process and instances. Our test set is defined below.

Point distribution: The Gabor noise method is costly firstly because it requires many splats: a pixel must receive enough contributions for expected statistical properties to arise. The problem with a Poisson point distribution is that it tends to produce clusters and

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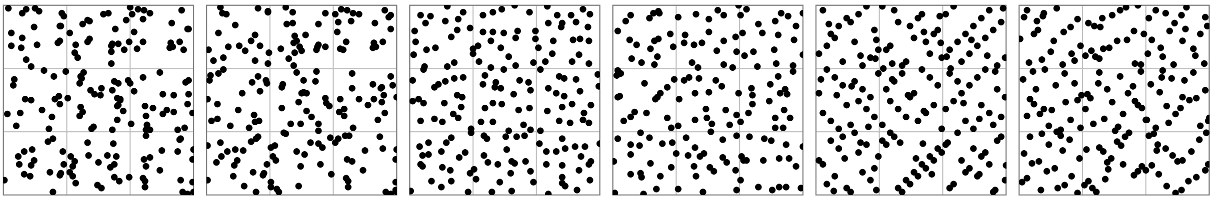


(a) *Poisson* **(b)** *Strat. Poisson* **(c)** *Rect. jittered* **(d)** *Hex. jittered* **(e)** *Sobol* **(f)** *Scramb. Sobol*

Figure 1: *Studied point distributions for $N = 16$. (a) Seminal Poisson point process. (b) Stratified Poisson, i.e. constant number of impulses per cell. (c),(d) Jittered grid distribution from N sub-cells along a square (respectively, hexagonal) regular grid. (e),(f) Sobol and scrambled Sobol sequences generating point coordinates in each cell.*

voids, which wastes samples. However, there is no a priori reason to consider Poisson exclusively. Many alternative point distributions could be used instead, with better coverage uniformity over the texture space. In particular, blue noises and low discrepancy distributions provide more regular coverage. Among all possible point distributions we therefore study and compare the original Poisson process to a stratified Poisson process, a jittered grid process (for both rectangular and hexagonal grids), and point distributions sampled using low-discrepancy Sobol sequences (see fig. 1).

Weights: Splats are modulated by uniform random weights on $[-1, 1]$ inherited from the shot noise model in electrical engineering [Wik18]. But the weighting role, effect and usefulness have not been fully studied in the case of sparse convolution. Splats with a low weight count for little in the sum, which seems like a waste of computation. We therefore compare the original uniform weighting on $[-1, 1]$ to the minimalist Bernoulli distribution on $\{-1, 1\}$.

Kernel: The true Gabor kernel [Gab46] is a Gaussian times $\exp(i\omega x)$. The Gabor noise seminal paper uses the real part of it, *i.e.* a cosine, multiplied by a Gaussian truncated to $\approx 5\%$ value. The choice of a cosine rather than a sine has never been discussed, while the later would enforce a strict zero mean. We thus compare these two kernels for all our test cases.

3. Evaluation

Whatever the Gabor noise algorithm variant, the seminal properties must be respected: producing an instance of a Gaussian texture process (*i.e.* random uncorrelated phases) reproducing the target PSD with a normalized variance. We thus measure and compare the quality of the process for each alternative as a function of the number of impulses per cell N, looking for the fastest convergence. As measuring the randomness of Fourier phases is particularly difficult [Lec15], we evaluate an equivalent property that is the *Gaussianity* of the process (1).

However, the statistics of the process are not enough for a Computer Graphics application where the user wants to produce one image *instance* out of the process while still hoping for "nice" textural properties. First, the average and contrast of a whole texture instance could differ from the one of the process, thus breaking the normalization. Second, not only the overall statistics or PSD are important but also the *spatial stationarity* of textures properties has to be ensured. The latter includes *process* stationarity (*i.e.* no spatial bias) (2) and windowed texture properties in *instances* (3) : a texture is about spatial variations of values, but above a given scale its statistics (*e.g.* average, contrast) should not vary. Windowed statistics allow to measure this. Note that if the large-scale windowed statistics are stationary and equal to the process statistics then it's also the case for the whole image. Therefore there is no need of a specific test for the whole image.

