

Supplemental Materials for “A Unified Discrete Collision Framework for Triangle Primitives”

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1 Detailed Examination of Cases Where the Projected Triangle Degenerates.

Examples are provided for cases where two triangles intersect perpendicularly and degenerate upon projection, categorized according to their corrected contact states: Point-Triangle, Edge-Triangle, Point-Point, Edge-Edge, and Point-Edge cases.

The figure shows the step-by-step process for each example where two triangles intersect perpendicularly. The normal vector of the red-bordered triangle is $(0,1,0)$, and the normal vector of the blue-bordered triangle is $(0,0,1)$. (1) Initial state. (2) Project the triangles along the normal vector of the second triangle (corresponding to Step 1 in Section 3.3 of the paper). (3) Compute candidate points, calculate signed distances at each point, and determine the penetration depth for the first triangle (corresponding to Steps 2, 3, and 7 in Section 3.3 of the paper). (4)-(5) Perform the same processing for the second triangle (corresponding to Steps 4-6 and 8 in Section 3.3 of the paper). (6) Use Equation (6) from the paper to calculate the penetration depths of both triangles, as obtained in Steps 7 and 8 in Section 3.3 of the paper. Select the candidate point of the second triangle with the smallest penetration depth magnitude. (7) Apply the gradient, which is the normal vector of the first triangle, to resolve the collision (corresponding to Step 9 in Section 3.3 of the paper). (8) Top view of the figure: blue indicates the state before resolution and yellow indicates the state after resolution. In this figure, a simple push-out correction is applied to one of the triangles. The meaning of the step numbers is consistent across all figures.

1.1 Point-Triangle case

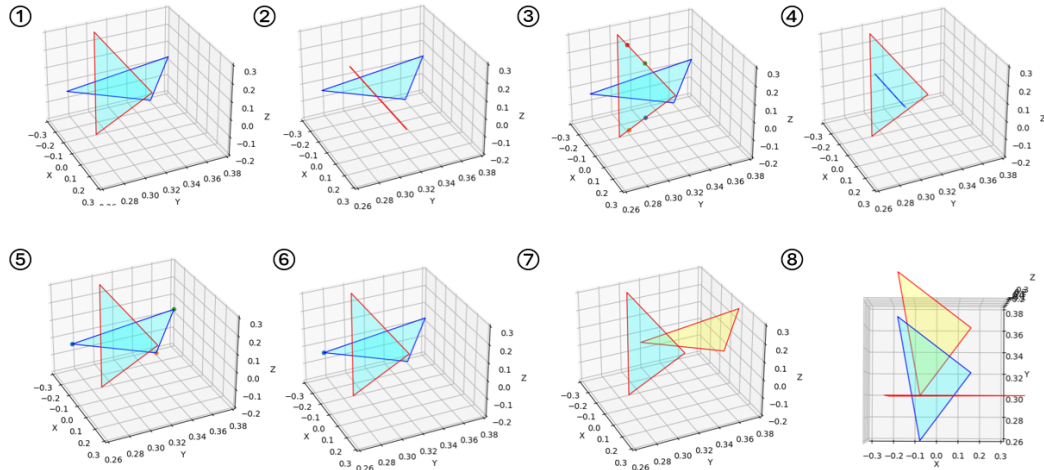


Figure 1: Point-Triangle case when the projected line segment fits inside the other triangle.

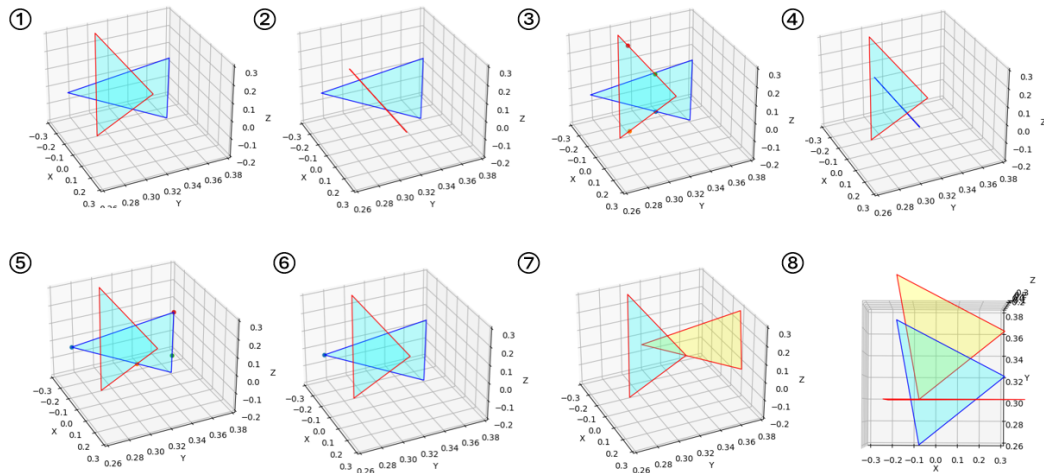


Figure 2: Point-Triangle case when the projected line segment does not fit inside the other triangle.

