

A Simple Validity Condition for B-Spline Hyperpatches

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Abstract

The use of hyperpatches as a method for solid modelling has a problem: the validity of the model is not guaranteed. The problem of ensuring the validity of hyperpatch representations of solids is discussed in this work, and a validity condition for cubic uniform b-spline hyperpatches is presented. Our validity condition is based on comparisons among points, and it is robust and easy to implement.

1. Introduction

A volumetric dataset is a special kind of non-homogeneous solid in which each point of its interior has a different property value varying continuously. Volumetric datasets can be used to model volumetric solids with smooth shapes.

Hyperpatches can be used as a modelling tool with the advantage of being able to model the interior as well as the boundary surfaces. Also, hyperpatches can be edited easily by moving control points that make them suitable for interactive design.

In any representation scheme, validity is a very important property^{1, 2, 3}, as it ensures that we can not obtain representations of meaningless objects. To apply algorithms to an invalid representation of a solid may produce bad results. It is not a good idea to rely on human operators to ensure the validity of the model, because the complexity of the model can make it impossible to check its validity without the help of a computer. On the other hand, models are not always created by humans and, in this case, visual checking of validity is impossible.

As validity is a very important property of a representation scheme, it is essential to develop an automatic condition to ensure the validity of our hyperpatches. This paper explains a simple condition of validity for uniform b-spline tri-cubic hyperpatches.

2. Previous work

There are several references dealing with the problem of checking if a hyperpatch is invalid. In this section we outline them.

The early work of Sederberg and Parry⁴ focused on free-form deformation states that the jacobian is a very interesting tool to determine the inner distribution of points in the interior of a hyperpatch. They don't use the jacobian for checking validity, but to ensure that a deformation is volume preserving.

Joy and Duchaineau⁵, in the context of finding the surfaces that represent the boundary of a trivariate tensor-product b-spline solid, propose a method based on the Jacobian that can be seen as a validity test.

They are not interested in checking if a solid is or is not valid, but in finding the correct boundary surfaces of that solid. Using the implicit function theorem, they state that the real boundary of the solid is a subset of the union of a set of parametric b-spline patches and the isosurface where the determinant of the jacobian is zero. They approximate the jacobian by using interval techniques and adaptive subdivision of the domain space.

Gain⁶, in the context of free-form deformation derives a validity test for hyperpatches that is very similar to the work of Joy and Duchaineau. Gain, like Joy, realizes that the jacobian is zero if and only if the three partial derivatives of the solid are linearly dependent, and they use a conic-hull hodograph to bound these derivatives.

The proposed method works by comparing relative positions of the control points of the hyperpatch. His method is a sufficient condition and so, it restricts the range of allowable free-form deformations, but it can be used repeatedly in short steps to achieve the same result without obtaining an invalid deformation.

Another slightly different approach is provided by Choi

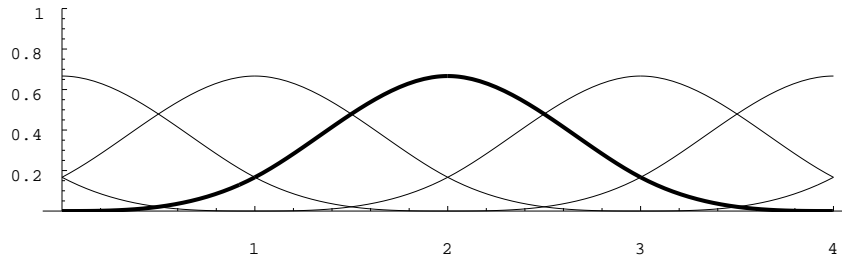


Figure 1: Uniform cubic b-spline blending functions. Each function is nonzero only in four intervals of length 1. Moreover, each function is a shifted copy of one another.

and Lee⁷. They are interested in developing an injectivity (validity) condition to be used in the context of 3D morphing. And so, their condition is based on control points displacement rather than in positions of that control points.

Choi and Lee do not use a conic-hull hodograph to bound the columns of the jacobian determinant, instead they use 3D regions to bound its rows. That allows them to derivate a sufficient condition to determine if the hyperpatch is invalid in terms of points displacement.

Another interesting work dealing with the validity of swept geometric entities based of the jacobian is⁸.

3. Mathematical preliminaries

Hyperpatches are well known mathematical tools, and they have been successfully applied to solid modelling⁹. Prior to exposing our work, let us explore some known concepts related to hyperpatches (a more detailed explanation can be found in ¹⁰), and define some useful terminology that we employ later in this paper.

