

Dynamic 2D/3D Registration

Sofien Bouaziz Andrea Tagliasacchi Mark Pauly

Presenters



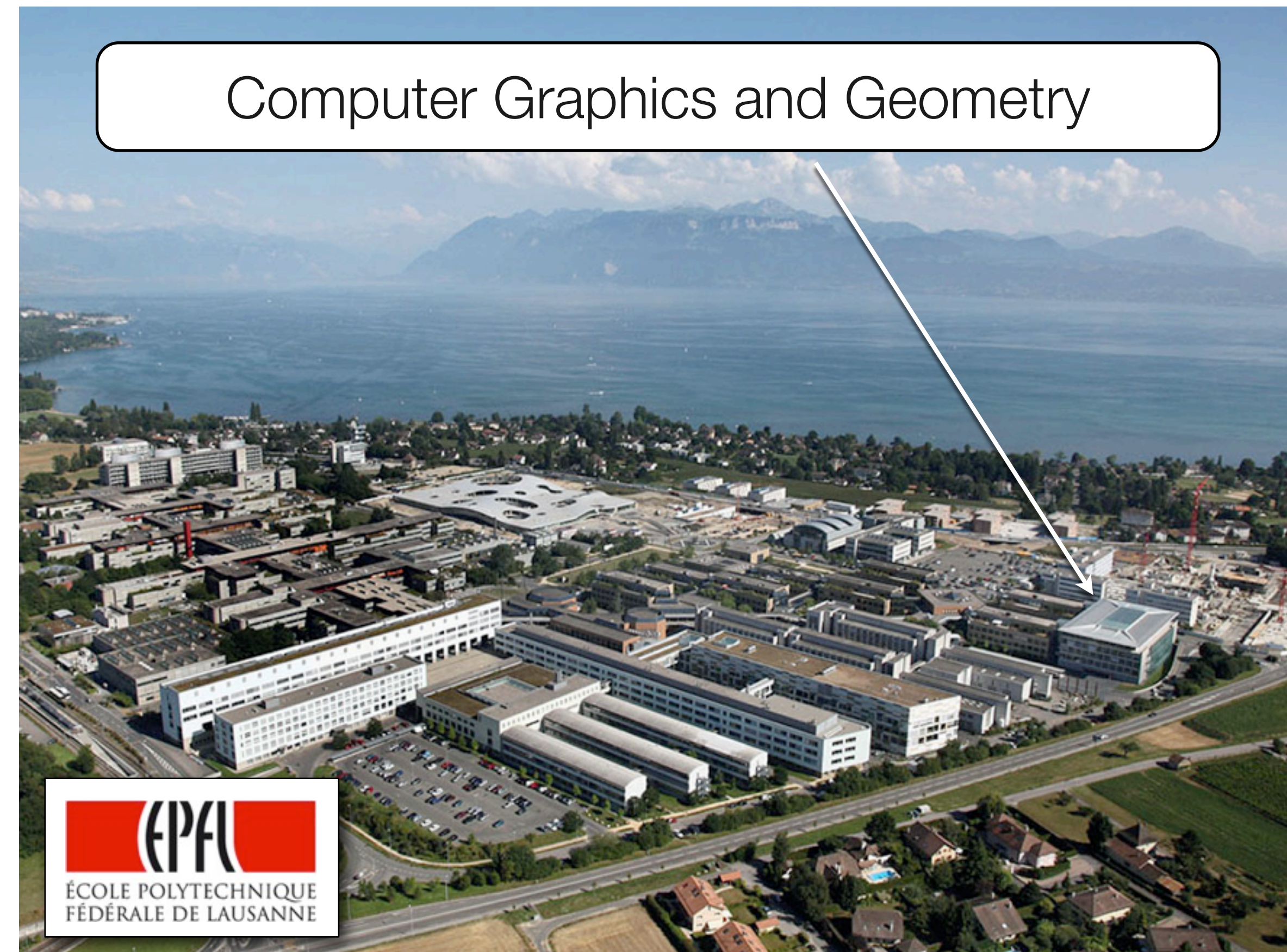
Sofien Bouaziz



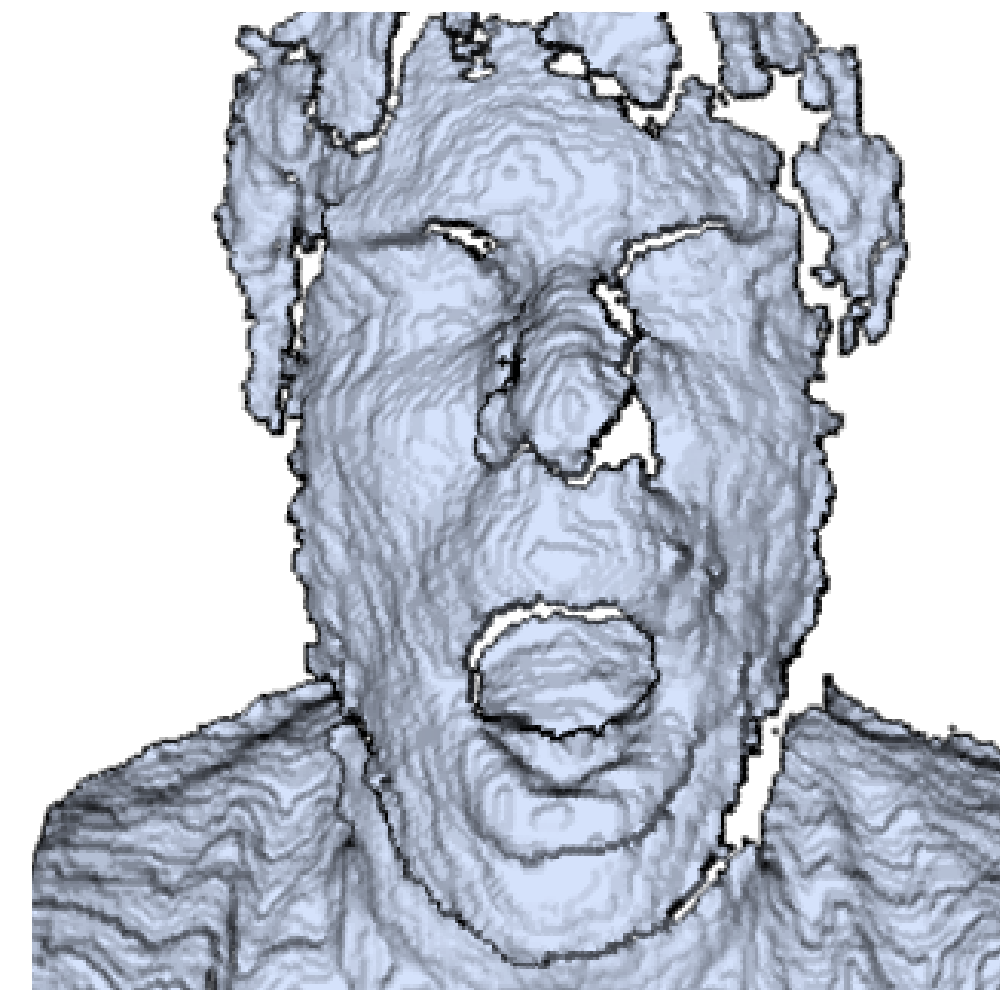
Andrea Tagliasacchi



Mark Pauly

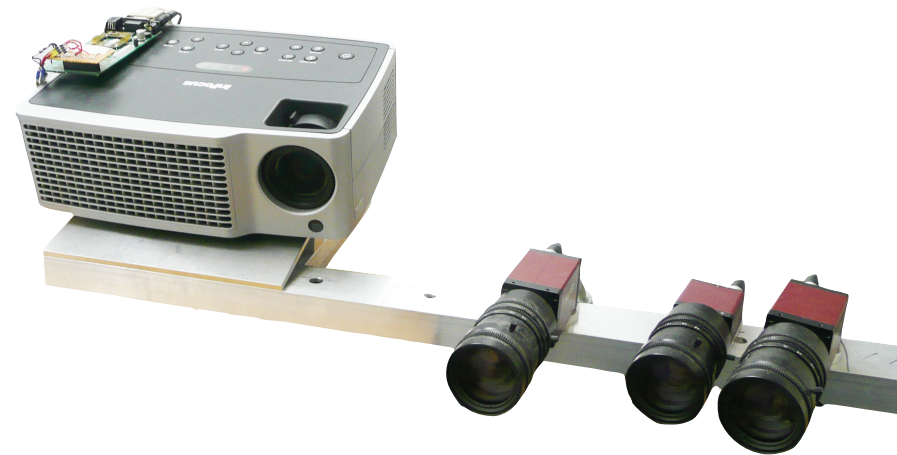


RGB-D Sensors



RGB-D Sensors

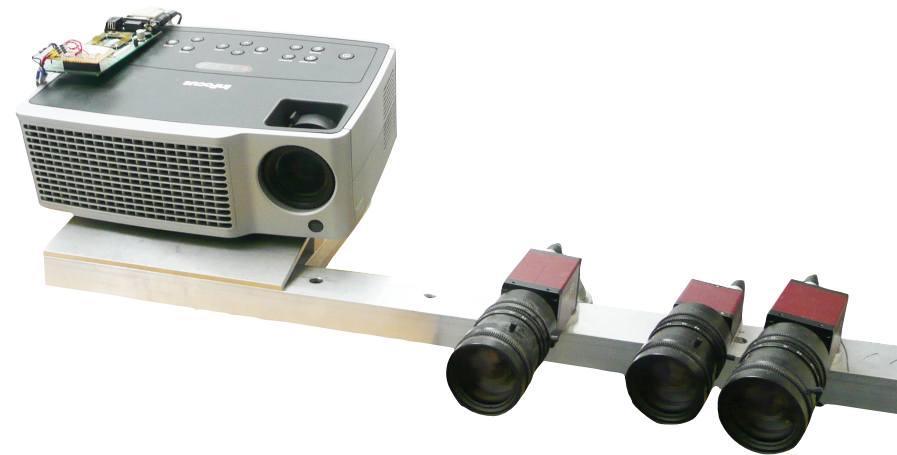
2008



RGB-D Sensors



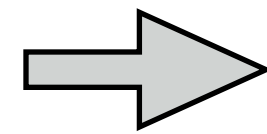
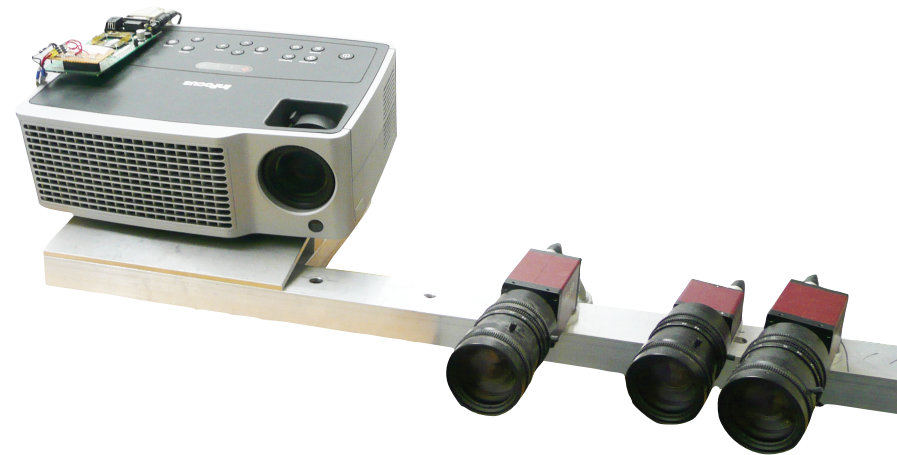
2008



RGB-D Sensors



2008



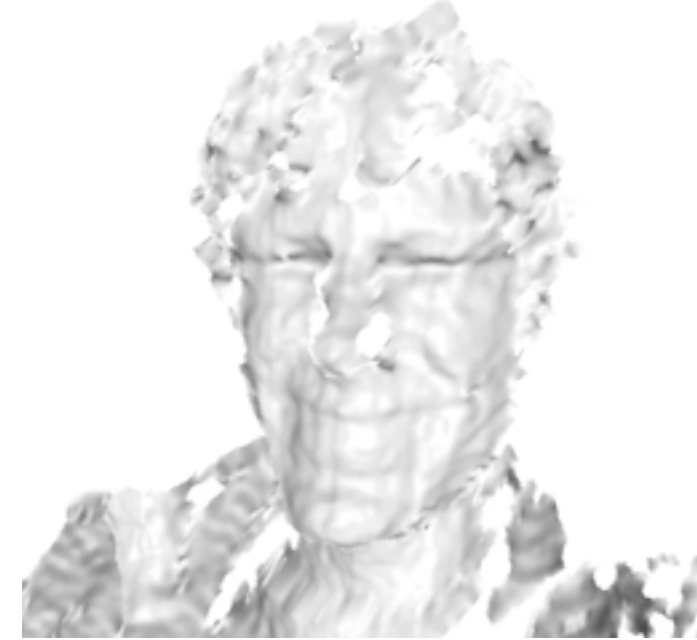
2010



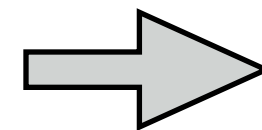
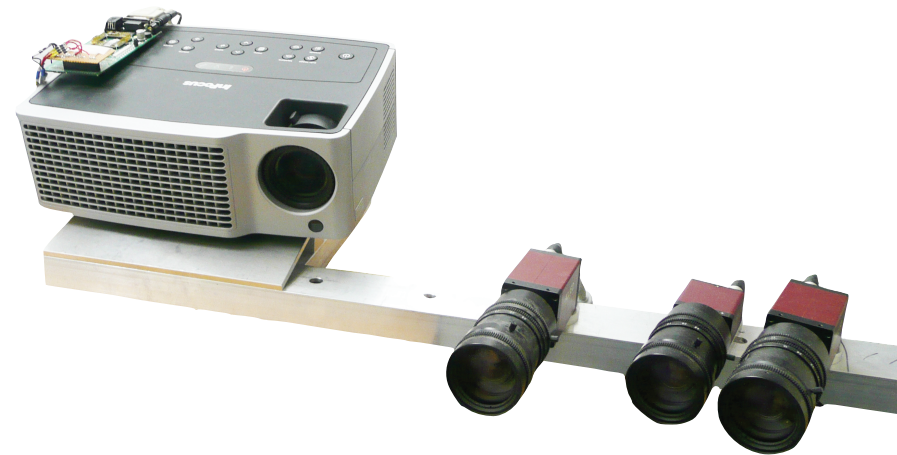
RGB-D Sensors



2008



2010



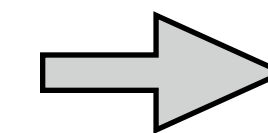
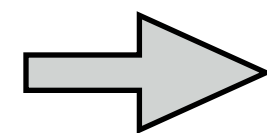
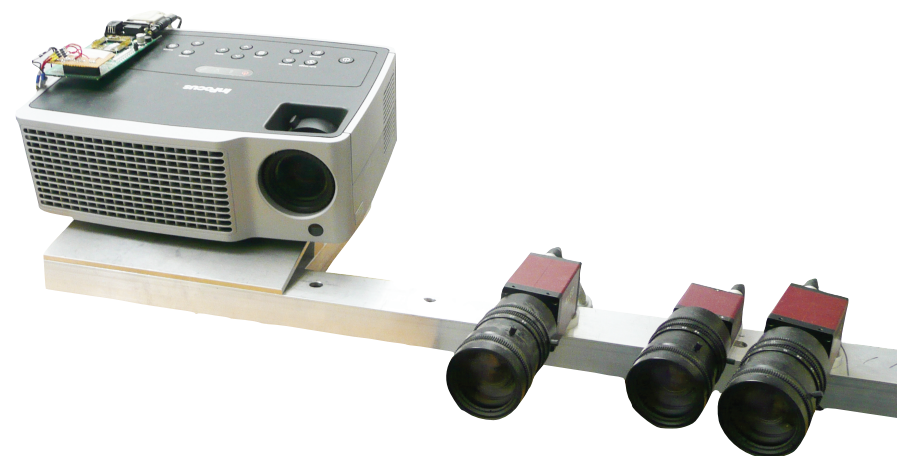
RGB-D Sensors



2008



2010



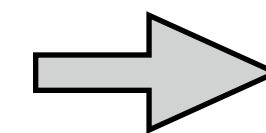
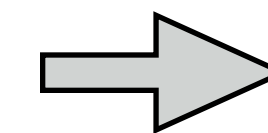
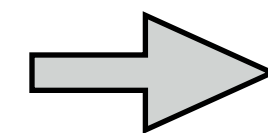
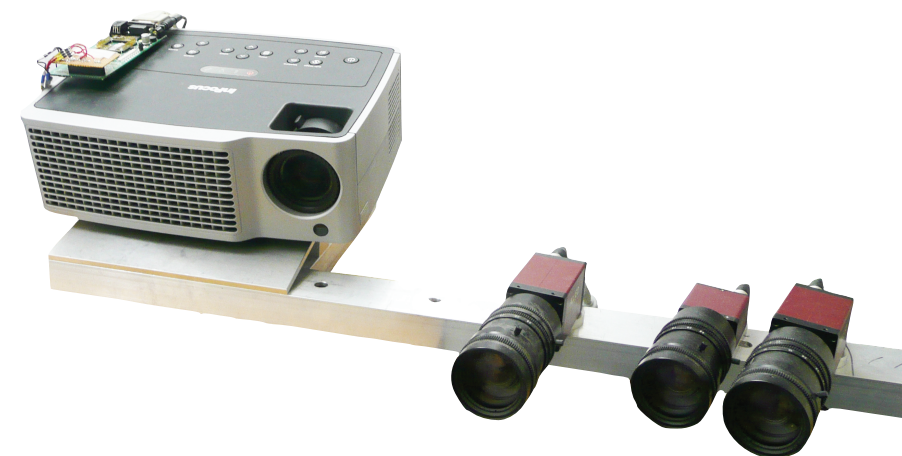
RGB-D Sensors



2008



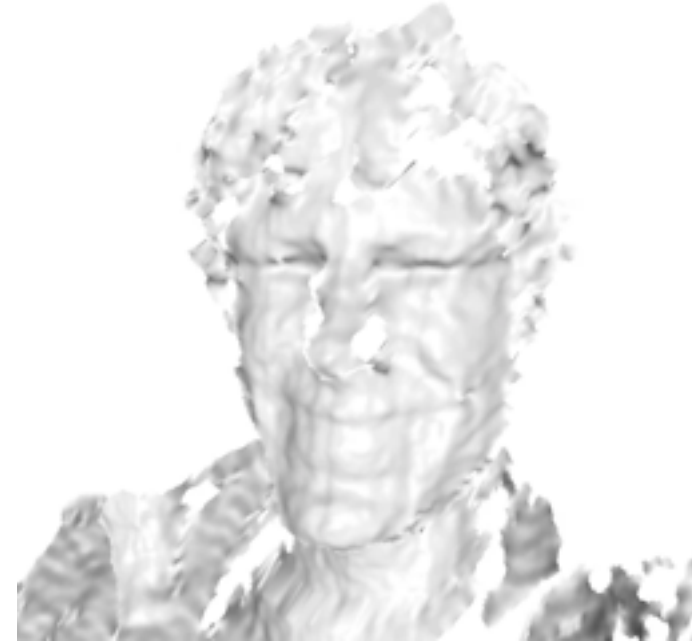
2010



RGB-D Sensors

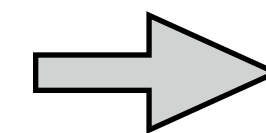
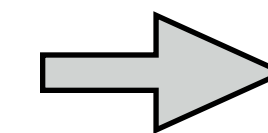
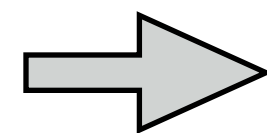
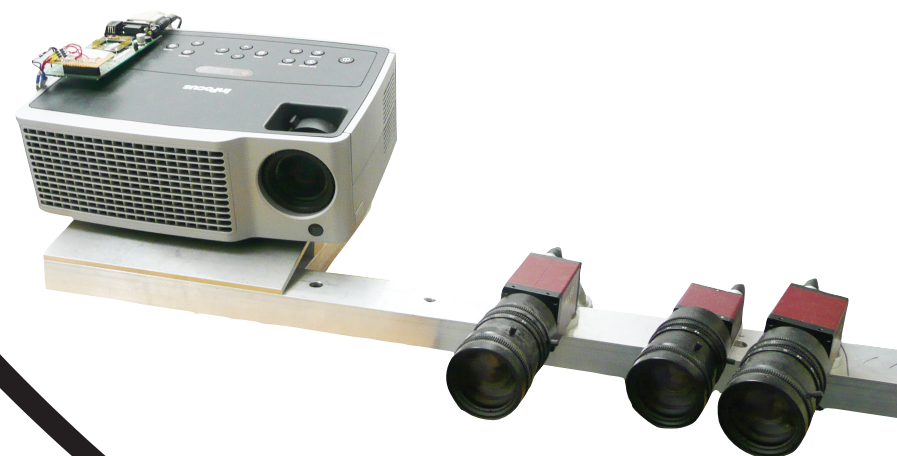


2008



2010

2013



size/cost



Overview

Introduction

- Registration
 - 3D Geometry
 - 2D Images
 - Combined 2D / 3D
- Applications
 - Rigid Scanning
 - Non-rigid Modeling
 - Realtime Face Tracking

Conclusions

RGB-D Sensors



<http://www.openni.org>

<http://www.openni.org/openni-programmers-guide>

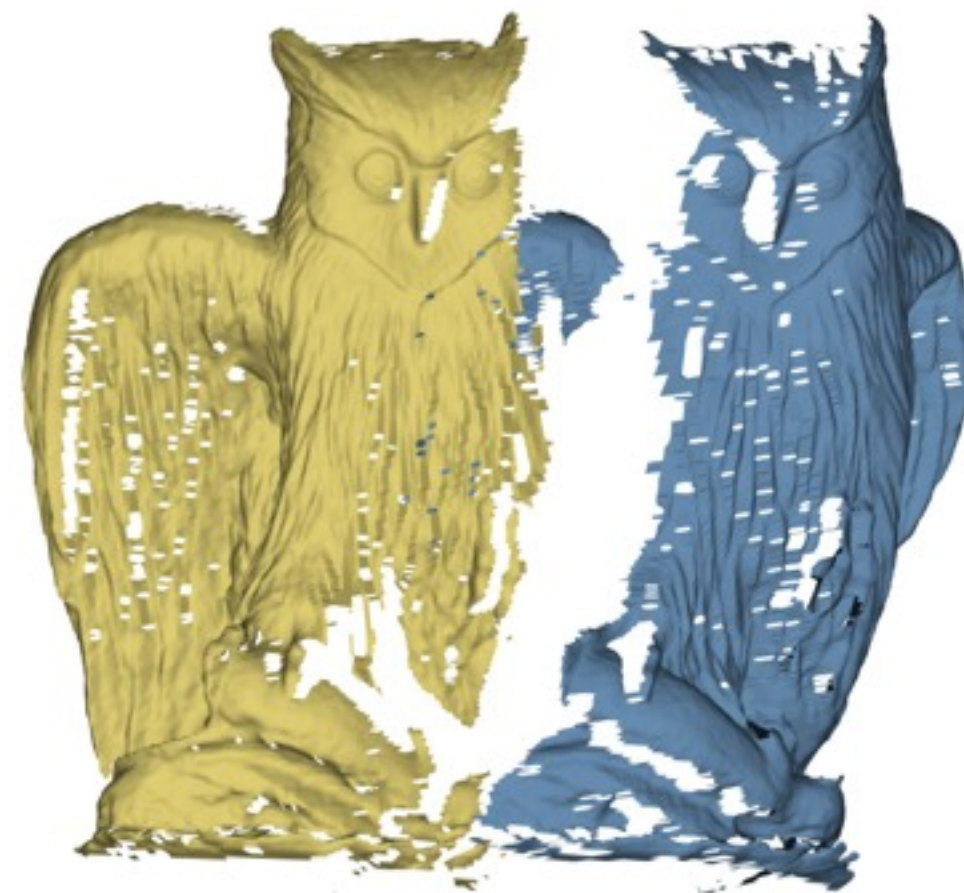
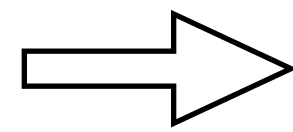
Registration - Examples

- Scan to scan



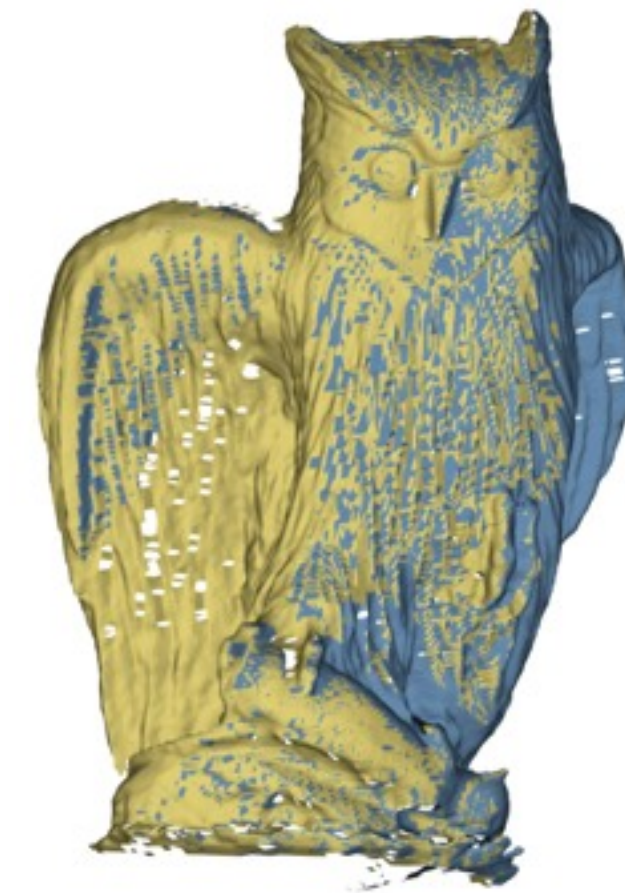
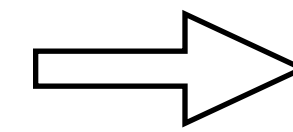
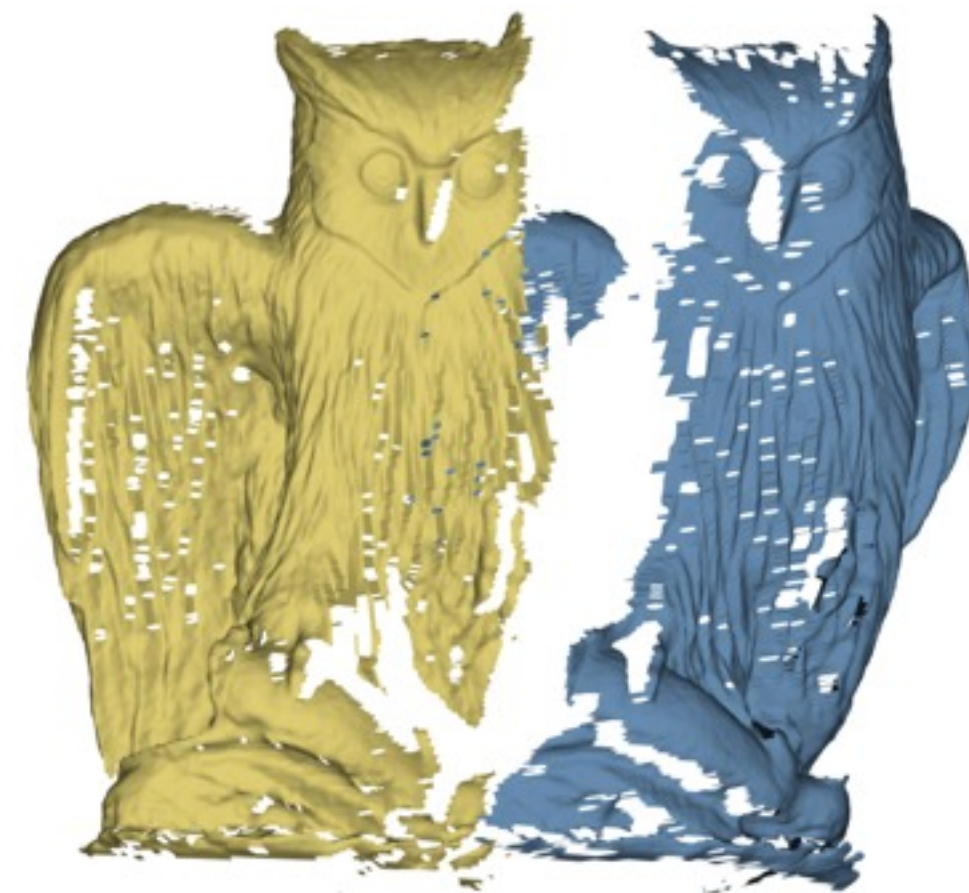
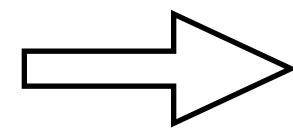
Registration - Examples

- Scan to scan



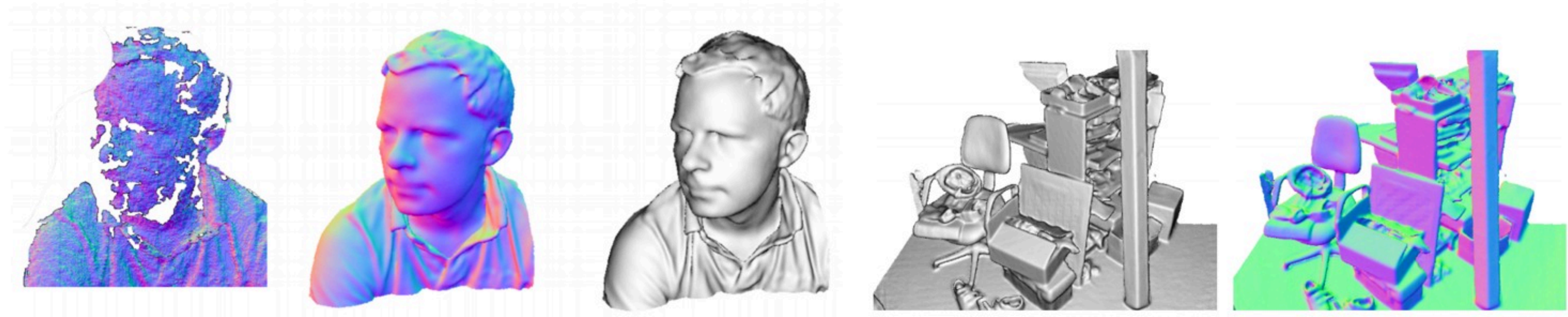
Registration - Examples

- Scan to scan



Registration - Examples

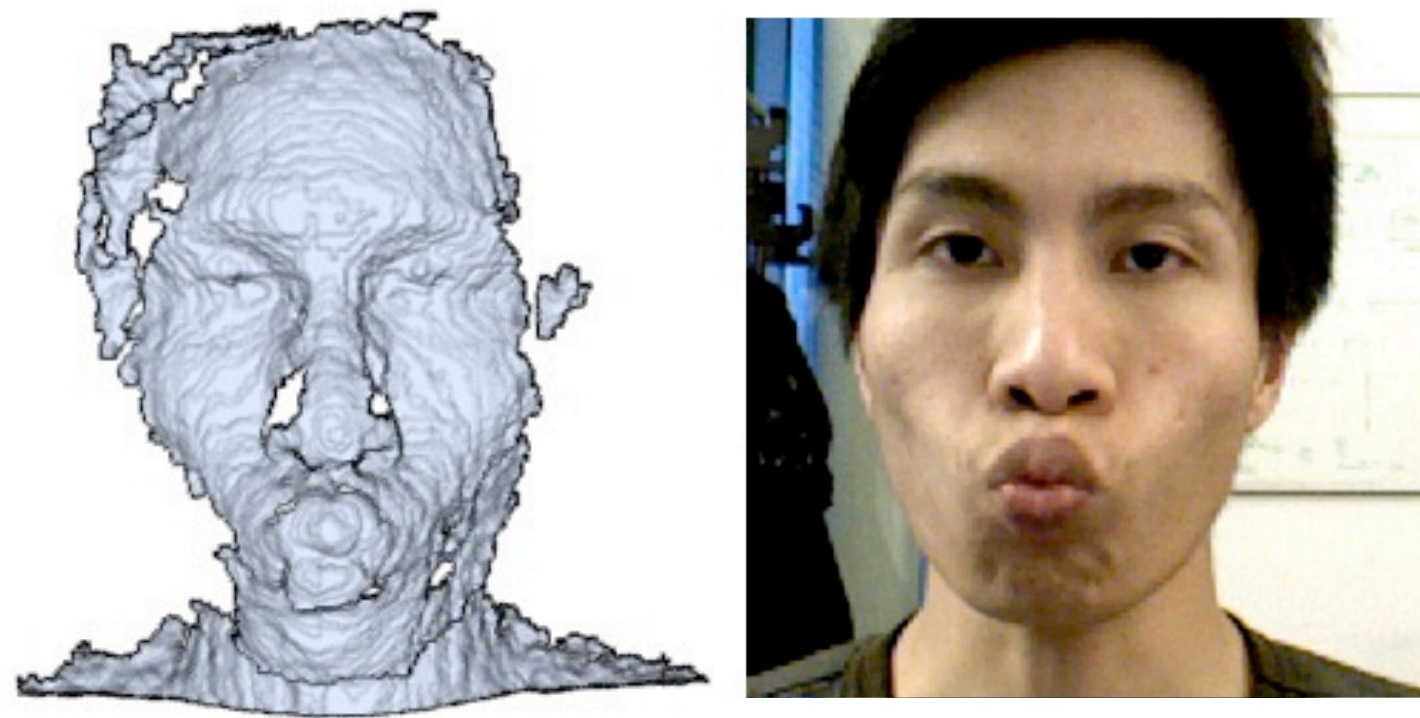
- Scan to scan



Newcombe, Izadi, Hilliges, Molyneaux, Kim, Davison, Kohli, Shotton, Hodges, Fitzgibbon:
KinectFusion: Real-Time Dense Surface Mapping and Tracking, *ISMAR 2011*

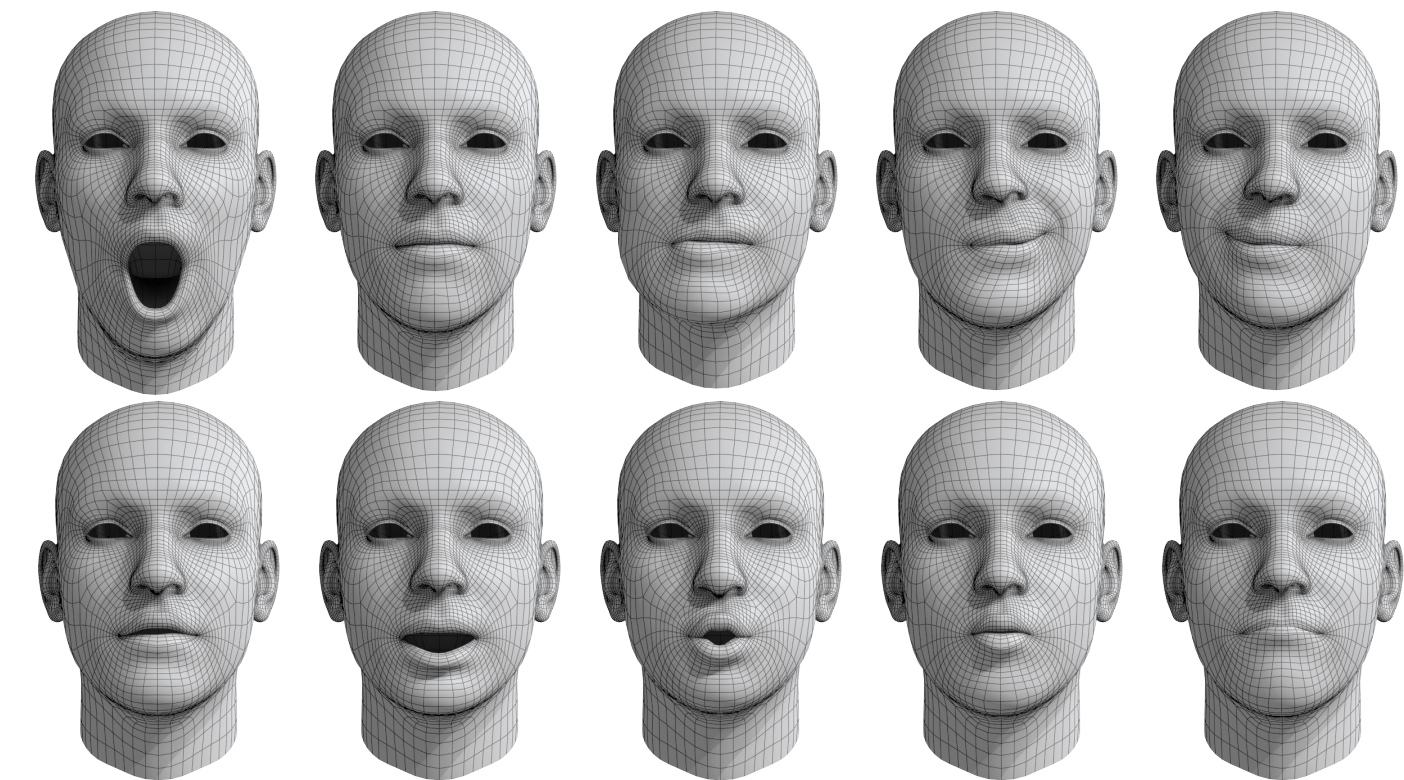
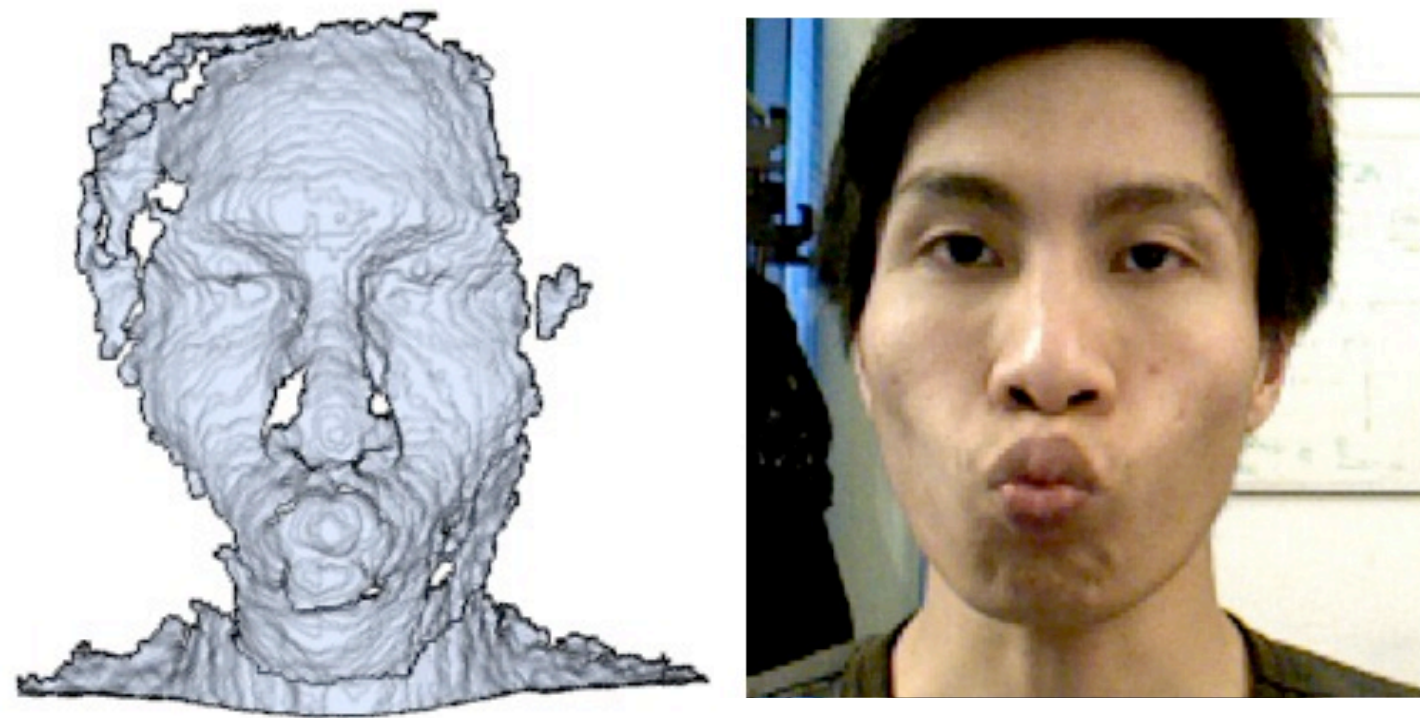
Registration - Examples