1.2 Edge-Triangle case

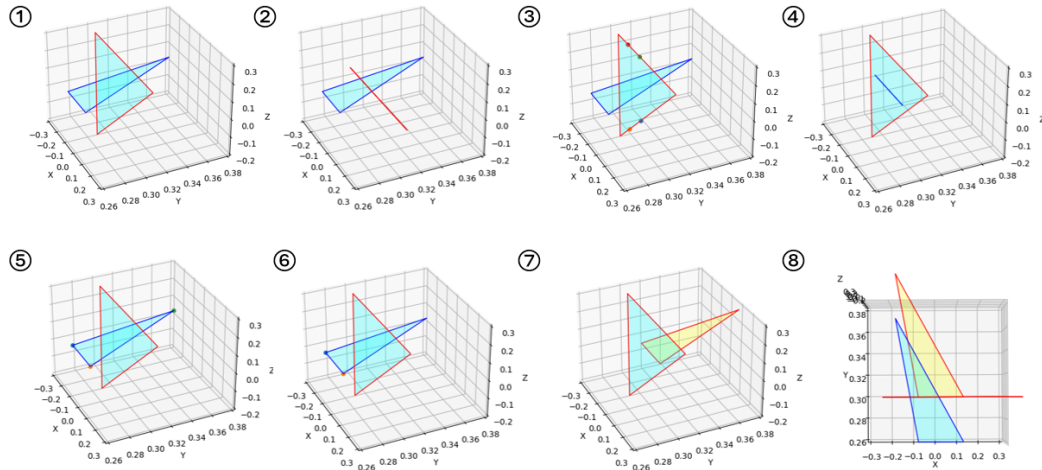


Figure 3: Edge-Triangle case when the projected line segment fits inside the other triangle. (The case is also included in the paper.)

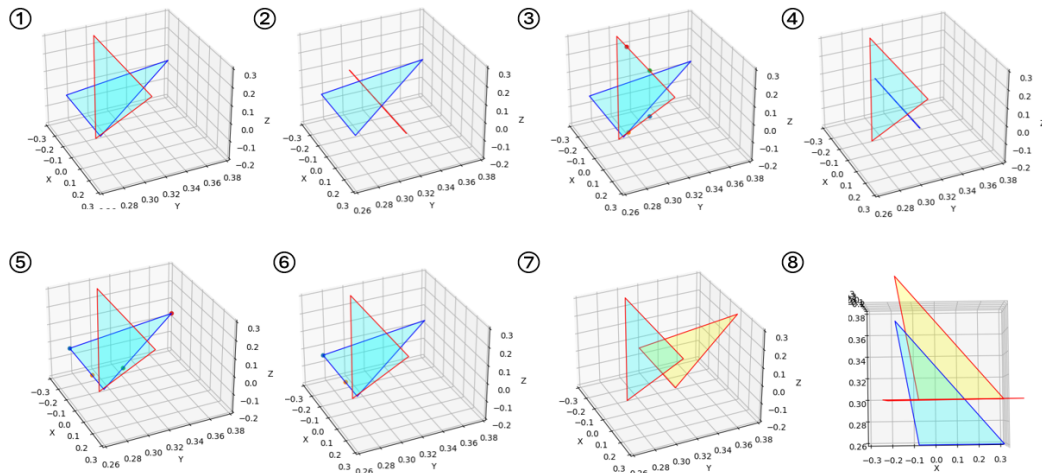


Figure 4: Edge-triangle case when the projected line segment does not fit inside the other triangle.

