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Modelling and animation of stalactite growth for virtual caves

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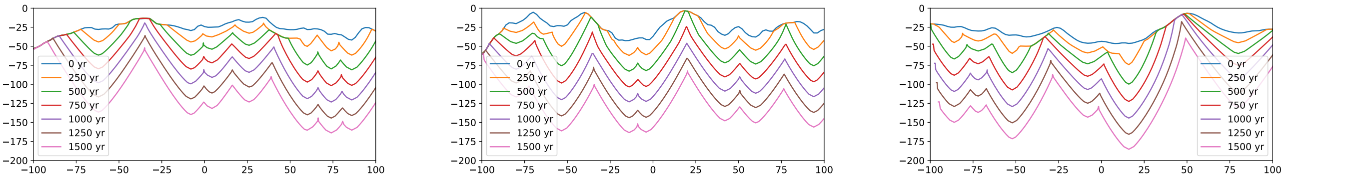

Figure 1: *The evolution stalactites formed from three random ceilings during 1500 years. Stalactites grow from the existing protuberances without user intervention. The proposed model allows the animation of the evolution of stalactite growth. Distances are in cm.*

Abstract

Producing large-scale scenarios in video games, animation, and the VFX industry is inherently complex and time-consuming. As a consequence, procedural modelling of landscapes, either natural or generated by human activity, is a recurring topic in computer graphics. In this paper we propose a model for the growth of stalactites that is based on physical-chemical models. The model, which is presented here for 2D landscapes, is able to generate animations of the evolution of these speleothems along time. The resulting formations are consistent with the theoretical models and with observed speleothems, and provide realistic stalactite appearance in a cave ceiling.

CCS Concepts

• ***Computing methodologies*** → ***Computer graphics;*** *Shape modeling; Mesh models; Mesh geometry models;*

1. Introduction

Procedural modelling of complex scenarios is an active research topic in computer graphics [STBB14]. Algorithmic generation of natural [GGP*19, GCRR20] or urban landscapes [RPPGP17, MMP20] provides faster production times, specially for large scale scenes, and, in many cases, more realistic results that are difficult to achieve through manual modelling. In this work we address the problem of procedural modelling of stalactites in caves, through the simulation of their growth along time (Figure 1). Our contribution is a procedure to simulate the growth of stalactites, that enables the animation of the formation process and that uses physics based criteria to their formation in a particular cave ceiling. This dynamic approach offers advantages over static representations by illustrating the evolving process, enhancing understanding of stalactite formation and cave environments. The model enables spontaneous stalactite growth in an irregular cave ceiling and is able to simulate the evolution of complex scenes without manual intervention.

2. Related Work

The two key processes responsible for the formation of many cave types are the solution of the calcium carbonate present in rocks, usually limestone, that leaves voids, and the deposition of the carbonate on the walls of these cavities, generating complex formations called speleothems, such as stalactites and stalagmites [SH03, MJ10]. Generation of cave scenarios can be done by modelling and reproducing these natural processes. To obtain cavities that resemble real caves, a common strategy is to simulate water flow to create channels across rock substrate. This approach ranges from the use of L-systems [MBMT15] or the computation of paths across an existing cavity [BC09], to more complex and realistic models to generate large cave networks that take into account different rock layers [PM15].

Several authors have also focused on stalactite formation. Stalactite growth is due to carbonate precipitation as water flows on the surface of rock. Short et al. [SBB*05] develop a governing equation for the growth of stalactite surface, and prove that stalactites tend to an asymptotic shape using a polynomial law for the radius

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along the formation. This asymptotic shape has been used to generate stalactites in voxel based procedural caves [CCZ11], and also in caves represented with triangle meshes that describe the wall surface [TW09, FM22]. These works present a good degree of control on the manipulation of stalactite morphology, but have the limitation that the stalactite geometry is generated in its final shape, overlooking the formation process. Moreover, in these models, the location of the stalactites is random or user defined, and does not consider the natural process that decides where a stalactite is formed on the cave ceiling. Our work differs with their approach in that we use Short's growth model to compute time evolution, instead of the resulting asymptotic shape.

3. Stalactite growth model

In this work we present the model for two-dimensional scenes. We will assume that the ceiling of the cave, from which stalactites will grow, is discretised by a series of points and edges that link them. That is, our ceiling is defined as $C = \{\mathbf{p}_i \in \mathbb{R}^2\}_{i=0,\ldots,n}$, with $\mathbf{p}_{i-1}$ connected to $\mathbf{p}_i$, as depicted in Figure 2. Since we aim to build a model that simulates the evolution of the speleothem along time, we need a governing equation for the position of each point, $\mathbf{p}_i$.