- Parameterized template to scan



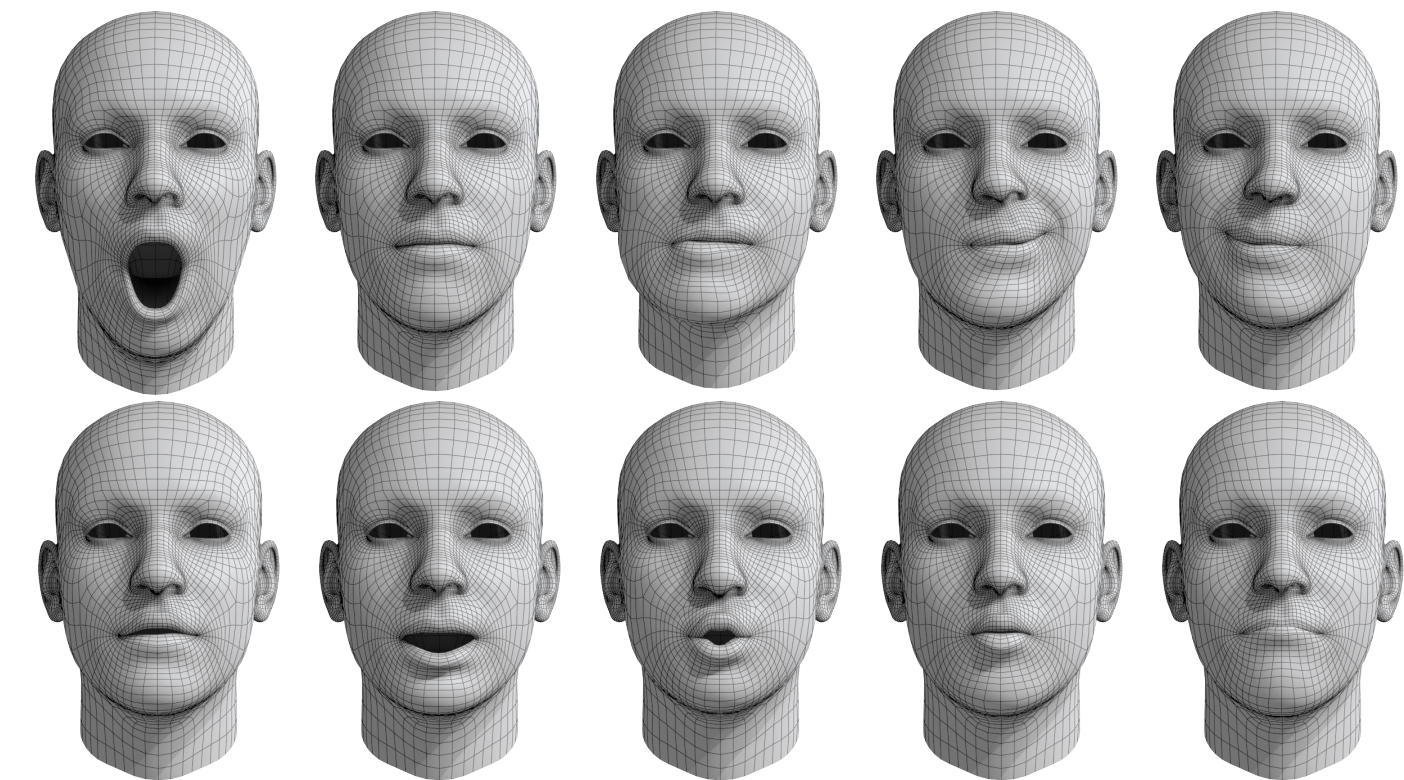
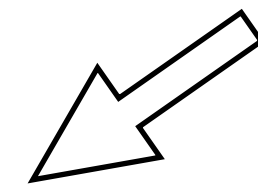
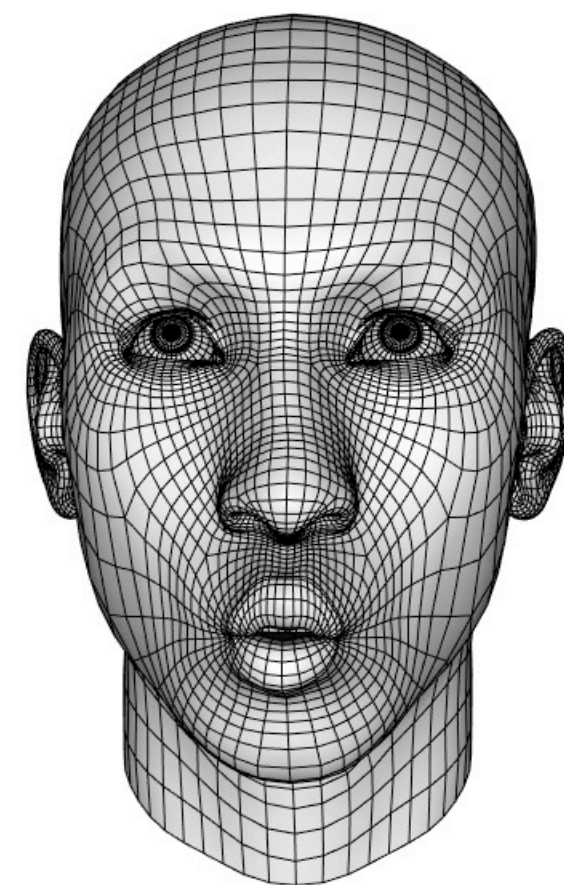
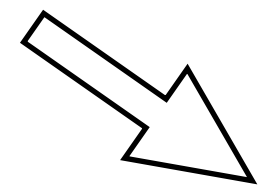
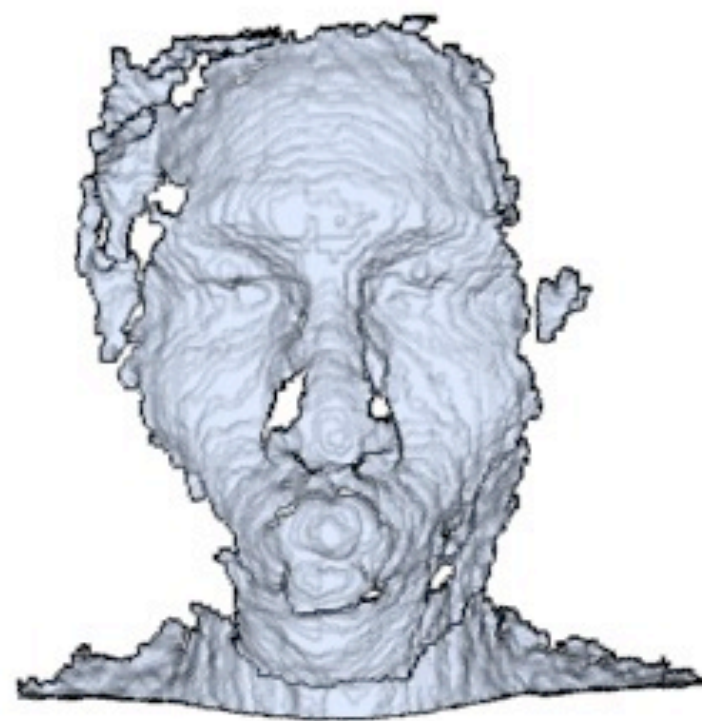
Registration - Examples

- Parameterized template to scan



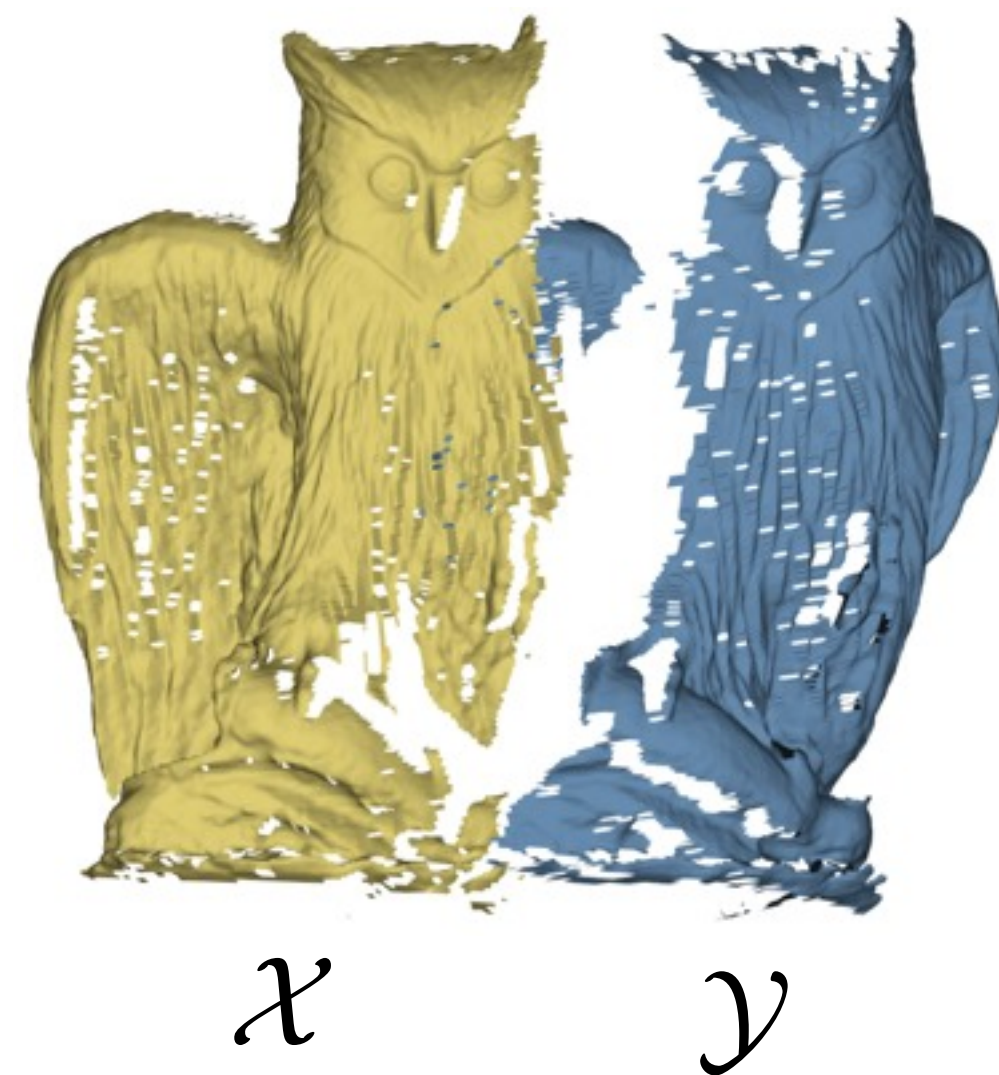
Registration - Examples

- Parameterized template to scan



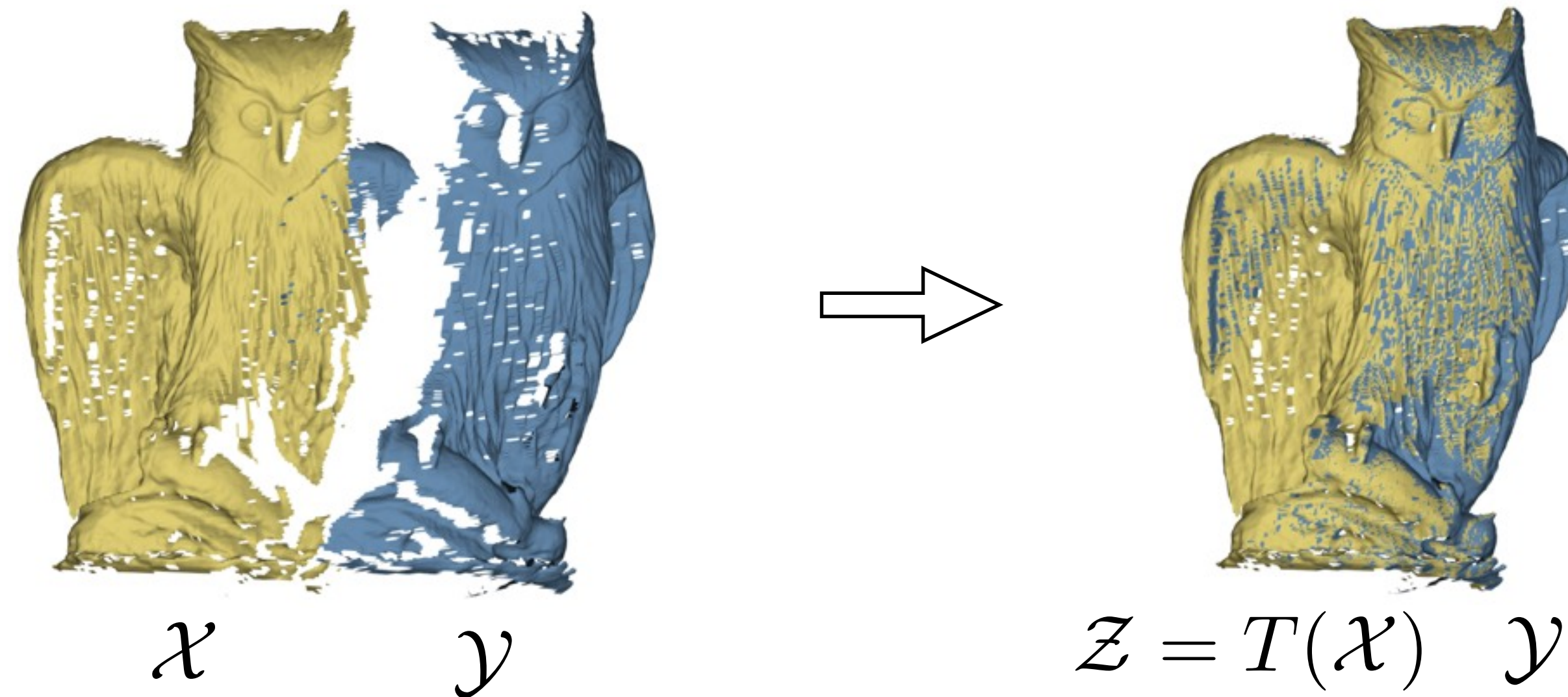
Pairwise Registration

- Align a *source model* \mathcal{X} onto a *target model* \mathcal{Y}



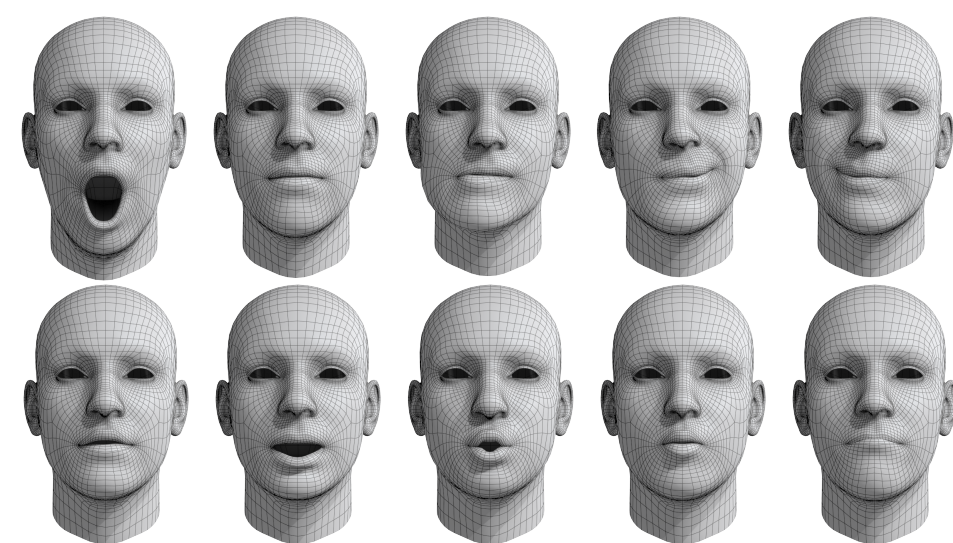
Pairwise Registration

- Align a *source model* \mathcal{X} onto a *target model* \mathcal{Y}
 - find a *transformation* $T(\mathcal{X})$ that brings \mathcal{X} into alignment with \mathcal{Y}

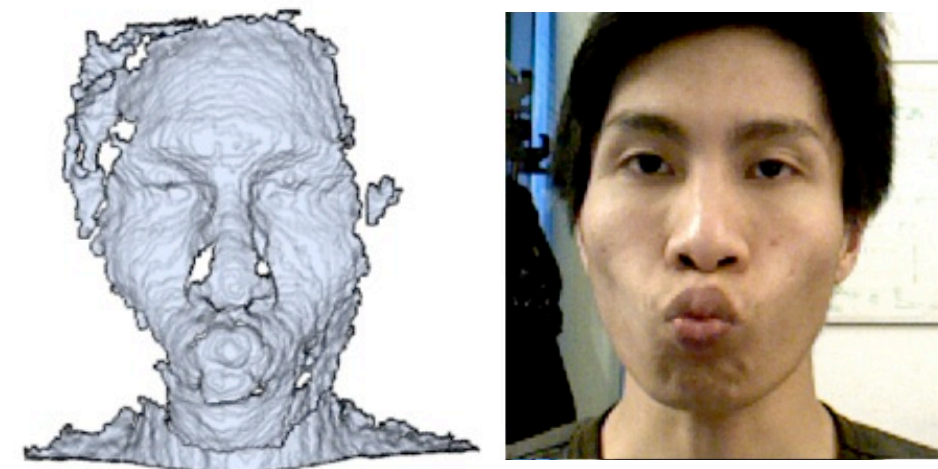


Pairwise Registration

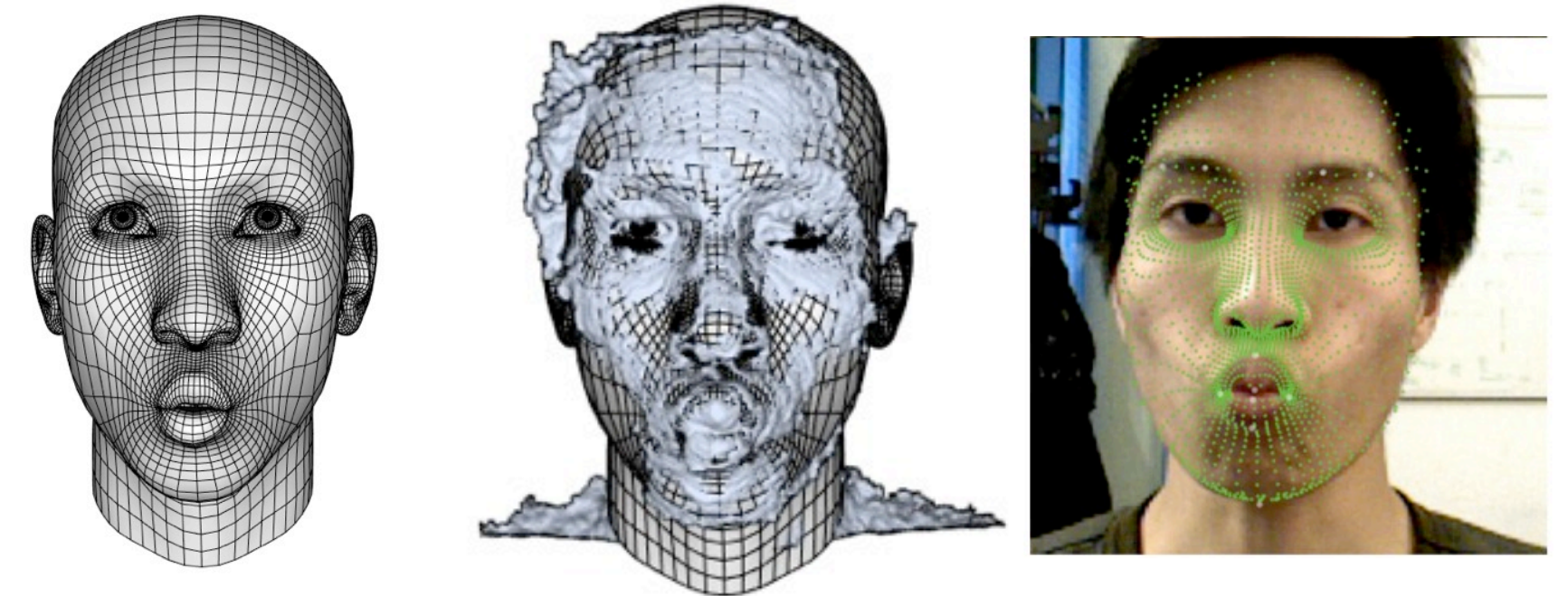
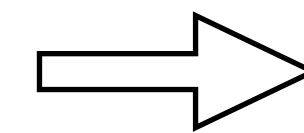
- Align a *source model* \mathcal{X} onto a *target model* \mathcal{Y}
 - find a *transformation* $T(\mathcal{X})$ that brings \mathcal{X} into alignment with \mathcal{Y}



\mathcal{X}



\mathcal{Y}



$\mathcal{Z} = T(\mathcal{X})$

Pairwise Registration

- Align a *source model* \mathcal{X} onto a *target model* \mathcal{Y}
 - find a *transformation* $T(\mathcal{X})$ that brings \mathcal{X} into alignment with \mathcal{Y}
- Two main questions:

Pairwise Registration

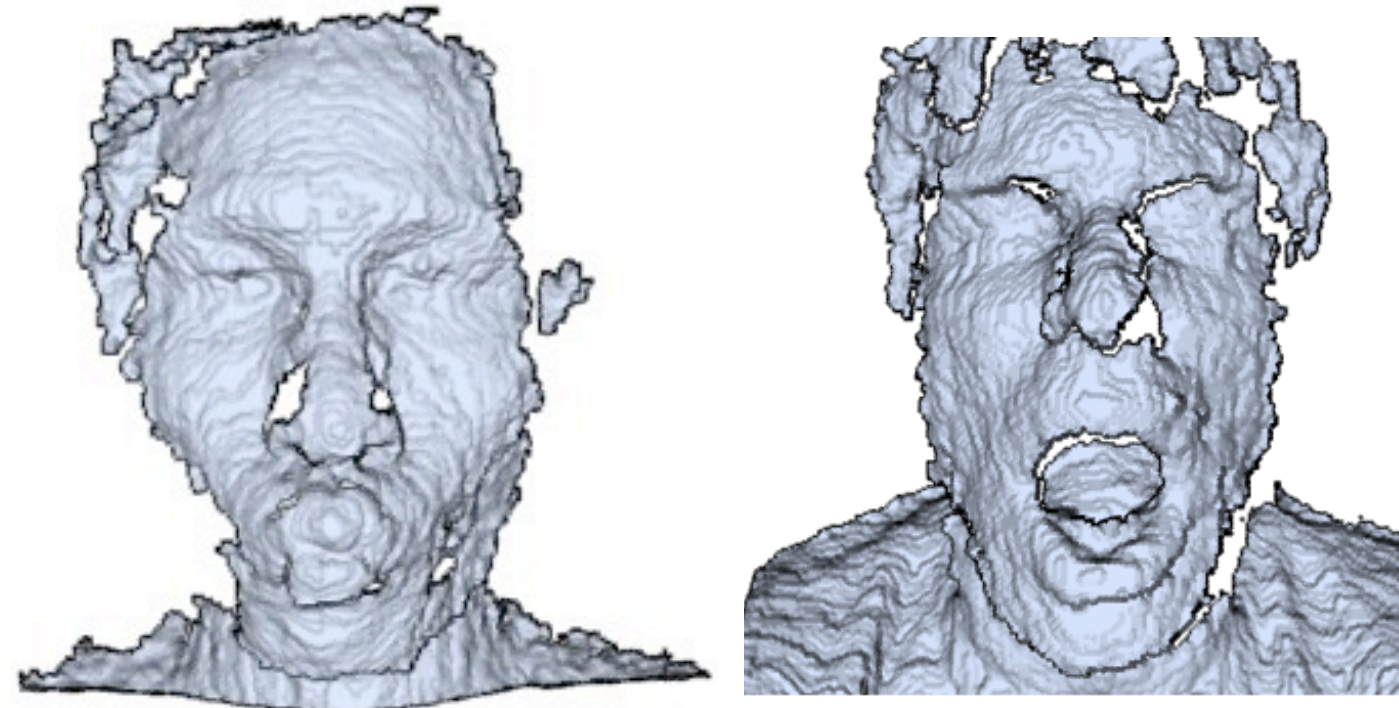
- Align a *source model* \mathcal{X} onto a *target model* \mathcal{Y}
 - find a *transformation* $T(\mathcal{X})$ that brings \mathcal{X} into alignment with \mathcal{Y}
- Two main questions:
 - How do we measure the quality of the alignment?

Pairwise Registration

- Align a *source model* \mathcal{X} onto a *target model* \mathcal{Y}
 - find a *transformation* $T(\mathcal{X})$ that brings \mathcal{X} into alignment with \mathcal{Y}
- Two main questions:
 - How do we measure the quality of the alignment?
 - What transformations are allowed / good?