A hyperpatch (also called parametric solid) is a set of points limited by patches. The coordinates of these points are given by three continuous, single-valued functions of the form:

$$\mathbf{x} = x(u, v, w) \quad \mathbf{y} = y(u, v, w) \quad \mathbf{z} = z(u, v, w)$$

where the parametric variables u , v and w are constrained to the interval $[0, 1]$, therefore, a hyperpatch is a parametric mapping of a solid domain (a unit cube) into three-dimensional space. The domain of the mapping is called parametric space \mathbf{U} and has axes U , V and W , and its range is called modelling space \mathbf{R}^3 and has axes X , Y and Z .

The shape of the hyperpatch is given by a three-dimensional grid Γ of arranged three-dimensional points $\mathbf{g}_{i,j,k} \in \mathbf{R}^3$ called *geometric coefficients* or *control points*, and a set of *blending functions* that applies over the control points.

The coefficients in Γ are ordered according to its indices,

thus, we can establish a coordinate system (called *index space*) \mathbf{I} with axes I , J and K in which the coefficients are ordered. Let $\mathbf{g}_{i_1,j,k}$ and $\mathbf{g}_{i_2,j,k}$ be two points in Γ , we say that $\mathbf{g}_{i_1,j,k}$ is less than $\mathbf{g}_{i_2,j,k}$ with respect to the axis I , and note $\mathbf{g}_{i_1,j,k} <_I \mathbf{g}_{i_2,j,k}$, if $i_1 < i_2$. Similarly we can establish an order in the other two axis J and K .

Fixing the value of one of the parametric variables results in a surface on the hyperpatch in terms of the other two variables which remain free. These surfaces are called *iso-surfaces*. There are three families of iso-surfaces which are obtained by fixing each one of the three parametric variables. If the hyperpatch $\mathbf{S}(u, v, w)$ is valid, only one surface of each family passes through each point $\mathbf{p} = \mathbf{S}(u, v, w)$. Two iso-surfaces that have fixed the same coordinate are called *parallel*, and they are called *perpendicular* if the fixed coordinate is different in each one.

The *iso-curves* are functions of one of the three parametric variables. We obtain them by fixing two of the variables while the third one remains free. Again, there are three families of iso-curves and, if the hyperpatch $\mathbf{S}(u, v, w)$ is valid, only one iso-curve of each family passes through each point in the solid $\mathbf{p} = \mathbf{S}(u, v, w)$. We call $\mathbf{C}_D(d)$ to an iso-curve which has the parametric direction $D \in \mathbf{U} = \{U, V, W\}$ free, $d \in [0, 1]$, $d \in \{u, v, w\}$.

The tangent vector of a hyperpatch $\mathbf{S}(u, v, w)$ in one parametric direction $D \in \mathbf{U} = \{U, V, W\}$ evaluated in a point $\mathbf{u} = (u, v, w)$, $\mathbf{S}^D(u, v, w)$, is the vector that we obtain by evaluating the parametric derivative:

$$\mathbf{S}^D(u, v, w) = \frac{\partial \mathbf{S}(u, v, w)}{\partial d}$$

where $d \in \{u, v, w\}$. There are three families of tangent vectors which are obtained by derivating with respect to each one of the three parametric coordinates.

We can define different blending functions to obtain a hyperpatch with different features. We are interested in hexagonal uniform tricubic b-spline functions because they have the following features:

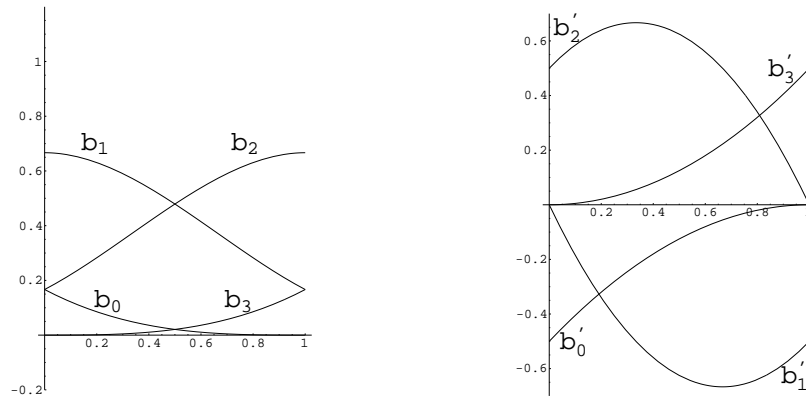


Figure 2: In each unit interval only four uniform cubic b-spline blending functions are nonzero. This figure shows the portion of each blending function that applies in a given interval (left), and the derivatives of these functions (right).