According to Short et al. [SBB*05], stalactites grow due to precipitation of carbonate as saturated water flows through the surface of stone. By solving a free surface flow problem, they deduce that the normal velocity of point $\mathbf{p}$ on a stalactite can be obtained as

$$\mathbf{v} = \dot{\mathbf{p}} \cdot \mathbf{n} = \alpha \cdot l_Q \cdot \left(\frac{l_Q}{r \sin\theta} \right)^{1/3}, \tag{1}$$

where r is the radius of the stalactite cross-section at $\mathbf{p}$, θ is the angle between the wall surface and the horizontal, l_Q is a characteristic length and α is a constant that depends on the chemical environment inside the cave. Typical values that lead to stalactite growth are $l_Q \sim 0.1mm$ and $\alpha \sim 1(yr)^{-1}$, resulting in a stalactite length velocity $v_c = \alpha \cdot l_Q \sim 0.1mm/yr$. Our proposal is to use this differential equation to determine the evolution of the stalactite wall points along time. In order to use Eq. (1), we will proceed in several steps. First, for each $\mathbf{p}_i \in C$ we will determine whether it is part of a stalactite or not. Then, we will estimate the radius r_i and the angle θ_i of the stalactite at $\mathbf{p}_i$. Next, we will compute $\mathbf{v}_i$ applying Eq. (1) and integrate the location of $\mathbf{p}_i$. Since the equation is not determined when $\theta = 0$, we will treat separately the evolution of points that are close to the stalactite tip.

3.1. Characterisation of stalactite points

Our first step is to identify which ceiling points can be considered as belonging to a stalactite. These will be the points that will displace using Eq. (1). We will consider as a stalactite any sequence $\mathbf{p}_k, \mathbf{p}_{k+1}, \ldots, \mathbf{p}_{k+n}$ of ceiling points for which the curvature of ceiling height is $\mathbf{p}_j'' > \varepsilon_s$, with $\varepsilon_s \geq 0$ a small threshold (in Figure 2 a sequence of points that form a stalactite have been highlighted). To prevent small ceiling oscillations to be considered as stalactites, the length of the sequence n can be forced to have a minimum value. The curvature is computed assuming that our ceiling representation is a discretisation of a differentiable ceiling profile.

3.2. Estimation of local stalactite growth

Once we have identified the regions that correspond to stalactites, we need to compute r and θ for each point. Given a point $\mathbf{p}_i$, the angle θ_i and the normal direction $\mathbf{n}_i$ are computed using central differences, as shown in Figure 2, inset. Then, for each sequence of points that represent a stalactite, we take the point with minimum height $\mathbf{p}_t = (x_t, h_t)$ which, by construction, will be unique. We consider the radius of the stalactite at $\mathbf{p}_i$, r, as the horizontal distance from point $\mathbf{p}_i$ to $\mathbf{p}_t$. After all these values have been computed, Eq. (1) can be computed for every point where $\theta \neq 0$, and its position can be integrated using an explicit Euler scheme.

3.3. Stalactite tip growth

At points close to the stalactite tip, both the angle θ and r will nearly vanish and, thus, the growth velocity defined by Eq. (1) has a singularity. For this reason, we will compute the tip shape by interpolating the ceiling function at these points. Let's consider points $\mathbf{p}_k, \mathbf{p}_{k+1}, \ldots, \mathbf{p}_{k+n}$ that form a stalactite, with coordinates $\mathbf{p}_i = (x_i, h_i)$. Let's take $T = \{\mathbf{p}_t, \mathbf{p}_{t+1}, \ldots, \mathbf{p}_{t+m}\}$ as the points that have a value of $r \cdot \sin\theta \leq \varepsilon_t$, being $\varepsilon_t > 0$ a threshold to identify tip points, with $t > k$ and $t + m < k + n$. We will proceed as follows. In the first place, we compute the evolution of the points that do not belong to T using the differential equation. Then we interpolate the value of h for the points in T, using Hermite interpolation on the height and derivative of the two immediate neighbours of the tip, $\mathbf{p}_{t-1}$ and $\mathbf{p}_{t+m+1}$. Figure 3 shows an scheme of this process.

3.4. Ceiling remeshing

As stalactite grows, the set of points that represent its surface tend to separate from each other, specially close to the tip. On the contrary, the points where the stalactite meets the ceiling will displace almost horizontally, moving towards the next ceiling point. In order to maintain a proper resolution and a well defined ceiling function, we propose adding and removing points in these two situations.