Pairwise Registration

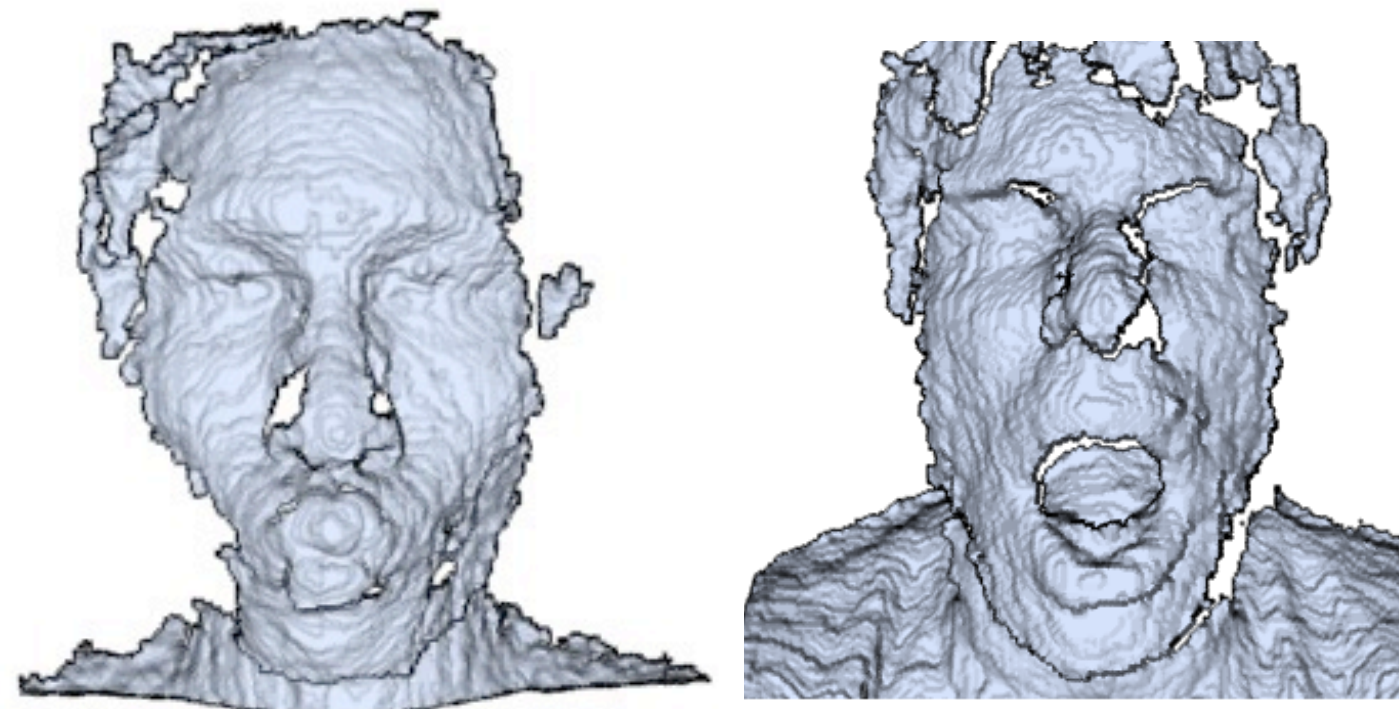
- Issues



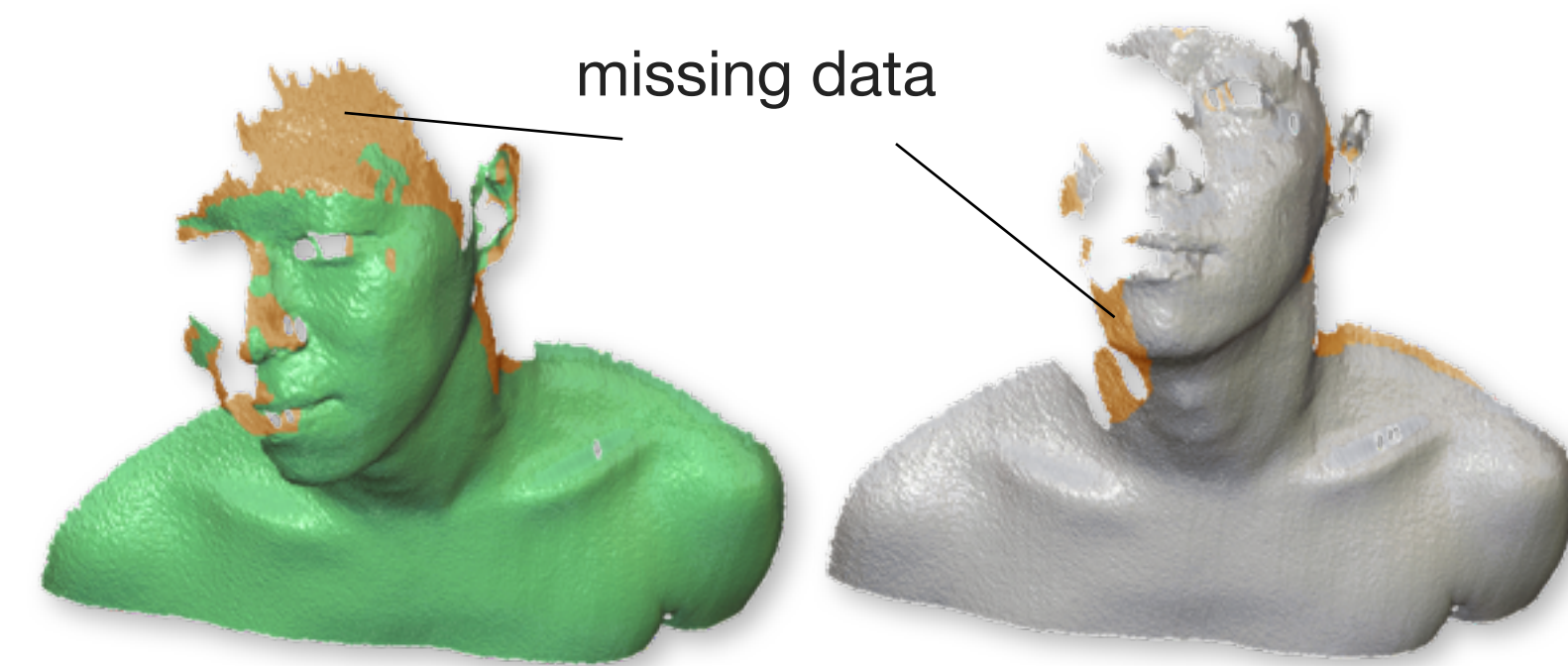
Noise

Pairwise Registration

- Issues



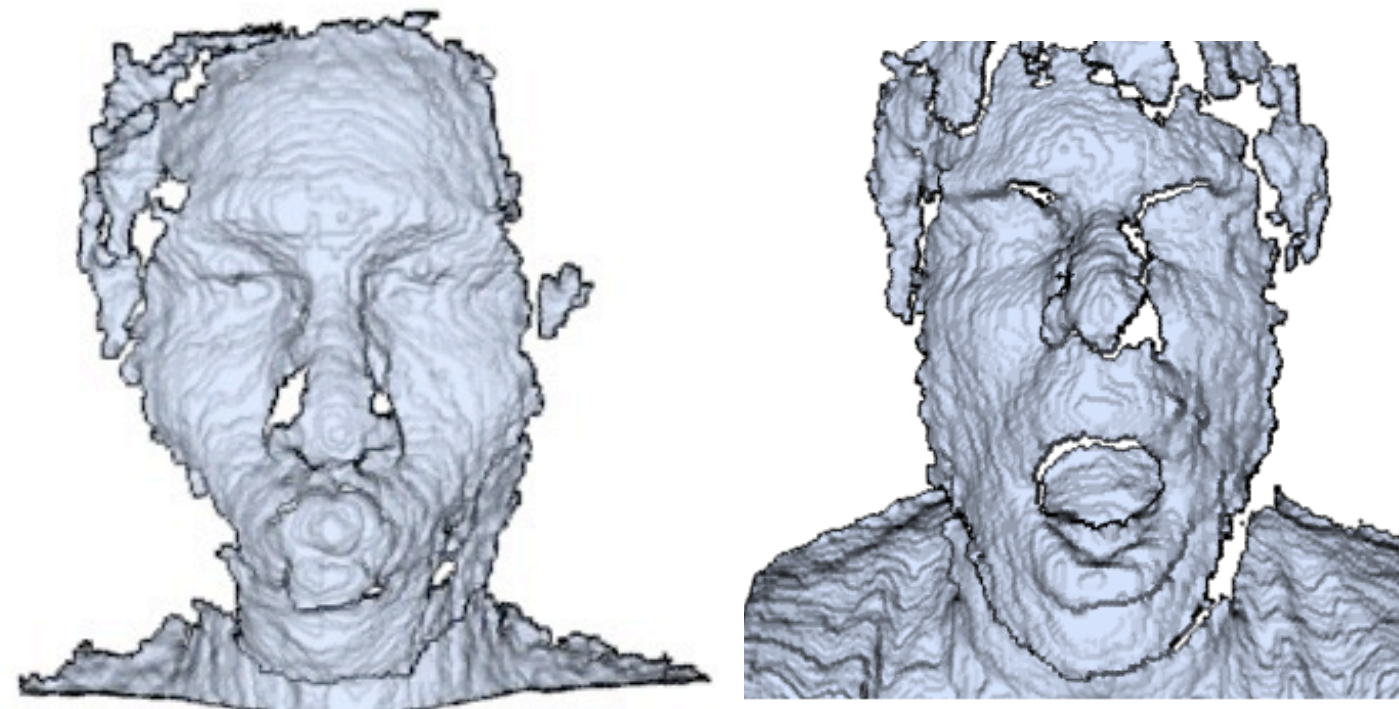
Noise



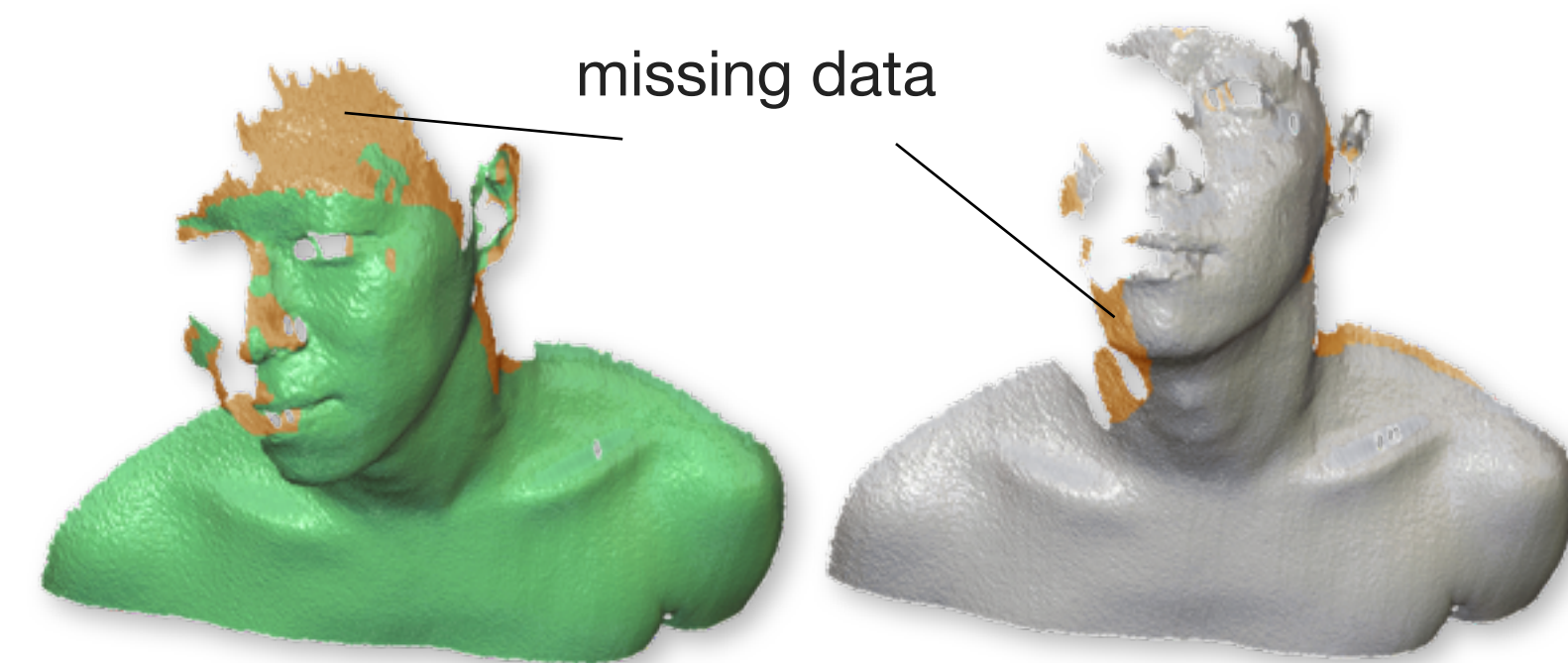
Partial matching

Pairwise Registration

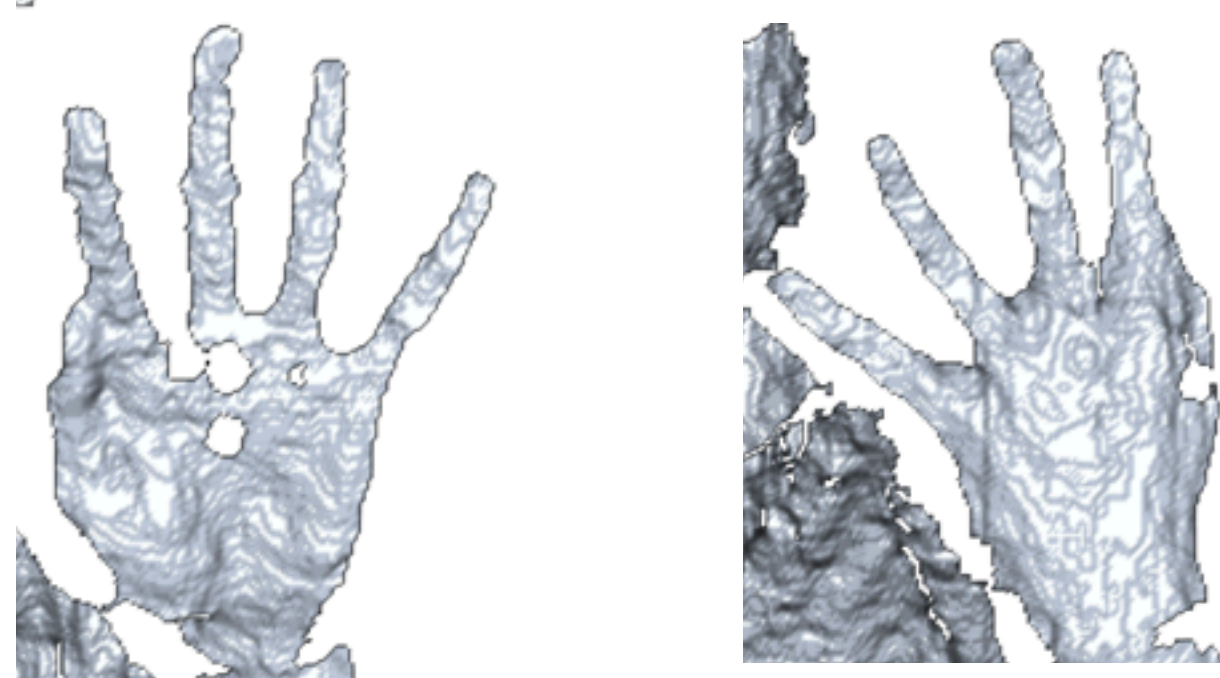
- Issues



Noise



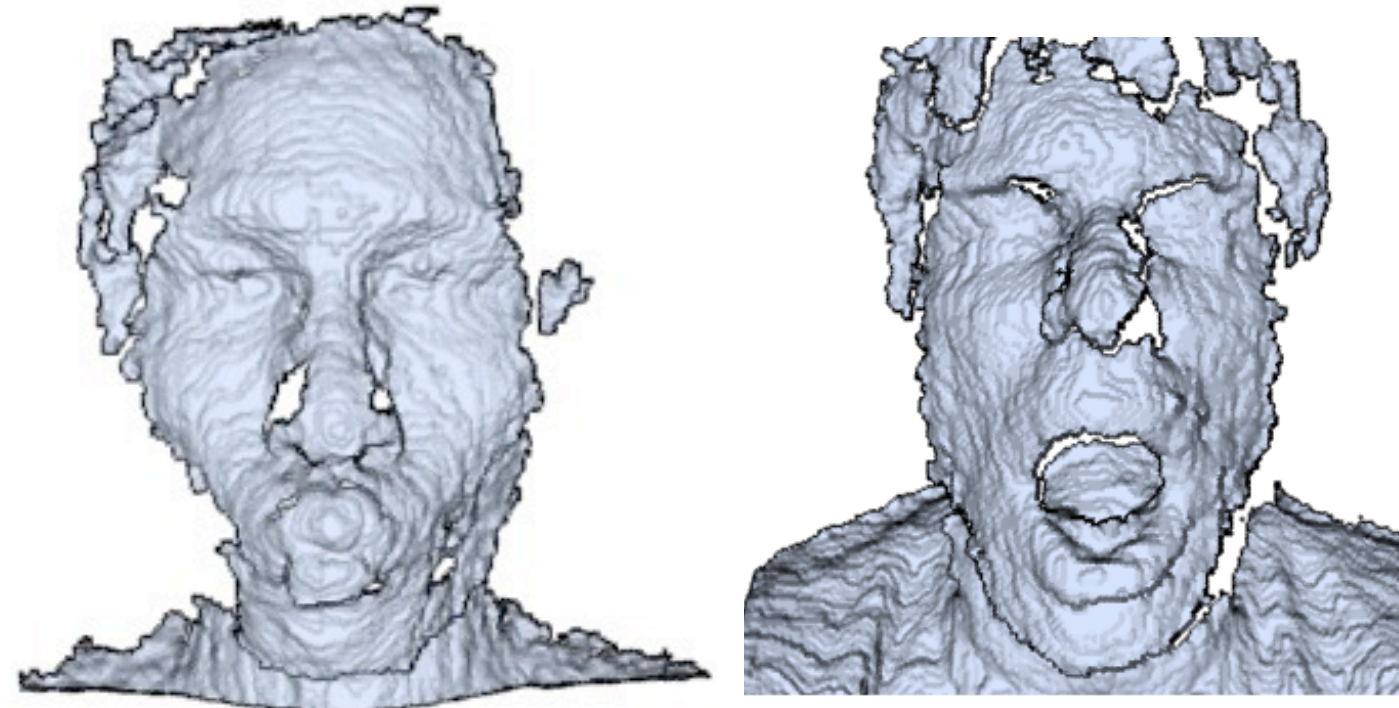
Partial matching



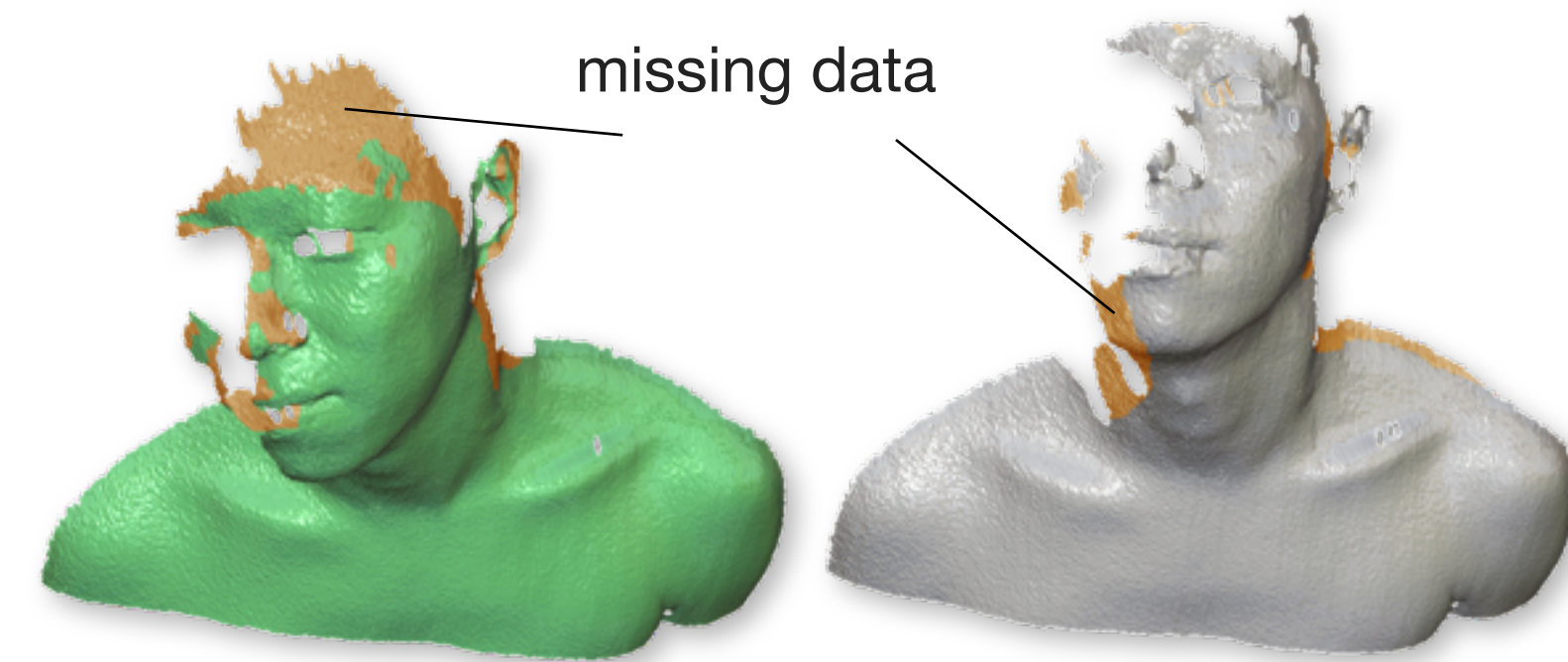
Ambiguity

Pairwise Registration

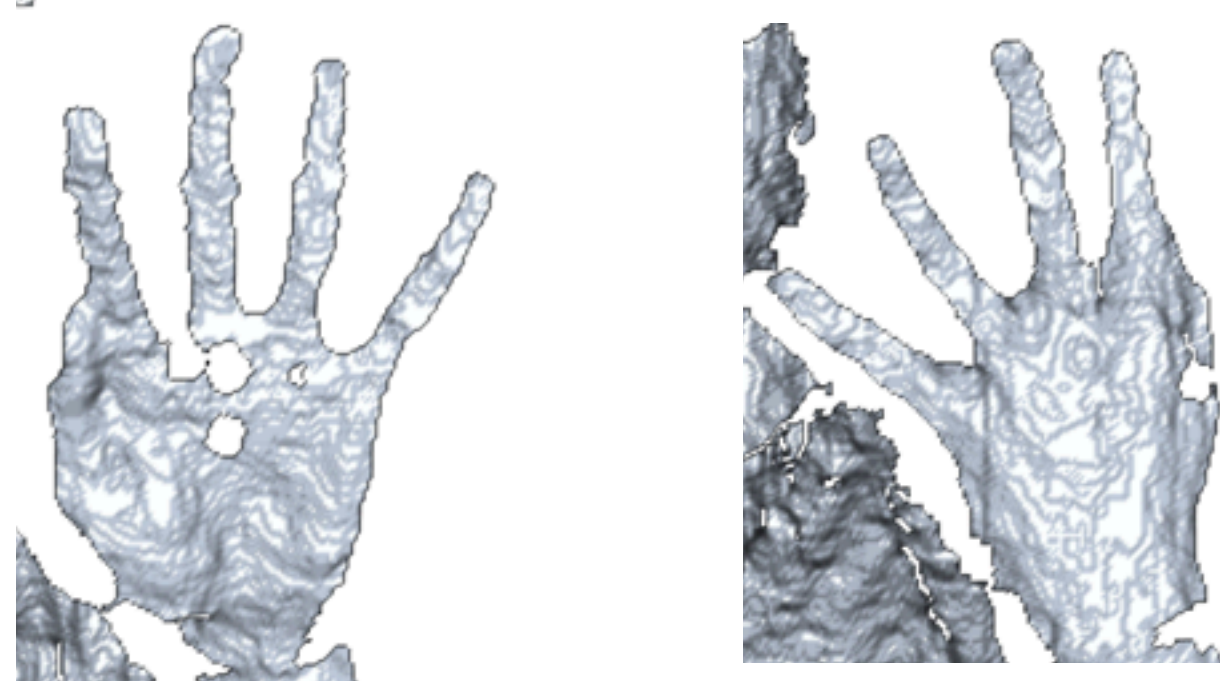
- Issues



Noise



Partial matching



Ambiguity



Illumination changes

Overview

Introduction

- **Registration**

- 3D Geometry
- 2D Images
- Combined 2D / 3D

- Applications

- Rigid Scanning
- Non-rigid Modeling
- Realtime Face Tracking

Conclusions

Registration

- Registration as energy minimization

$$\arg \min E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

Diagram illustrating the registration process as energy minimization. The equation $\arg \min E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$ is shown, with arrows pointing from the variables to their respective labels: T points to "transformation", \mathcal{X} points to "source", and \mathcal{Y} points to "target".

Registration

- Registration as energy minimization

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

*How do we measure the
quality of the alignment?*

Registration

- Registration as energy minimization

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

$$E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y}) = E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) + E_{\text{prior}}(T)$$

Alignment Error

How do we measure the quality of the alignment?

Transformation Error

What transformations are allowed / good?

Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y})$$

Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function

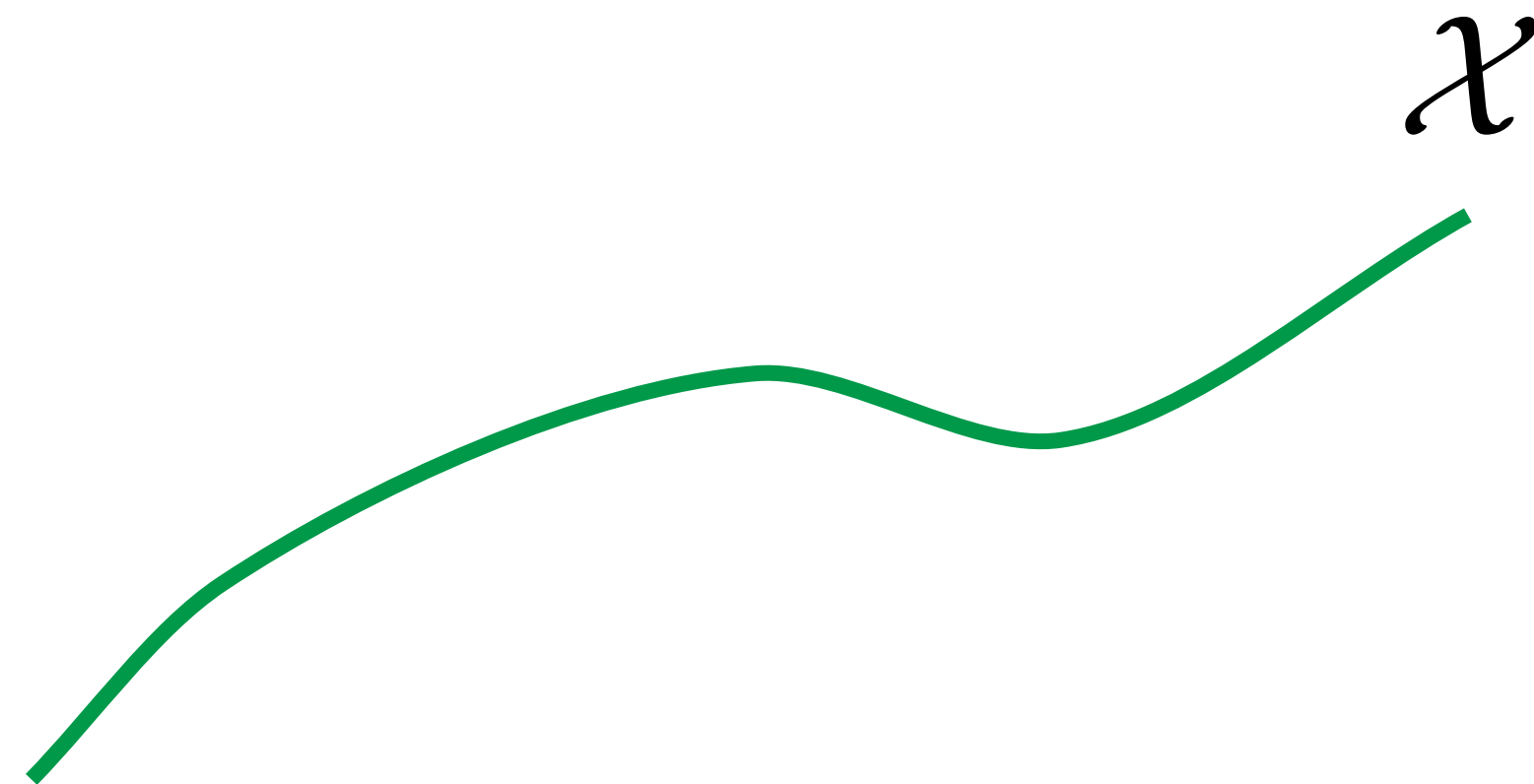
Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



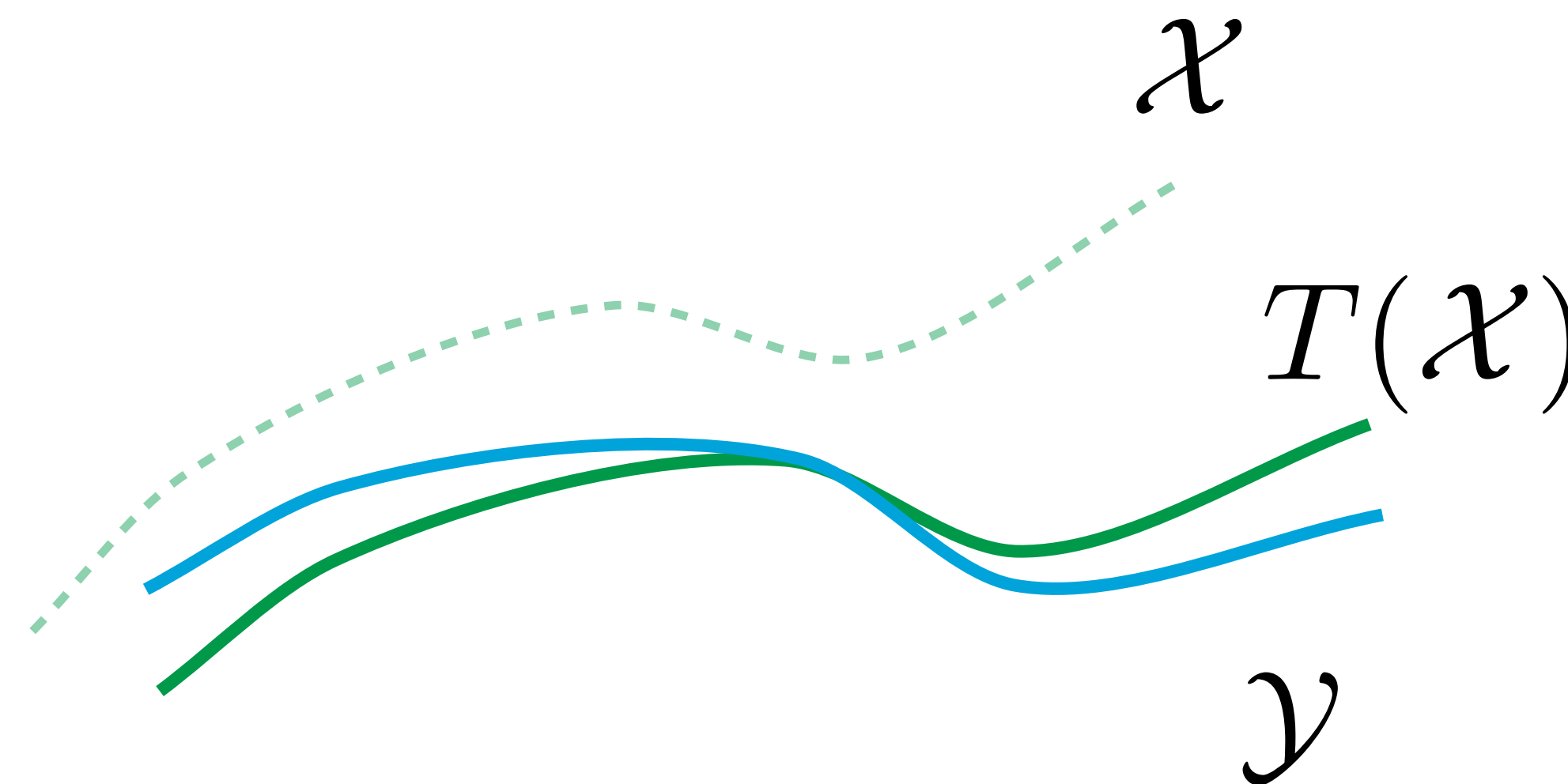
Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



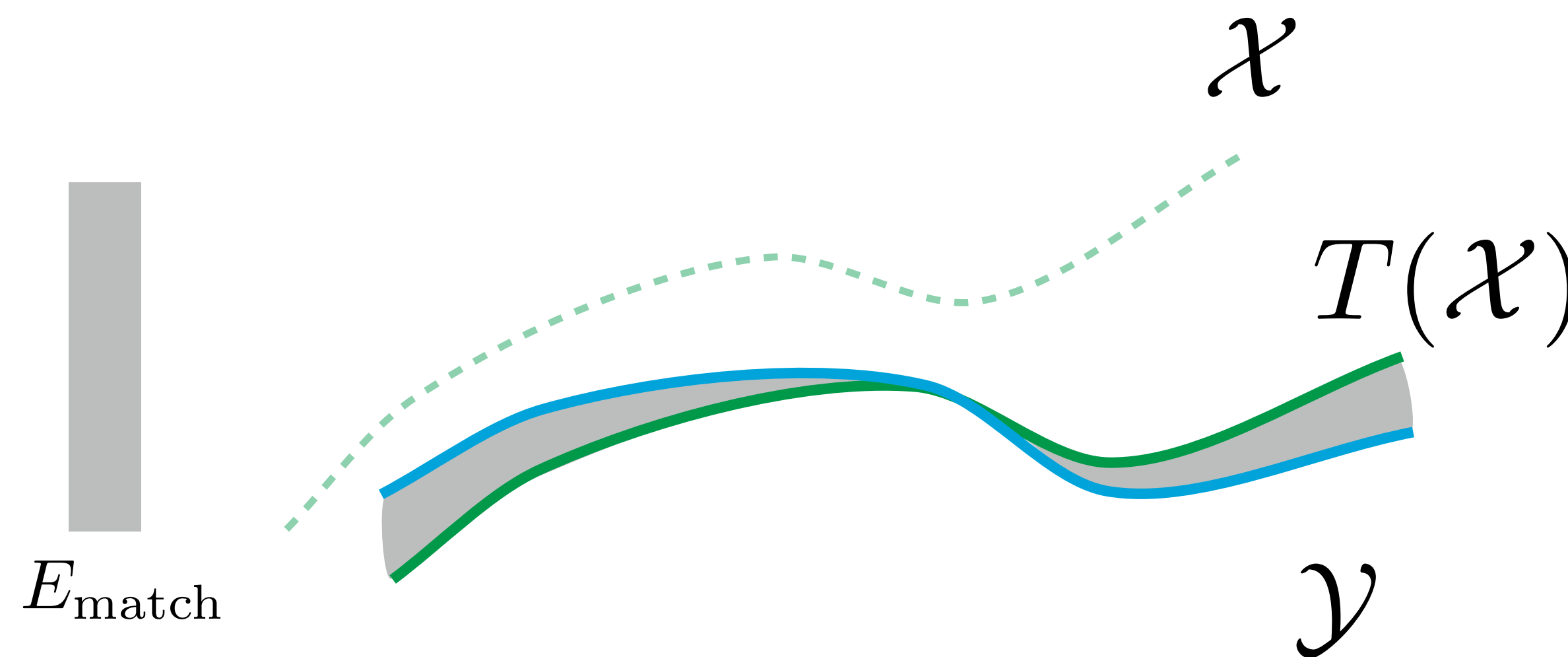
Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



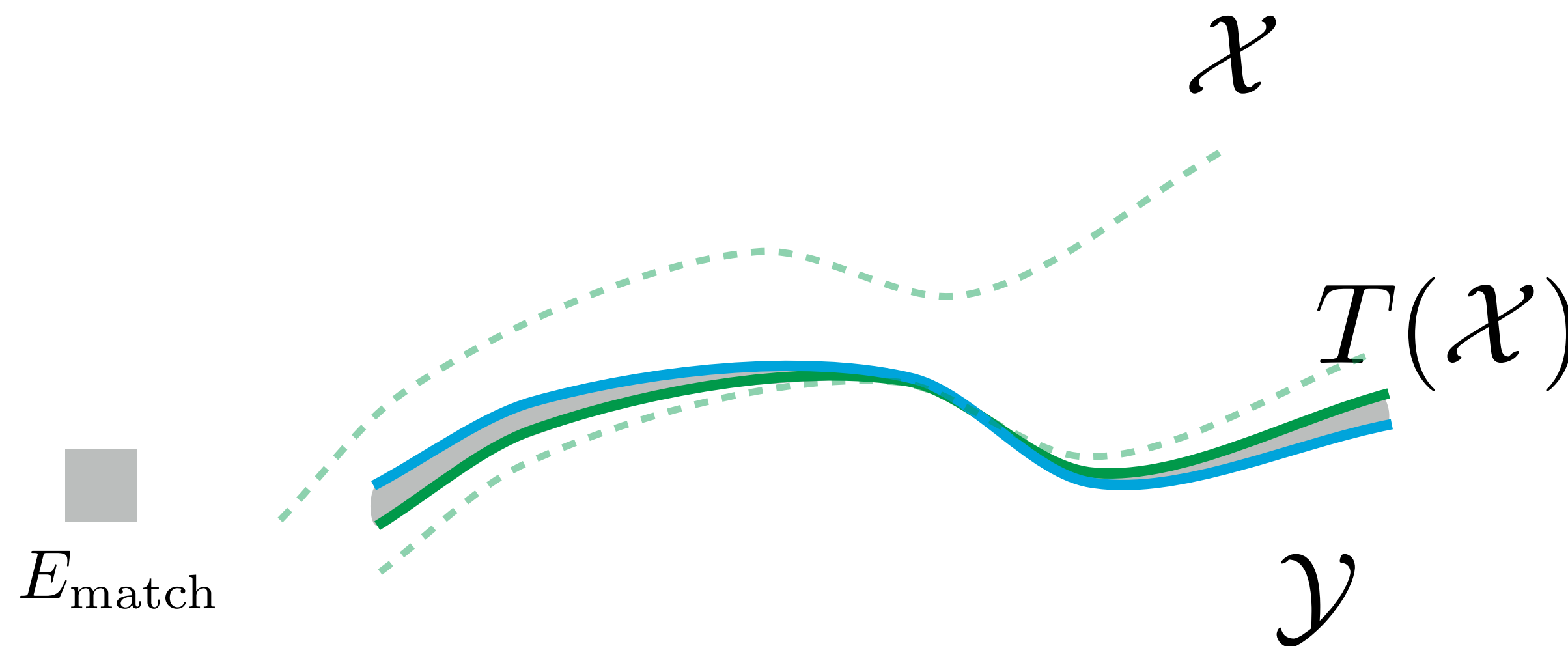
Registration

- Alignment Error

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(T, \mathcal{X}, \mathcal{Y}) = \int_{\mathcal{X}} \varphi(T(\mathbf{x}), \mathcal{Y}) d\mathbf{x}$$

distance function



Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic



articulated

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic



articulated



composite

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic



articulated



composite



fluid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion
prior



rigid



elastic



articulated



composite



fluid

Registration

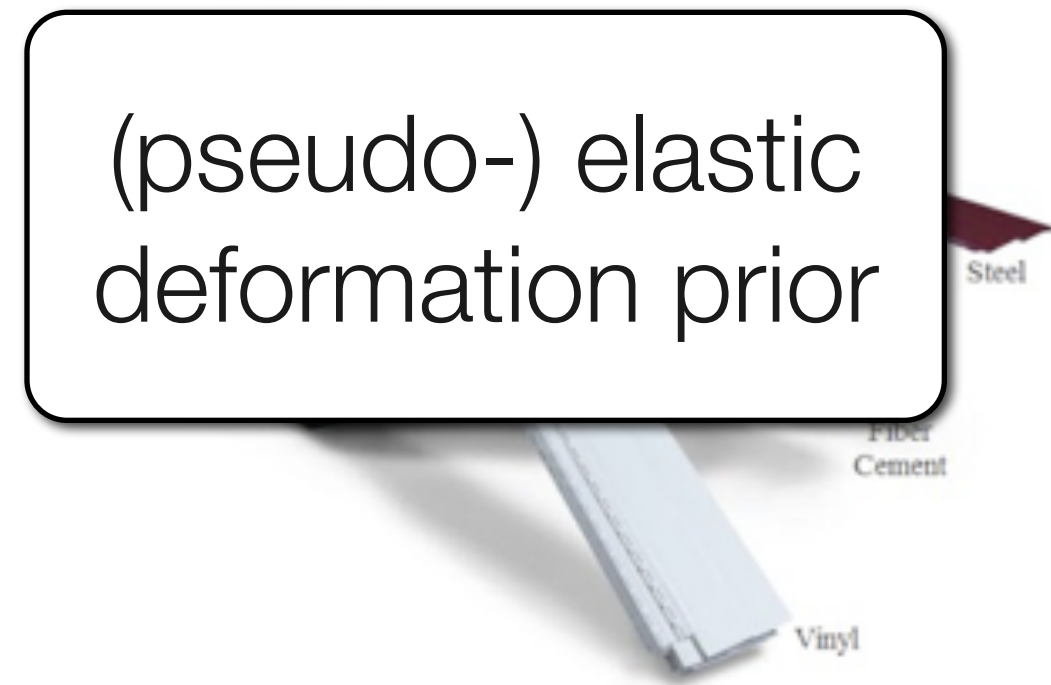
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion
prior



rigid

(pseudo-) elastic
deformation prior



elastic



articulated



composite



fluid

Registration

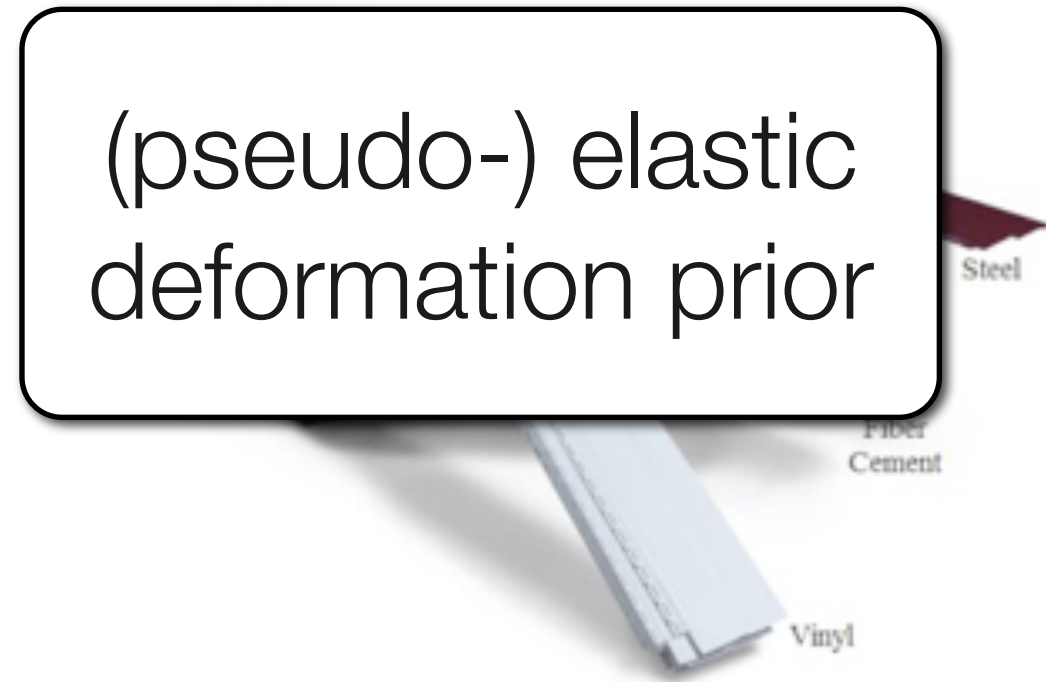
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion
prior



rigid

(pseudo-) elastic
deformation prior



elastic

skeletal model
prior



articulated



composite



fluid

Registration

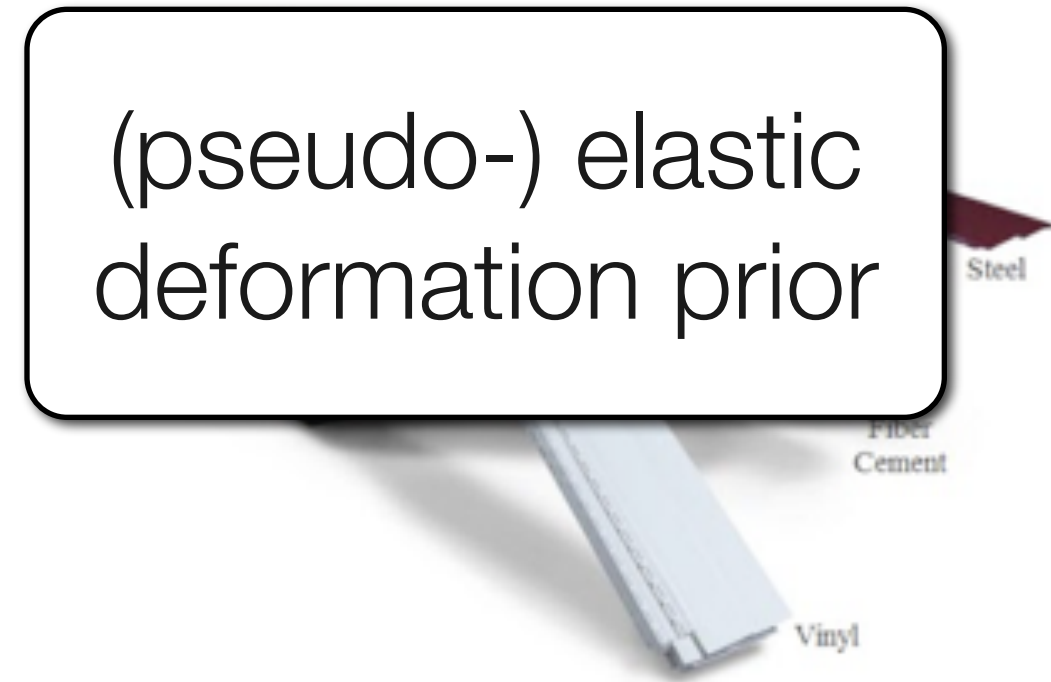
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion
prior



rigid

(pseudo-) elastic
deformation prior



elastic

skeletal model
prior



articulated

data-driven
prior



composite



fluid

Registration

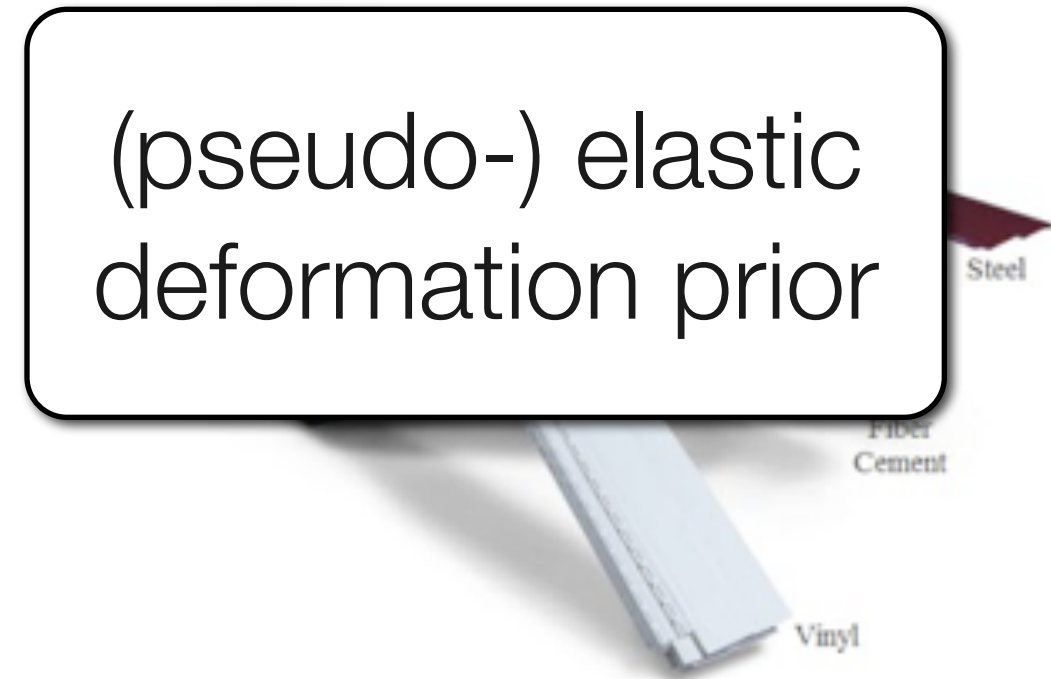
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion
prior



rigid

(pseudo-) elastic
deformation prior



elastic

skeletal model
prior



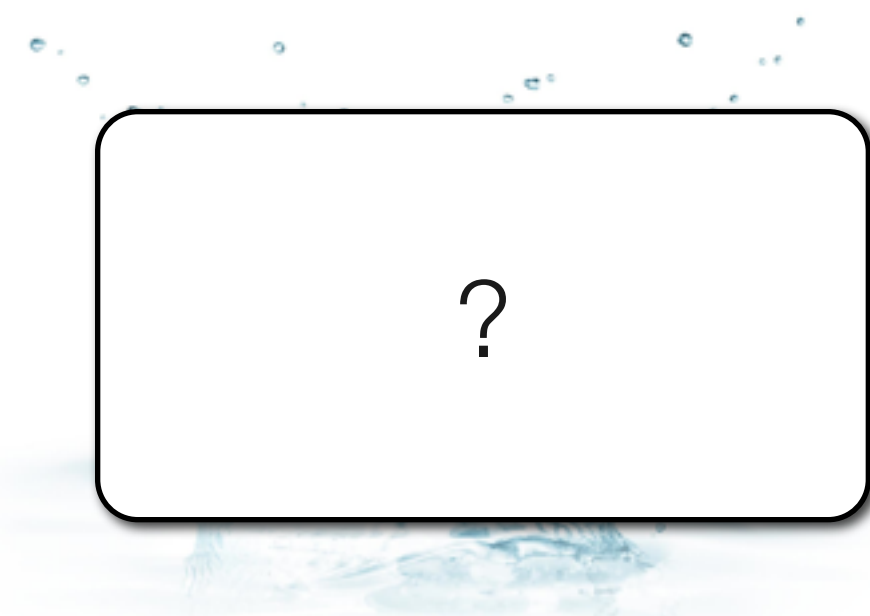
articulated

data-driven
prior



composite

?



fluid

Registration

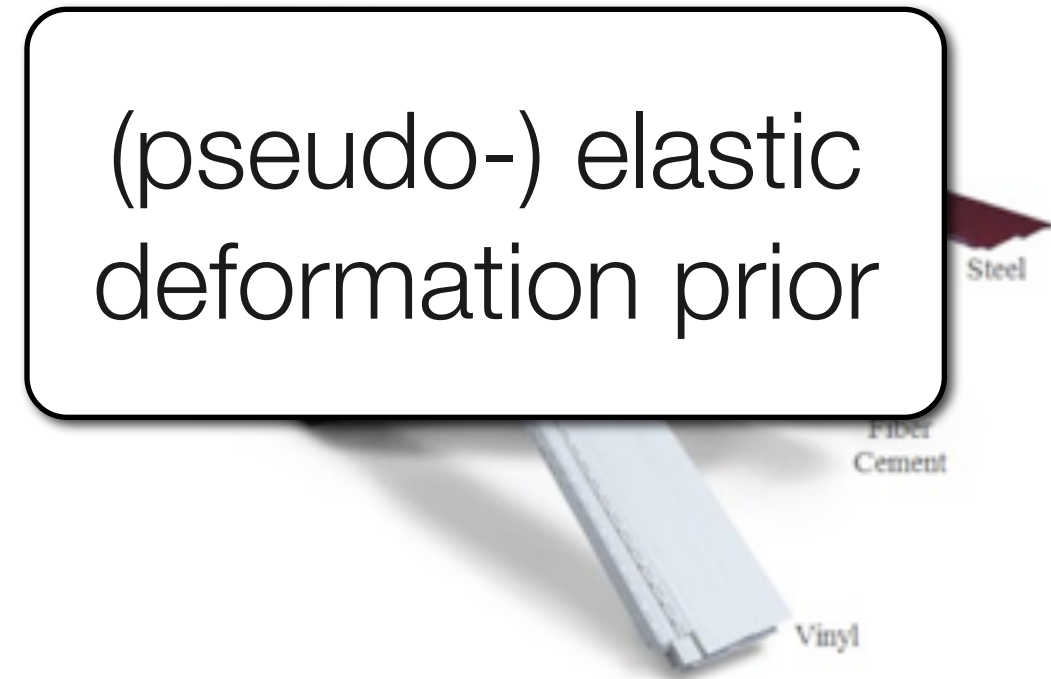
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

rigid motion
prior



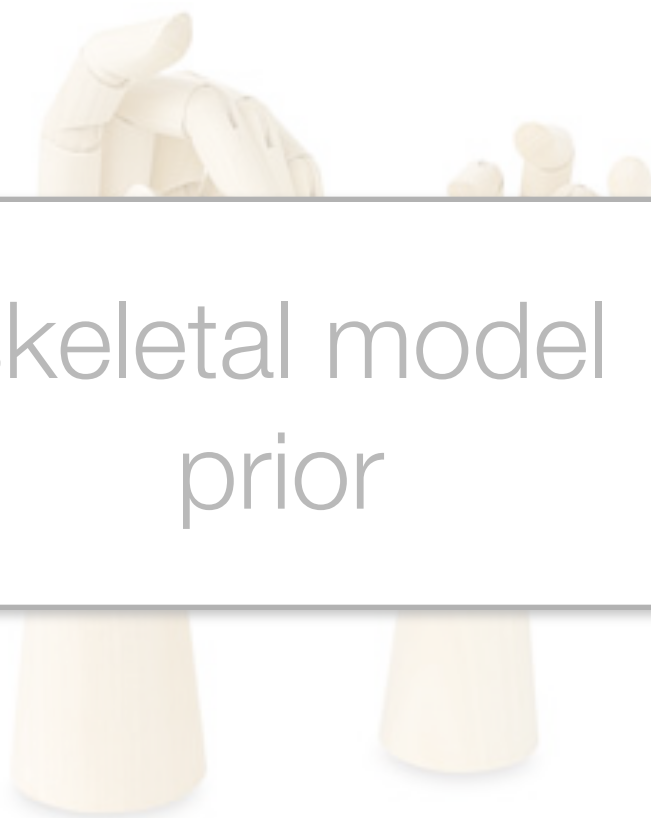
rigid

(pseudo-) elastic
deformation prior



elastic

skeletal model
prior



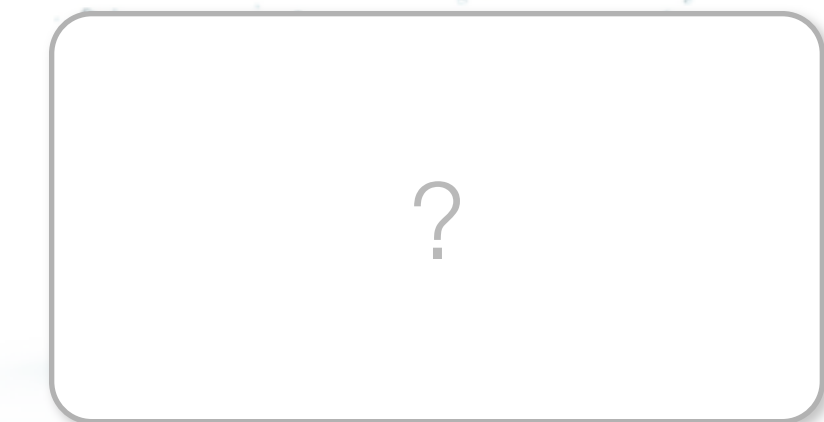
articulated

data-driven
prior



composite

?



fluid

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source

target

transformation

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

The diagram illustrates the energy minimization process. The expression $\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$ is shown. Three arrows point to the variables in the function: one from 'source' to \mathcal{X} , one from 'target' to \mathcal{Y} , and one from 'transformation' to T .

Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear
 - common solution: linearize, iterate

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

The diagram illustrates the variables in the registration energy minimization equation. It shows the expression $\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$. Three arrows point to the variables: one from the word 'source' to \mathcal{X} , one from the word 'target' to \mathcal{Y} , and one from the word 'transformation' to T .

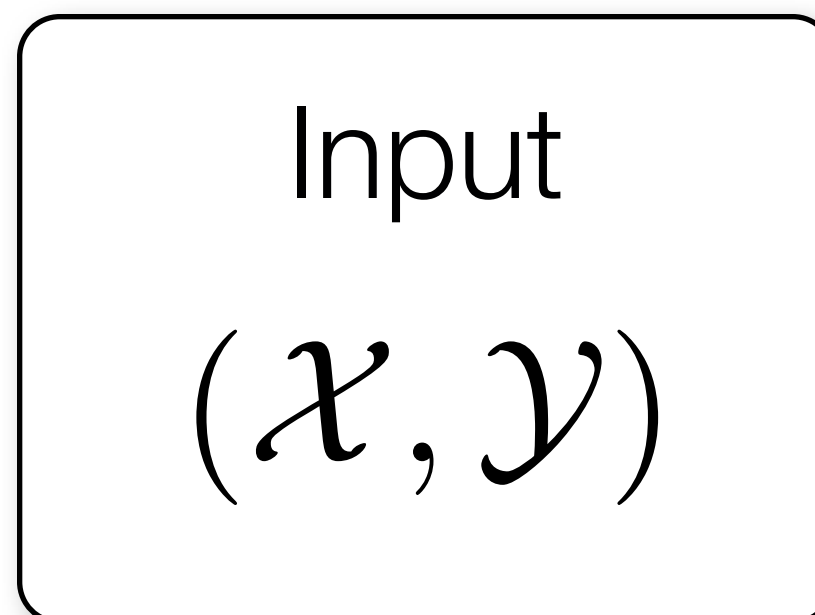
Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear
 - common solution: linearize, iterate

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

Diagram illustrating the registration process: The expression $\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$ is shown. Arrows point from the labels "source" and "target" to the variables \mathcal{X} and \mathcal{Y} respectively. An arrow points from the label "transformation" to the variable T .