- The degree of the mapping is cubic in each parametric variable, so the hyperpatch equation is a tricubic function. A set of cubic blending functions combines the effects of $l \times m \times n$ geometric coefficients \mathbf{g}_{ijk} , with $l, m, n \geq 4$ to obtain the shape of the hyperpatch. We have chosen cubic blending functions because they have enough flexibility to model complex shapes at the same time as they maintain numeric stability.
- The b-spline blending functions are piecewise polynomial, this means that we have local control over the shape of the hyperpatch. Let $\mathbf{S}(u, v, w)$ be a uniform tricubic hyperpatch defined by $l \times m \times n$ geometric coefficients, it is formed by $(l-3) \times (m-3) \times (n-3)$ polynomial segments $\mathbf{S}_{ijk}(u, v, w)$, $i = 0, \dots, l-3$, $j = 0, \dots, m-3$, $k = 0, \dots, n-3$, called *sub-hyperpatches*. Each blending function is nonzero only over four successive segments, therefore the effects of each coefficient is limited to only four segments, (see figure 1). The shape of each sub-hyperpatch is given by only 64 ($4 \times 4 \times 4$) of the coefficients in Γ . We call Γ_{ijk} to the subgrid of coefficients that define the shape of the sub-hyperpatch $\mathbf{S}_{ijk}(u, v, w)$.
- Uniform blending functions imply that the functions are all translates of one another (see figure 1), this convention allows us to use the same formulas in computing each hyperpatch segment (sub-hyperpatch). In each segment four shifted blending functions are applied for each parametric variable. The equations of the blending functions which are applied in a given segment are:

$$\begin{aligned} b_{0,3}(u) &= \frac{1}{6}(1-u)^3 \\ b_{1,3}(u) &= \frac{1}{6}(4-6u^2+3u^3) \\ b_{2,3}(u) &= \frac{1}{6}(-3u^3+3u^2+3u+1) \\ b_{3,3}(u) &= \frac{1}{6}u^3 \end{aligned}$$

and their derivatives are:

$$\begin{aligned} b'_{0,3}(u) &= -\frac{1}{2}(1-u)^2 \\ b'_{1,3}(u) &= \frac{1}{2}(-4u+3u^2) \\ b'_{2,3}(u) &= \frac{1}{2}(1+2u-3u^2) \\ b'_{3,3}(u) &= \frac{1}{2}u^2 \end{aligned}$$

- The hexagonal uniform tricubic b-spline functions provide a wide set of solids for design purposes. One can easily define many polygonal solids and close approximations to spheres⁵. This is sufficient for the kind of solids we are interesting on. see figure 2.

For convenience we suppose that all are constrained to the interval $[0, 1]$, thus, the equation of a sub-hyperpatch $\mathbf{S}_{ijk}(u, v, w)$ is:

$$\mathbf{S}_{ijk}(u, v, w) = \sum_{r=0}^3 \sum_{s=0}^3 \sum_{t=0}^3 b_{r,3}(u)b_{s,3}(v)b_{t,3}(w)\mathbf{g}_{i+r,j+s,k+t}$$

where $\mathbf{G}_{ijk} = \{\mathbf{g}_{i+r,j+s,k+t} / r, s, t = 0, \dots, 3\}$.

Definition 1 Span of coefficients

Let $\mathbf{S}_{ijk}(u, v, w)$ be a sub-hyperpatch defined by the 64 coefficients in Γ_{ijk} . We call *span of coefficients* σ to each subset of four coefficients that we obtain by fixing the value of two of the indices in \mathbf{I} while the third index remains free, (see figure 3). For example: σ_{r-t}^{ijk} is the span of coefficients of Γ_{ijk} formed by the points:

$$\sigma_{r-t}^{ijk} = \{\mathbf{g}_{i+r,j+s,k+t}, s = 0, \dots, 3\}$$

the minus sign (-) indicates the index that remains free.

We consider that a span of coefficients σ is *locally ordered* if the coordinates in the modelling space \mathbf{R}^3 of the coefficients in σ are ordered in the same way than the coordinates in the

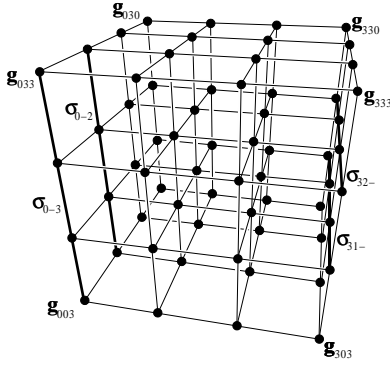


Figure 3: This picture shows the grid Γ of geometric coefficients of a hyperpatch and some of the spans defined in it (thick lines).