This leads to three statistical tests: (1) the Gaussianity of the process, (2) its spatial stationarity along the texture space, and (3) the large-scale stationarity inside an image instance. We define precisely these tests and apply them to the alternative ingredients in the following sections. More plots and details are given in supplemental material.

3.1. Global process Gaussianity

To choose the number N of splats sufficient for an acceptable "convergence" to a Gaussian texture, previous work only visually estimated the quality of obtained instance images and periodograms. Here, convergence is not in terms of asymptotic values: while adding more splats, an instance will never converge and the pattern will keep evolving (just more and more slowly, as illustrated in supplemental). However, in terms of probability law, Gabor noise converges as a process to a Gaussian distribution as N increases, because it inherits from shot noise [Pap71]. We thus evaluate the Gaussianity by measuring the *Cramer von Mises criterion* (CvM) of the noise process compared to the normal distribution using 7 million samples. This metric tends to zero as the process converges, and we use the seminal Gabor noise process as a reference to calibrate visual noise quality. Empirically we found that the CvM should be less than 0.5 for a reasonable visual quality, coherently corresponding to the estimation of $N = 30$ for the seminal Gabor noise [LLDD09].

Result. The seminal process based on a Poisson point distribution slowly converges to a Gaussian distribution as N increases. As fig. 2 shows, alternative point distributions (stratified Poisson and jittered grids) greatly speed up the process convergence, achieving the same Gaussianity criterion as the seminal high quality Gabor noise with up to 6 times fewer splats and as few as $N = 4$.

In addition, fig. 3 shows that the use of Bernoulli weights (solid curves) results in even faster convergence of the noise process for Poisson and Stratified Poisson. This makes the stratified Poisson point distribution using Bernoulli weighting the best alternative, reaching a constant 15× convergence speed-up compared to the seminal Gabor noise process.

Our experiments also revealed that changing the kernel from cosine to sine does not change the process convergence.

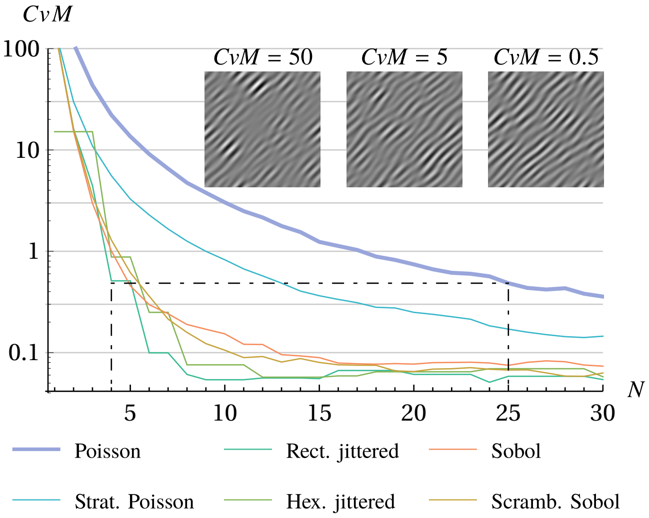


Figure 2: *Compared Gaussianity-wise convergence. Cramer von Mises criterion value for the alternative point distributions for increasing N. The inset images show instances of seminal Gabor noise for CvM values of* 50, 5 *and* 0.5*. Alternative point distributions converge faster: for example, rectangular jittered grid reaches the same CvM value for N = 4 as Poisson for N = 25 (mixed black).*

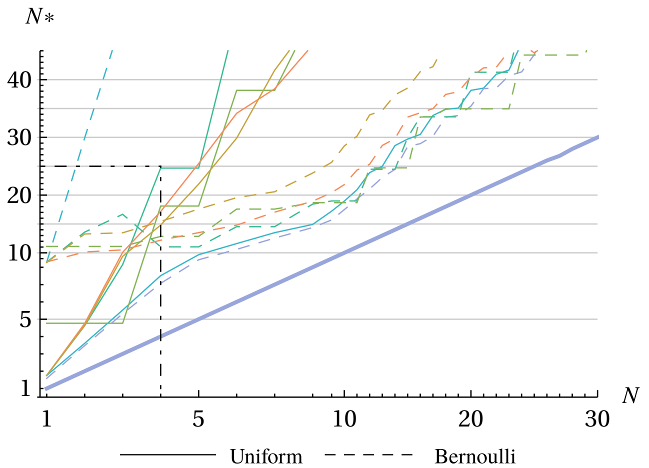


Figure 3: *Convergence gain. Same as fig. 2, but plotting the seminal Gabor impulse count N∗ required to get the same CvM quality as each tested alternative using an impulse count of N. In addition to the uniform weights (solid curves), we also plot the gains using Bernoulli weights (dashed). Coordinates below 10 are zoomed in.*