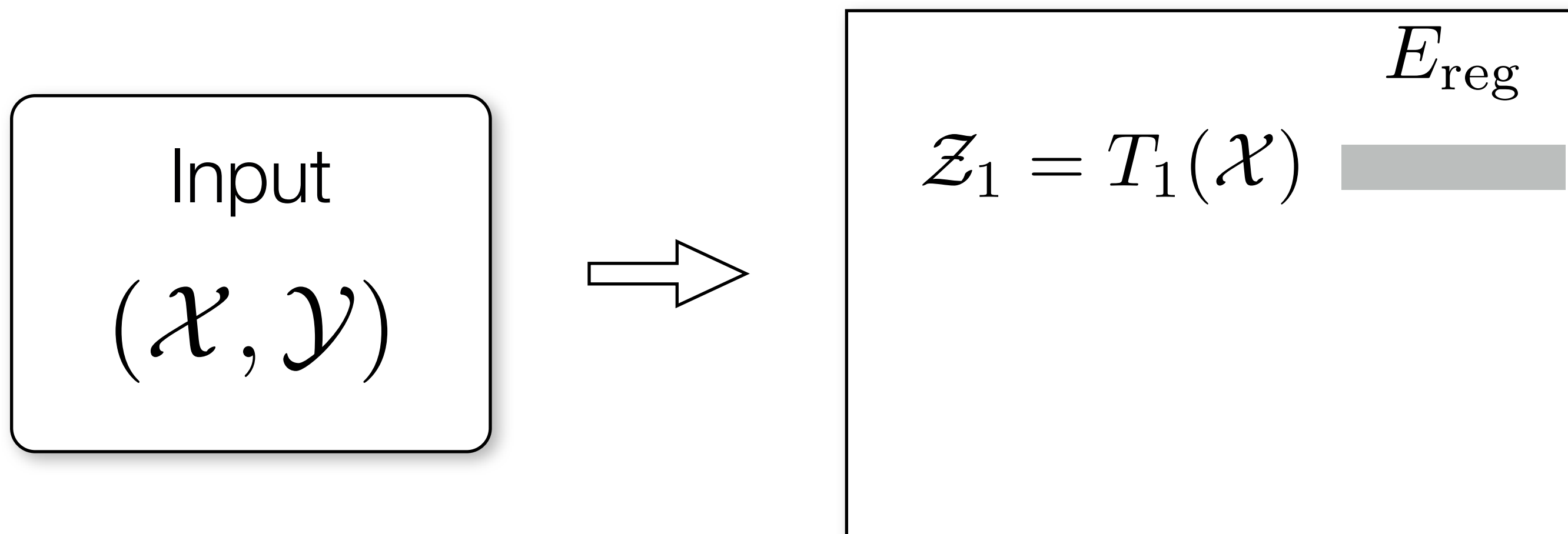
Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear
 - common solution: linearize, iterate

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source → \mathcal{X}
target → \mathcal{Y}
transformation → T



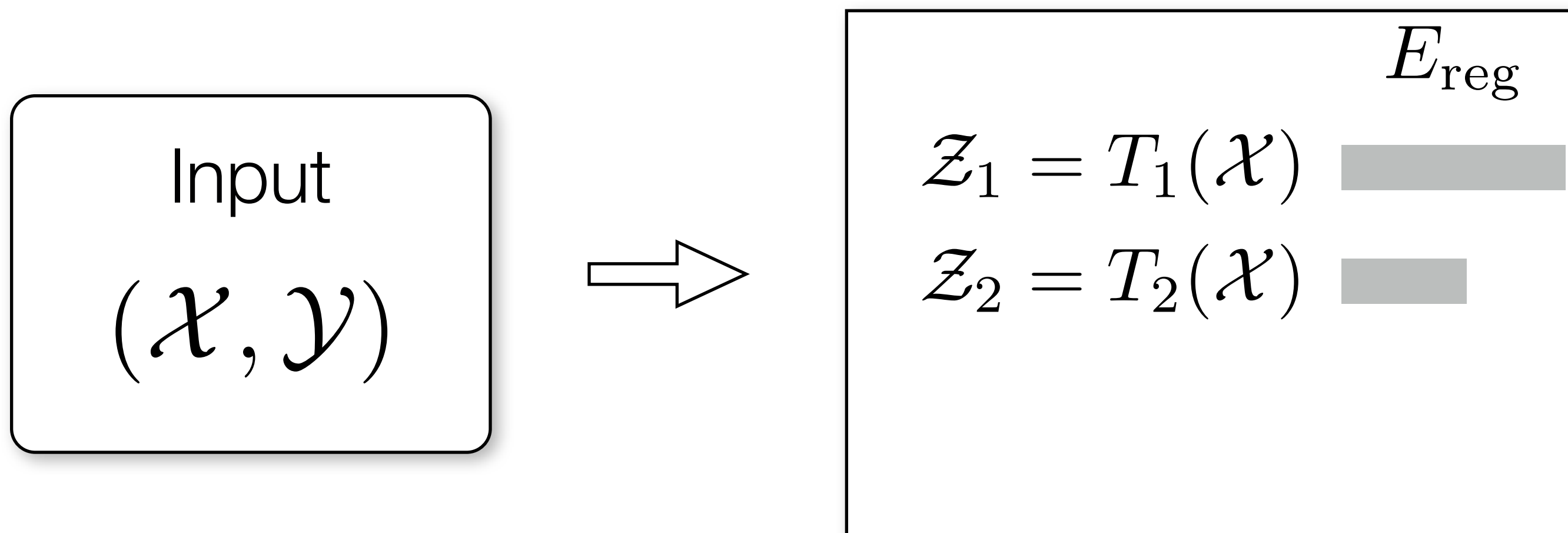
Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear
 - common solution: linearize, iterate

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source → \mathcal{X}
target → \mathcal{Y}
transformation → T



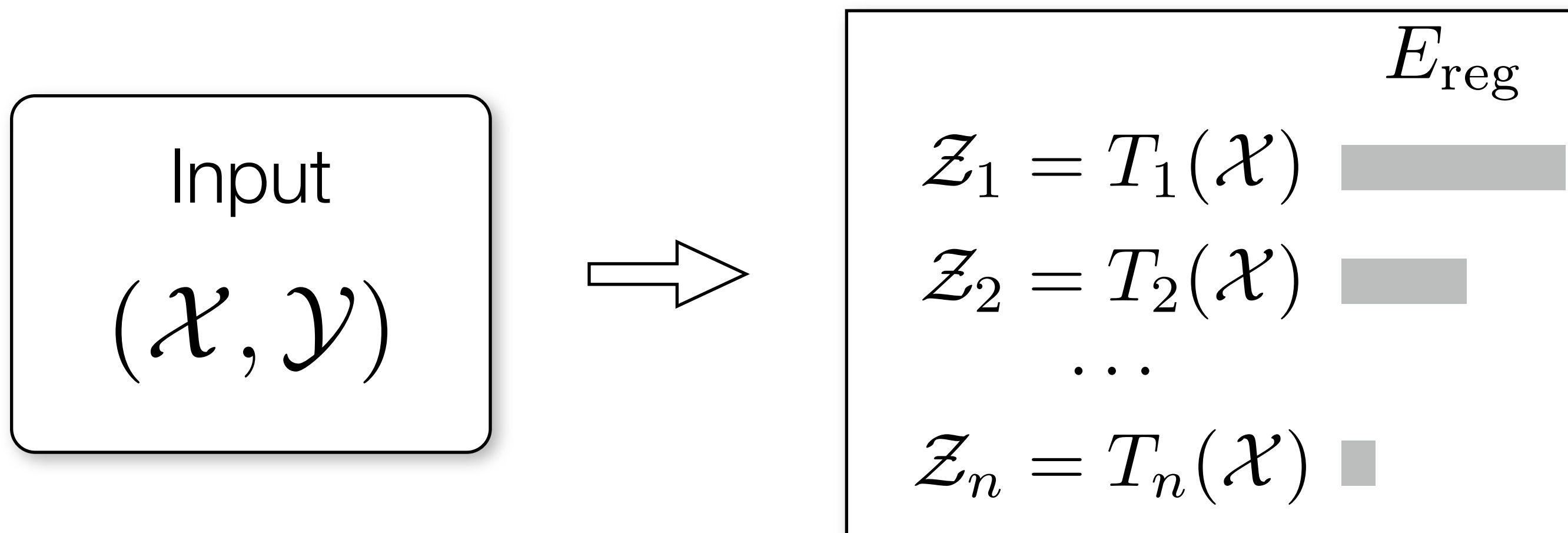
Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear
 - common solution: linearize, iterate

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source → \mathcal{X}
target → \mathcal{Y}
transformation → T



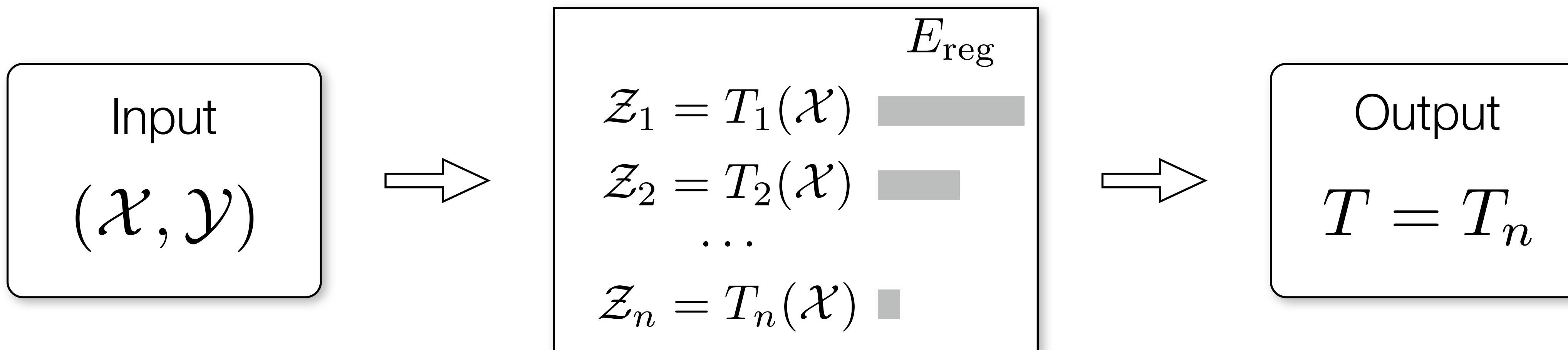
Registration

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Registration as energy minimization
 - typically non-linear
 - common solution: linearize, iterate

$$\arg \min_T E_{\text{reg}}(T, \mathcal{X}, \mathcal{Y})$$

source
target
transformation



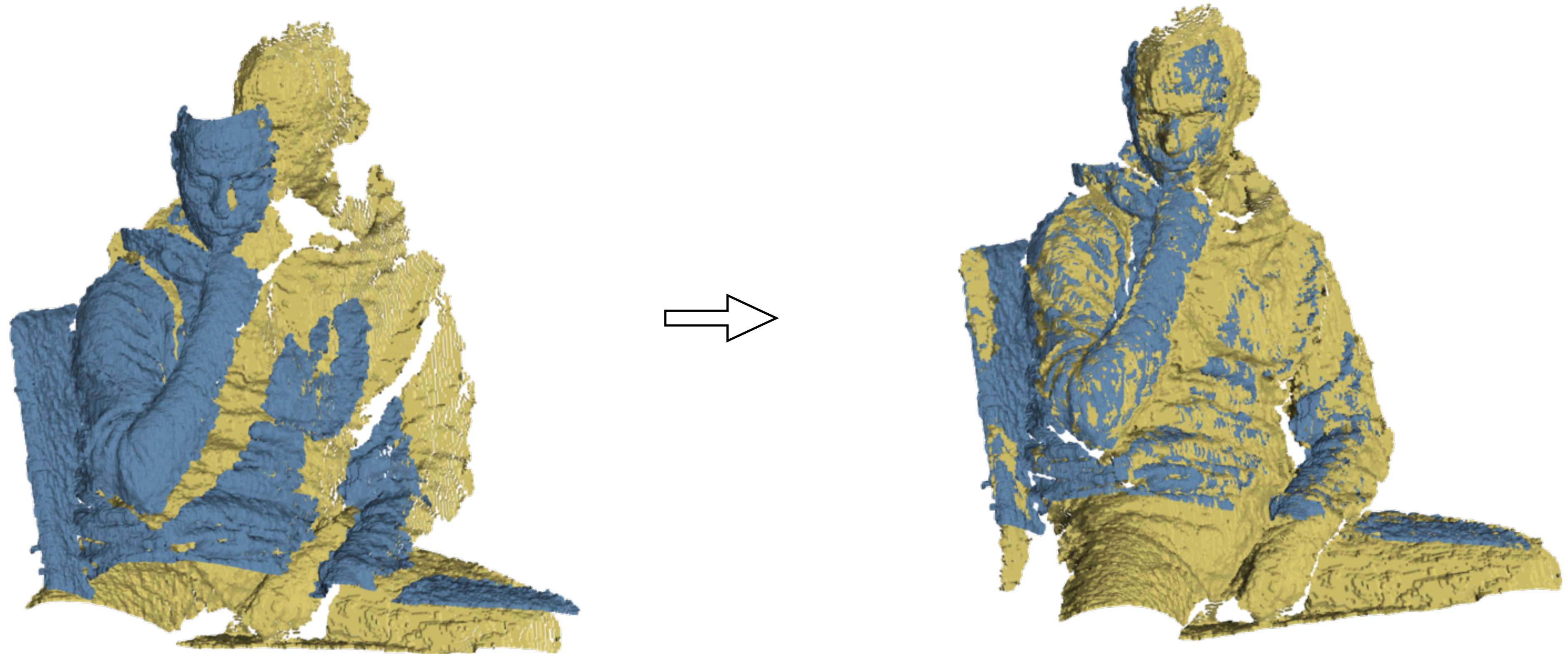
Overview

Introduction

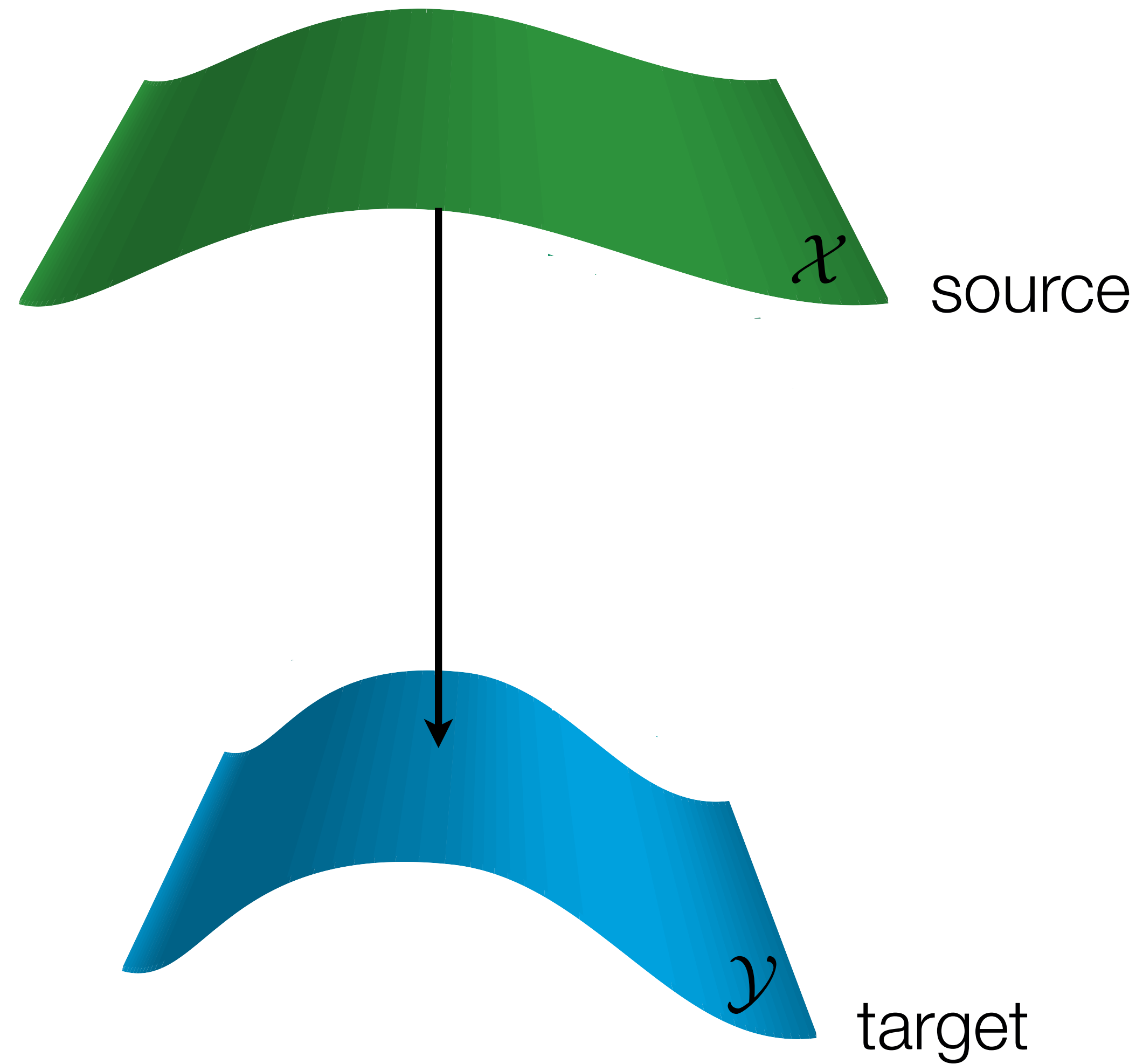
- Registration
 - **3D Geometry**
 - 2D Images
 - Combined 2D / 3D
- Applications
 - Rigid Scanning
 - Non-rigid Modeling
 - Realtime Face Tracking

Conclusions

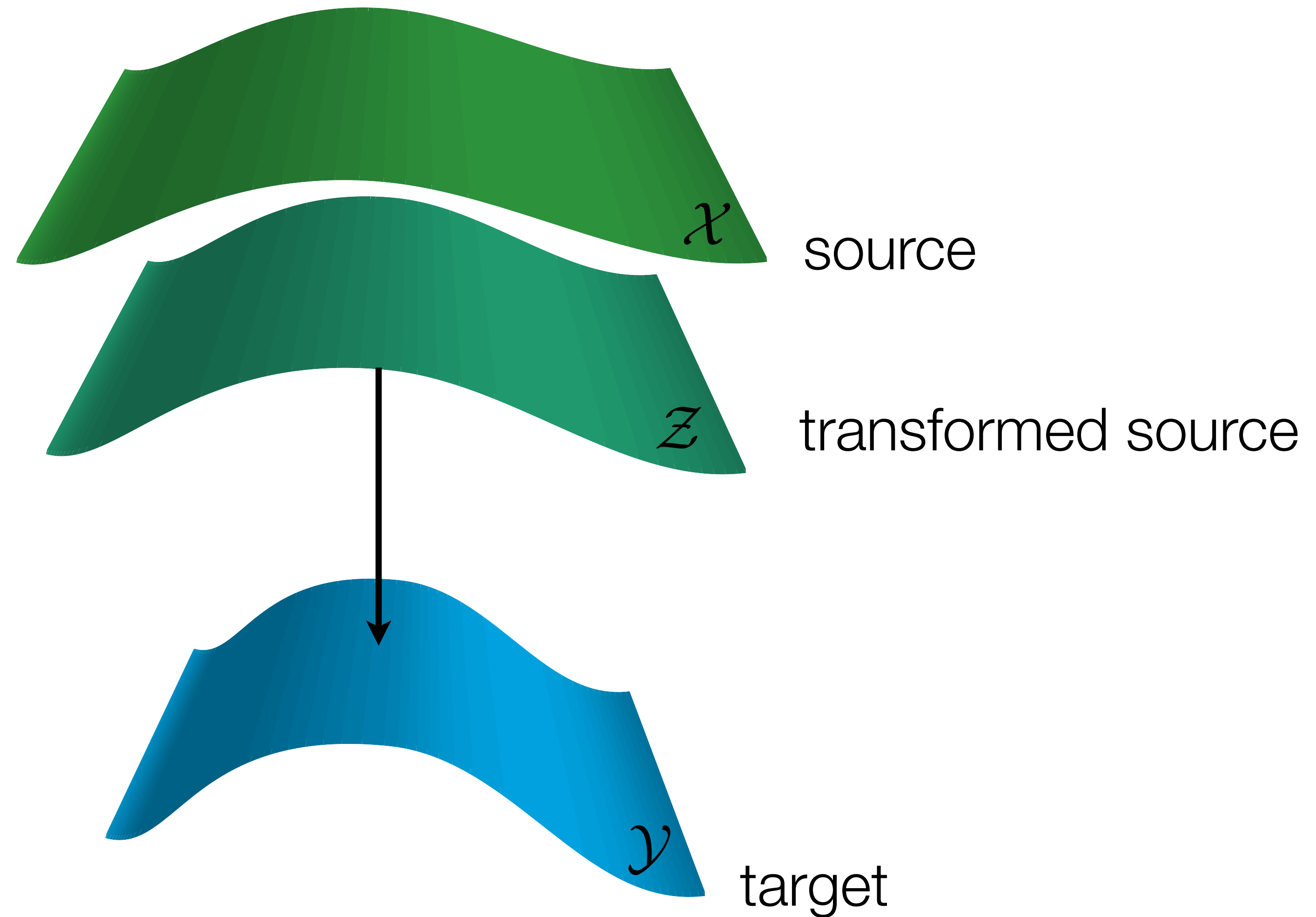
3D Registration



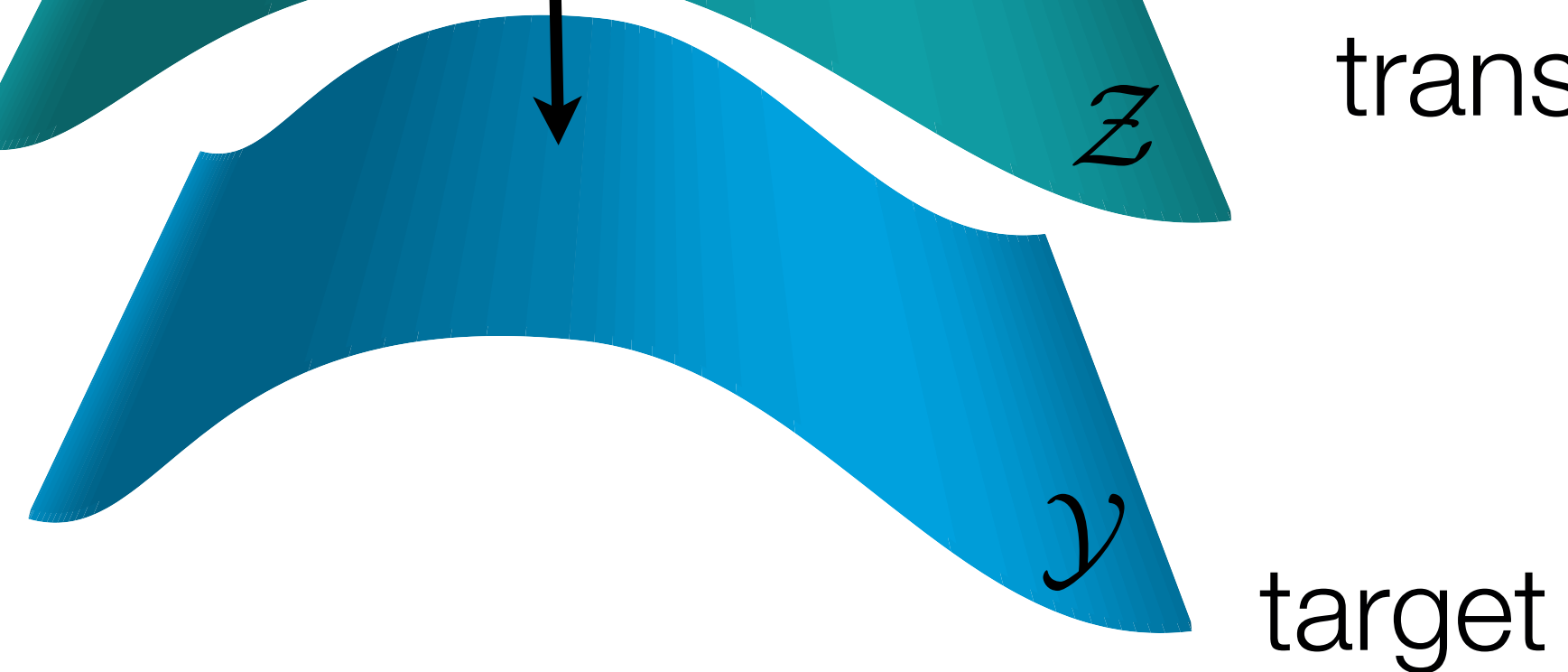
3D Registration



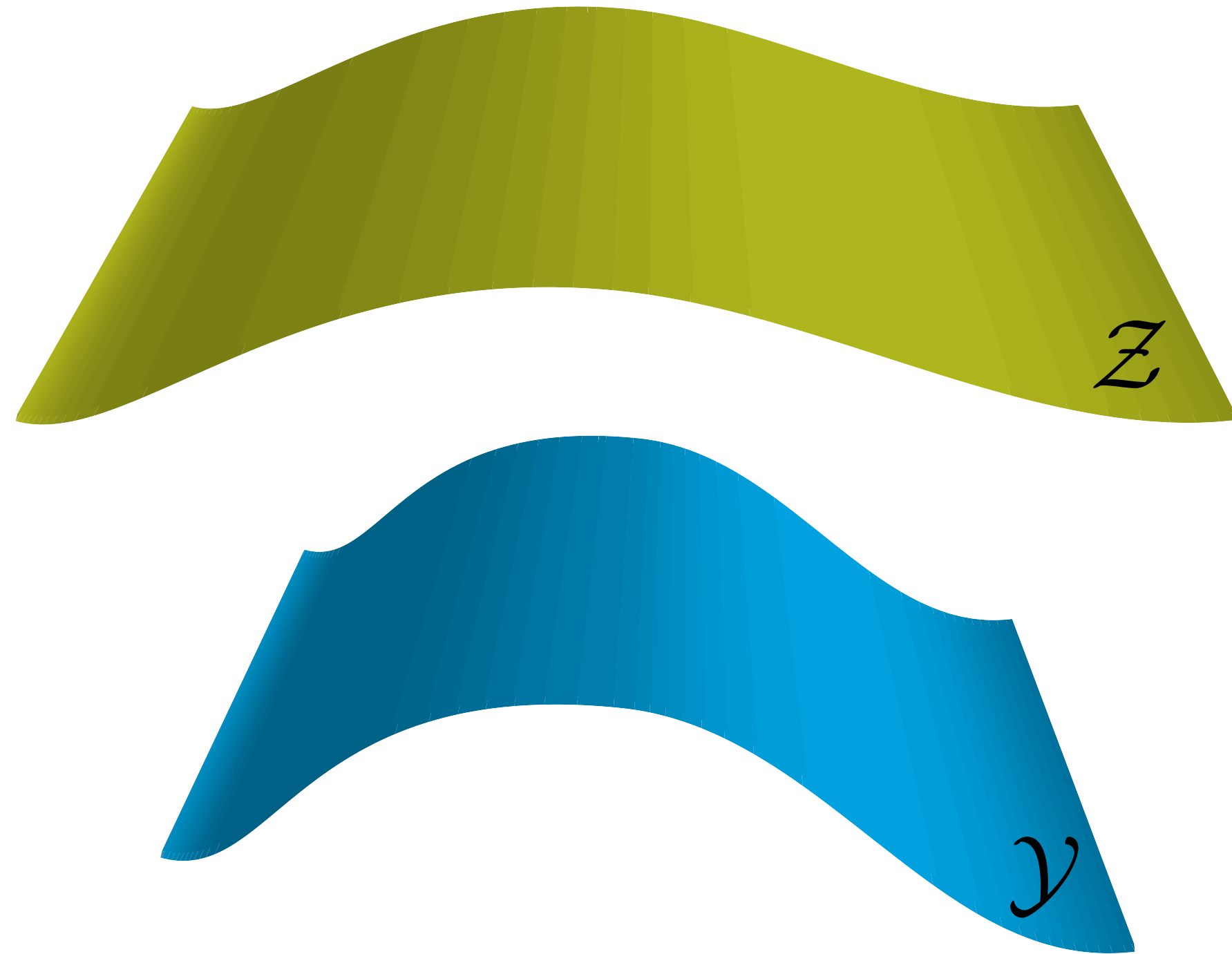
3D Registration



3D Registration



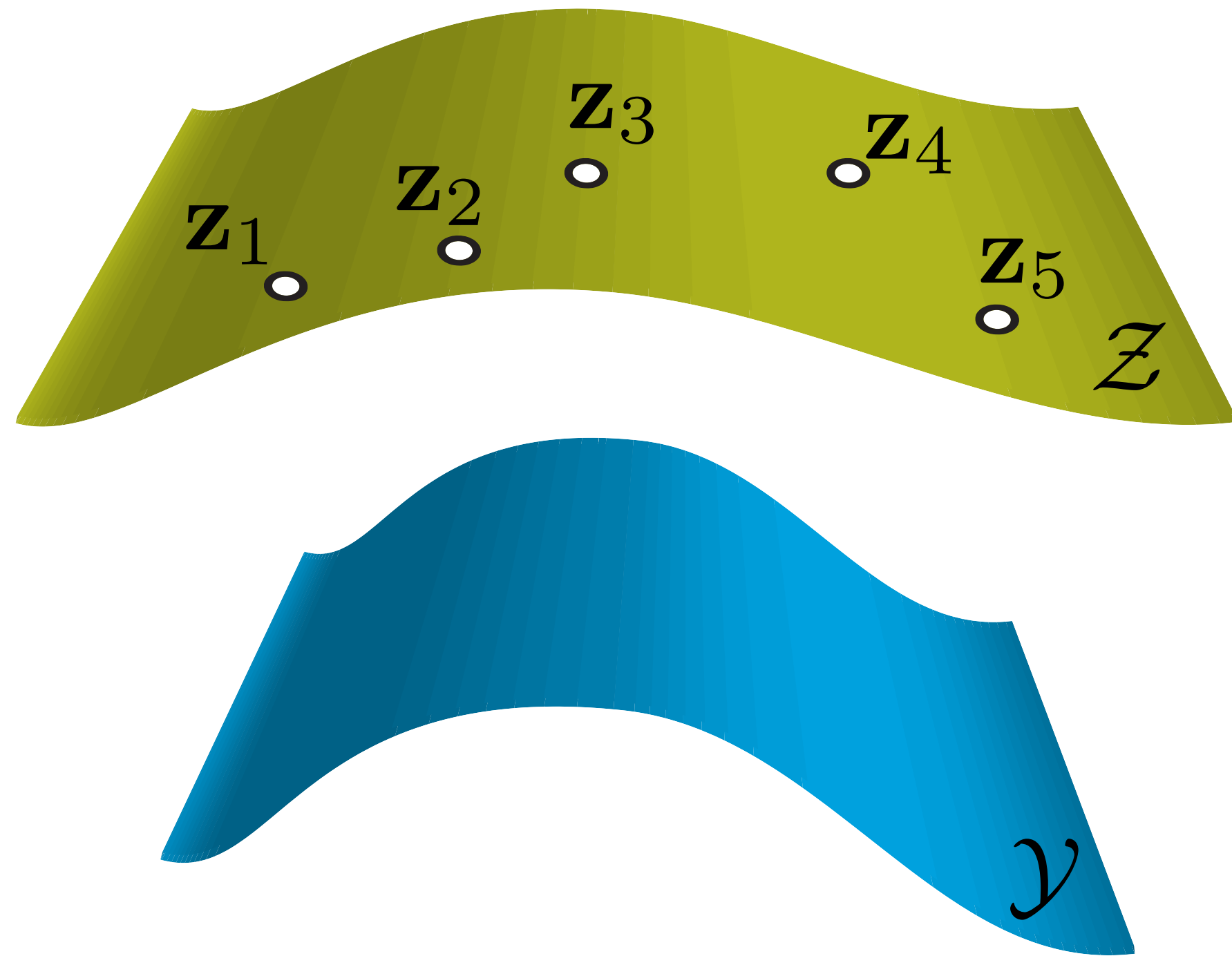
3D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$

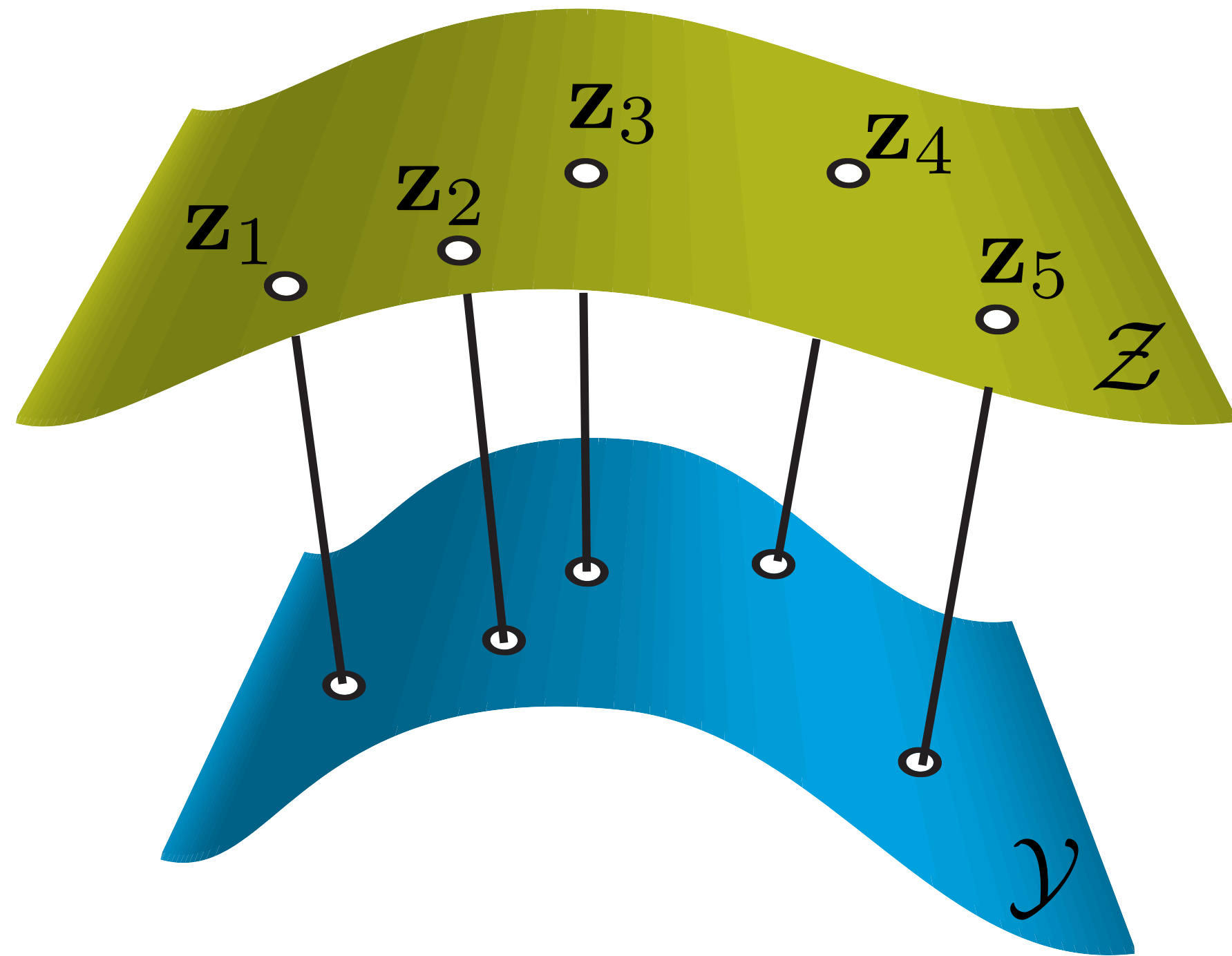
3D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$

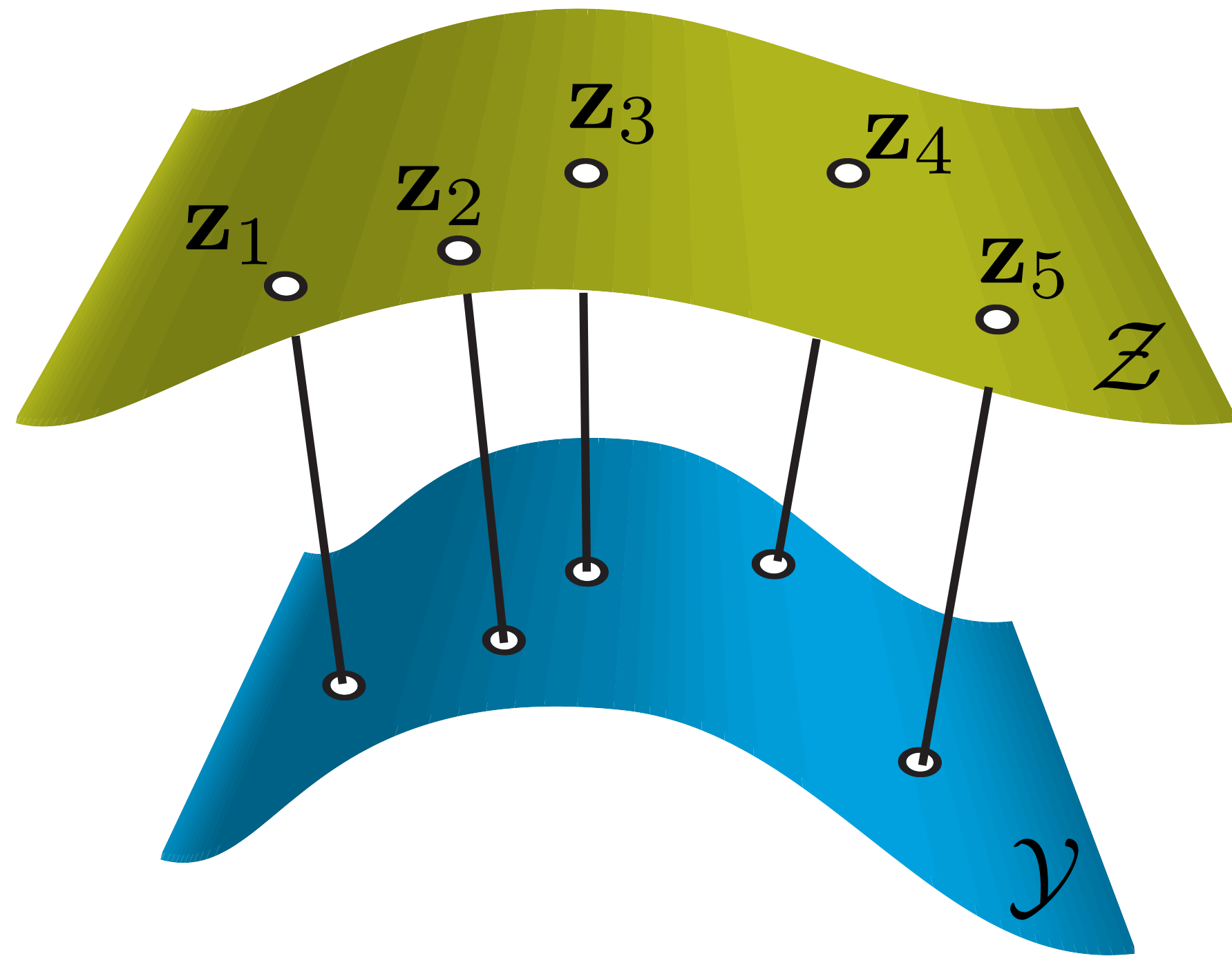
3D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$

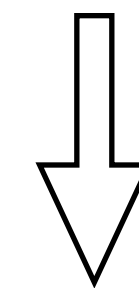
3D Registration - Matching



discretized matching cost based on point correspondences

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(\mathcal{Z}) = \int_{\mathcal{Z}} \varphi(\mathbf{z}, \mathcal{Y}) d\mathbf{z}$$



$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

3D Registration - Matching

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

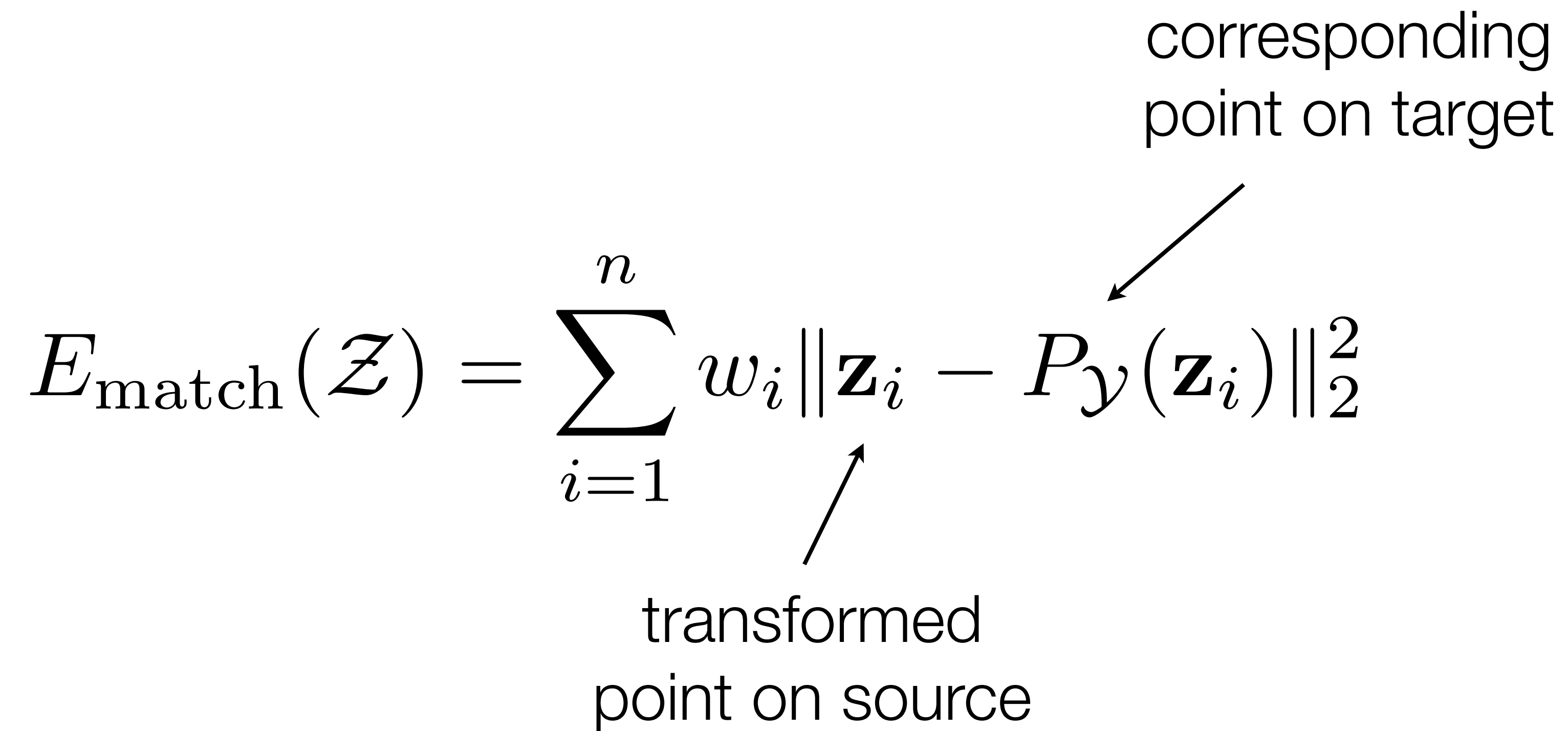
3D Registration - Matching

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

↑
transformed
point on source

3D Registration - Matching

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$



transformed
point on source

corresponding
point on target

3D Registration - Matching

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

correspondence weight
corresponding point on target

transformed point on source

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
 - Step 1: find correspondences using closest points for fixed transformation