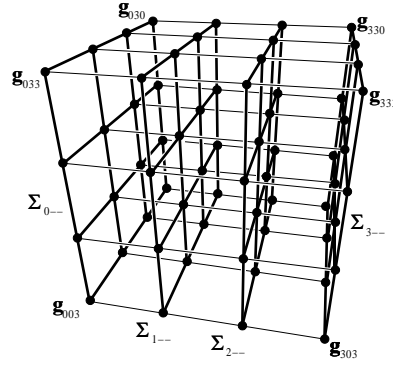


Figure 4: This picture shows the grid Γ of geometric coefficients of an hyperpatch. The four slices with the index I fix are marked with thick lines.

index space \mathbf{I} . For simplicity we call \mathbf{g}_q to the four coefficients of the span σ , thus we say that σ is locally ordered if:

$$\forall q \in \{0, 1, 2\} \begin{aligned} &(x_q \leq x_{q+1} \vee x_q \geq x_{q+1}) \wedge \\ &(y_q \leq y_{q+1} \vee y_q \geq y_{q+1}) \wedge \\ &(z_q \leq z_{q+1} \vee z_q \geq z_{q+1}) \end{aligned}$$

Definition 2 Slice of coefficients

Let $\mathbf{S}_{ijk}(u, v, w)$ be a sub-hyperpatch defined by the 64 coefficients in Γ_{ijk} . We call *slice of coefficients* Σ to each subset of sixteen coefficients that we obtain by fixing the value of one of the indices in \mathbf{I} while the other two indices remain free, (see figure 4). For example: $\Sigma_{-s_1}^{ijk}$ is the slice of coefficients of Γ_{ijk} formed by the points:

$$\Sigma_{-s_1}^{ijk} = \{\mathbf{g}_{i+r, j+s, k+t} \mid r, s = 0, \dots, 3\}$$

the minus sign (-) indicates the indices that remain free. Using the order defined in the index space \mathbf{I} , we can order the slices of coefficients. Thus we say that:

$$\Sigma_{-s_1}^{ijk} <_I \Sigma_{-s_2}^{ijk}$$

if $s_1 < s_2$. And if the distance $|s_1 - s_2| = 1$ we say that slices $\Sigma_{-s_1}^{ijk}$ and $\Sigma_{-s_2}^{ijk}$ are *contiguous*.

Two slices that have fixed the same index are called *parallel*, and two slices that have different fixed indices are called *perpendicular*.

Each slice defines a uniform bicubic b-spline surface \mathbf{S} . The geometric coefficients of \mathbf{S} are the sixteen points that form the slice Σ .

If we take the four surfaces defined by the four slices that have fixed the same index, and we evaluate those surfaces by using the same values for its two parametric coordinates, we obtain four points that are the geometric coefficients of an iso-curve \mathbf{C} . This iso-curve is perpendicular to the four surfaces, that is, the free coordinates in \mathbf{C} correspond with

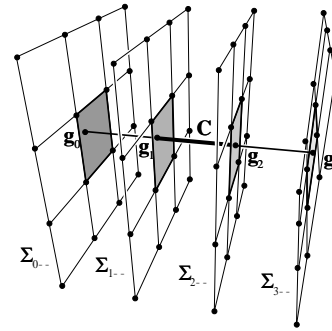


Figure 5: This figure depicts four slices of coefficients, the four uniform bicubic b-spline surfaces that they define (shaded in grey), and one iso-curve \mathbf{C} (thick line) defined by four points obtained by evaluating the surfaces using the same parametric values.

the fix indices in the slices that define the surfaces (see figure 5).

Definition 3 Slice skin of coefficients

A *slice skin*, Σk , is the surface defined by the nine bilinear patches that we obtain by taking groups of four contiguous coefficients in Σ , (see figure 6). For example: $\Sigma k_{-s_1}^{ijk}$ is the bilinear surface defined by the nine patches with coefficients:

$$\begin{aligned} &\mathbf{g}_{i+r, j+s, k+t}, \mathbf{g}_{i+r+1, j+s, k+t}, \\ &\mathbf{g}_{i+r, j+s, k+t+1}, \mathbf{g}_{i+r+1, j+s, k+t+1} \end{aligned}$$

with $r, t = 0, \dots, 2$.

Two slice skins that have the same fixed indices are called *parallel*, and two slice skins that have different fixed indices are called *perpendicular*.

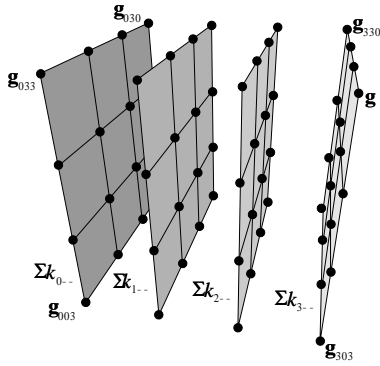


Figure 6: Each slice skin is a bilinear surface formed by nine bilinear patches. This picture shows the four slice skins with the index I fix.

4. Validity condition

A uniform tricubic b-spline hyperpatch is invalid when the parametric function $\mathbf{S}(u, v, w)$ is not single-valued (not injective) over the domain \mathbf{U} . There are two different situations in which $\mathbf{S}(u, v, w)$ is not injective⁷: First, when two different injective sub-hyperpatches intersect, and second, when an individual sub-hyperpatch is not injective. Let us explore these situations separately.