3.2. Process at a given location

The seminal Gabor noise process is stationary by construction since it draws on Poisson distribution, which is translation-invariant and thus has no location bias, but the point distributions we bench are not (*e.g.* with jittering the statistic differs between cells center vs border regions). Splat convolution smooths differences but we must still ensure that such bias does not show up in the final texture. The global process statistics presented above are blind to these local effects, so we must also study stationarity explicitly.

For this, we study the mean and variance of the process *at a given location* within a cell. In the absence of bias, these statistics should be the same as those of the global process and be spatially constant. By characterizing their spatial fluctuations using median and interdecile statistics, our experiments show that this is the case for the proposed alternatives with a relative interdecile under 5%. This shows that these methods preserve the translation invariance of the original Gabor noise process despite all the benched point processes themselves being more or less spatially biased.

3.3. Instance stationarity and normalization

Satisfactory global statistics can be obtained with well spread local artifacts, while users expect *spatial stationarity* of the local texture properties above some macroscopic "pattern" scale. For instance the Poisson distribution tends to have clustered and void areas despite the process being homogeneous. More generally, unbiased *processes* can still cause macro-fluctuations along the *instances*.

For Gabor noise, the natural texture scale on which considering instance large-scale stationarity is the diameter of a kernel because, by construction, pixels are uncorrelated above this size (*i.e.* they can't be influenced by the same splat). We thus consider instance image statistics (in practice, average and contrast) windowed at this scale, and we study the fluctuations of the spatial statistics of small independent image instances (thanks to ergodicity resulting from uncorrelation). These span onto 3×3 cells: the minimum cell span required to observe one full kernel splatted around an impulse randomly placed in the central cell.

Global and local statistics fluctuations are expected in instances of

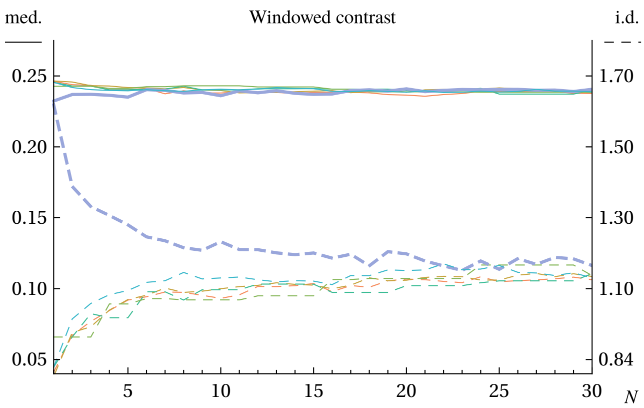


Figure 4: *Measure of image stationarity as fluctuations in windowed contrast. Windowed contrast is the* spatial *variance in a given image window. We measure its stationarity along the image as the median (solid curves, left axis) and the interdecile normalized by the median (dashed curves, right axis) of its fluctuations from window to window. Median and interdecile are more robust than mean and variance since these variance samples are not normally distributed. We compute them for each point distribution and each N and show only the case of Bernoulli weights. The uniform case is quite similar, only with worse interdecile ranges for small N values. Note that the expected median of contrast is 0.25, since we scale and offset the noise so as the [0,1] value range includes 95% of the histogram.*

a stochastic procedural texture, despite the a priori normalization of the process. If undesired, a second pass is required to normalize the instance (which would be costly, complicated or impractical in many real-world use cases). But if we can ensure that local image statistics are stationary and close to process statistics by construction, then an acceptably normalized instance is directly obtained.

