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CNCUR : A simple 2D Curve Reconstruction Algorithm based on constrained neighbours

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Abstract

Given a planar point set $S \in \mathbb{R}^2$ (where $S = \{v_1, \ldots, v_n\}$) sampled from an unknown curve Σ, the goal is to obtain a piece-wise linear reconstruction the curve from S that best approximates Σ. In this work, we propose a simple and intuitive Delaunay triangulation(DT)-based algorithm for curve reconstruction. We start by constructing a Delaunay Triangulation (DT) of the input point set. Next, we identify the set of edges, EN_p in the natural neighborhood of each point p in the DT. From the set of edges in EN_p, we retain the first two shorter edges connected to each point. To take care of open curves, one of the retained edges has to be removed based on a parameter δ. Here, δ is a parameter used to eliminate the longer edge based on the allowable ratio between the maximum and minimum edge lengths. Our algorithm inherently handles self-intersections, multiple components, sharp corners, and different levels of Gaussian noise, all without requiring any parameters, pre-processing, or post-processing.

CCS Concepts
• ***Computing methodologies*** $\rightarrow$ ***Shape modeling;***

1. Introduction

Curve reconstruction has received significant attention in computational geometry and computer graphics over the past two decades. Various algorithms [OPP*21] have been proposed to tackle this problem under different assumptions and conditions.

An input point set may have one or more of the following characteristics: noise, outliers, self-intersections, sharp corners, open curves and disconnected components. To handle many of them, algorithms often have to tune multiple parameters which is a challenging task. Algorithms that do not use any parameter or use only a single parameter have difficulties to handle all the mentioned characteristics, as can be seen from Table 1 in [OPP*21]. In particular, handling multiple components and sharp corners is challenging for algorithms that use only a single parameter (refer Table 1 in [OPP*21]). Even the recent work [MOW22] does not handle open, multiple components and non-manifold curves.

The proposed algorithm is Delaunay triangulation (DT) based and has the following features:

- No parameter is used to handle self-intersections, noise, and multiple components.
- To handle open curves, a single parameter is used.

A comprehensive survey and benchmarking of various algorithms for curve reconstruction are discussed by Ohrhallinger et. al. [OPP*21]. Figure 1 shows a sample output from our proposed algorithm.

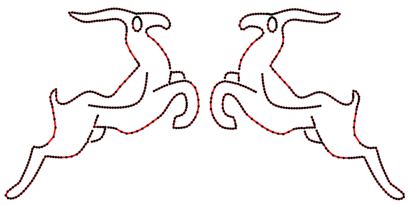

Figure 1: *Our output for the input point set with following characteristics - closed/open curves, multiple components, multiple holes, self-intersections and sharp corners.*

2. Method

Algorithm 1 details the procedure for reconstruction. For an input point set (Figure 2), we start by constructing a DT (Figure 2(a)). Identify the set of edges, EN_p in the natural neighborhood of a point p in the DT and retain only the two shorter edges (Figure 2(b)). Figure 2(c) shows the procedure for another point in the set. Figure 2(d) shows the reconstructed edges in green along with DT. The rightmost figure shows the reconstructed output for the input. For a manifold curve as input point set, this procedure is guaranteed to generate manifold output under a well-sampled condition (this statement can be shown to be true for ϵ sampling [ABE98]).

In case, if open curves are to be handled, we compare the minimum-length edges $e1$ and $e2$ connected at each point. If

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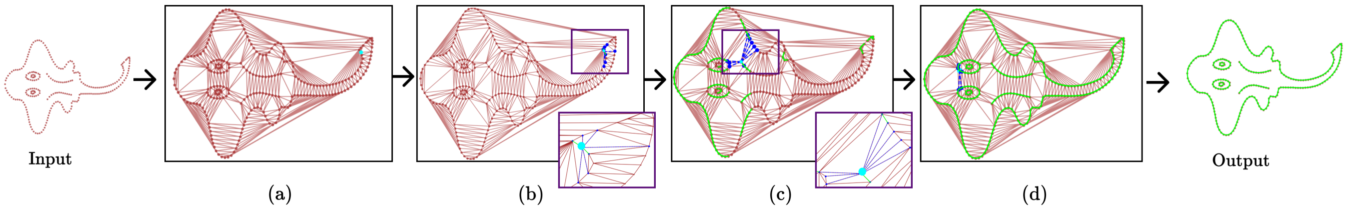