4.1. Intersection of two valid sub-hyperpatches

Two valid sub-hyperpatches $\mathbf{S}_{i_1 j_1 k_1}$ and $\mathbf{S}_{i_2 j_2 k_2}$ intersect if their border iso-surfaces intersect or if one sub-hyperpatch is completely inside the other.

If the sub-hyperpatches are *contiguous*, that is, if:

$$\begin{aligned} & (|i_2 - i_1| = 1 \wedge j_1 = j_2 \wedge k_1 = k_2) \vee \\ & (i_1 = i_2 \wedge |j_2 - j_1| = 1 \wedge k_1 = k_2) \vee \\ & (i_1 = i_2 \wedge j_1 = j_2 \wedge |k_2 - k_1| = 1) \end{aligned}$$

it is clear that they do not intersect because they share three slices of coefficients. Thus, we can avoid to check the intersection of a sub-hyperpatch with its six contiguous sub-hyperpatches.

If they are not contiguous we can perform the intersection test very fast if we use the following hierarchy of tests:

1. If the intersection of the bounding boxes aligned to the main axes X , Y and Z of the geometric coefficients in $\mathbf{G}_{i_1 j_1 k_1}$ and $\mathbf{G}_{i_2 j_2 k_2}$ is empty, then the sub-hyperpatches $\mathbf{S}_{i_1 j_1 k_1}$ and $\mathbf{S}_{i_2 j_2 k_2}$ do not intersect.

This happens because the uniform tricubic b-spline hyperpatches hold the strong convex-hull property, (that states that each sub-hyperpatch is completely inside of

the convex-hull formed by the 64 coefficients that define its shape), and because the convex-hull of a sub-hyperpatch is completely inside of the bound box of its coefficients.

In most cases this is the only test that we need to perform. If this test fails, we perform the next test.

2. If the intersection of the convex-hulls of the geometric coefficients in $\mathbf{G}_{i_1 j_1 k_1}$ and $\mathbf{G}_{i_2 j_2 k_2}$ is empty, then the sub-hyperpatches $\mathbf{S}_{i_1 j_1 k_1}$ and $\mathbf{S}_{i_2 j_2 k_2}$ do not intersect.

Again this is true due to the strong convex-hull property of the uniform tricubic b-spline hyperpatches.

3. If tests 1 and 2 fail, we have to calculate the intersection of the border iso-surfaces.

4.2. Validity of a sub-hyperpatch

The validity of a sub-hyperpatch is not easy to check. Although we can think that if none of the slice skins of coefficients of a sub-hyperpatch intersect any other parallel slice skin, then, the sub-hyperpatch is injective, this is not true. A sub-hyperpatch can lose its injectivity even although its parallel slice skins don't intersect (see figure 7), so a validity condition is needed.

Theorem 1 The validity condition

Let \mathbf{S}_{ijk} be a uniform tricubic b-spline sub-hyperpatch. the sub-hyperpatch \mathbf{S}_{ijk} is valid if it holds the following conditions:

1. All its spans σ of geometric coefficients are locally ordered.
2. None of its parallel slice skins Σk intersect to each other.

Proof

We are going to prove the theorem by contradiction. Let us suppose that the sub-hyperpatch \mathbf{S}_{ijk} holds the conditions stated by the theorem but it is invalid.

Applying the implicit function theorem, the jacobian determinant of the function \mathbf{S}_{ijk} is zero in some point of the parametric domain \mathbf{U} .

$$\exists \mathbf{u} \in \mathbf{U} / |J(\mathbf{S}_{ijk}(u, v, w))| = \begin{vmatrix} \frac{\partial x_{ijk}}{\partial u} & \frac{\partial x_{ijk}}{\partial v} & \frac{\partial x_{ijk}}{\partial w} \\ \frac{\partial y_{ijk}}{\partial u} & \frac{\partial y_{ijk}}{\partial v} & \frac{\partial y_{ijk}}{\partial w} \\ \frac{\partial z_{ijk}}{\partial u} & \frac{\partial z_{ijk}}{\partial v} & \frac{\partial z_{ijk}}{\partial w} \end{vmatrix} = 0$$

This jacobian determinant can only be zero at \mathbf{u} if and only if in the point $\mathbf{u} = (u, v, w)$ of the parametric space \mathbf{U} the vectors $(\frac{\partial x_{ijk}}{\partial u}, \frac{\partial y_{ijk}}{\partial u}, \frac{\partial z_{ijk}}{\partial u})$, $(\frac{\partial x_{ijk}}{\partial v}, \frac{\partial y_{ijk}}{\partial v}, \frac{\partial z_{ijk}}{\partial v})$ and $(\frac{\partial x_{ijk}}{\partial w}, \frac{\partial y_{ijk}}{\partial w}, \frac{\partial z_{ijk}}{\partial w})$ are linearly dependent.