For points moving apart, we define a threshold $\varepsilon_d > 0$. Whenever two adjacent points of a stalactite separate a distance larger than this value, a point is added between them. To preserve convexity, Hermite interpolation is used as in the case of the stalactite tip. For points next to the ceiling, stalactite growth will push them towards neighbour ceiling points. In Figure 4, if point $\mathbf{p}_{i+1}$ is added to the stalactite, it will eventually collapse against the next ceiling point, $\mathbf{p}_{i+2}$. To prevent this, if the horizontal distance between points $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$ falls below a threshold, point $\mathbf{p}_{i+1}$ is removed.

4. Results and discussion

We have tested our model in two scenarios: a straight horizontal ceiling with a protuberance, and a ceiling with a random profile. The aim of the first test is to determine how a single stalactite evolves. We have set four different initial conditions, changing the shape (conic or parabolic) and the width of the starting protuberance. The results are shown in Figure 5. It is very noticeable that stalactites with a narrower shape grow faster than wider stalactites. We have observed small velocity increases when the tip is remeshed in narrower shapes, which could explain this behaviour. All the

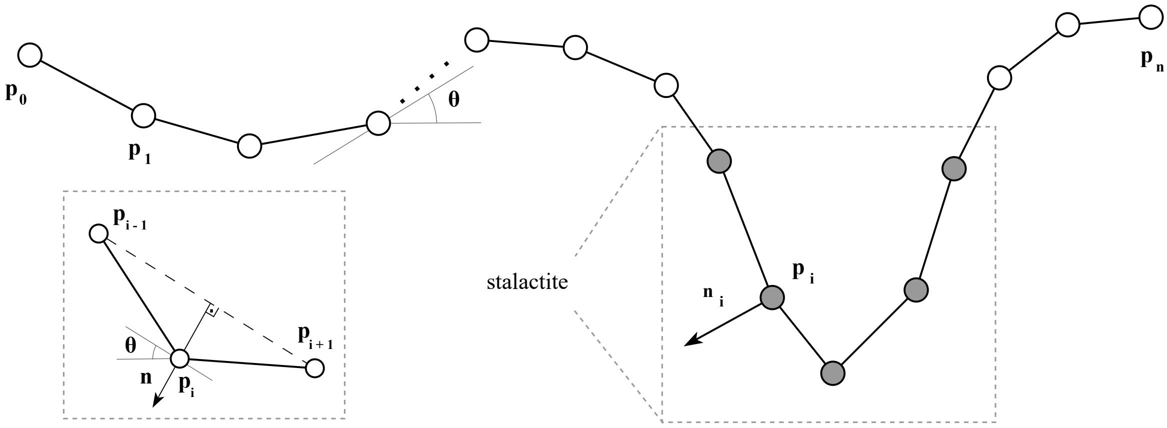

Figure 2: *We consider a ceiling represented by a series of points $\mathbf{p}_i$. We characterise a stalactite as a connected sequence of convex points (grey dots). The angle of the ceiling surface, and the normal direction, are computed from this discretisation to approximate a growth model. Inset figure: the normal vector $\mathbf{n}_i$ and the angle to the horizontal, θ_i at each point $\mathbf{p}_i$ are obtained by subtracting the two adjacent points, $\mathbf{p}_{i-1}$ and $\mathbf{p}_{i+1}$, which also give a direction orthogonal to the ceiling surface at p_i.*

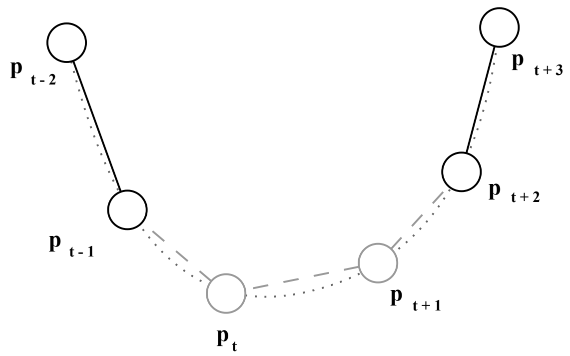

Figure 3: *The stalactite growth described by Equation 1 is undefined at the stalactite tip. The tip is interpolated using a cubic Hermite spline to preserve stalactite continuity. In the figure, height at $\mathbf{p}_t$ and $\mathbf{p}_{t+1}$ is interpolated from the location of points $\mathbf{p}_{t-1}$ and $\mathbf{p}_{t+2}$ and the tangent at these two points defined by $\mathbf{p}_{t-2}$ and $\mathbf{p}_{t+3}$ respectively.*