1.3 Point-Point case

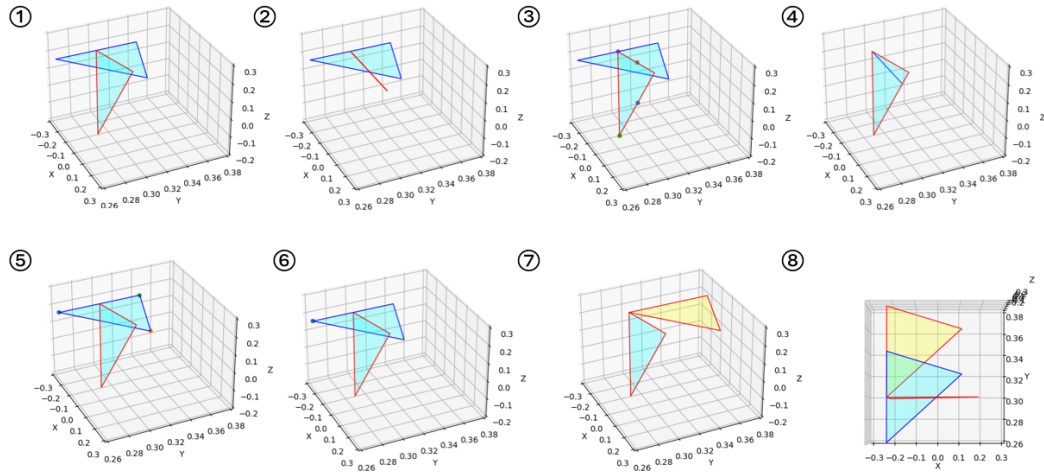


Figure 5: Point-Point case when the projected line segment fits inside the other triangle.

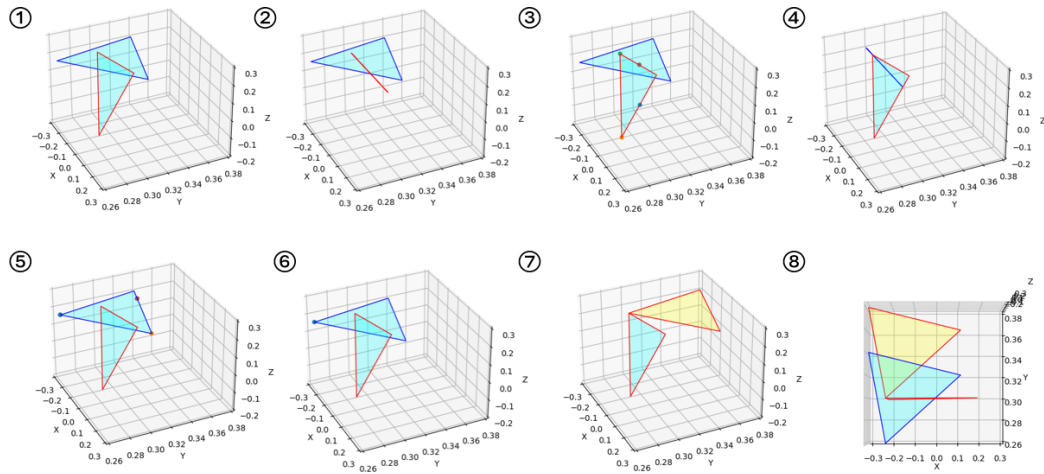


Figure 6: Point-Point case when the projected line segment does not fit inside the other triangle.

1.4 Edge-Edge case

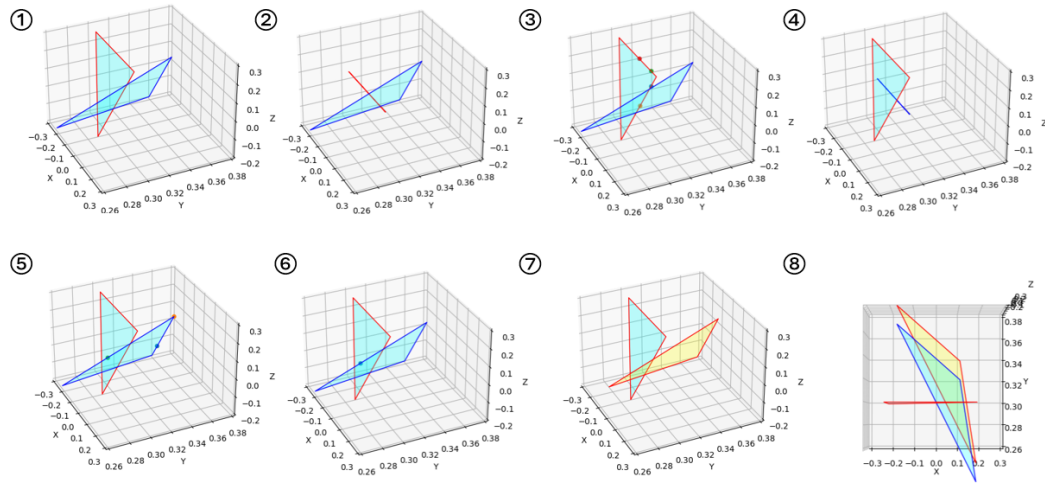


Figure 7: Edge-Edge case

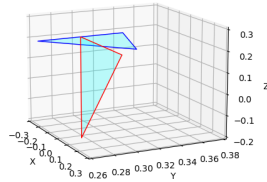


Figure 8: When a projected triangle degenerates onto an edge of the other triangle, it is considered to be in a touching state, and thus it is determined that there is no intersection when intersecting test.