As these vectors are the tangent vectors of the sub-hyperpatch evaluated in \mathbf{u} , $\mathbf{S}_{ijk}^D(\mathbf{u})$, in each parametric direction $D \in \mathbf{U} = \{U, V, W\}$, this means that the jacobian determinant can only be zero if some of the following conditions

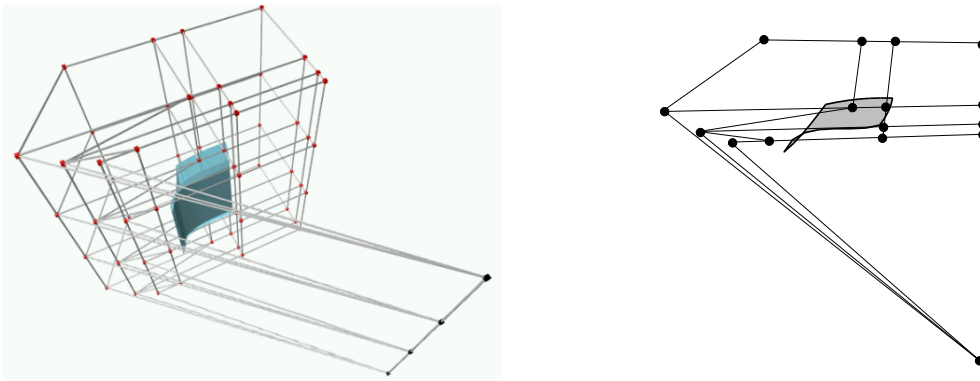


Figure 7: This figure shows an example of a sub-hyperpatch that is invalid although none of their slice skins intersect any other parallel slice skin. Left we can see the entire sub-hyperpatch and right we can see only a slice of sixteen coefficients for clarity.

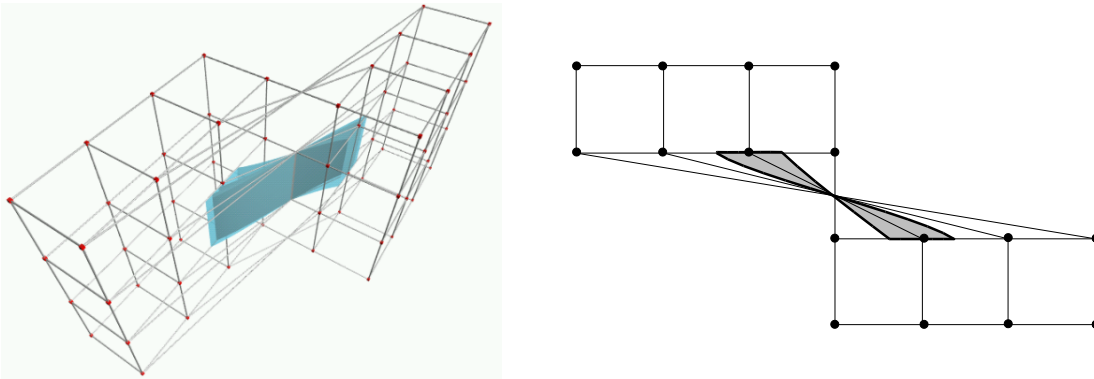


Figure 8: This figure shows an example of a sub-hyperpatch that is invalid although all their spans of coefficients are locally ordered. Left we can see the entire sub-hyperpatch and right we can see only a slice of sixteen coefficients for clarity.

occurs: one tangent is zero, or the three tangents lie in the same plane. Let us study these situations separately.

1. One tangent vector $\mathbf{S}^D(\mathbf{u})$ is zero.

This means that the first derivative of the iso-curve \mathbf{C}_D passing through \mathbf{u} is also zero, (\mathbf{C}_D is not regular).

The derivative of an iso-curve $\mathbf{C}_D(d)$ can be expressed as:

$$\frac{\partial \mathbf{C}_D}{\partial d} = \lim_{\Delta d \rightarrow 0} \frac{\mathbf{C}_D(d + \Delta d) - \mathbf{C}_D(d)}{\Delta d}$$

If $\partial \mathbf{C}_D / \partial d$ is zero, this implies that close to \mathbf{u} , a little increment of d , Δd in the parametric space cannot produce an increment in the modelling space \mathbf{R}^3 .

The points over an iso-curve \mathbf{C}_D are the blend of the geometric coefficients of that iso-curve by using the uniform cubic b-spline blending functions. As we can see in figure 2 (left), all the portions of the blending functions that apply in an iso-curve are monotonic.