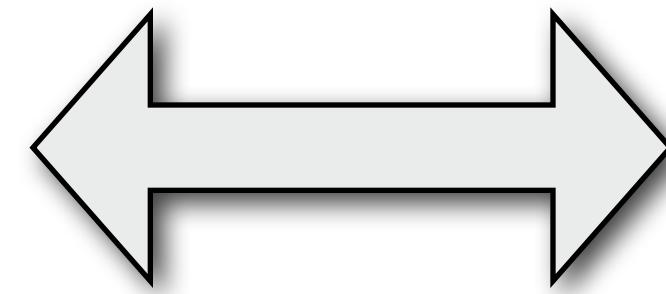
3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
 - Step 1: find correspondences using closest points for fixed transformation
 - Step 2: find best rigid transformation for fixed correspondences

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

- Step 1: find correspondences using closest points for fixed transformation



- Step 2: find best rigid transformation for fixed correspondences

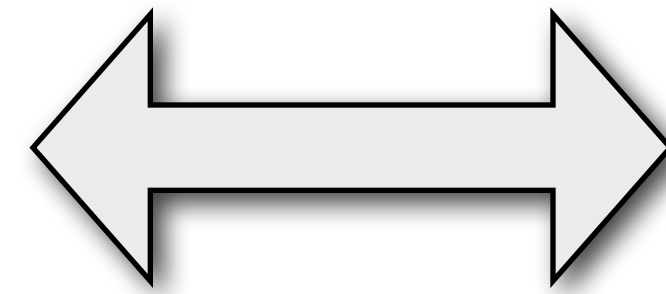
Iterate until
convergence

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

- Step 1: find correspondences using closest points for fixed transformation

→ efficient data structures



Iterate until
convergence

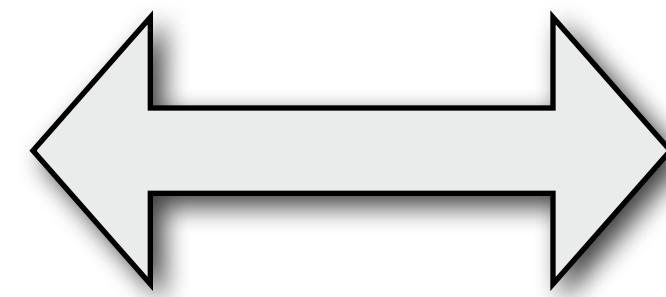
- Step 2: find best rigid transformation for fixed correspondences

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

- Step 1: find correspondences using closest points for fixed transformation

→ efficient data structures



Iterate until
convergence

- Step 2: find best rigid transformation for fixed correspondences

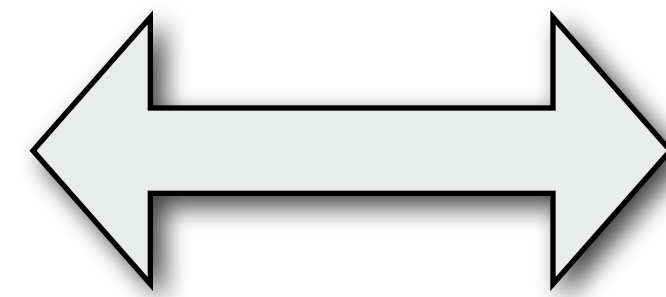
→ closed form solution

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

- Step 1: find correspondences using closest points for fixed transformation

→ efficient data structures



Iterate until
convergence

- Step 2: find best rigid transformation for fixed correspondences

→ closed form solution

DEMO

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
 - (approximate) closest points → (more) efficient data structures

3D Registration - Example

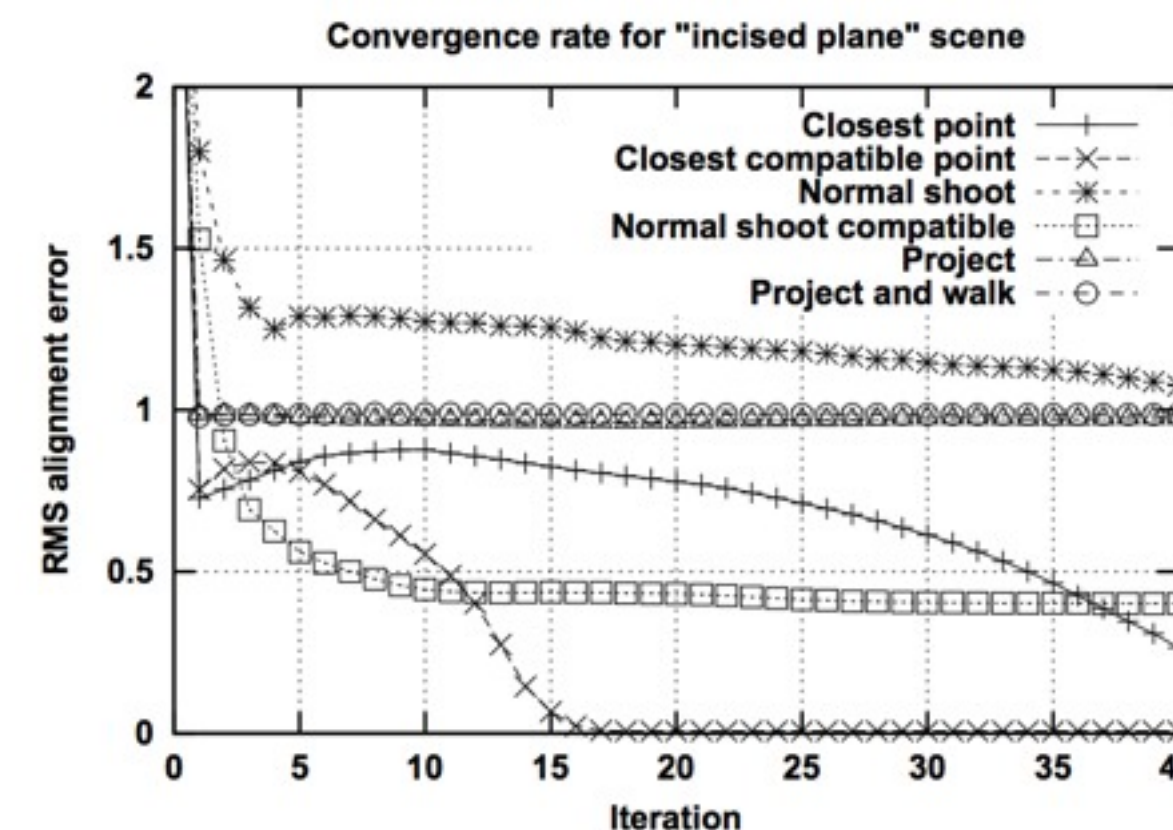
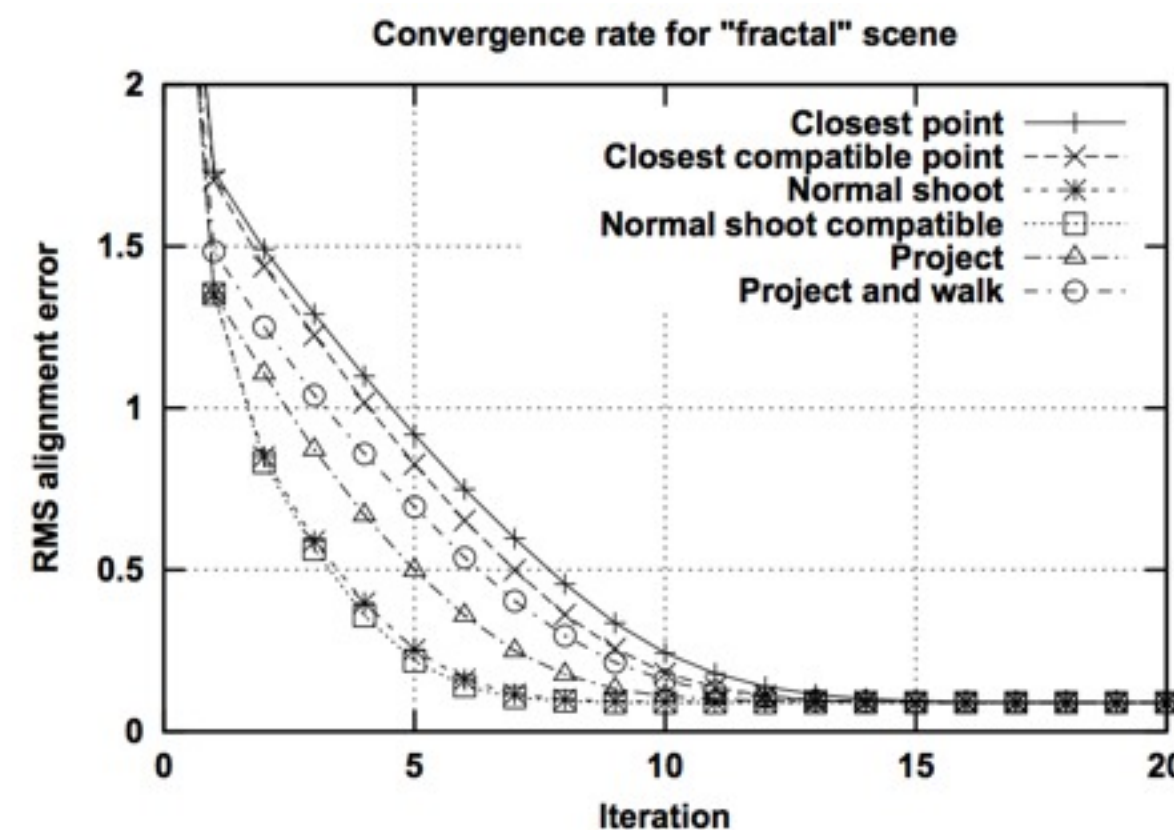
- Iterative Closest Point (ICP) Algorithm
 - (approximate) closest points → (more) efficient data structures
 - weight accounts for importance (area measure) and confidence

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
 - (approximate) closest points → (more) efficient data structures
 - weight accounts for importance (area measure) and confidence
 - heuristics to prune or down-weight bad correspondences

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm
 - (approximate) closest points → (more) efficient data structures
 - weight accounts for importance (area measure) and confidence
 - heuristics to prune or down-weight bad correspondences



Rusinkiewicz, Levoy: **Efficient Variants of the ICP Algorithm**, *3D Digital Imaging and Modeling, 2001*

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

- Side Remark: Error norm

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

- Side Remark: Error norm
 - squared Euclidean distance is sensitive to outliers

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

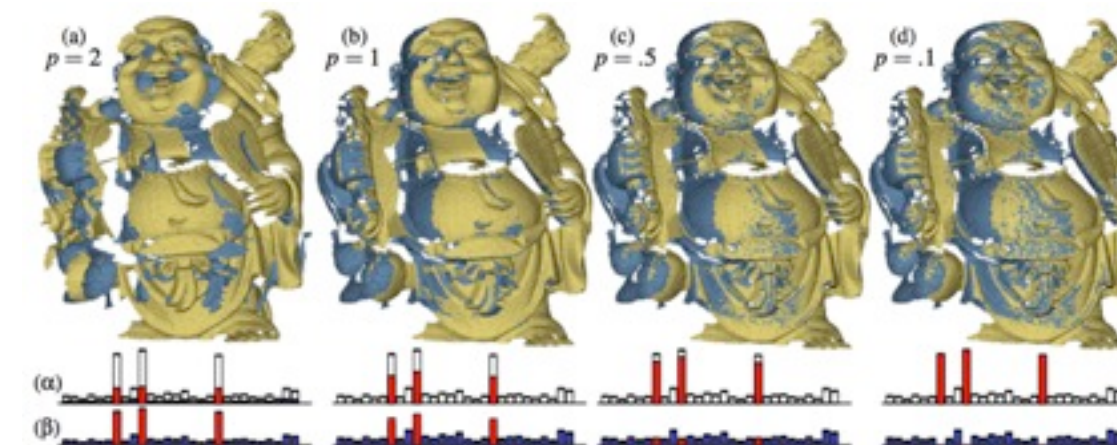
- Side Remark: Error norm
 - squared Euclidean distance is sensitive to outliers
 - robust norms reduce this sensitivity

3D Registration - Example

- Iterative Closest Point (ICP) Algorithm

$$E_{\text{match}}(\mathcal{Z}) = \sum_{i=1}^n w_i \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2$$

- Side Remark: Error norm
 - squared Euclidean distance is sensitive to outliers
 - robust norms reduce this sensitivity



Bouaziz, Tagliasacchi, Pauly: **Sparse Iterative Closest Point**, *SGP 2013*

3D Registration - Prior

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



rigid



elastic



articulated



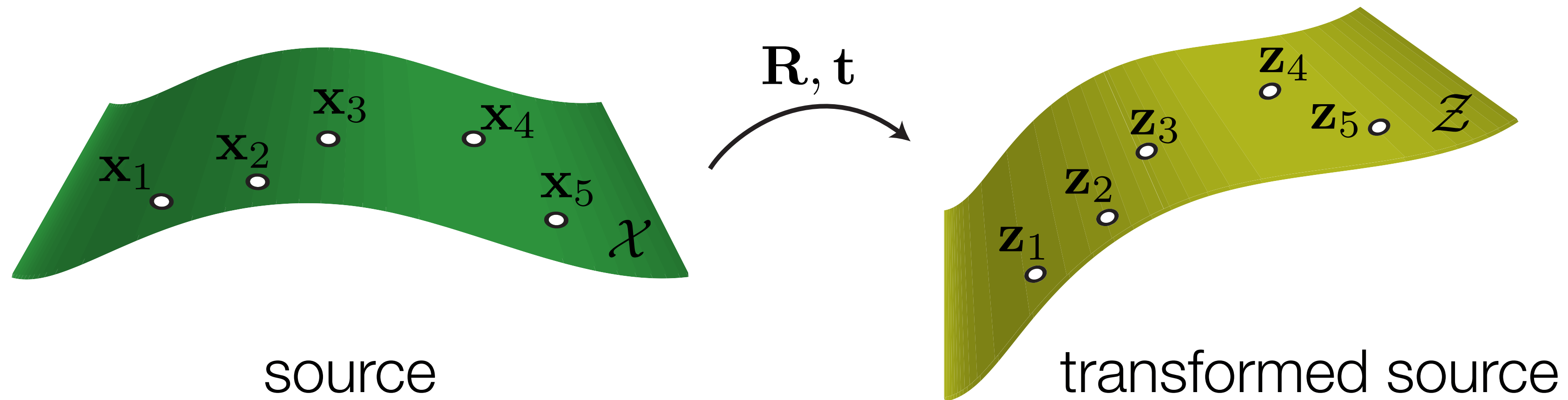
composite



fluid

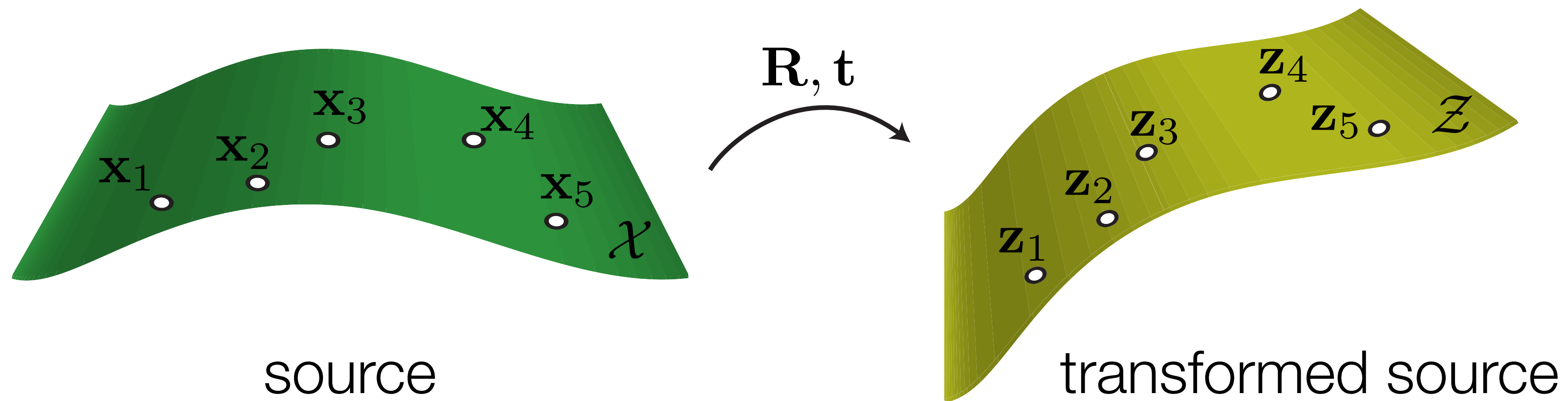
3D Registration - Prior

- Global Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



3D Registration - Prior

- Global Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



$$E_{\text{prior}}(\mathcal{Z}, \mathbf{R}, \mathbf{t}) = \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

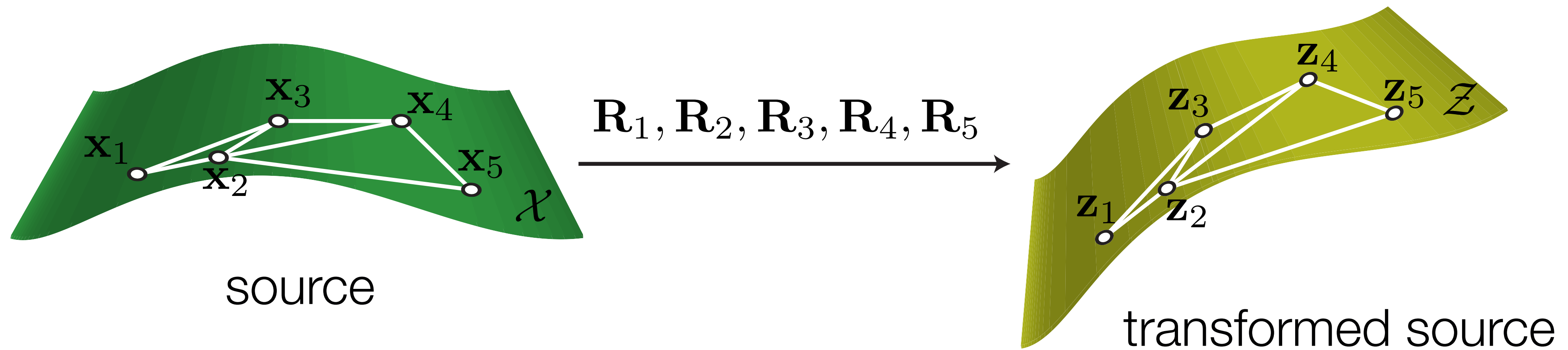
3D Registration - Prior

- Local Rigidity $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

3D Registration - Prior

- Local Rigidity

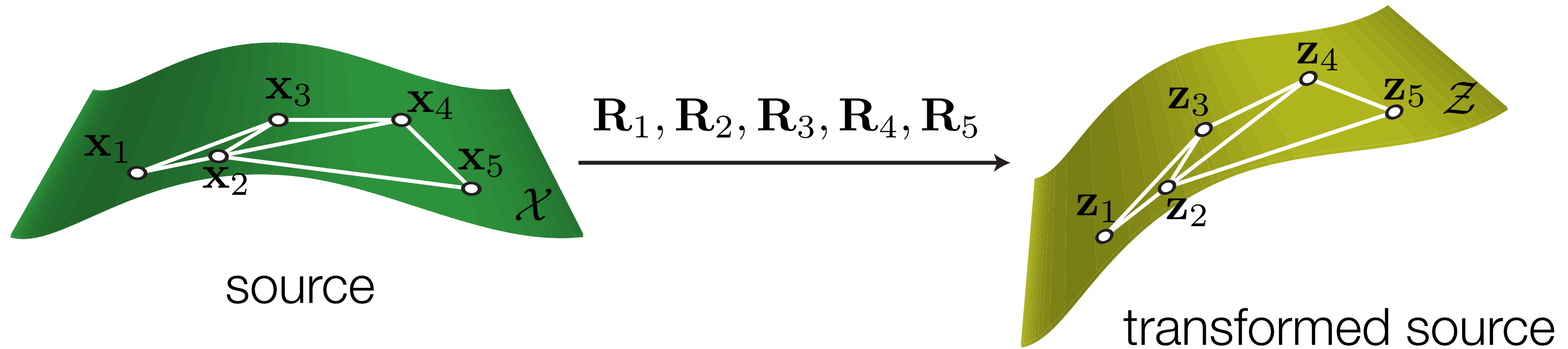
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



3D Registration - Prior

- Local Rigidity

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

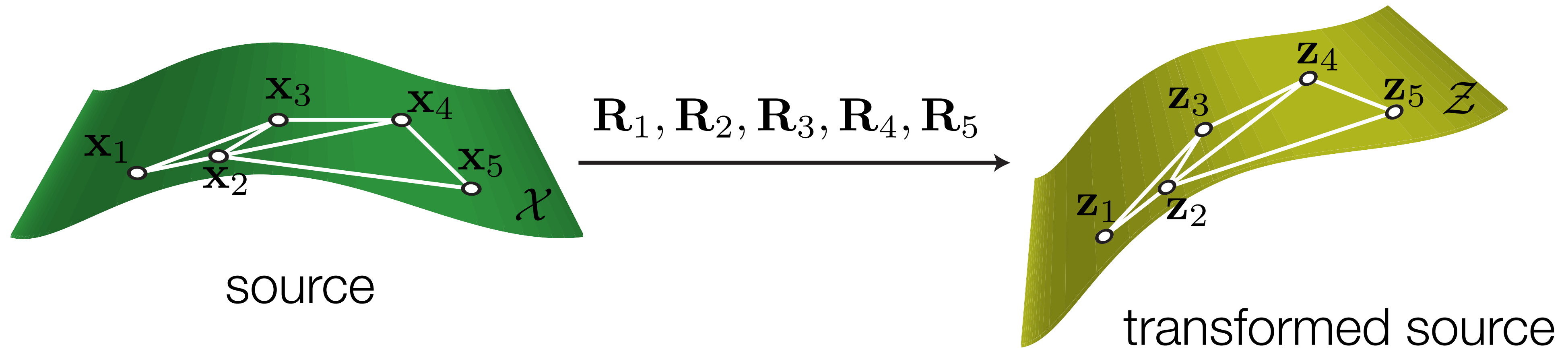


$$E_{\text{prior}}(Z, \mathbf{R}_i |_{i=1}^n) = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

3D Registration - Prior

- Local Rigidity

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



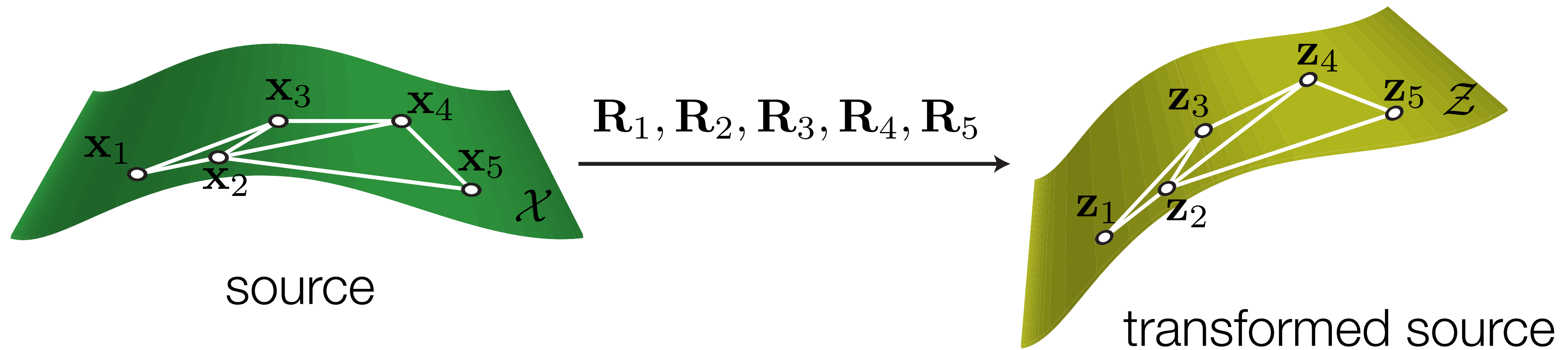
$$E_{\text{prior}}(Z, \mathbf{R}_i |_{i=1}^n) = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2$$

Sorkine, Alexa: **As-Rigid-As-Possible Surface Modeling**, SGP 2007

3D Registration - Prior

- Local Rigidity

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



DEMO

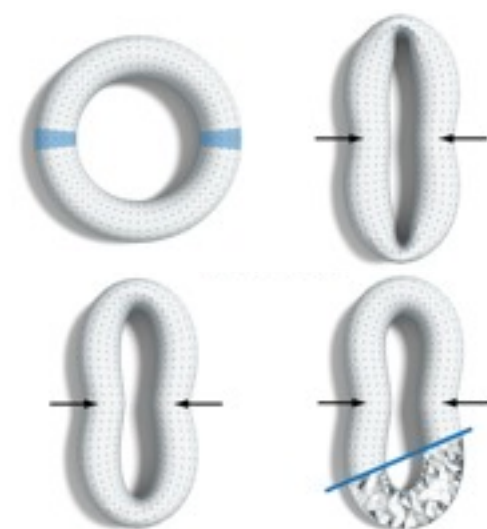
3D Registration - Prior

- Local Rigidity

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



Sumner, Schmid, Pauly:
Embedded Deformation for Shape Manipulation
ACM SIGGRAPH 2007



Bouaziz, Deuss, Schwartzburg, Weise, Pauly:
Shape-Up: Shaping Discrete Geometry With Projection
SGP 2012

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



m



p₁



p₂



p₃



p₄

3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



m



p₁



p₂



p₃



p₄

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

3D Registration - Prior

- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



m



p₁



p₂



p₃



p₄

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

mean shape

3D Registration - Prior

- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



m



p₁



p₂



p₃



p₄

$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

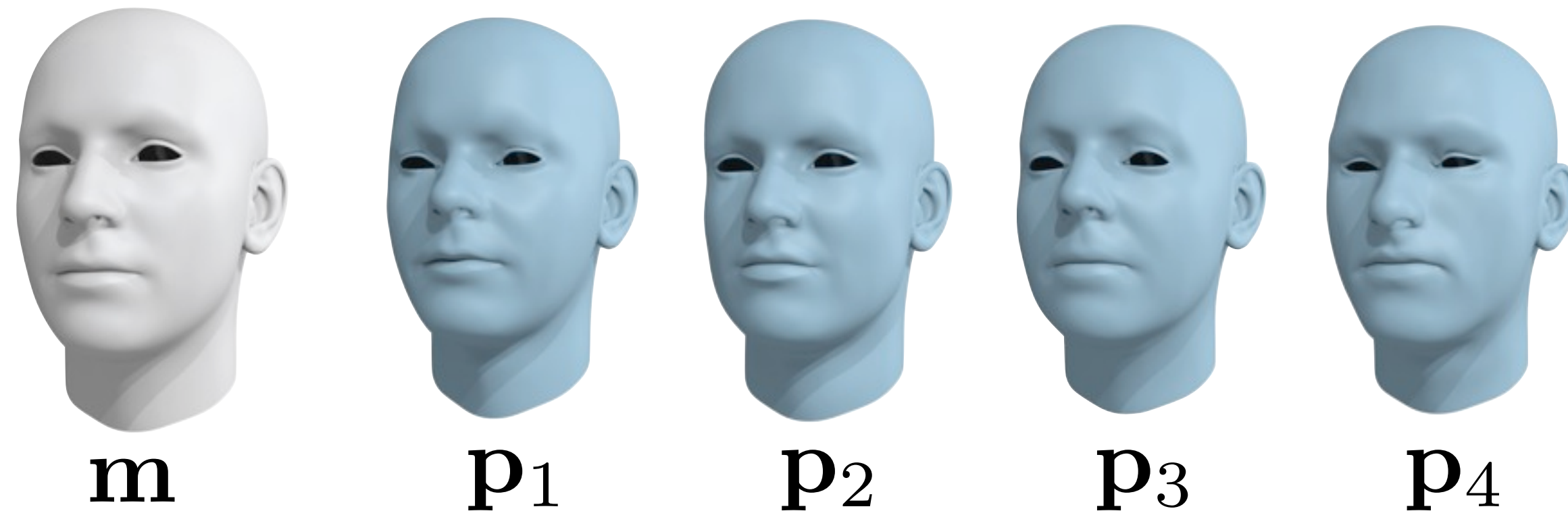
basis

mean shape

3D Registration - Prior

- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



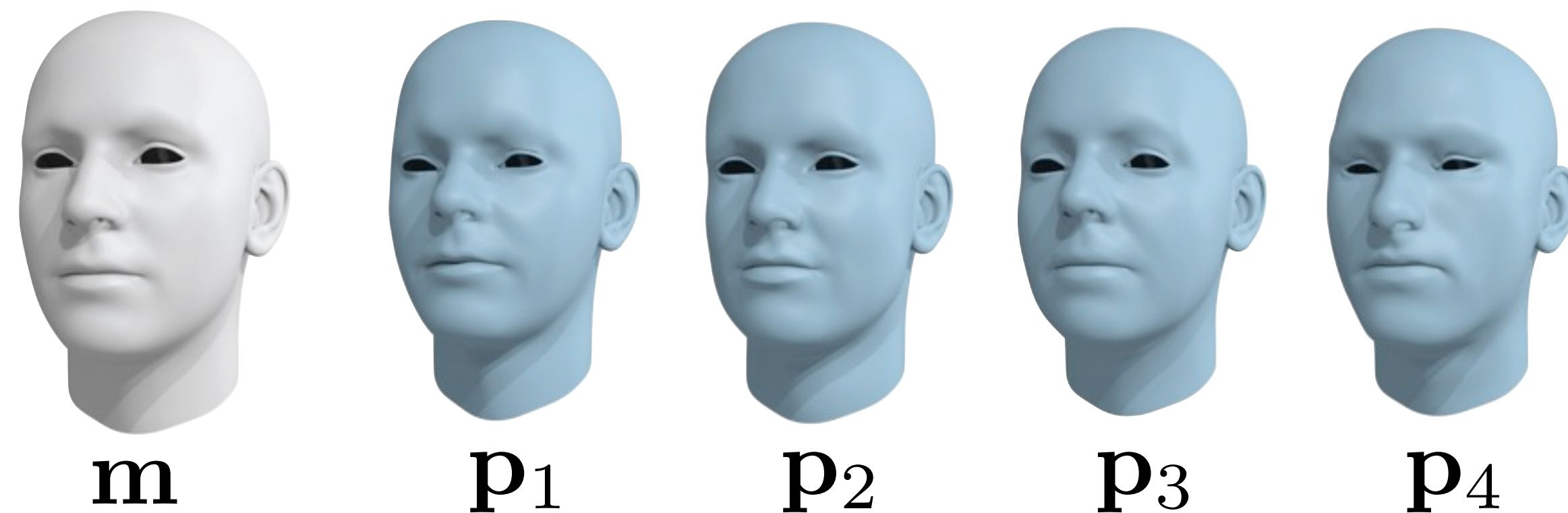
$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

coefficients (pointing to \mathbf{d})
 basis (pointing to \mathbf{P})
 mean shape (pointing to \mathbf{m})

3D Registration - Prior

- Linear Model

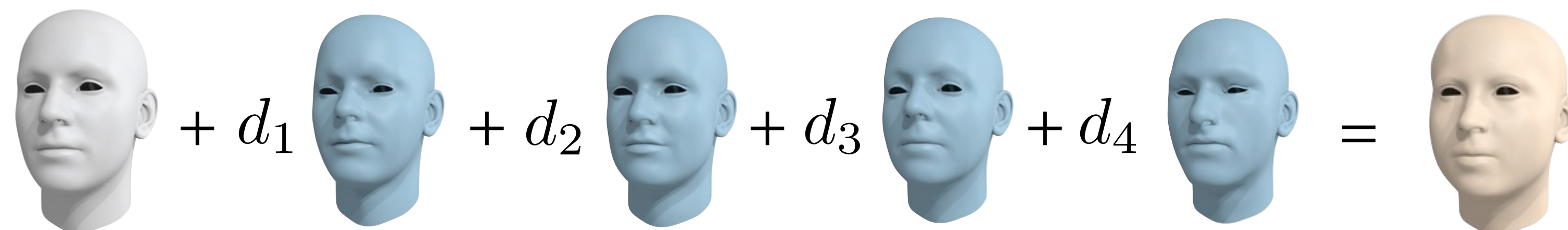
$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

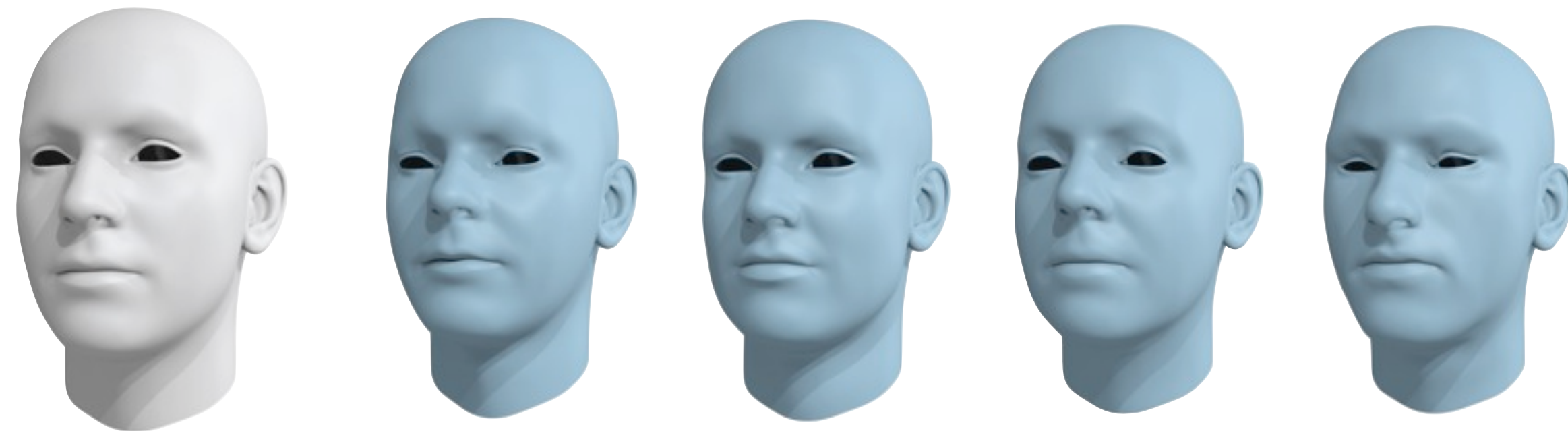
Diagram illustrating the linear model equation $\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$. Arrows point from the labels to the corresponding terms in the equation:

- coefficients** points to \mathbf{d} .
- basis** points to \mathbf{P} .
- mean shape** points to \mathbf{m} .



3D Registration - Prior

- Linear Model $E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$



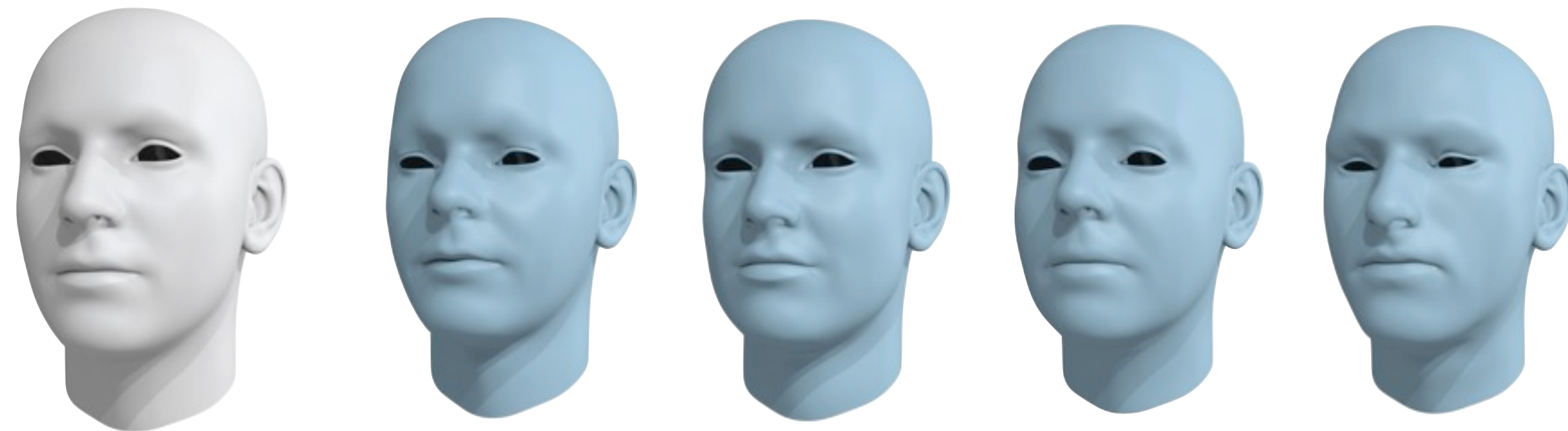
$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

$$E_{\text{prior}}(\mathbf{Z}, \mathbf{d}) = \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{P}_i\mathbf{d} + \mathbf{m}_i)\|_2^2$$

3D Registration - Prior

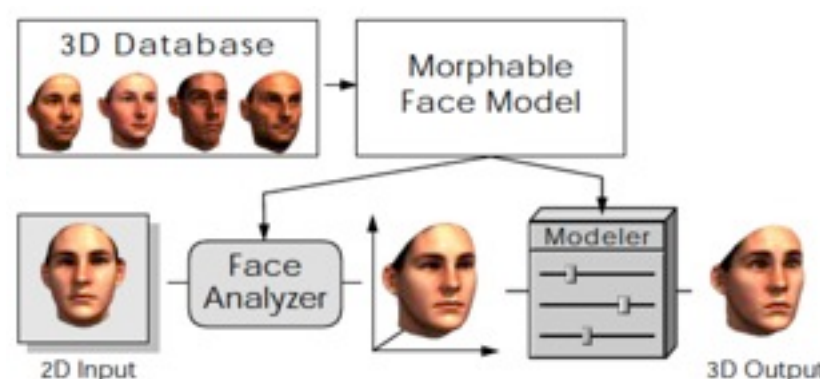
- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

$$E_{\text{prior}}(\mathbf{Z}, \mathbf{d}) = \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{P}_i\mathbf{d} + \mathbf{m}_i)\|_2^2$$

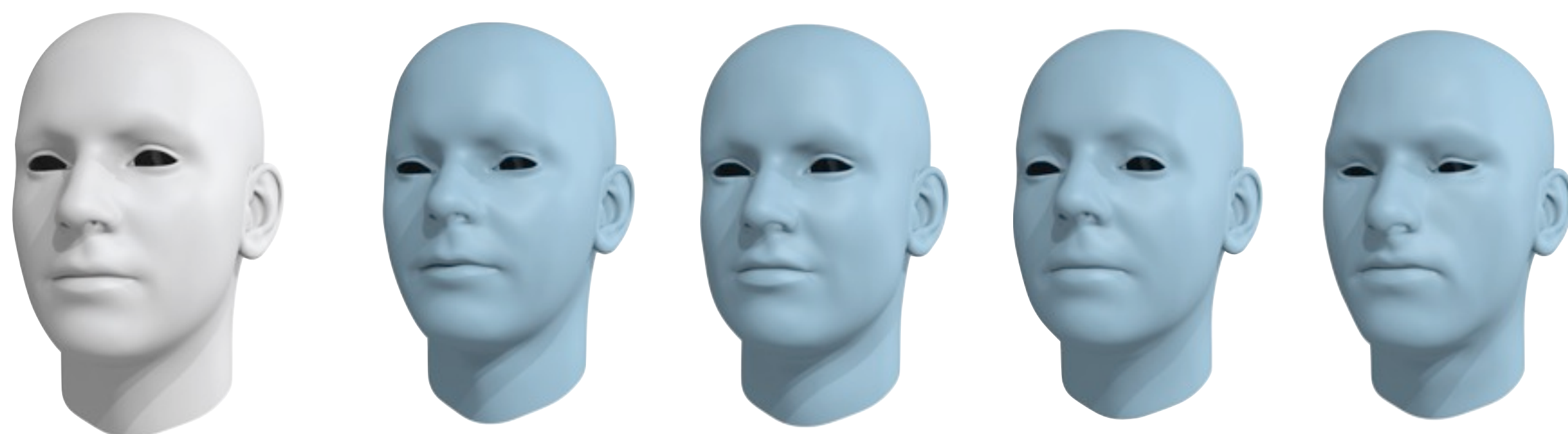


Blanz, Vetter: A Morphable Model
for the Synthesis of 3D Faces
ACM SIGGRAPH 1999

3D Registration - Prior

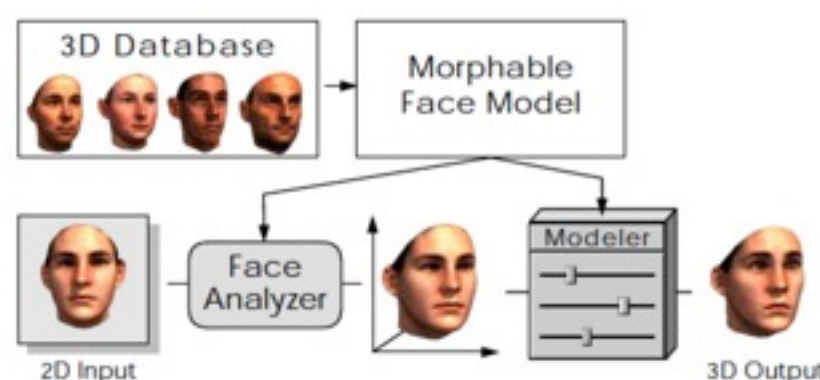
- Linear Model

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



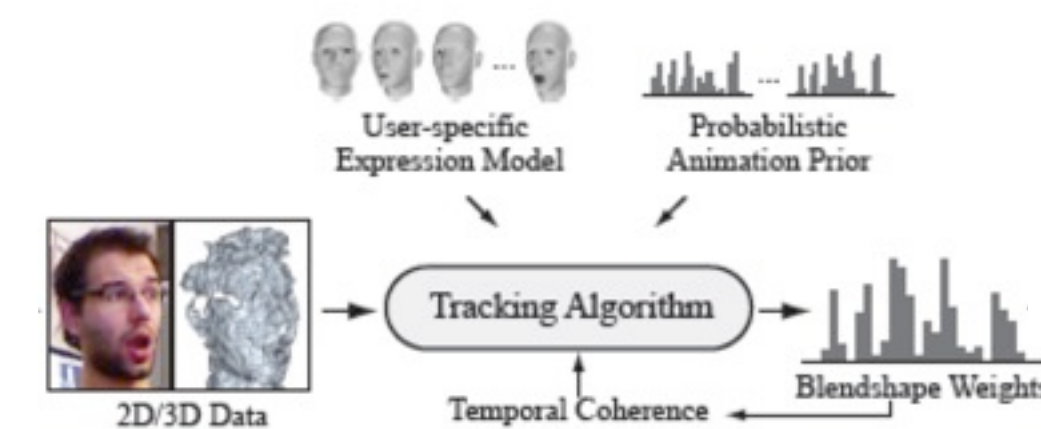
$$\mathbf{s} = \mathbf{P}\mathbf{d} + \mathbf{m}$$

$$E_{\text{prior}}(\mathbf{Z}, \mathbf{d}) = \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{P}_i\mathbf{d} + \mathbf{m}_i)\|_2^2$$