1.5 Point-Edge case

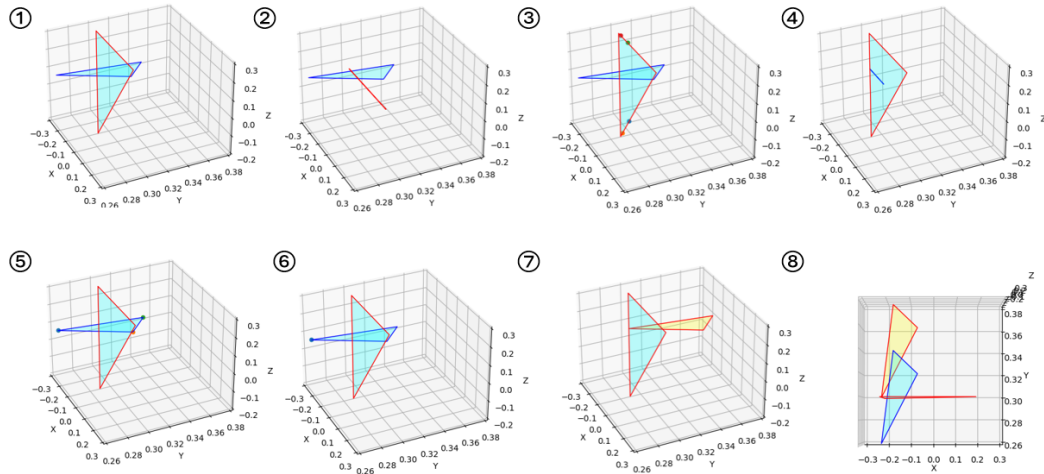


Figure 9: Point-Edge case when the projected line segment fits inside the other triangle.

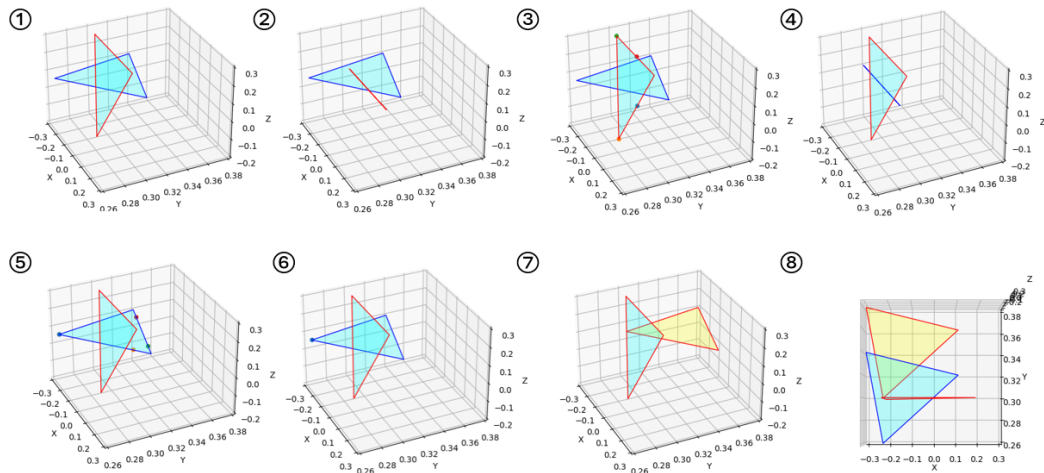


Figure 10: Point-Edge case when the projected line segment does not fit inside the other triangle.

2 Erleben Test

We conducted fundamental collision tests proposed by Erleben. We successfully completed the test without causing breakdowns. There are six cases: Spikes, Spike and wedge, Wedges, Spike in hole, Spike in crack, and Wedge in crack. In some cases, edge collisions occur during the dropping process.