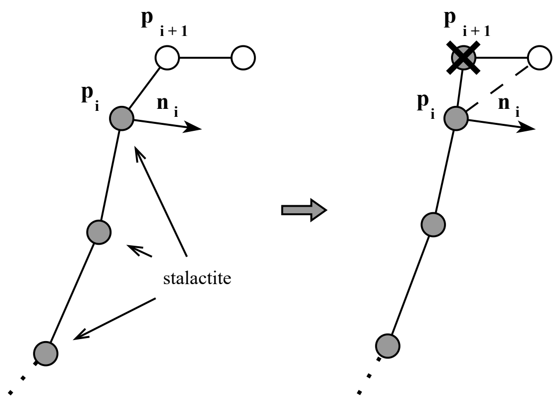

Figure 4: *As stalactite grows, point $\mathbf{p}_{i+1}$ will eventually run through its neighbour $\mathbf{p}_{i+1}$. To prevent instabilities, when $\mathbf{p}_i$ reaches horizontally the location of $\mathbf{p}_{i+1}$, this vertex is removed.*

growth rates are, however, within the velocities observed in natural caves. In the figure, the final shape of all four stalactites after 1000 years is also compared. We observe that, regardless the initial configuration, the stalactites tend to adopt a similar asymptotic shape, as expected according to the theoretical model [SBB*05]. Compared to previous computer graphics research [TW09, CCZ11], this means that our model is able to achieve equivalent results, with the advantage that we are able to provide a physics-based animation of the stalactite growth, although further comparison is still to be done. The second scenario is run upon a random initial ceiling, in order to evaluate our model in a more general situation, similar to its potential usage in a production environment. The goal is to test whether stalactites grow naturally from the existing local minima and how different stalactites interact during their growth. Figure 1 presents the results for three simulations of 1500 years long. In these simulations we see how the proposed model is able to generate stalactites at the locations where they would naturally appear, and how several stalactites can grow and evolve together. It is noteworthy how two stalactites can merge or split during their evolution, as it happens in the second and third scenarios. In this sense, our model is more general than previous proposals [TW09, CCZ11], in which predefined stalactite shapes are just placed at prescribed locations. The model, however, has several limitations. The most obvious is that it currently applies only to 2D scenarios. Also, tip remeshing causes velocity changes as

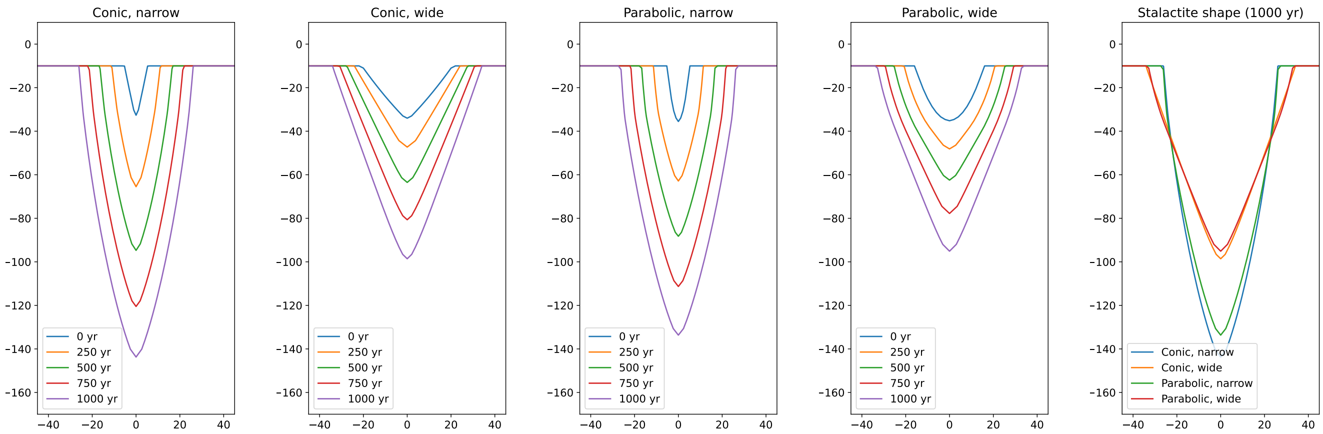

Figure 5: *The evolution of four stalactites with different initial shape (all distances in cm). From left to right, a narrow, conic stalactite; a wide, conic, stalactite; a narrow, parabolic stalactite; and a wide, parabolic stalactite. Growth rates are similar to those of real speleothems, although dependence on curvature is observed in our model.*

the speleothem grows, which could explain velocity dependence on shape. Moreover, we have observed that it is not straightforward to control the width to height ratio of stalactites, as it is in previous work [TW09, CCZ11]. This feature would be desirable for the development of authoring tools. The model is limited to the generation of stalactites, but it can be easily extended to stalagmite growth using equivalent governing equations [Kau03].