Blanz, Vetter: **A Morphable Model for the Synthesis of 3D Faces**
ACM SIGGRAPH 1999

Weise, Bouaziz, Li, Pauly: **Realtime Performance-based Facial Animation**
ACM SIGGRAPH 2011



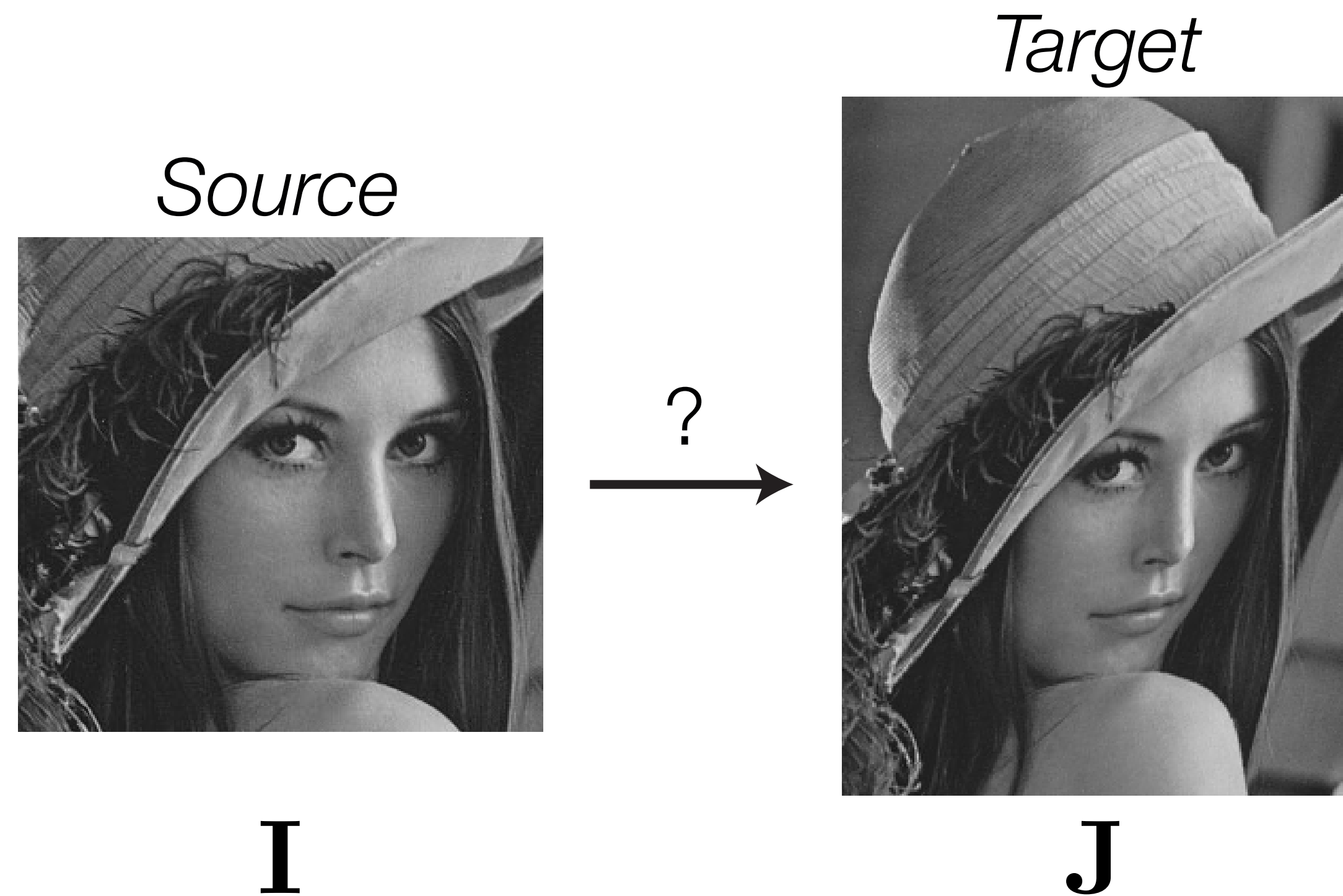
Overview

Introduction

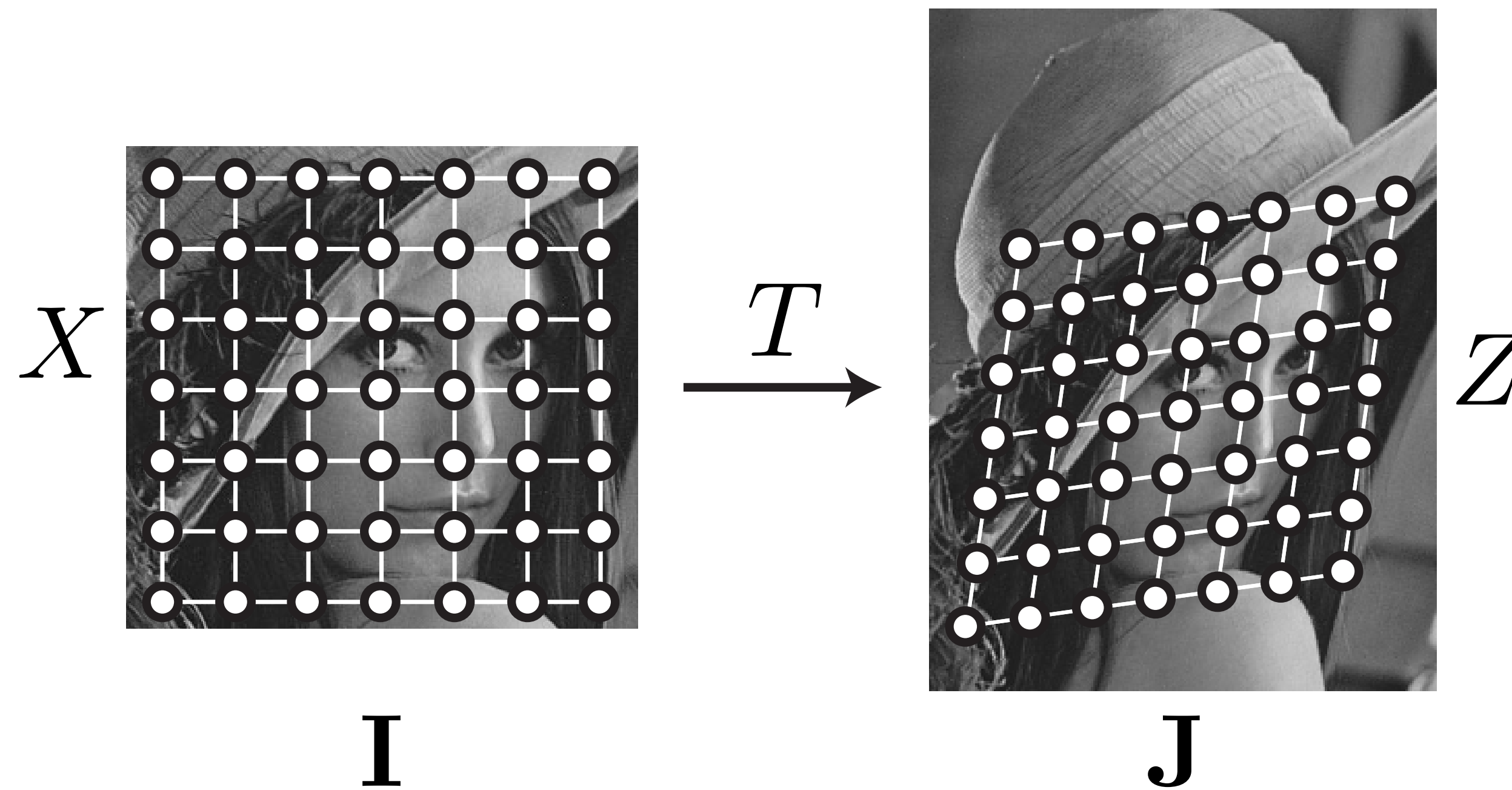
- Registration
 - 3D Geometry
 - **2D Images**
 - Combined 2D / 3D
- Applications
 - Rigid Scanning
 - Non-rigid Modeling
 - Realtime Face Tracking

Conclusions

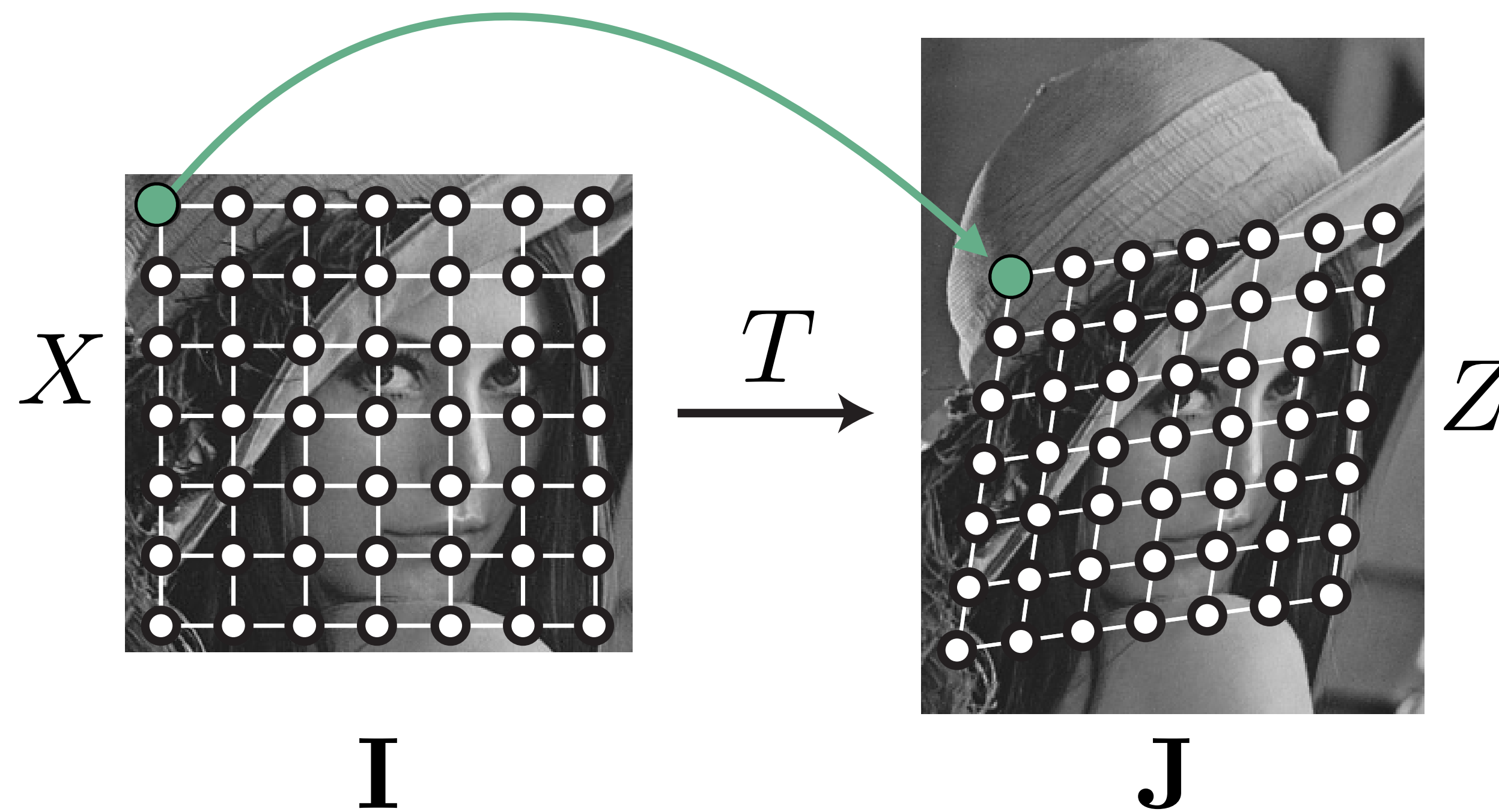
2D Registration



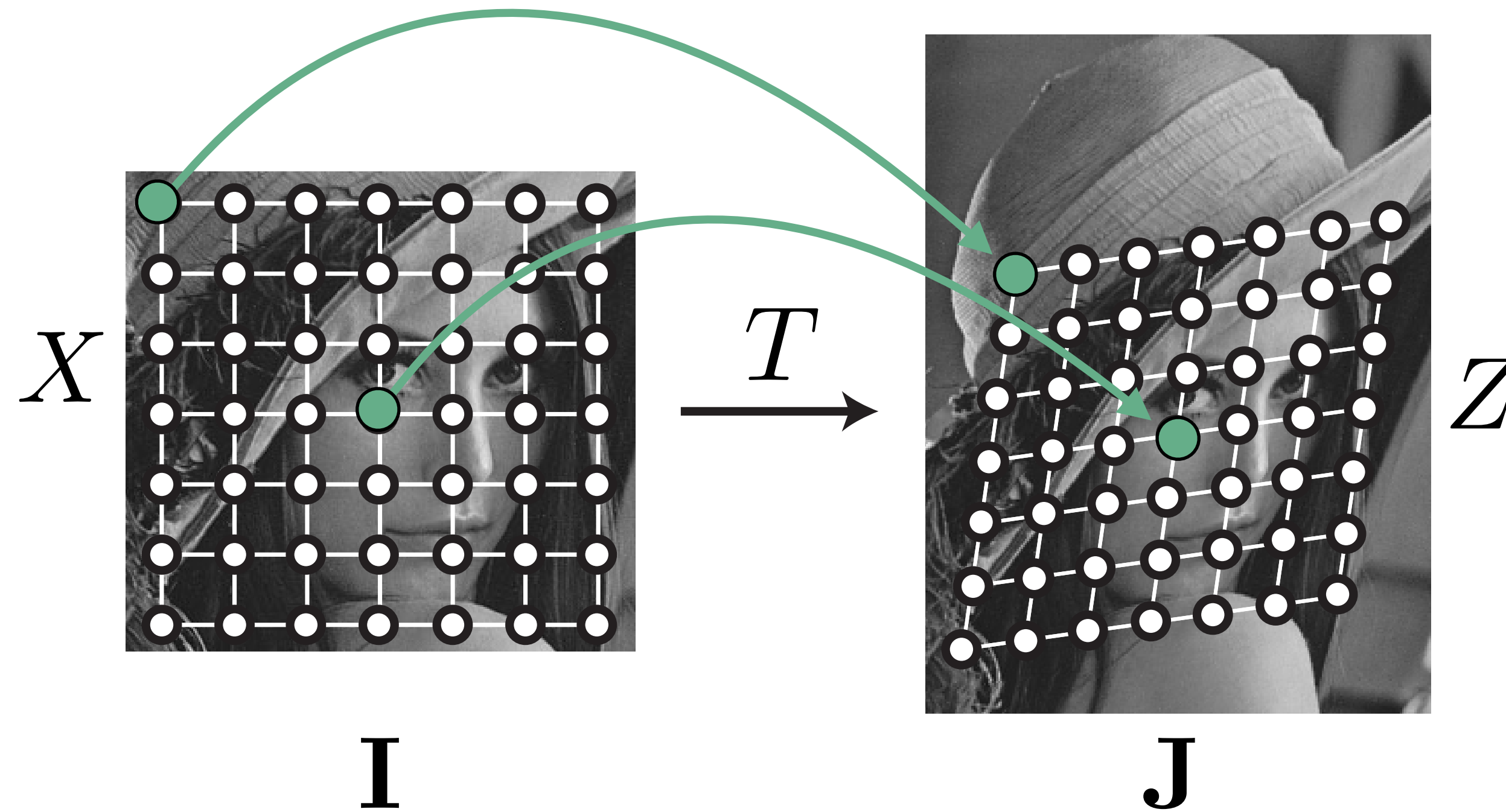
2D Registration



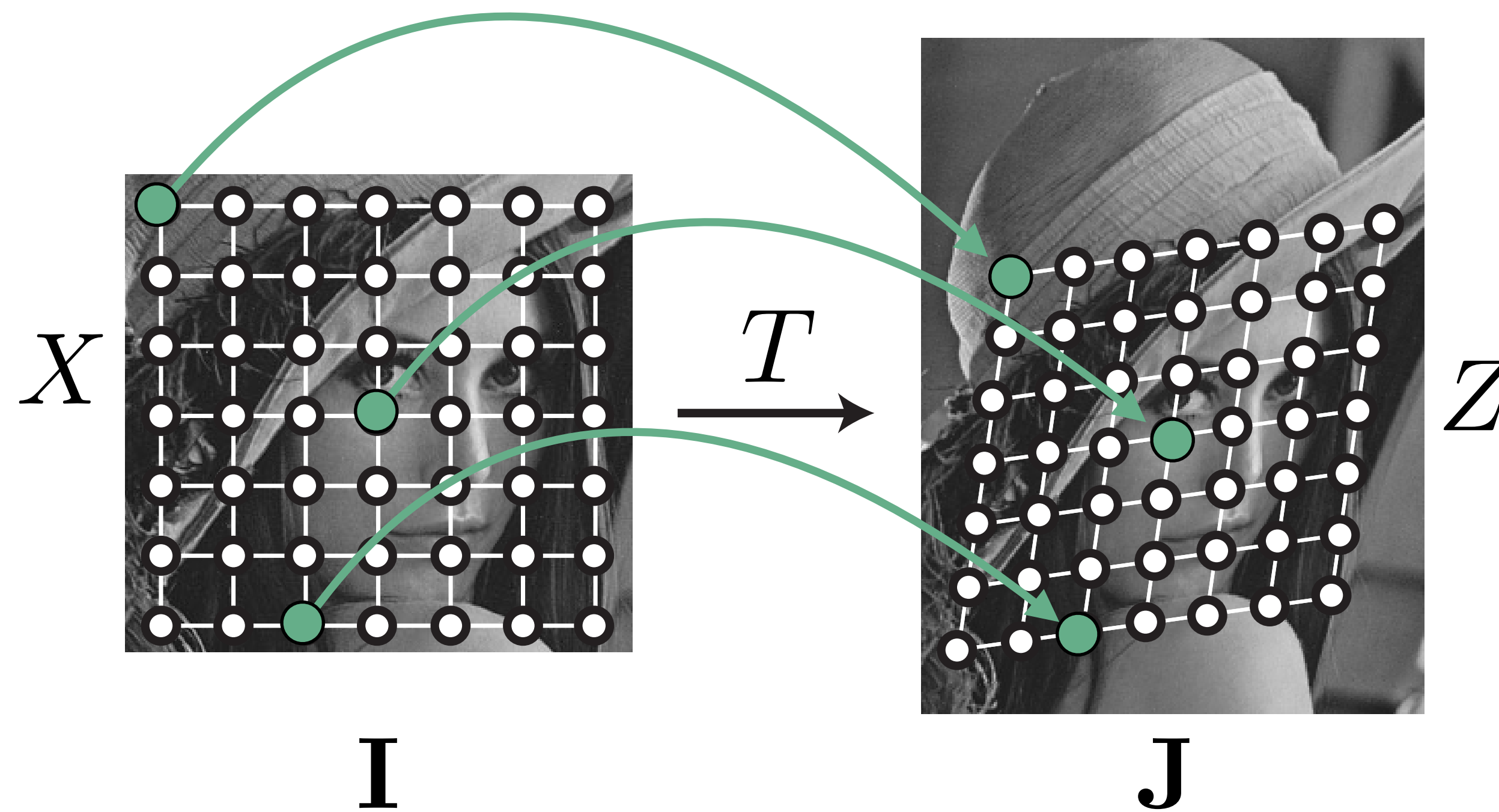
2D Registration



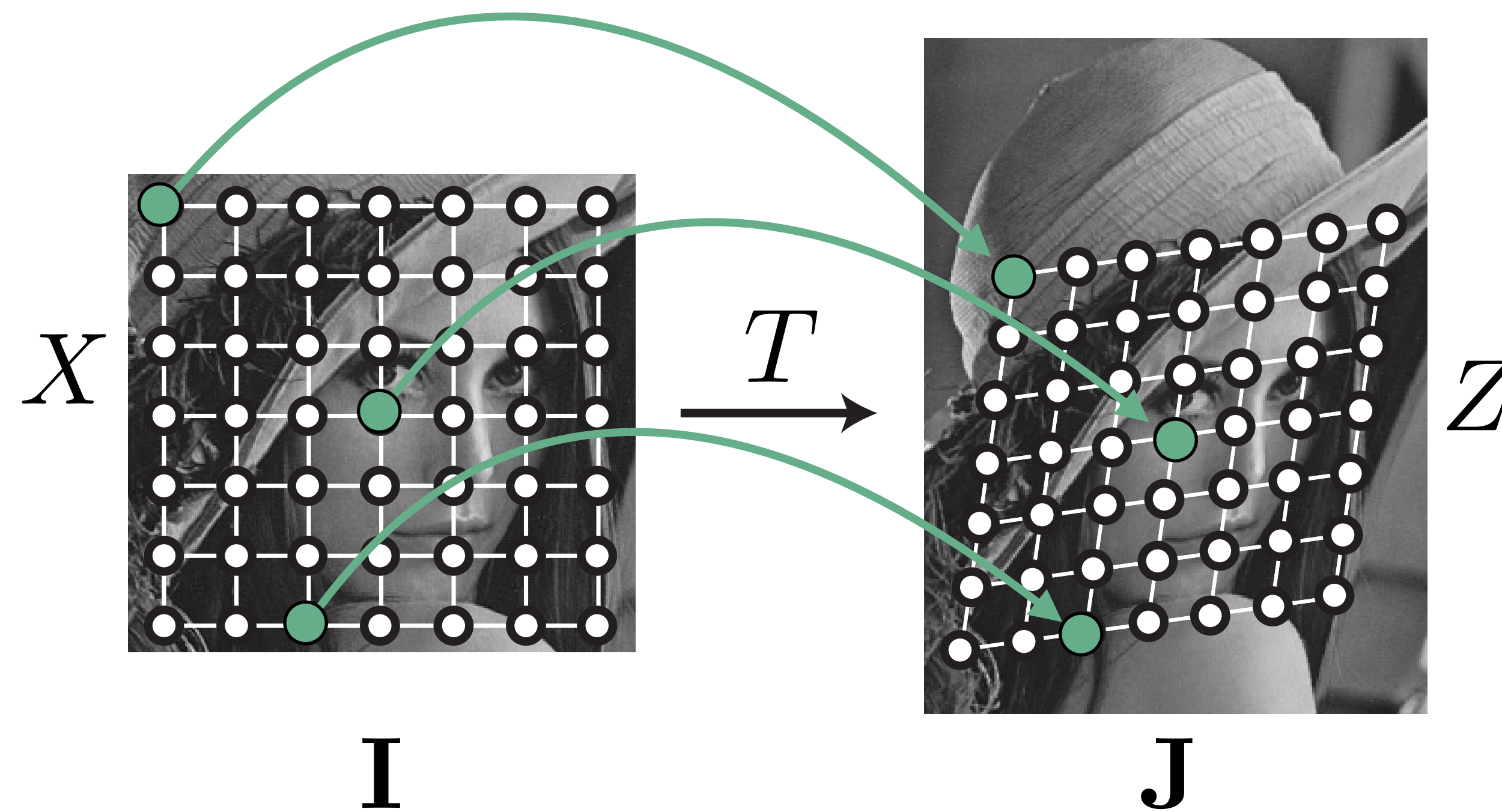
2D Registration



2D Registration

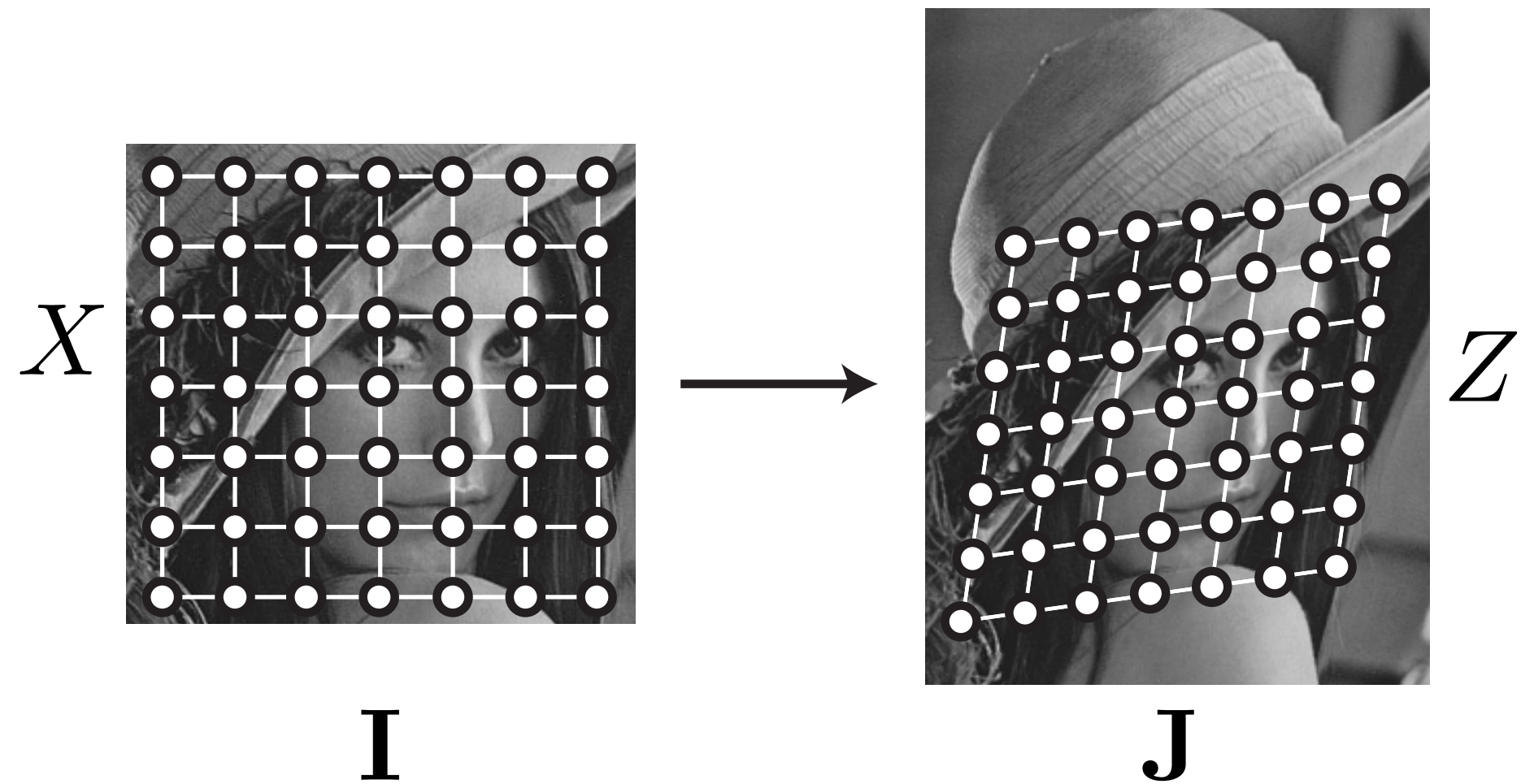


2D Registration



DEMO

2D Registration - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{z}_i)\|_2^2$$

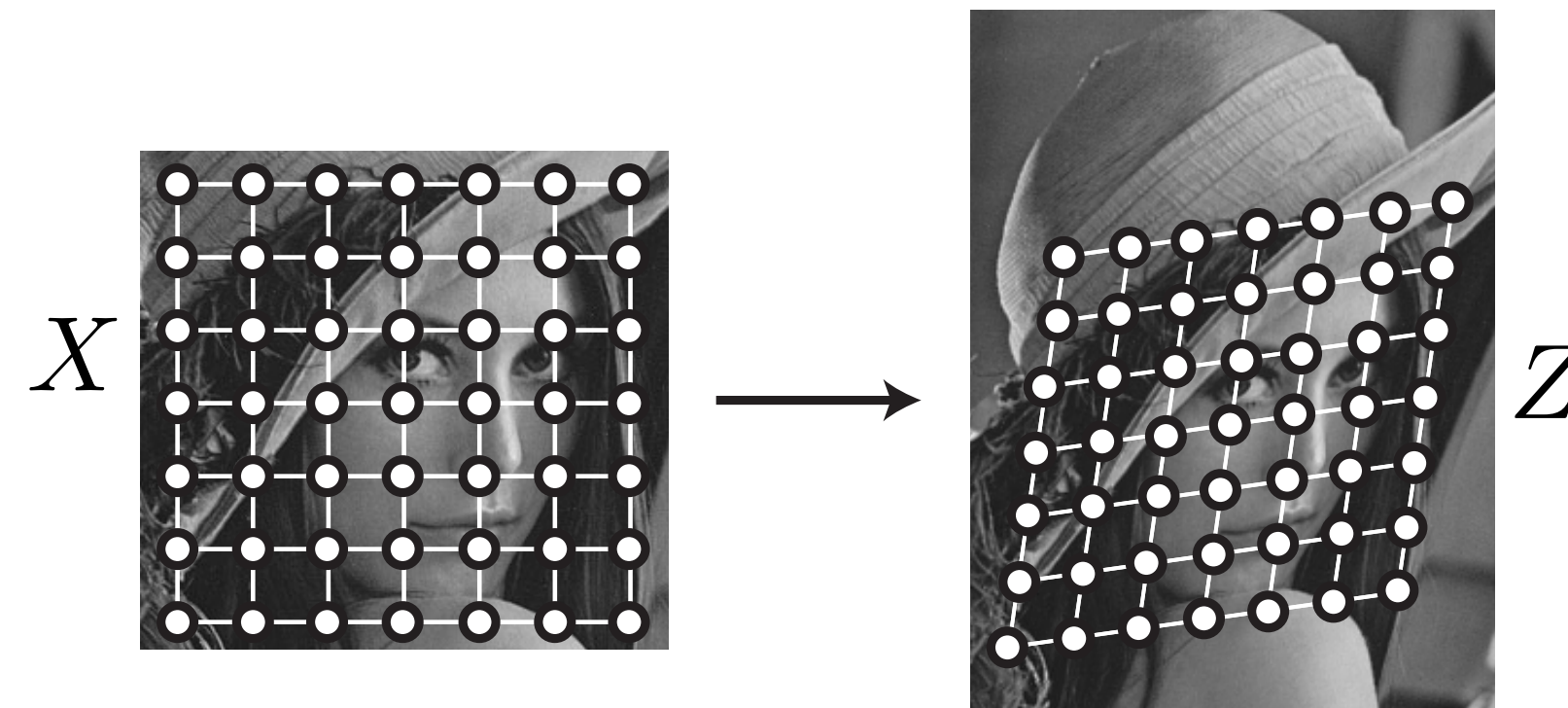
color value at
 \mathbf{x}_i of image **I**

color value at
 \mathbf{z}_i of image **J**

2D Registration - Prior

- Lucas-Kanade
- Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$E_{\text{prior}}(Z) = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \left\| (\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i) \right\|_2^2$$

Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

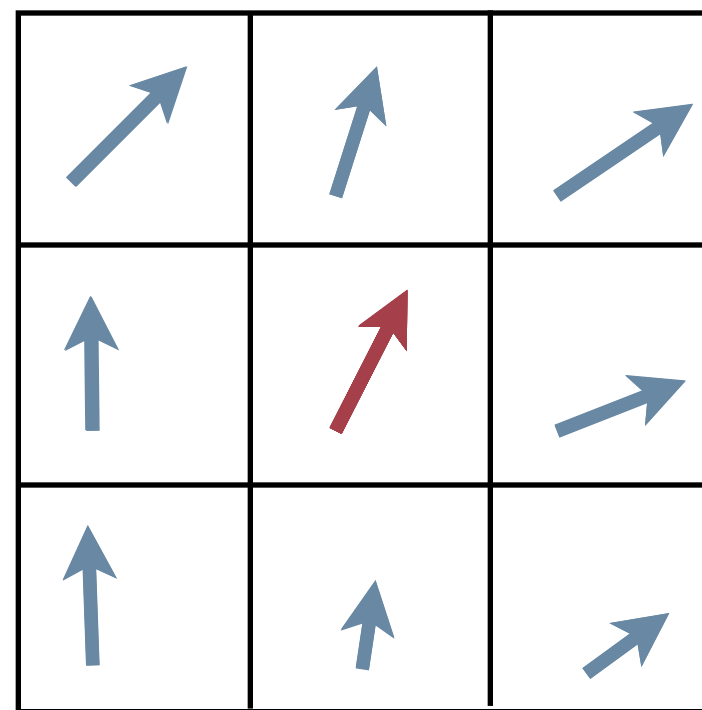
2D Registration - Prior

- Lucas-Kanade

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Constant displacement field

$$E_{\text{prior}}(Z) = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i)\|_2^2$$



Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

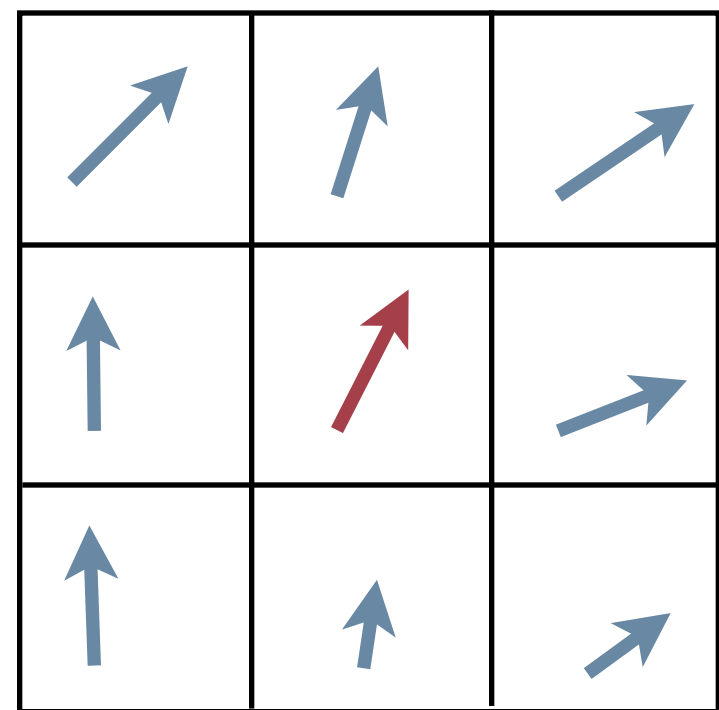
2D Registration - Prior

- Lucas-Kanade

- Constant displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{prior}}(Z) = \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{x}_j) - (\mathbf{z}_i - \mathbf{x}_i)\|_2^2$$



$$\longrightarrow \| \uparrow - \nearrow \| + \| \uparrow - \nearrow \| + \| \uparrow - \nearrow \| + \| \uparrow - \uparrow \| + \| \uparrow - \nearrow \| + \| \uparrow - \uparrow \| + \| \uparrow - \nearrow \| + \| \uparrow - \nearrow \|$$

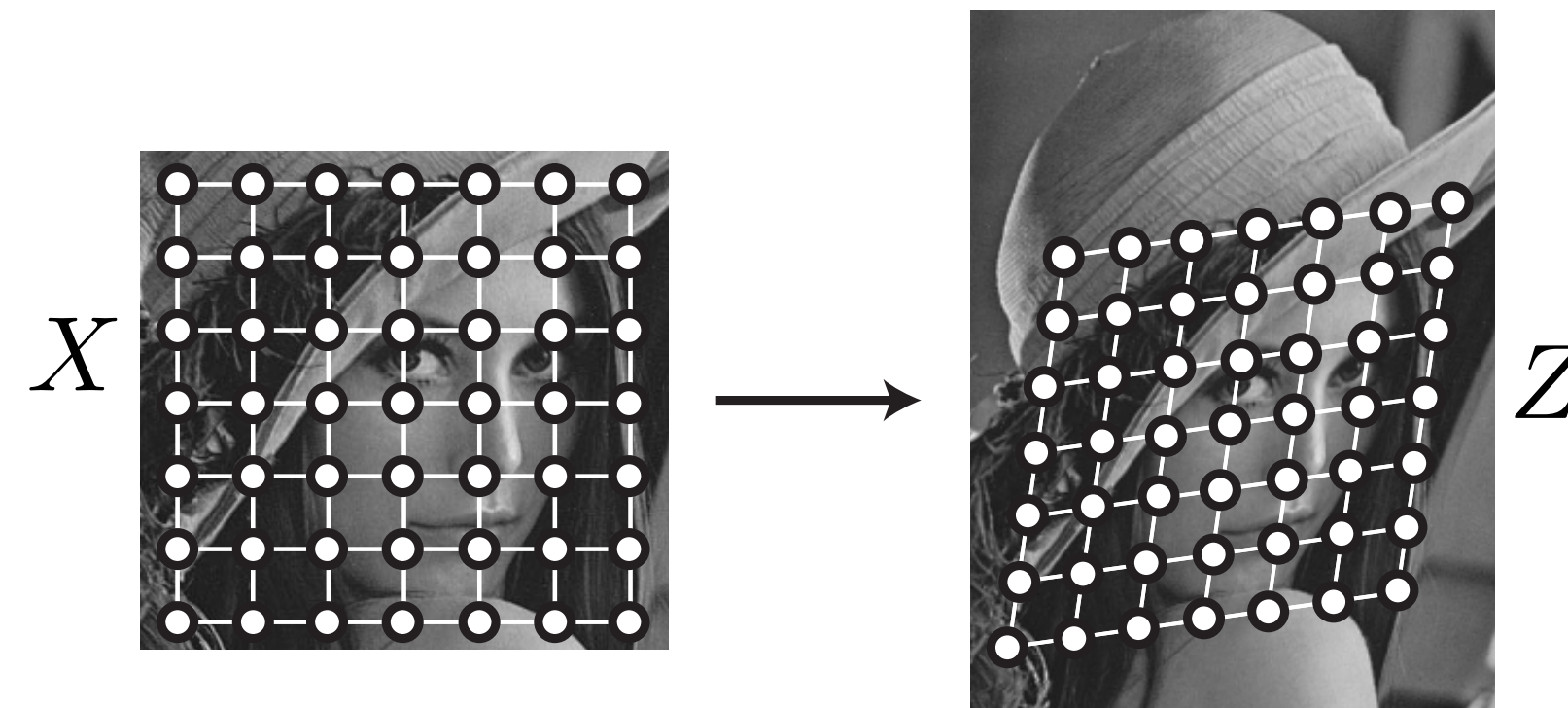
Lucas, Kanade: **An Iterative Image Registration Technique with an Application to Stereo Vision**, *IJCAI 1981*

2D Registration - Prior

- Horn-Schunck

- Smooth displacement field

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$



$$E_{\text{prior}}(Z) = \sum_{i=1}^n \left\| (\mathbf{z}_i - \mathbf{x}_i) - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mathbf{z}_j - \mathbf{x}_j) \right\|_2^2$$

Horn, Schunk: “**Determining Optical Flow**”, *Artificial Intelligence Journal* 1981

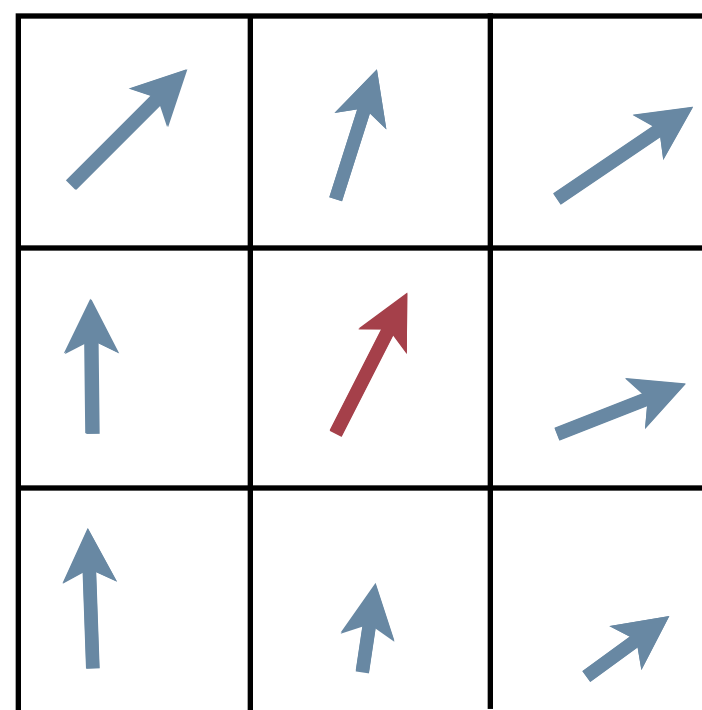
2D Registration - Prior

- Horn-Schunck

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Smooth displacement field

$$E_{\text{prior}}(Z) = \sum_{i=1}^n \left\| (\mathbf{z}_i - \mathbf{x}_i) - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mathbf{z}_j - \mathbf{x}_j) \right\|_2^2$$