Result. Our experiments show (see fig. 4) that for all point distributions the interdecile of windowed contrast fluctuations remains small (*i.e.* the stationarity is good) and the median conforms to the process variance normalized to 0.25 for all N (*i.e.* the instance normalization is not biased). Moreover, the benched alternative point distributions result in significantly lower contrast fluctuations over instances than the seminal Gabor noise, especially when using Bernoulli weighting. It also shows that the contrast relative interdecile range doesn't tend to zero: some contrast fluctuations are unavoidable. It stabilizes around 1.15 pretty early, which means that it's no use increasing N in the hope of getting better stationarity.

We also studied the fluctuations of the mean of instance images (of 3×3 cells). Its fluctuations are low (less than 0.5% of the noise value range around the analytic expected value 0). Still, using sine for the harmonic part of the kernel instead of cosine actually ensures that the image mean is perfectly 0 over all instances.

4. Conclusion and future work

Our study leads to the following recommendation: replace the Poisson point distribution with stratified Poisson (*i.e.* fixed number of splats per cells), and replace uniform weighting with Bernoulli $\{-1, 1\}$. The recommended CvM quality of 0.5 corresponding to $N = 30$ with the seminal method is now obtained with $N = 2$ (corresponding to 6.3 splats per pixel). In addition to the accelerated convergence, implementation performance is also improved by the fact that the Poisson point process requires the costly generation of Poisson random numbers to determine the varying N for each different cell. All this provides a total performance gain of 17× (or up to 24 for higher N, see fig. 5) with minimal modifications to the seminal algorithm. This is achieved without introducing spatial bias in instances, and indeed improves the usability by lowering the variability of instance image statistics. Once applied to a real case combining several kernels (see fig. 6), this gain is confirmed.

Moreover, a sine-based kernel ensures exact zero-mean texture instances. Also, using a C0 or C1 continuous envelope instead of truncated Gaussian would avoid slight artifacts at splat borders for very low N or if the noise is meant to be differentiated (*e.g.* bump).

For future work, we should first study the complex cases combining several kernel types and update our evaluation of optimal N and performance gain factor: close kernels introduce some redundancy between their splats that allows to decrease N. we would like to study removing the weights while using a sine-based kernel, since their main purpose is to ensure the zero-mean which is already guaranteed with this kernel. We could also study replacing point-location randomness by phase-randomness on regular grids (*i.e.* costing a scalar random value instead of a random vector).

Finally, our approach mostly relies on properties of sparse convolution noises, and thus could also apply to other derived methods that work on richer power spectra.

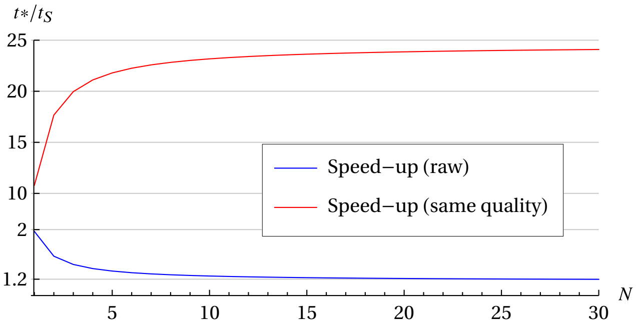
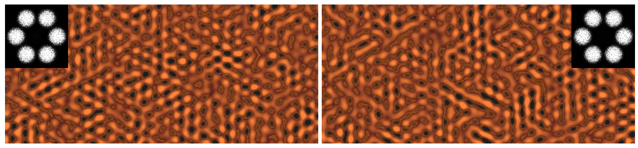


Figure 5: *Red curve: Runtime performance ratio of stratified Poisson with Bernoulli weighting $t_S(N)$ relative to the the same quality seminal Gabor noise t^*. Blue curve: for the same splat count we gain 20% by avoiding the costly random evaluation of N per cell.*



Seminal Gabor, $N = 3\times 30$ *Bernoulli+strat.+sin, $N = 3\times 2$*

Figure 6: *Real case with complex power spectrum (6 lobes obtained using 3 base kernels, cf. inset) and non-linear post-treatment. Our optimized set of ingredients achieves the same visual quality 17 times faster than the seminal method.*

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