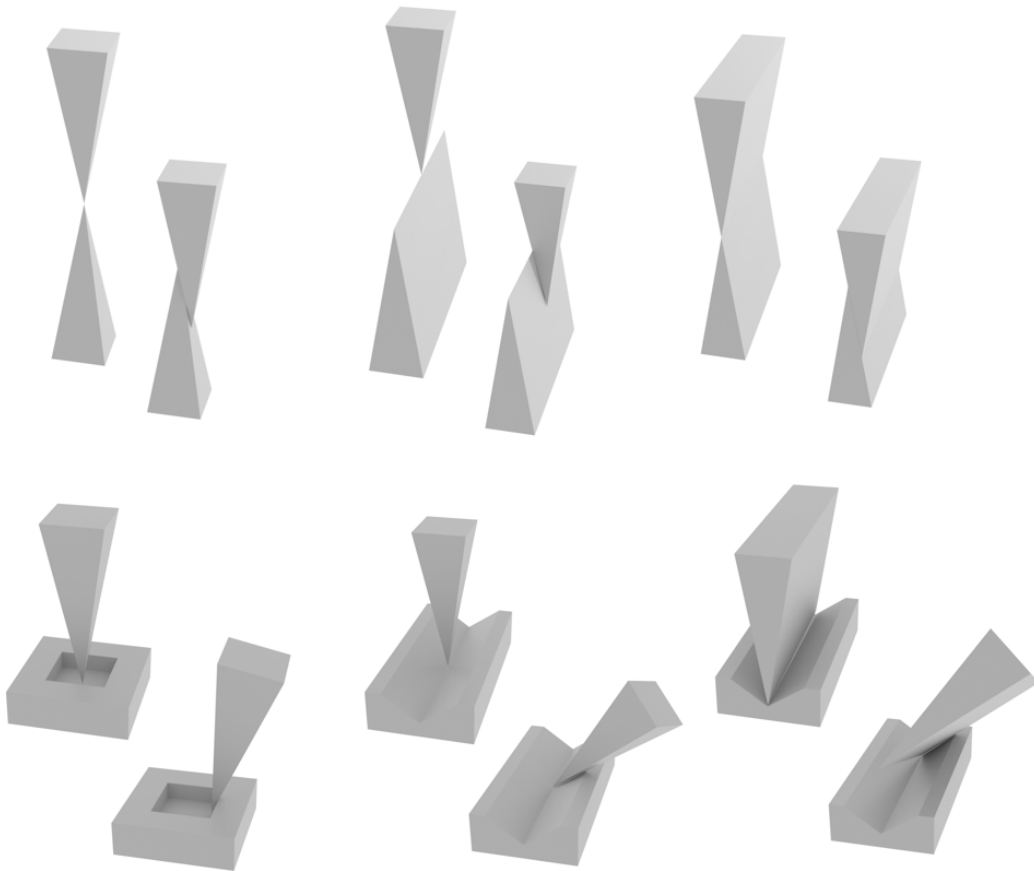


Figure 11: Erleben test results.

3 Derivation of the Gradient $-\nabla f$ from the Function f in Section 3.2 of the paper

In our method, the gradient $-\nabla f$ of the function f corresponds to the normal vector of either one of the triangles. We show the derivation through the following equations.

We begin with Equation 6 of the paper for the function f :

$$\begin{aligned} f &= \min(d_{\text{tri1}}, d_{\text{tri2}}), \\ d_{\text{tri1}} &= \max(0.0, -d_{Q_l^1}), \\ d_{\text{tri2}} &= \max(0.0, -d_{Q_m^2}). \end{aligned}$$

Here, f is determined by the minimum of d_{tri1} and d_{tri2} , which are themselves dependent on $d_{Q_l^1}$ and $d_{Q_m^2}$. These terms, $d_{Q_l^1}$ and $d_{Q_m^2}$, are expressed as:

$$d_{Q_l^1} = \mathbf{N}_2 \cdot \mathbf{Q}_l^1 + d_2,$$

and

$$d_{Q_m^2} = \mathbf{N}_1 \cdot \mathbf{Q}_m^2 + d_1,$$

where \mathbf{N}_1 and \mathbf{N}_2 are the normal vectors corresponding to triangle T_1 and T_2 , respectively. In the case where one of the triangles dominates the minimum, the function f takes the form:

$$f = \max(0.0, -(\mathbf{N} \cdot \mathbf{Q} + d)),$$

where \mathbf{N} is the normal vector of the corresponding triangle, and \mathbf{Q} is the selected point among the candidates.

Finally, the gradient of f with respect to \mathbf{Q} is given by the gradient of the dot product term. Since the gradient of $\mathbf{N} \cdot \mathbf{Q}$ with respect to \mathbf{Q} is \mathbf{N} , we conclude that:

$$-\nabla f = \mathbf{N}.$$