5. Conclusion

We have presented a physics-based model of stalactite growth that is consistent with theoretic stalactite models and with previous computer graphics literature. Our model enables the automatic generation of these sepeleothems on a predefined cave scenario and the animation of their evolution. Despite its current limitations, our proposal is a very promising base for the future development of a method to generate 3D stalactites and other speleothems.

References

[BC09] Boggus M., Crawfis R.: Procedural creation of 3d solution cave models. In *Proceedings of the 20th IASTED International Conference on Modelling and Simulation* (2009), pp. 180–186. 1

[CCZ11] Cui J., Chow Y.-W., Zhang M.: Procedural generation of 3d cave models with stalactites and stalagmites. *International Journal of Computer Science and Network Security 11*, 8 (2011), 94. 2, 3, 4

[FM22] Franke K., Müller H.: Procedural generation of 3d karst caves with speleothems. *Computers & Graphics 102* (2022), 533–545. URL: https://www.sciencedirect.com/science/article/pii/S0097849321002132, doi:10.1016/j.cag.2021.10.002. 2

[GCRR20] Gasch C., Chover M., Remolar I., Rebollo C.: Procedural modelling of terrains with constraints. *Multimedia Tools and Applications 79* (2020), 31125–31146. doi:s11042-020-09476-3. 1

[GGP*19] Galin E., Guérin E., Peytavie A., Cordonnier G., Cani M.-P., Benes B., Gain J.: A review of digital terrain modeling. *Computer Graphics Forum 38*, 2 (2019), 553–577. doi:10.1111/cgf.13657. 1

[Kau03] Kaufmann G.: Stalagmite growth and palaeo-climate: the numerical perspective. *Earth and Planetary Science Letters 214* (9 2003), 251–266. doi:10.1016/S0012-821X(03)00369-8. 4

[MBMT15] Mark B., Berechet T., Mahlmann T., Togelius J.: Procedural generation of 3d caves for games on the gpu. In *Foundations of Digital Games* (2015). URL: http://www.fdg2015.org/papers/fdg2015_paper_80.pdf. 1

[MJ10] Meakin P., Jamtveit B.: Geological pattern formation by growth and dissolution in aqueous systems. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 466* (3 2010), 659–694. doi:10.1098/RSPA.2009.0189. 1

[MMP20] Mas A., Martin I., Patow G.: Simulating the evolution of ancient fortified cities. *Computer Graphics Forum 39*, 1 (2020), 650–671. doi:10.1111/cgf.13897. 1

[PM15] Pytel A., Mann S.: Procedural modeling of cave-like channels. *Journal of Computer Graphics Techniques Vol 4*, 2 (2015). 1

[RPPGP17] Roglà Pujalt O., Pelechano Gómez N., Patow G. A.: Procedural semantic cities. In *- 2017: XXVII Spanish Computer Graphics Conference* (2017), European Association for Computer Graphics (Eurographics), pp. 113–120. 1

[SBB*05] Short M. B., Baygents J. C., Beck J. W., Stone D. A., Toomey R. S., Goldstein R. E.: Stalactite growth as a free-boundary problem: A geometric law and its platonic ideal. *Physical Review Letters 94* (1 2005), 018501. doi:10.1103/PHYSREVLETT.94.018501. 1, 2, 3

[SH03] Self C., Hill C.: *HOW SPELEOTHEMS GROW: AN INTRODUCTION TO THE ONTOGENY OF CAVE MINERALS*. Tech. rep., 2003. URL: https://api.semanticscholar.org/CorpusID:56119536. 1

[STBB14] Smelik R. M., Tutenel T., Bidarra R., Benes B.: A survey on procedural modelling for virtual worlds. *Computer Graphics Forum 33*, 6 (2014), 31–50. doi:10.1111/cgf.12276. 1

[TW09] Tortelli D. M., Walter M.: Modeling and rendering the growth of speleothems in real-time. In *GRAPP 2009 - Proceedings of the 4th International Conference on Computer Graphics Theory and Applications* (2009), pp. 27–35. 2, 3, 4