Horn, Schunk: “**Determining Optical Flow**”, *Artificial Intelligence Journal* 1981

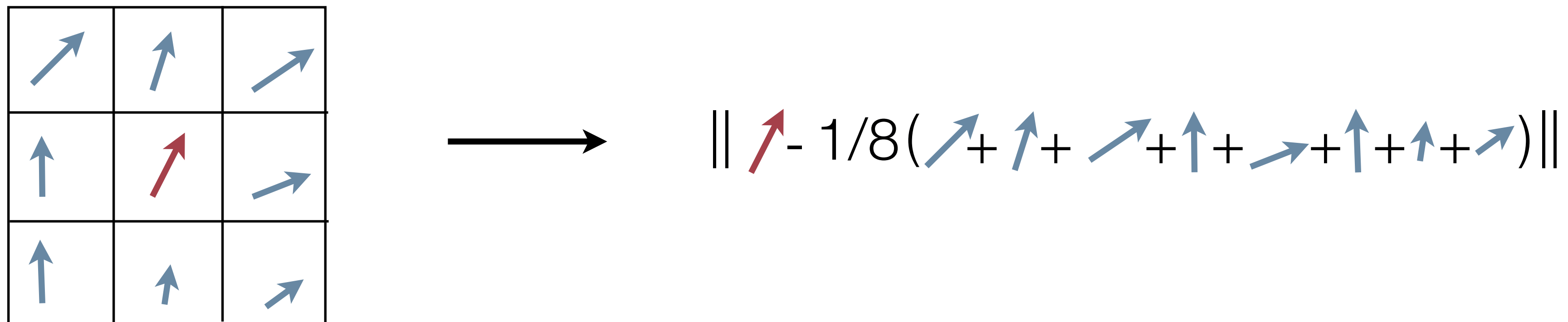
2D Registration - Prior

- Horn-Schunck

$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

- Smooth displacement field

$$E_{\text{prior}}(Z) = \sum_{i=1}^n \left\| (\mathbf{z}_i - \mathbf{x}_i) - \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mathbf{z}_j - \mathbf{x}_j) \right\|_2^2$$



Horn, Schunk: “**Determining Optical Flow**”, *Artificial Intelligence Journal* 1981

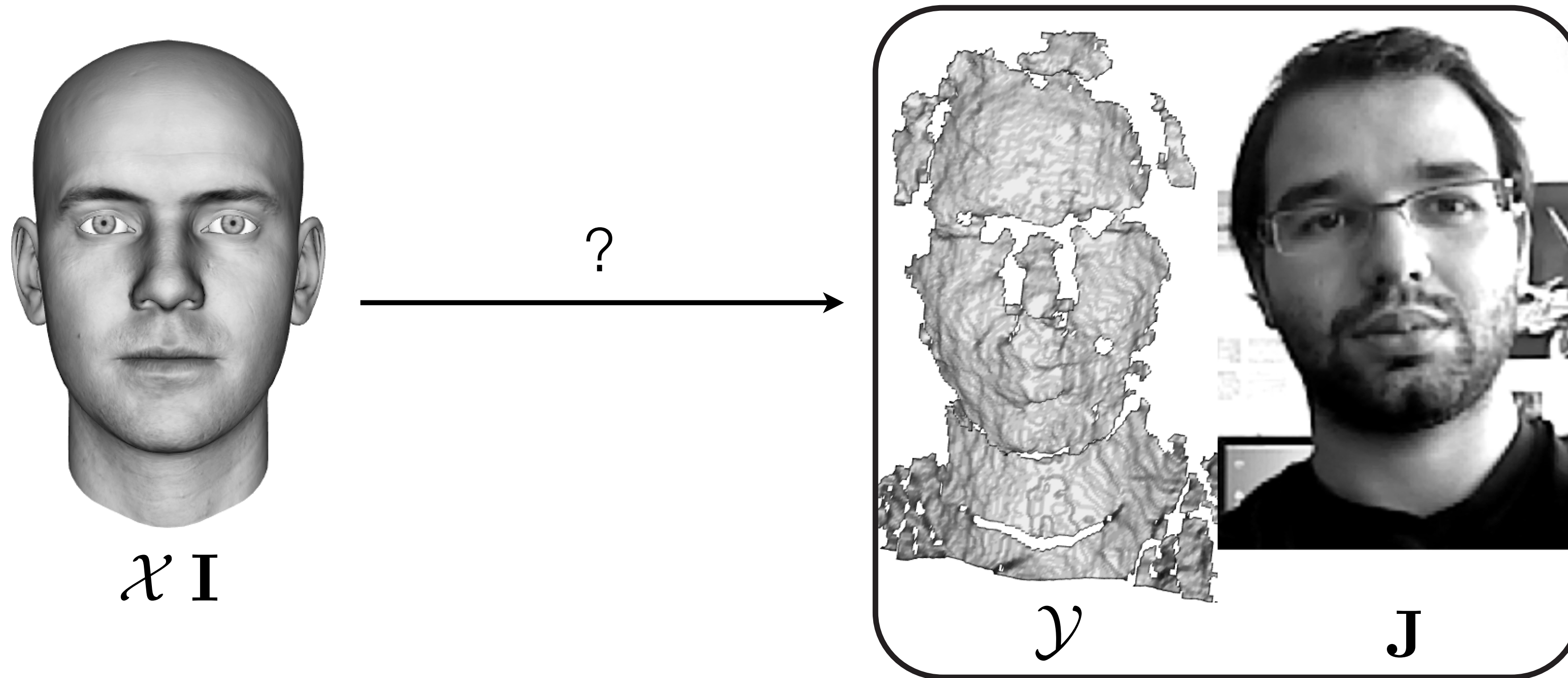
Overview

Introduction

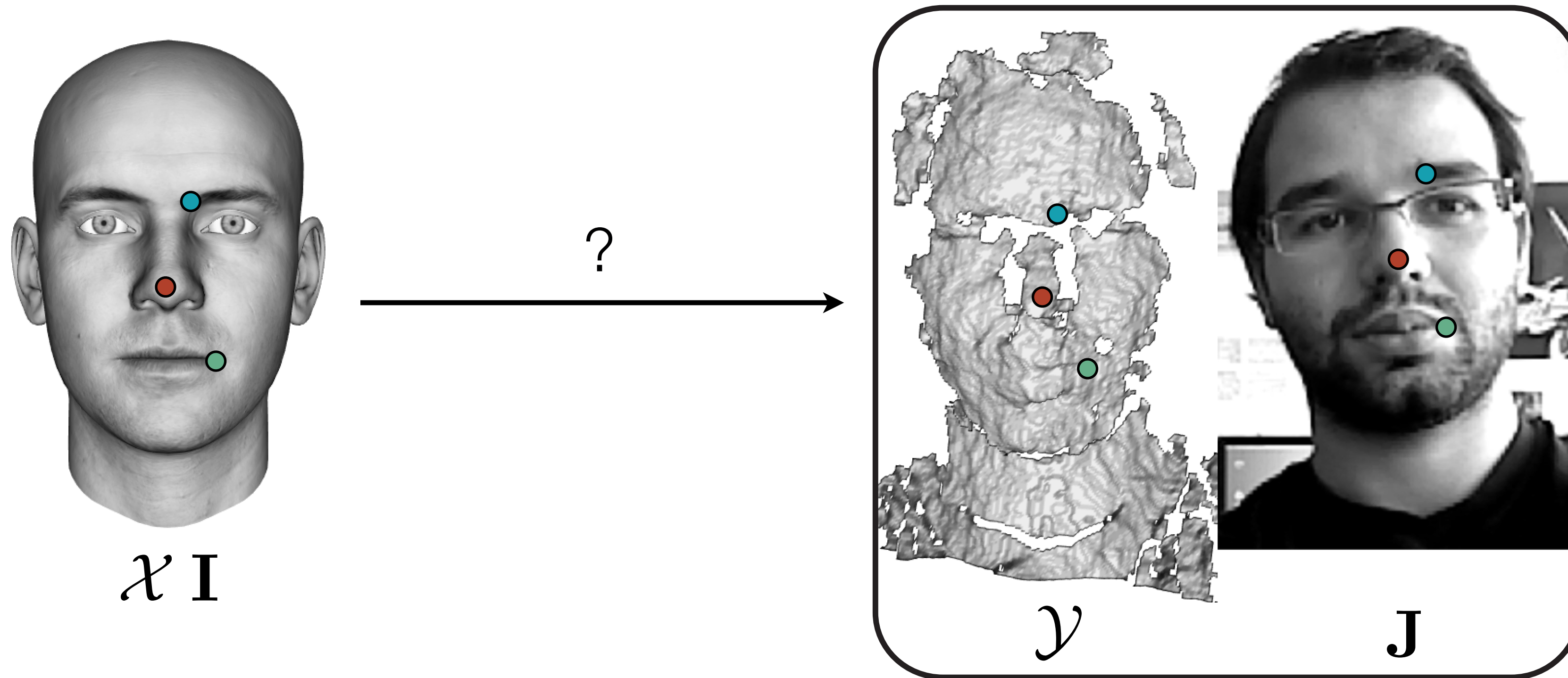
- Registration
 - 3D Geometry
 - 2D Images
 - **Combined 2D / 3D**
- Applications
 - Rigid Scanning
 - Non-rigid Modeling
 - Realtime Face Tracking

Conclusions

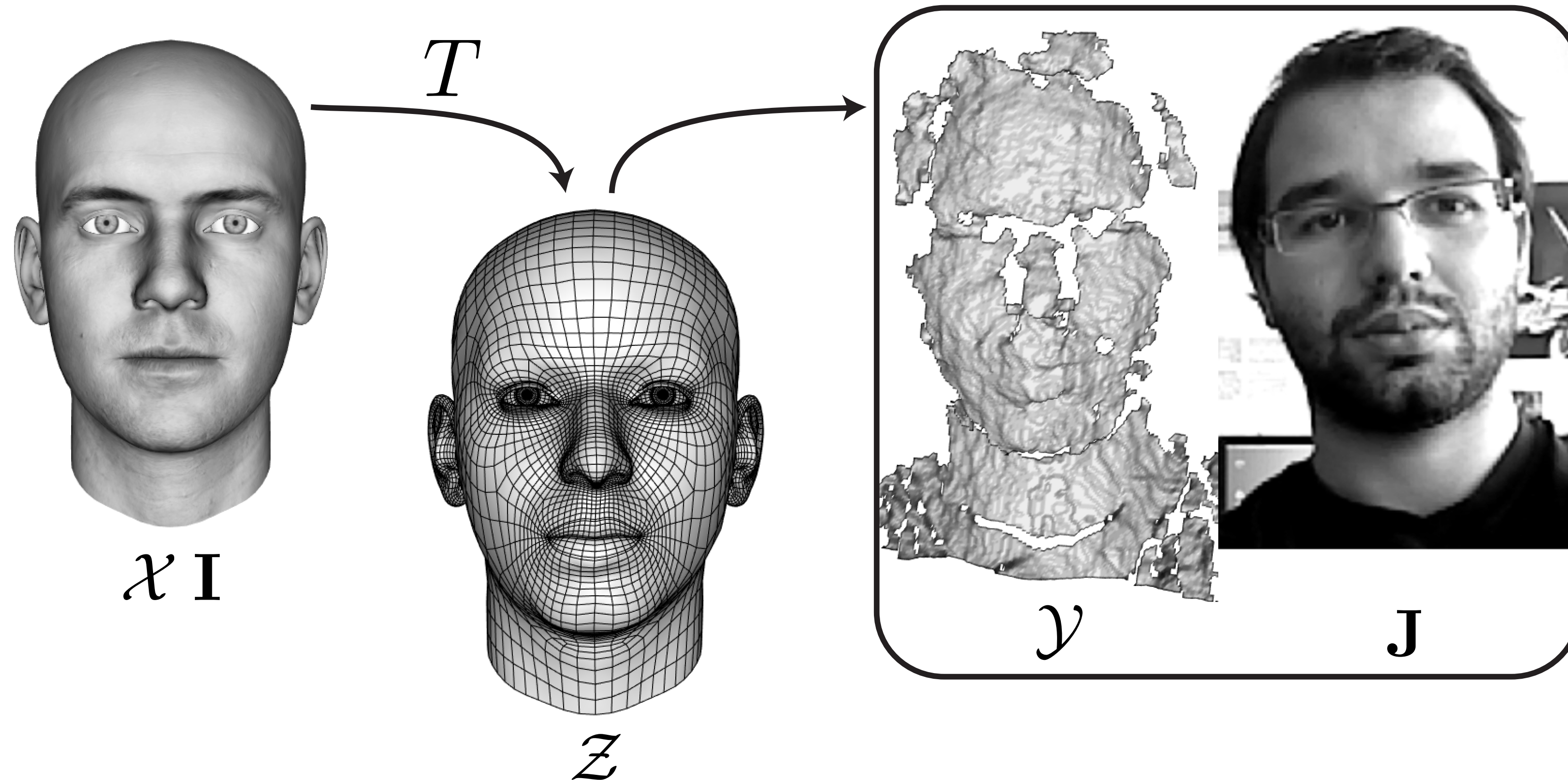
2D/3D Registration



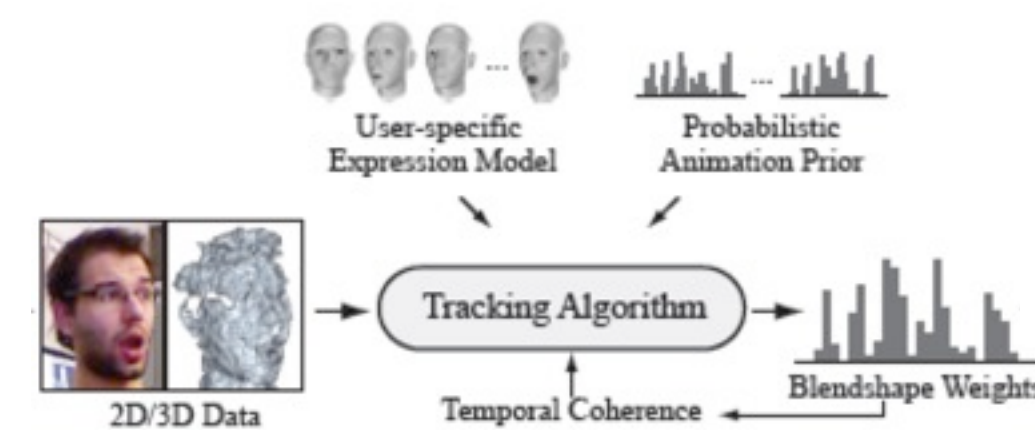
2D/3D Registration



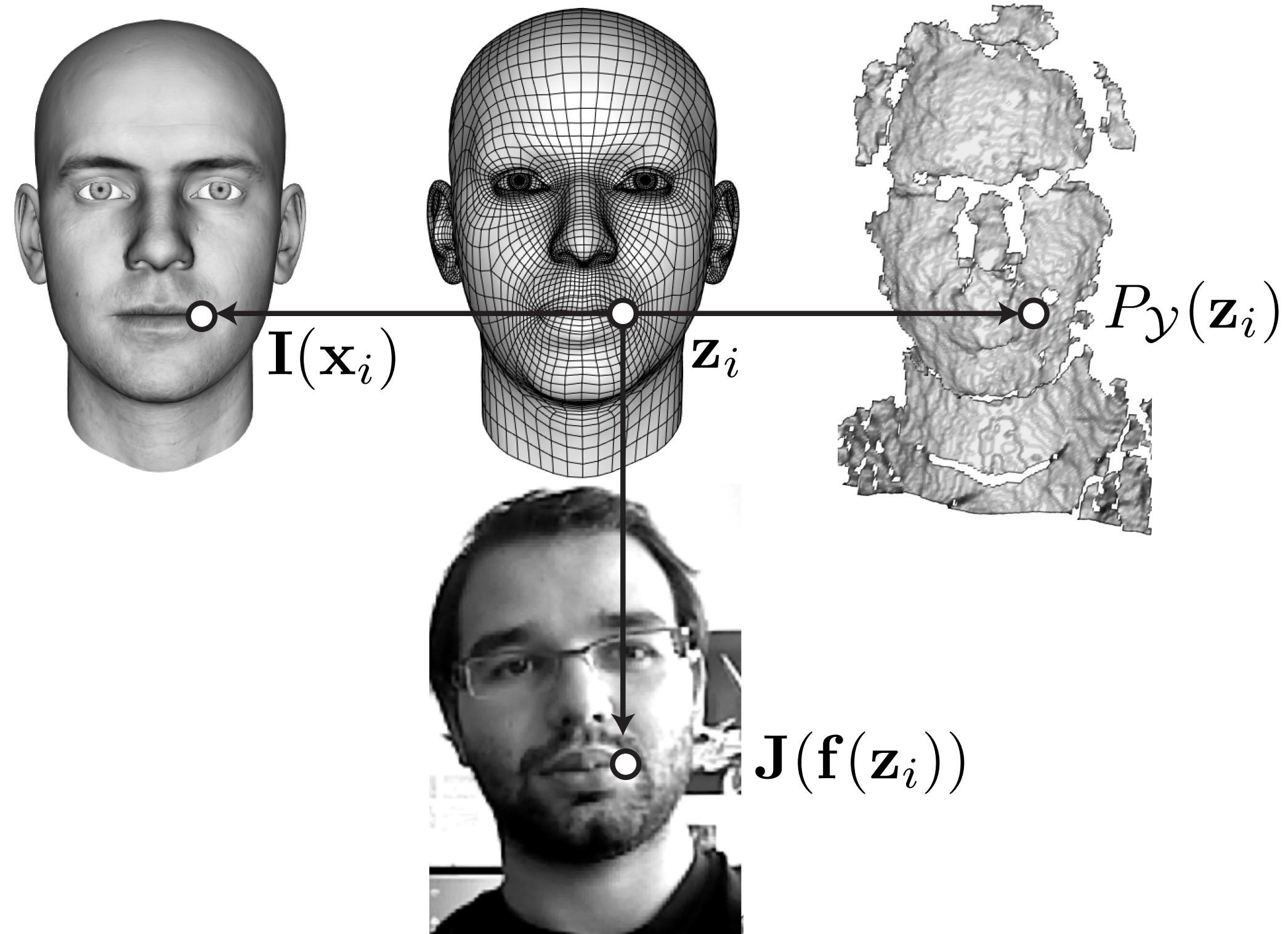
2D/3D Registration



Weise, Bouaziz, Li, Pauly: **Realtime Performance-based Facial Animation**,
ACM SIGGRAPH 2011



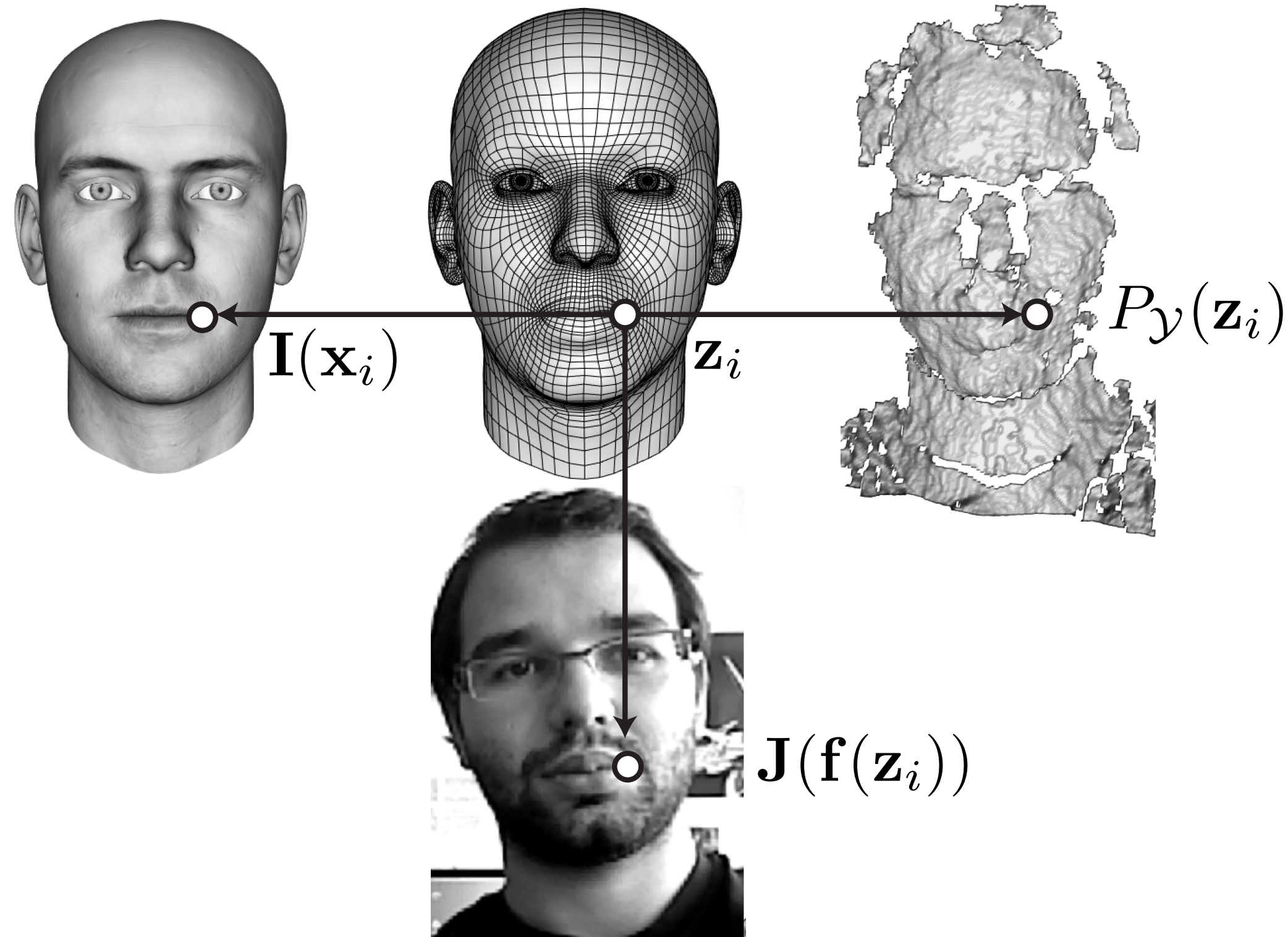
2D/3D - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_\gamma(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

2D/3D - Matching



$$E_{\text{reg}} = E_{\text{match}} + E_{\text{prior}}$$

projection of the 3D vertex
in the 2D image

$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

2D/3D - Matching

$$E_{\text{match}}(Z) = w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2$$

Orthographic projection $\mathbf{f}(\mathbf{z}_i) = [\mathbf{z}_{i,x} \quad \mathbf{z}_{i,y}]^T$

Perspective projection $\mathbf{f}(\mathbf{z}_i) = \left[\frac{f \mathbf{z}_{i,x}}{\mathbf{z}_{i,z}} \quad \frac{f \mathbf{z}_{i,y}}{\mathbf{z}_{i,z}} \right]^T$

Overview

Introduction

- Registration
 - 3D Geometry
 - 2D Images
 - Combined 2D / 3D
- Applications
 - **Rigid Scanning**
 - Non-rigid Modeling
 - Realtime Face Tracking

Conclusions

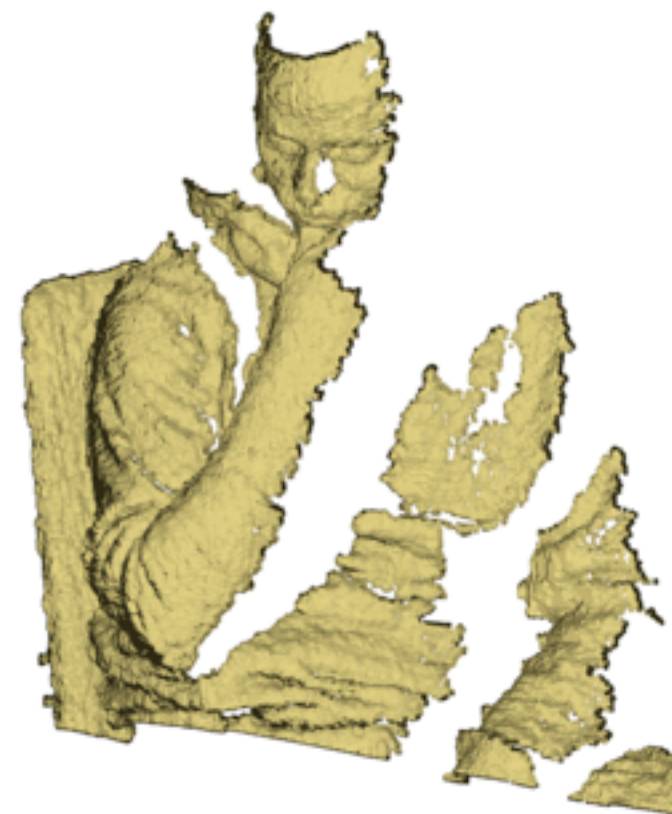
Rigid Scanning

- Popular Kinect application

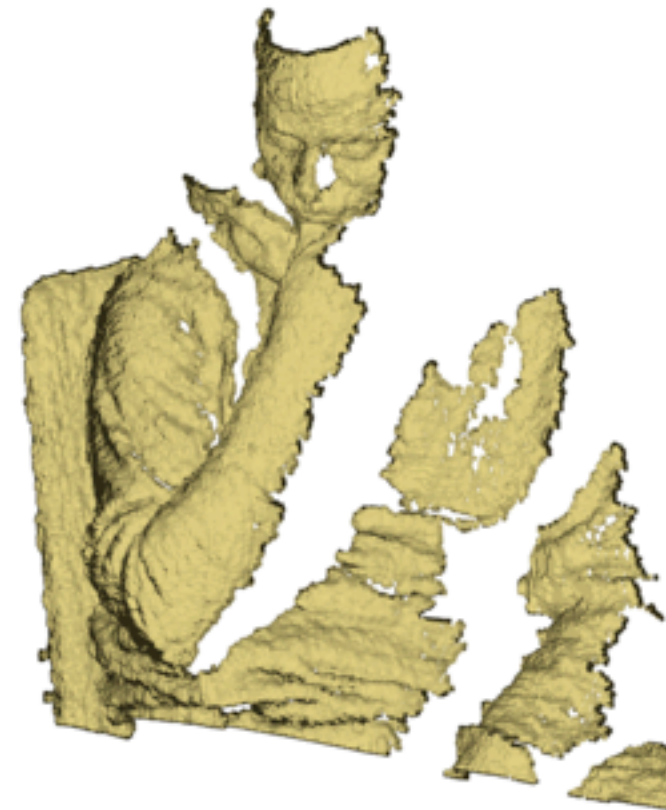
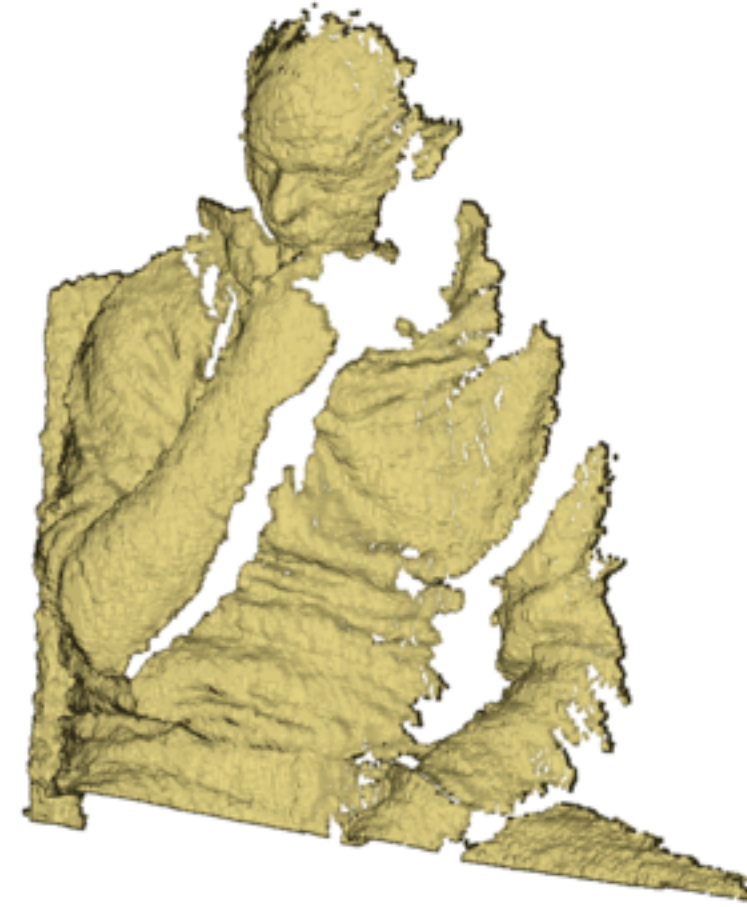