The first two blending functions, (those that apply to the first two geometric coefficients of the iso-curve), are decreasing, while the last two blending functions, (those that apply to the last two coefficients), are increasing. At the same time as d increases, the blending functions that apply over the two first coefficients have a minor value, while the blending functions that apply over the two last coefficients have a greater value.

The geometric coefficients that define the shape of the iso-curve are locally ordered because they are the result of blending of the coefficients in four slices of the sub-hyperpatch, that according to the theorem, are locally ordered and do not intersect. Therefore, the speed of that iso-curve cannot be zero since the curve is the blend of coefficients locally ordered blended by functions that are monotonic. The only way that the iso-curve be not regular is that the coefficients be not locally ordered.

Thus we reach a contradiction, due to the fact that we

suppose that one tangent vector \mathbf{S}^D is zero in the sub-hyperpatch.

- The three tangent vector evaluated in $\mathbf{u} \in \mathbf{U}$ lie in the same plane.

Let \mathbf{P} be the plane containing the three tangent vectors $\mathbf{S}^U(\mathbf{u})$, $\mathbf{S}^V(\mathbf{u})$ and $\mathbf{S}^W(\mathbf{u})$.

The three tangents should be linearly independent because they correspond to perpendicular parametric directions. If they are coplanaries in \mathbf{u} this is due to the fact that some tangent has been reoriented and is located in the plane \mathbf{P} formed by the other two tangents. We are going to call D to the direction of the reoriented tangent.

If \mathbf{S}^D is on the plane \mathbf{P} then the iso-curve \mathbf{C}_D that passes through \mathbf{u} intersects the plane \mathbf{P} more than once. The uniform cubic b-spline curves hold the variation diminishing property¹¹, hence it follows that the control polygonal of \mathbf{C}_D must intersect the plane \mathbf{P} also more than once. And this implies that the control polygonal of \mathbf{C}_D is not locally ordered.

But the control points that define the shape of \mathbf{C}_D are the blend of the coefficients in four slices that, applying the theorem, are locally ordered and do not intersect, so the control polygonal of \mathbf{C}_D is locally ordered.

Again we reach a contradiction, due to the fact that we suppose that one tangent vector \mathbf{S}^D is on the plane defined by the other two tangent vectors.

□

5. Conclusions

The condition presented is sufficient but not necessary. This means that some sub-hyperpatches that are valid do not hold it. Even so, it is a very useful condition since:

- It is robust because there are not special cases, and so, it works fine on all the sub-hyperpatches.
- It works by testing relative positions between control points and not testing displacement of control points. This makes it very useful not only to determine if the movement of a control point has violated the injectivity of the sub-hyperpatch, but also to determine if a given hyperpatch is valid or invalid.
- It is suitable for interactive modelling of solids because the uniform tricubic b-spline solids are local, and a modification over a little set of geometric coefficients has a limited effect in the shape of the solid. Therefore, the condition must be tested only on a limited set of coefficients.
- It is simple because it works only by comparing relative positions of geometric coefficients, and so, it is easy to implement and very fast. This allows us to manage ef-

ficiently solids defined by a great number of geometric coefficients.

- The condition restricts the shape that the sub-hyperpatches can take, but it doesn't restrict the expressive power of the representation model since we can model all the solids we need by increasing the number of sub-hyperpatches that form the hyperpatch. See the example of figures 9, 10 and 11.

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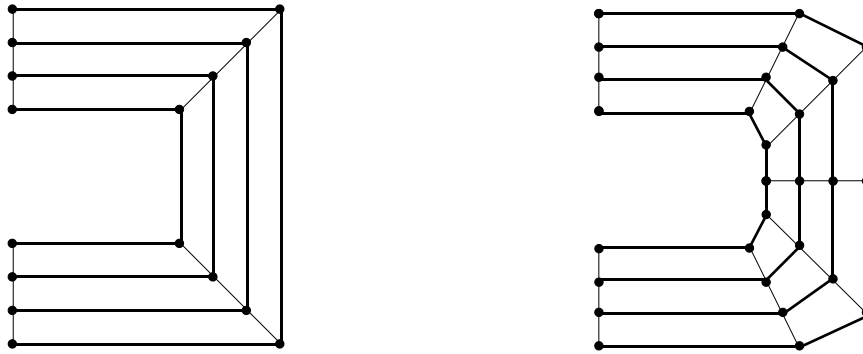


Figure 9: This figure shows how by increasing the number of sub-hyperpatches, we can model some shapes which are almost identical to those ones rejected by the validity condition. On the left, we can see a rejected lattice, (some of its spans are not locally ordered). On the right, a similar lattice to the one on the left, but accepted by the validity condition. For clarity, we show a simplified view here. In figures 10 and 11 we can see screenshots of the current application.