Figure 2: *Illustration of the proposed reconstruction algorithm. (a) DT of the input point set (b) For an input point, p (in cyan), natural neighbours (shown in blue) with the selected edges (in green) (c) Illustration for another point in the set. (d) DT with reconstructed edges (in green). Output (rightmost figure).*

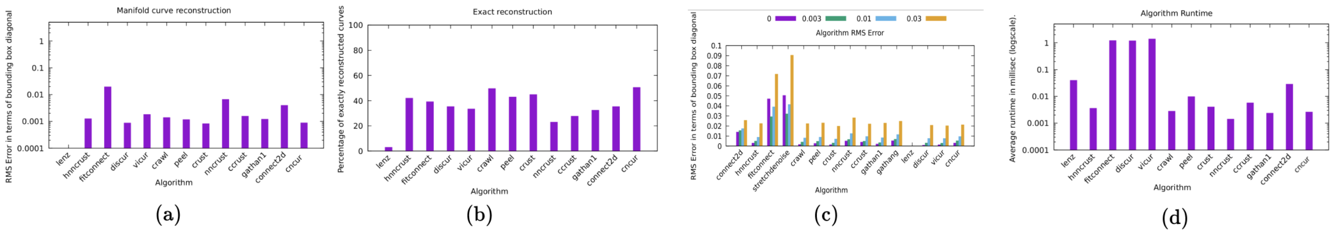

Figure 3: *Quantitative comparison on different evaluation metrics with various state of the art algorithms.*

Algorithm 1 CNCUR(**S**)

Input: A planar point set, $\mathbf{S} \subseteq \mathbb{R}^2$
Output: Reconstructed curve $\mathbf{C} \subseteq \mathbb{R}^2$.

Compute the 2D Delaunay triangulation, $DT(\mathbf{S})$.
for each point $\mathbf{p}_i \in \mathbf{S}$ **do**
  Collect the set of 1-ring vertices, $\mathcal{N}(\mathbf{p}_i)$, incident to $\mathbf{p}_i$ in $DT(\mathbf{S})$.
  Collect the set of edges $EN(\mathbf{p}_i)$ connecting $\mathbf{p}_i$ to each of the vertices in $\mathcal{N}(\mathbf{p}_i)$.
  Sort the edges in $EN(\mathbf{p}_i)$ by length in ascending order.
  Retain the first two shorter edges, e_1 and e_2, in $EN(\mathbf{p}_i)$.
end for
Combine all retained edges in $EN(\mathbf{p}_i)$ to form the edge set $\mathbf{C}'$.
Remove all duplicate edges in $\mathbf{C}'$ to obtain $\mathbf{C}$
return C

$max(e1,e2) \geq \delta \times min(e1,e2)$, then we keep only $min(e1,e2)$ in the list. Here δ is a parameter used to eliminate the longer edge and has to be fine-tuned.

3. Comparison and conclusions

We used the 2D benchmark code [OPP*21] for comparison with various start of the art algorithms. Figure 3 shows that our algorithm has performed well under different metrics - RMS error (Figure 3(a)), 'percentage of exactly reconstructed curves' (Figure 3(b)), noisy input (Gaussian levels of 0, 0.003, 0.01, 0.03 respectively)(Figure 3(c)), and average run time (Figure 3(d)). Figure 4 shows the qualitative comparison result.

We have proposed a simple curve reconstruction algorithm named *CNCUR* based on applying certain constraints on the edges of the DT of the point set. A parameter is needed only to handle open curves. Extensive comparison with the state-of-the-art algorithms shows that the algorithm has better overall performance on different measures like 'percentage of exactly reconstructed curves', RMS error, etc. The limitation of the algorithm is that it does not handle outliers.

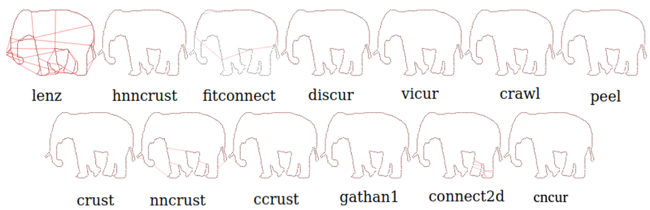

Figure 4: *Qualitative comparison result with various state of the art algorithms of a manifold model.*

References

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