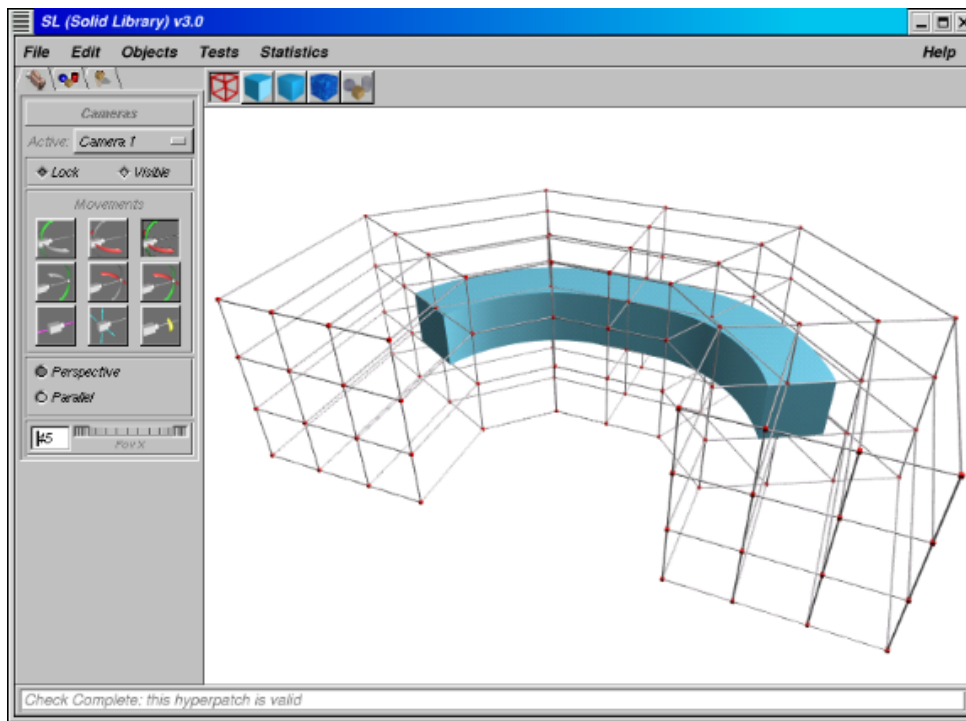


Figure 10: In this picture we show an actual application of our validity condition. This is a perspective view of figure 9 (right).

the variation diminishing property of b-spline approximation”, *Journal approximation theory*, **37**(1), pp. 1–4 (1983).

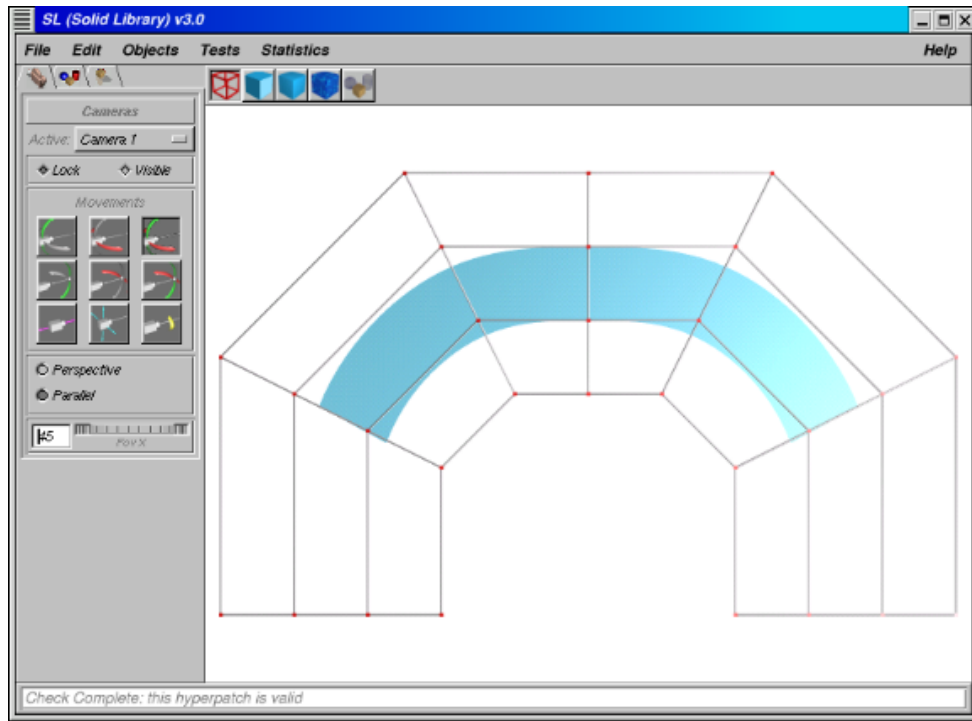


Figure 11: This is a parallel view of figure 9 (right).