Rigid Scanning

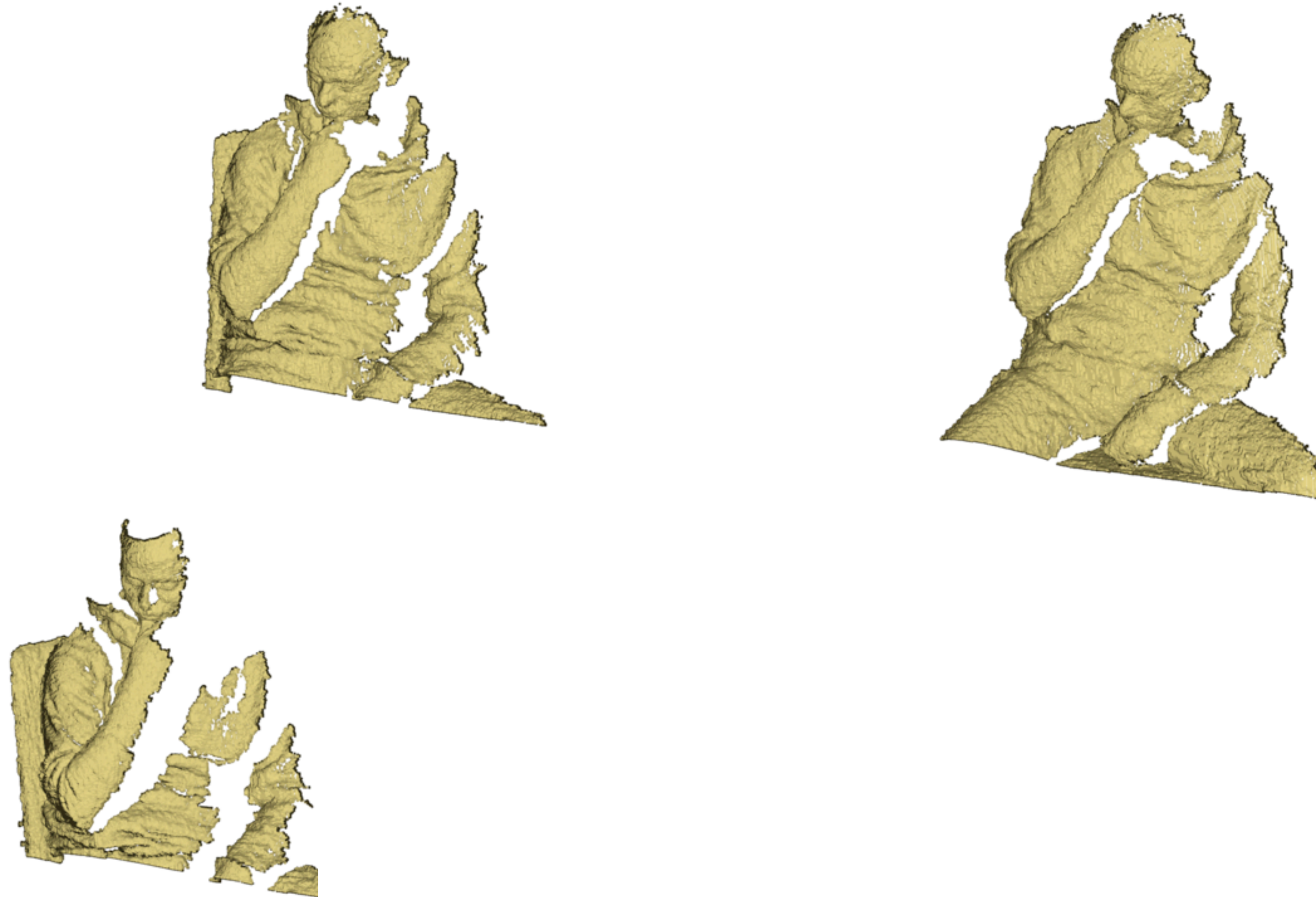
Rigid Scanning



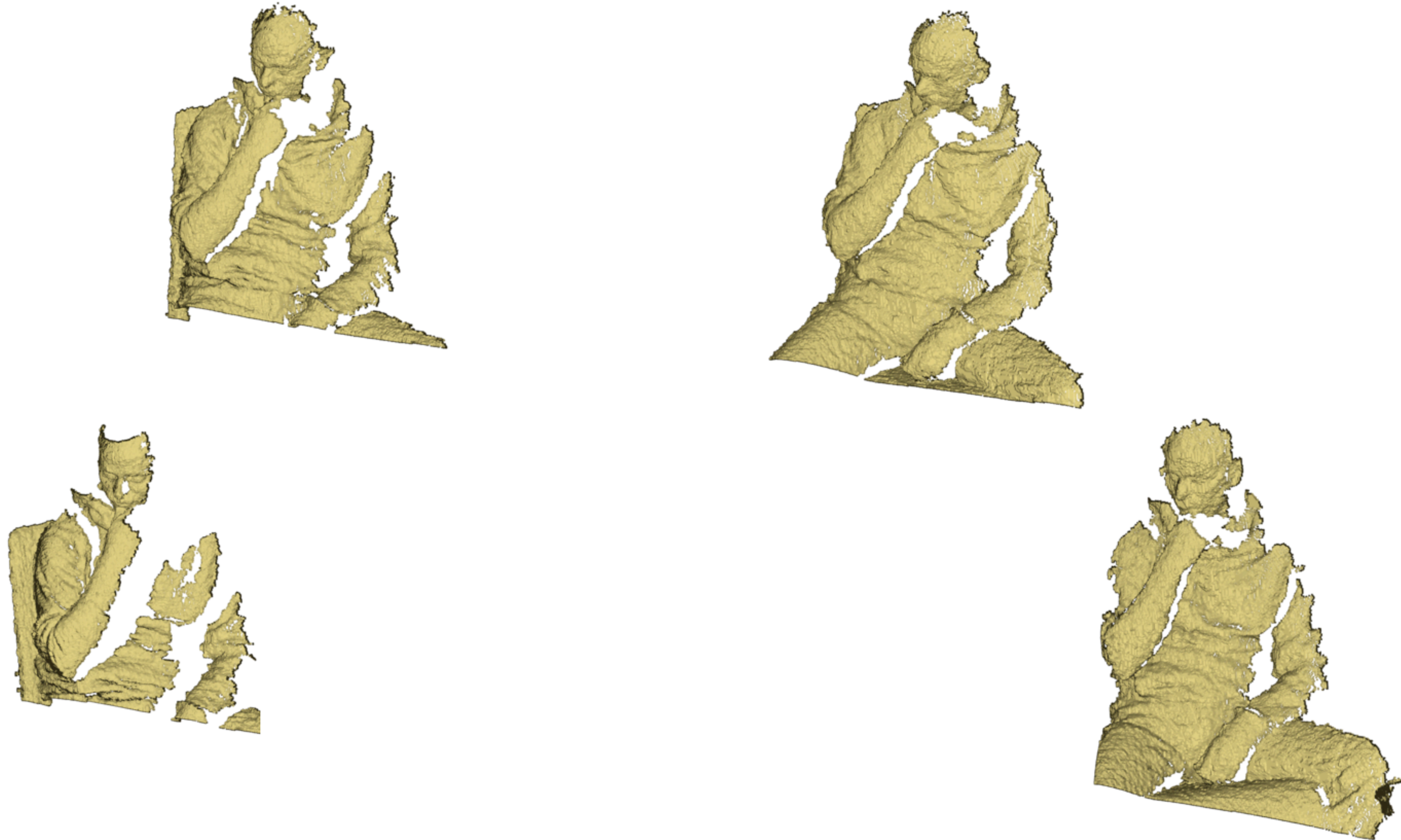
Rigid Scanning



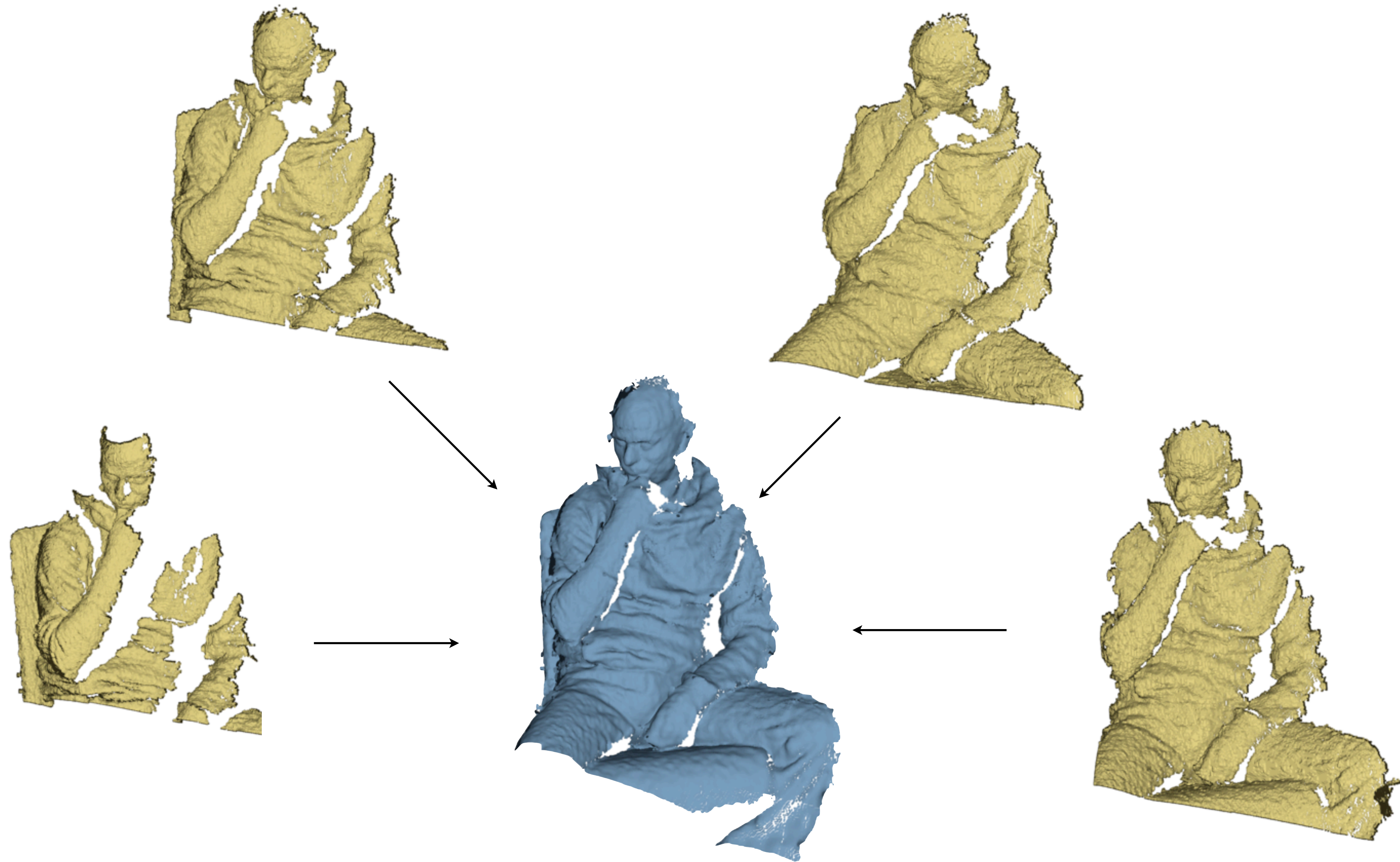
Rigid Scanning



Rigid Scanning



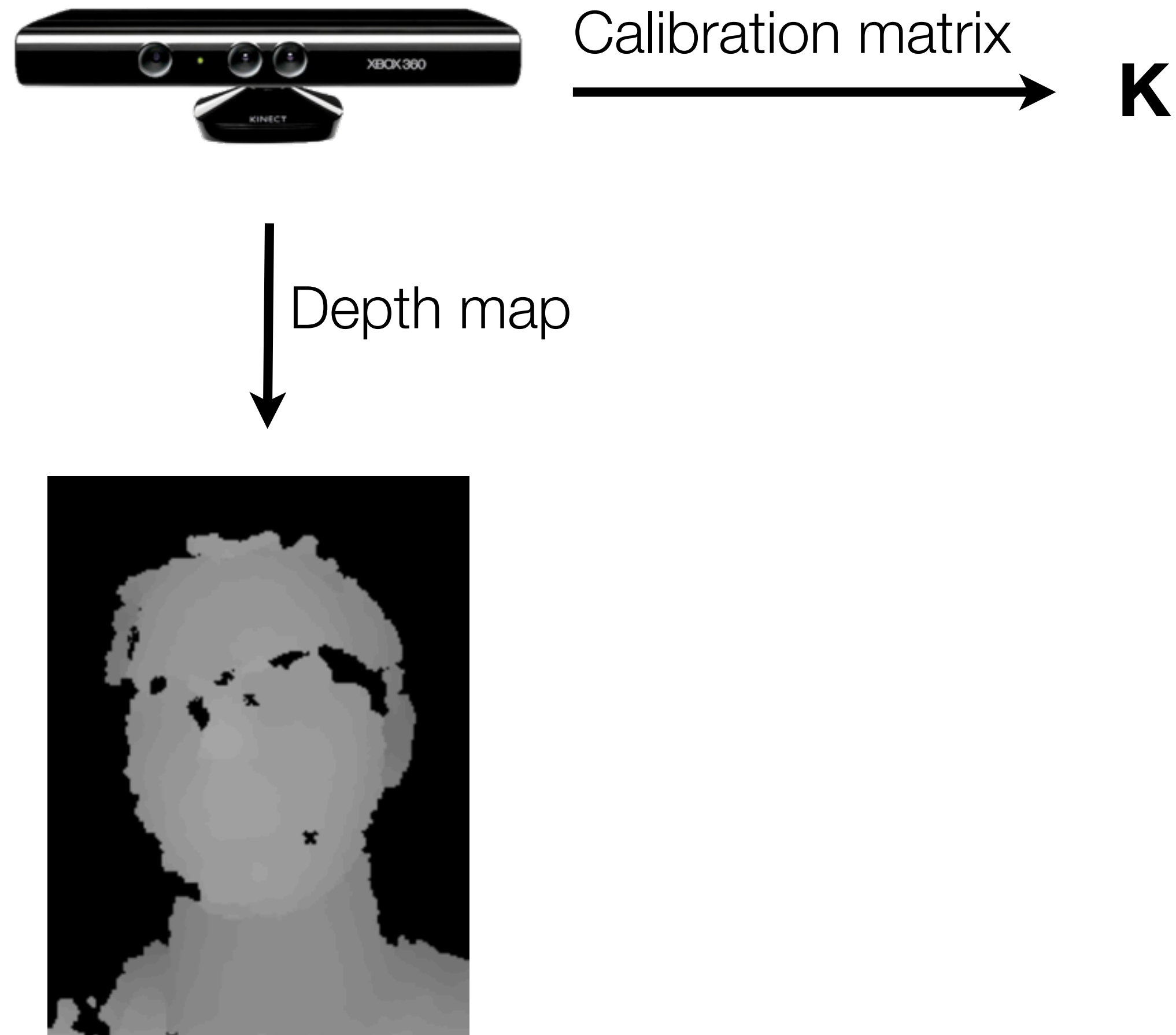
Rigid Scanning



Rigid Scanning - Pipeline



Rigid Scanning - Pipeline



Rigid Scanning - Pipeline

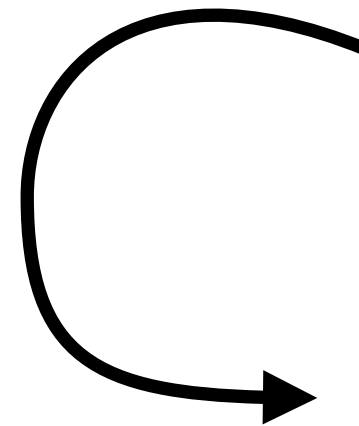


Calibration matrix
→ **K**

↓ Depth map



Smoothing



Rigid Scanning - Pipeline



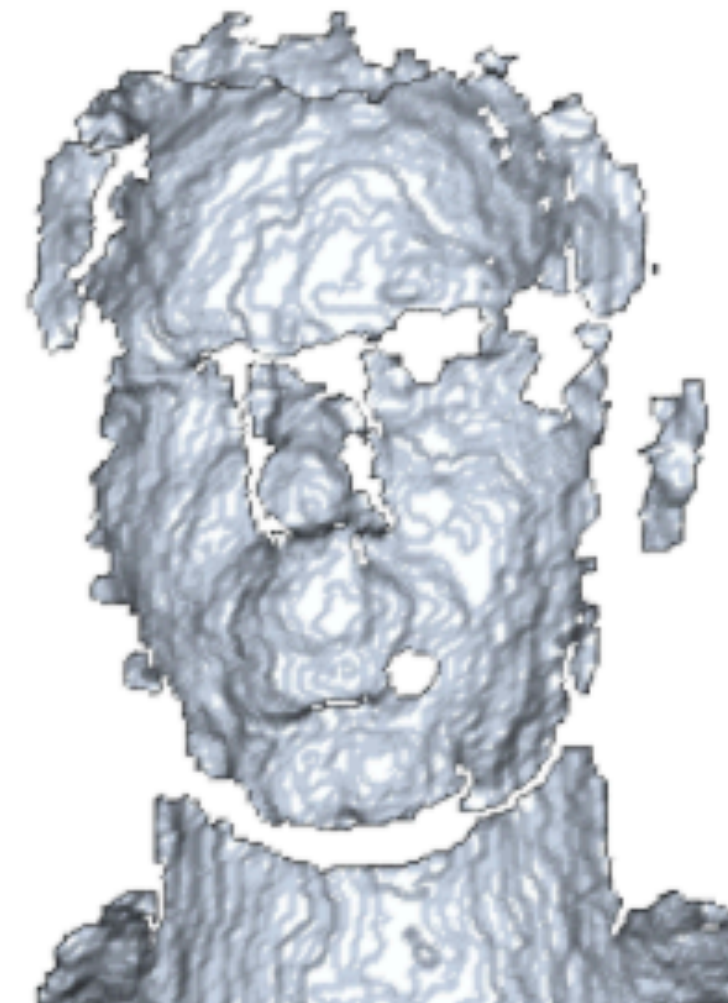
Calibration matrix

K

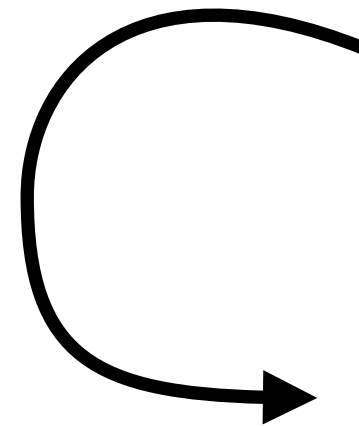
Depth map



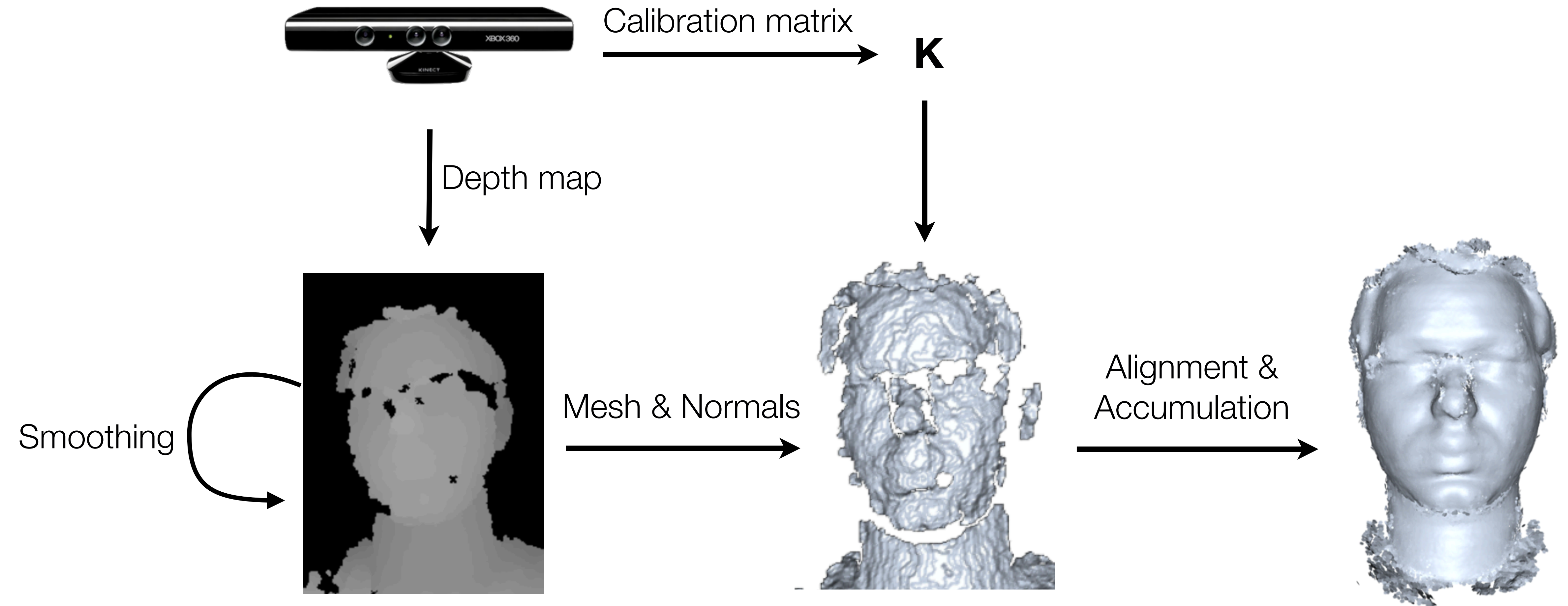
Mesh & Normals



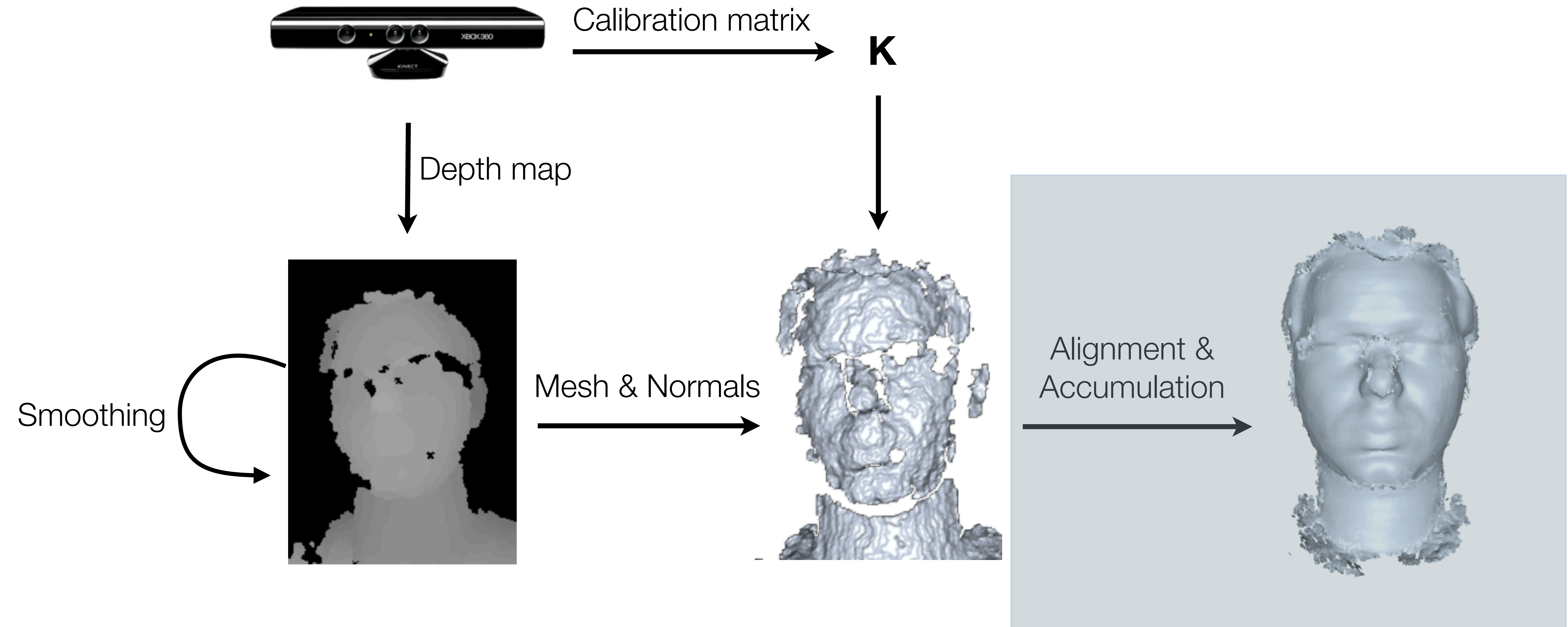
Smoothing



Rigid Scanning - Pipeline



Rigid Scanning - Pipeline



Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

\uparrow
 matching term

Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

↑
↑
 matching term rigid prior

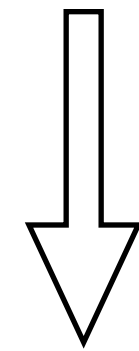
Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

↑
matching term
↑
rigid prior

Rigid Scanning - Optimization

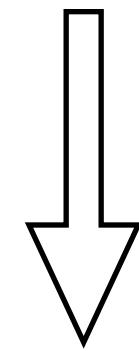
$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



$$\arg \min_{Z^{t+1}, \tilde{\mathbf{R}}, \tilde{\mathbf{t}}} w_1 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t \mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$

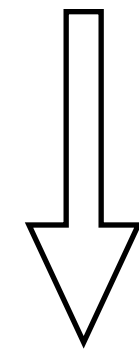


$$\arg \min_{Z^{t+1}, \tilde{\mathbf{R}}, \tilde{\mathbf{t}}} w_1 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t \mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

use previous estimate
= closest point

Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



$$\arg \min_{Z^{t+1}, \tilde{\mathbf{R}}, \tilde{\mathbf{t}}} w_1 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t \mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

use previous estimate
= closest point

linearize rotation matrix

$$\mathbf{R}^{t+1} = \tilde{\mathbf{R}}\mathbf{R}^t$$

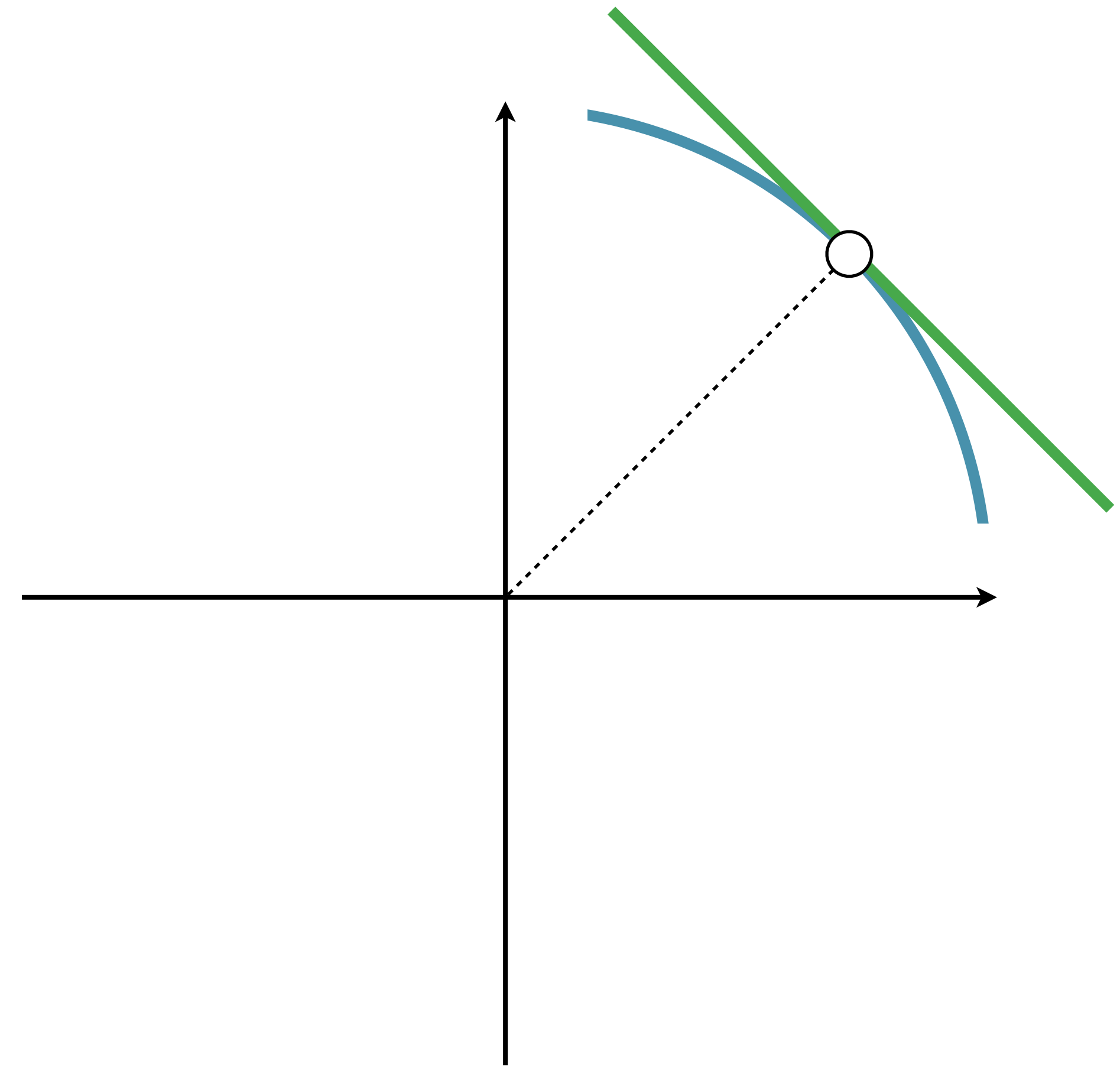
$$\mathbf{t}^{t+1} = \tilde{\mathbf{R}}\mathbf{t}^t + \tilde{\mathbf{t}}$$

Rigid Scanning - Optimization

Rotation matrix linearization

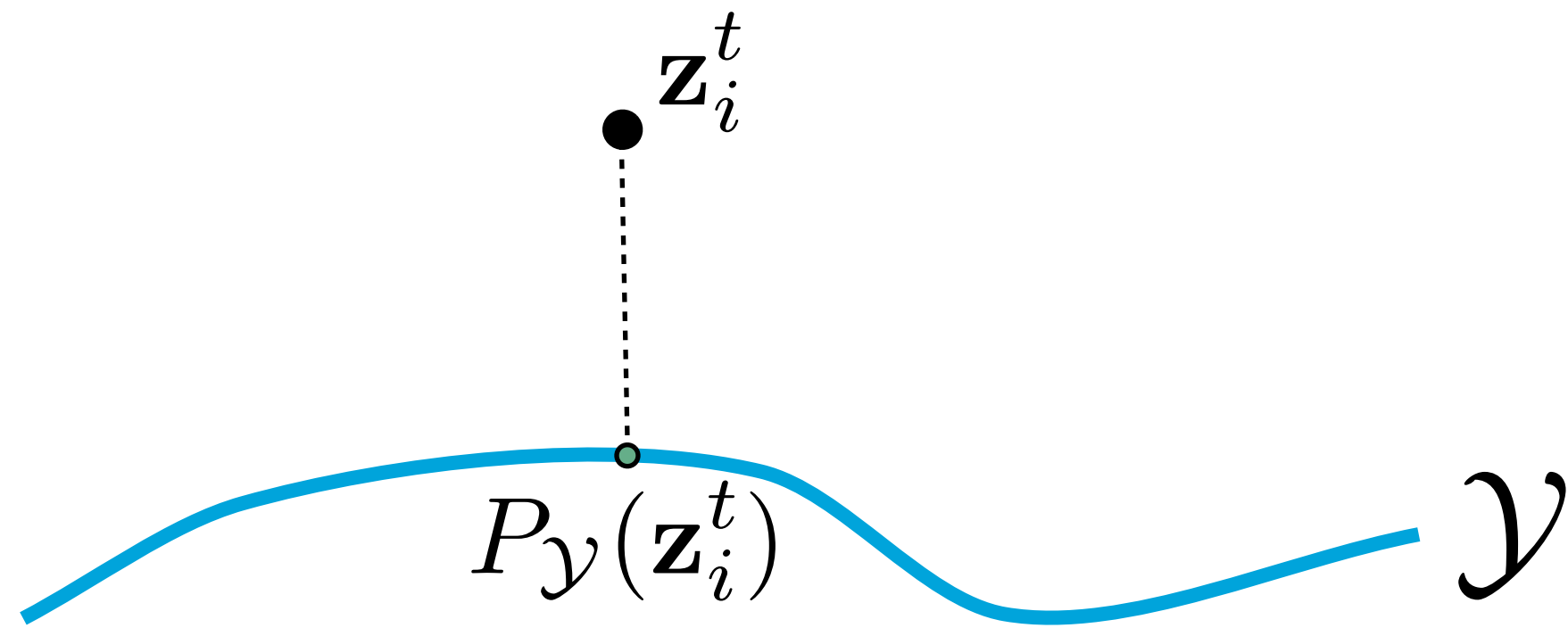
$$\sin(\theta) \approx \theta \quad \cos(\theta) \approx 1$$

$$\mathbf{R} \approx \tilde{\mathbf{R}} = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix}$$



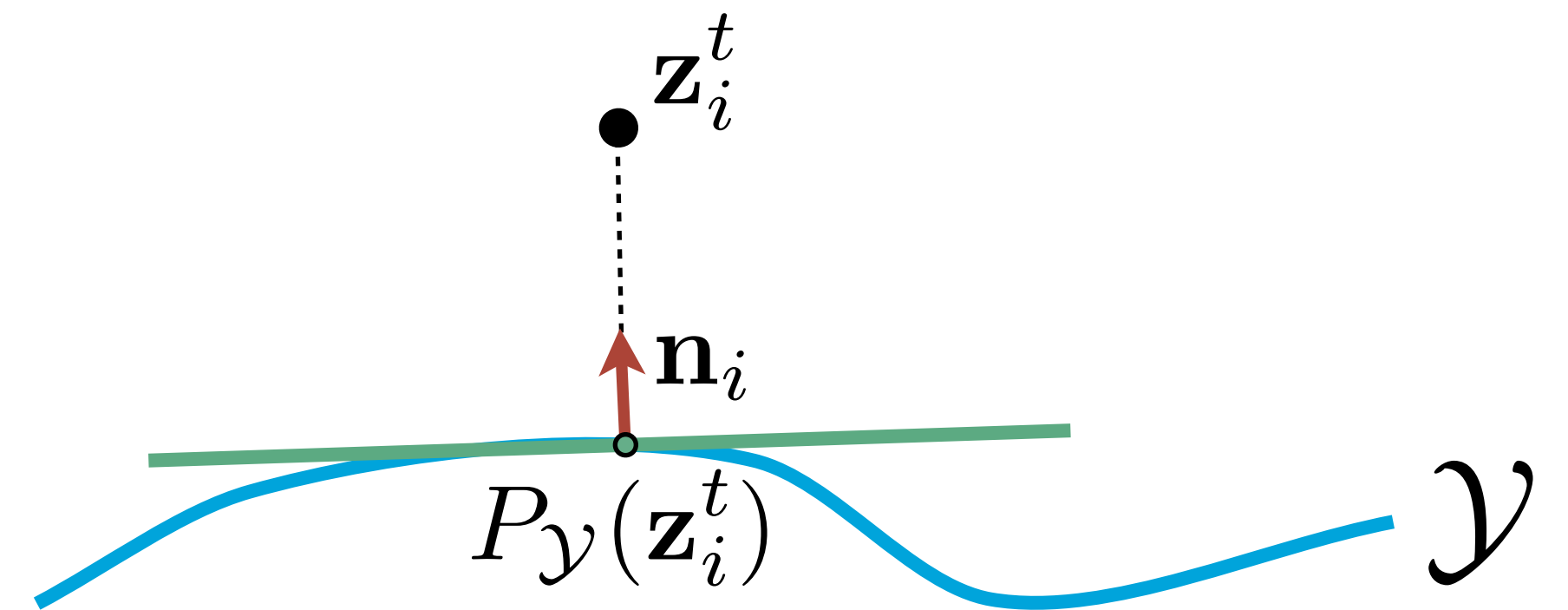
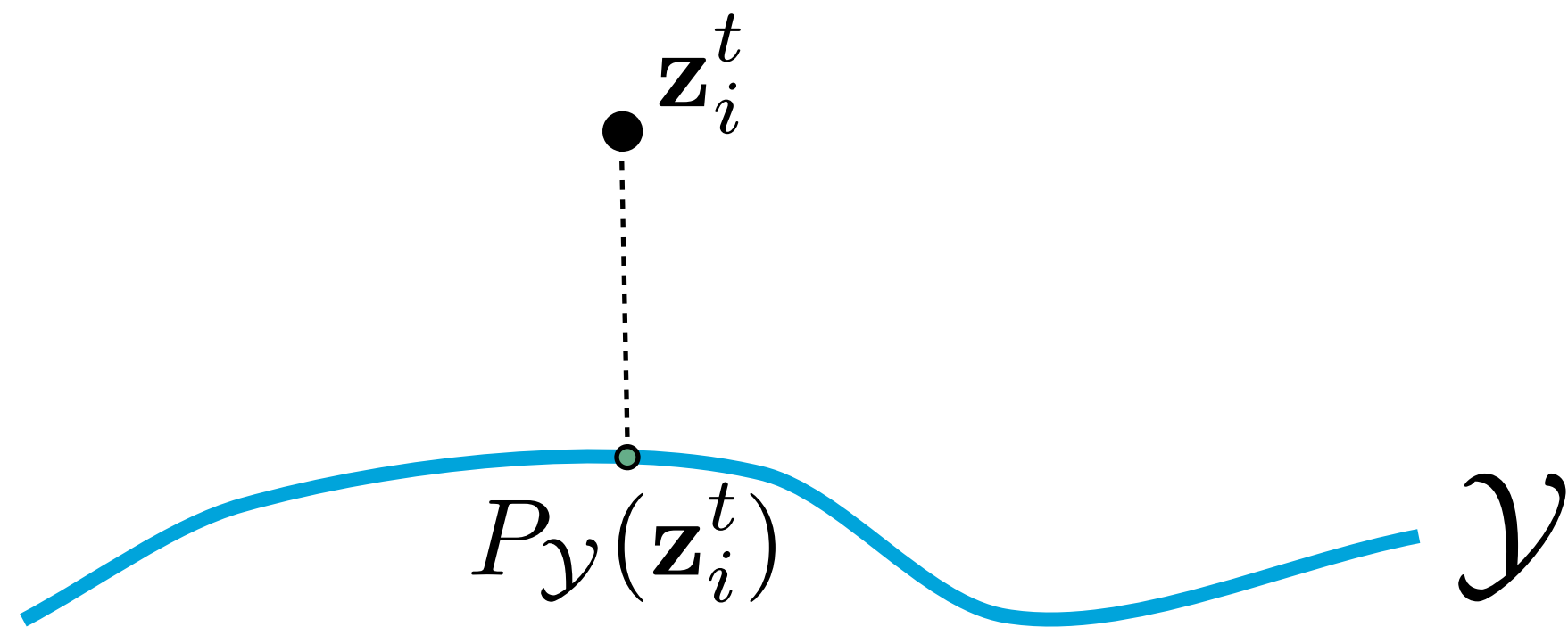
Rigid Scanning - Optimization

$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$



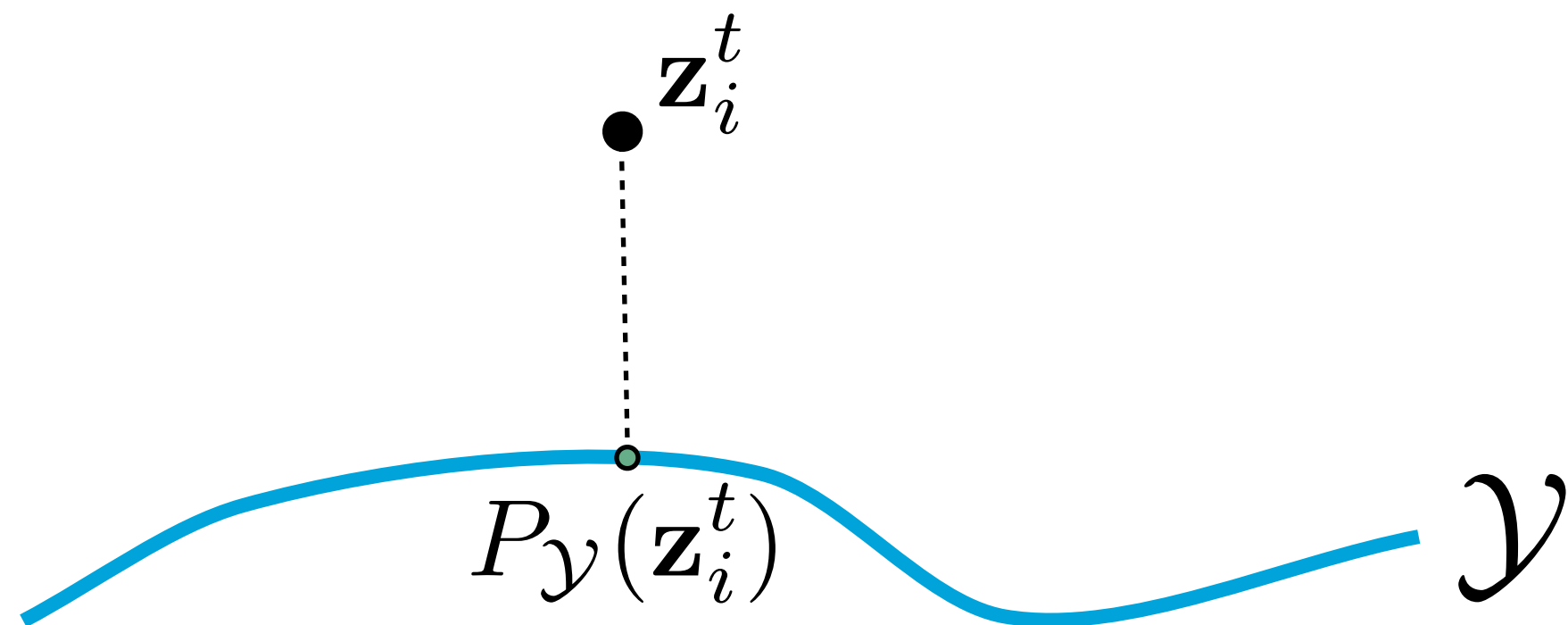
Rigid Scanning - Optimization

$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$

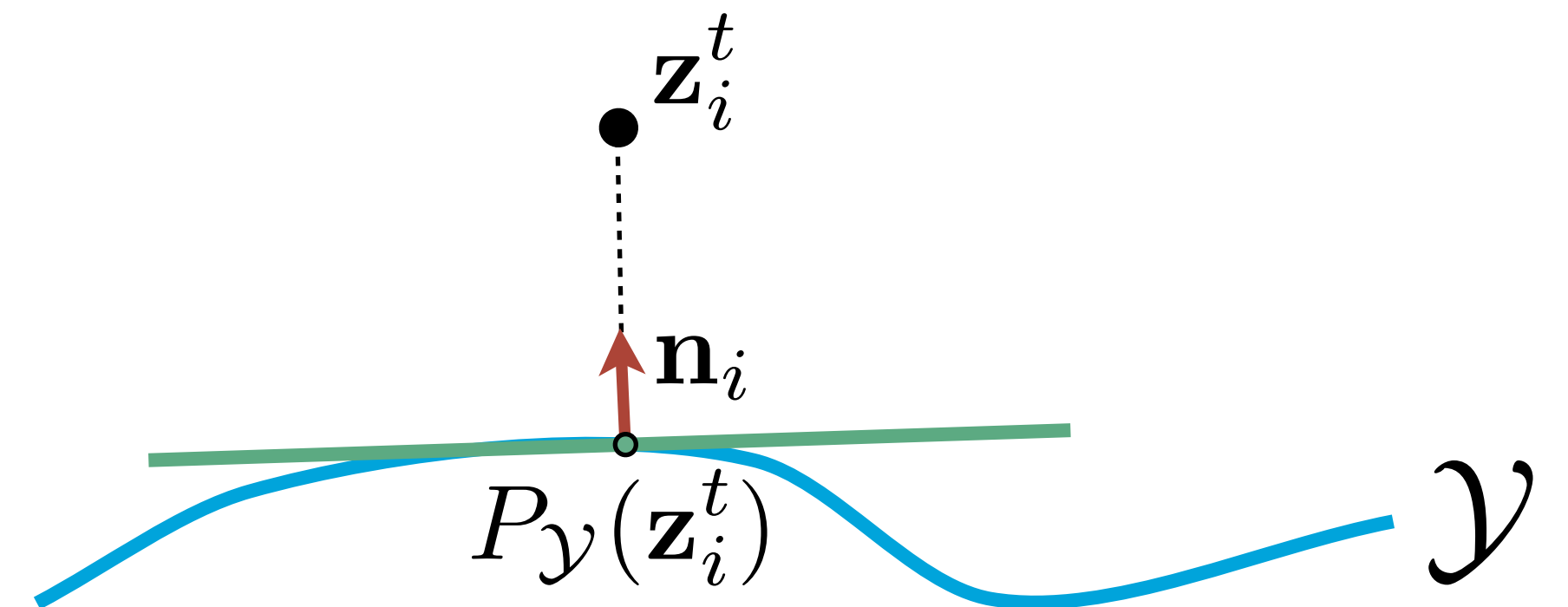


Rigid Scanning - Optimization

$$\|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2$$

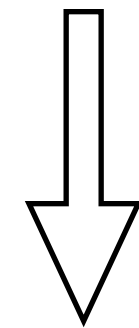


$$(\mathbf{n}_i^T (\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)))^2$$



Rigid Scanning - Optimization

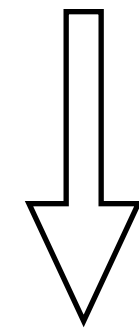
$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



$$\arg \min_{Z^{t+1}, \tilde{\mathbf{R}}, \tilde{\mathbf{t}}} w_1 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t \mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

Rigid Scanning - Optimization

$$\arg \min_{Z, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2$$



$$\arg \min_{Z^{t+1}, \tilde{\mathbf{R}}, \tilde{\mathbf{t}}} w_1 \sum_{i=1}^n (\mathbf{n}_i^T (\mathbf{z}_i^{t+1} - P_{\mathcal{Y}}(\mathbf{z}_i^t)))^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i^{t+1} - (\tilde{\mathbf{R}}(\mathbf{R}^t \mathbf{x}_i + \mathbf{t}^t) + \tilde{\mathbf{t}})\|_2^2$$

Rigid Scanning - Accumulation

Rigid Scanning - Accumulation

- Reduce the number of points

Rigid Scanning - Accumulation

- Reduce the number of points
- Reduce noise

Rigid Scanning - Accumulation

- Reduce the number of points
- Reduce noise

Volume

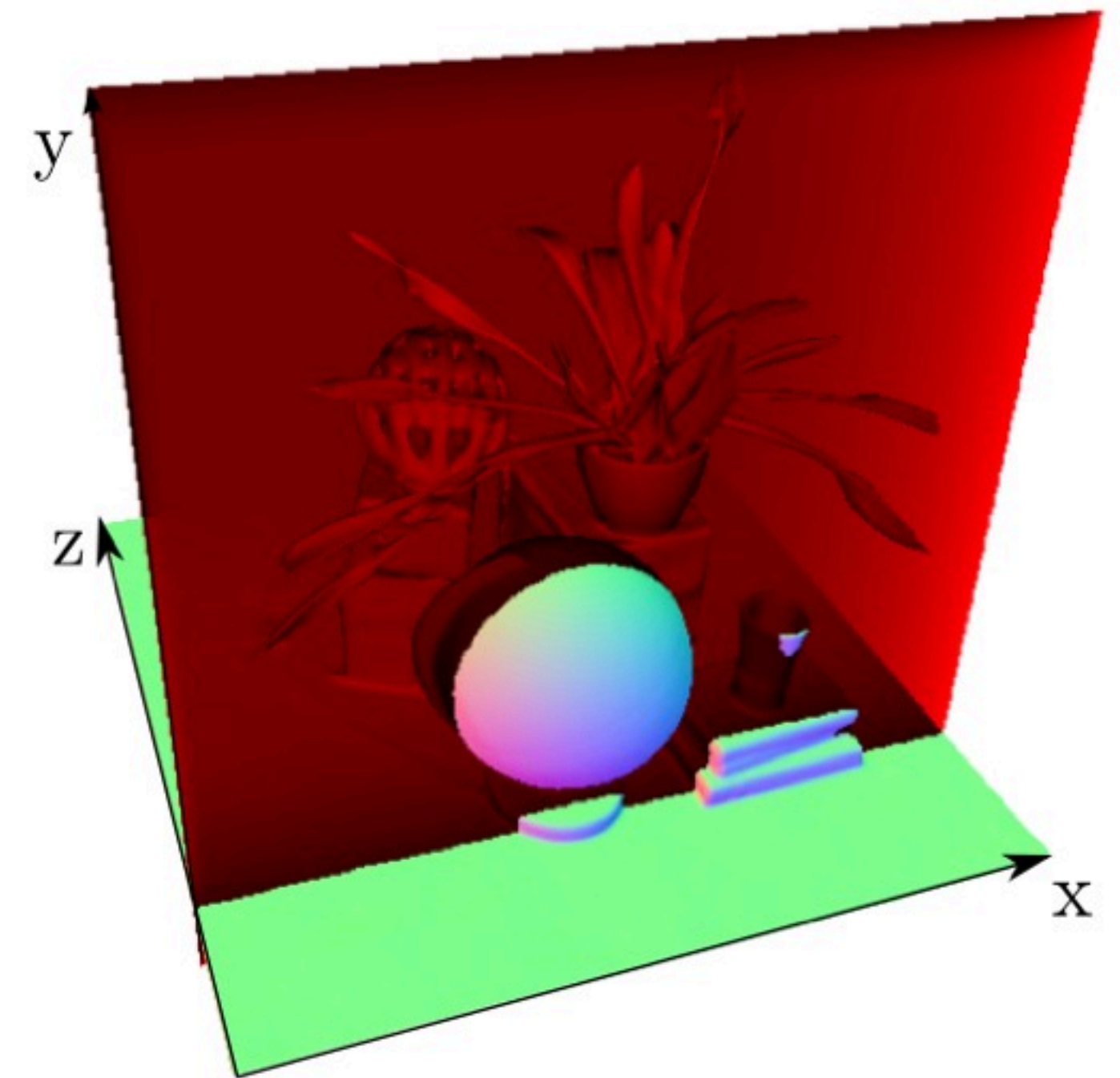
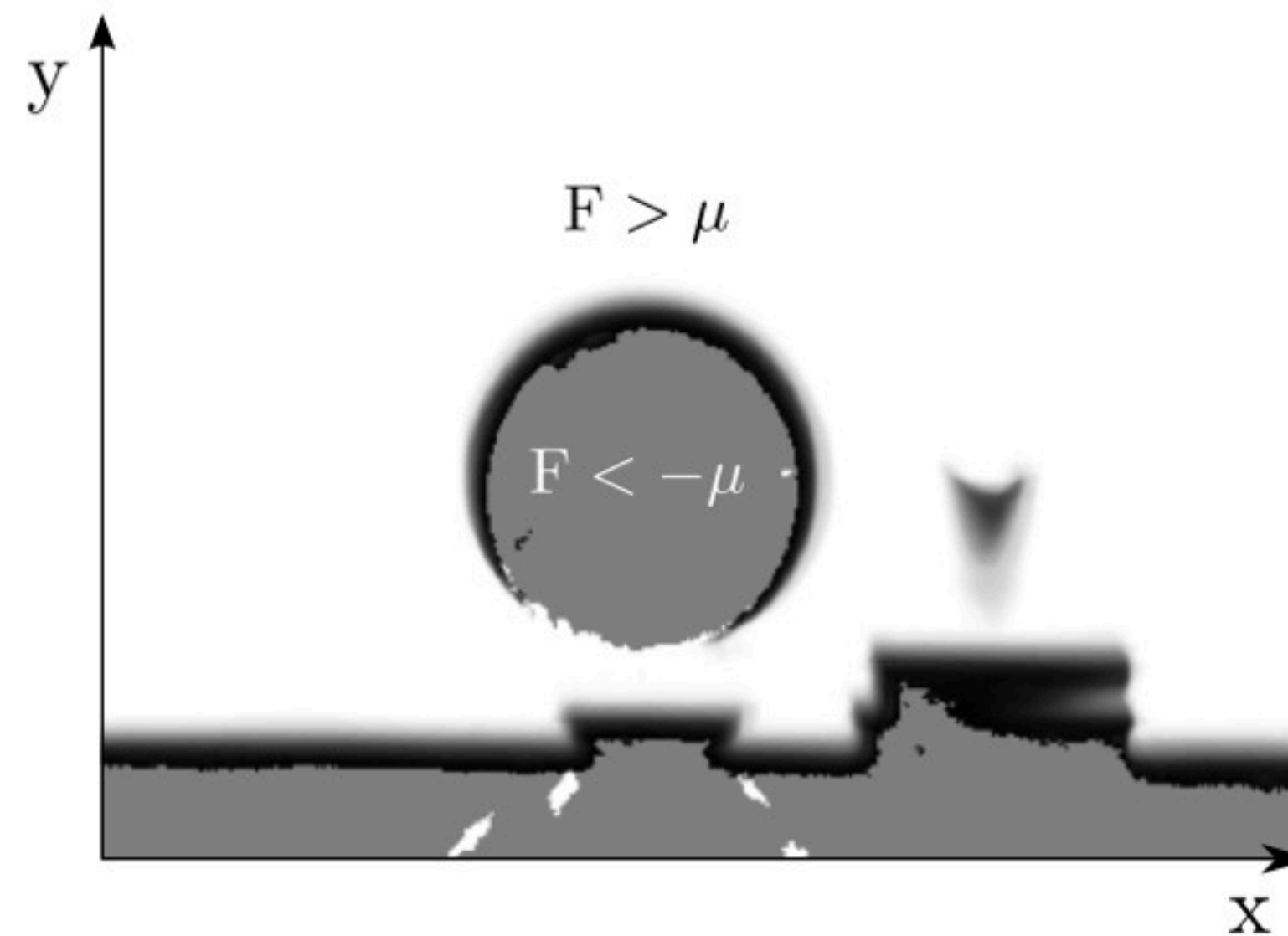
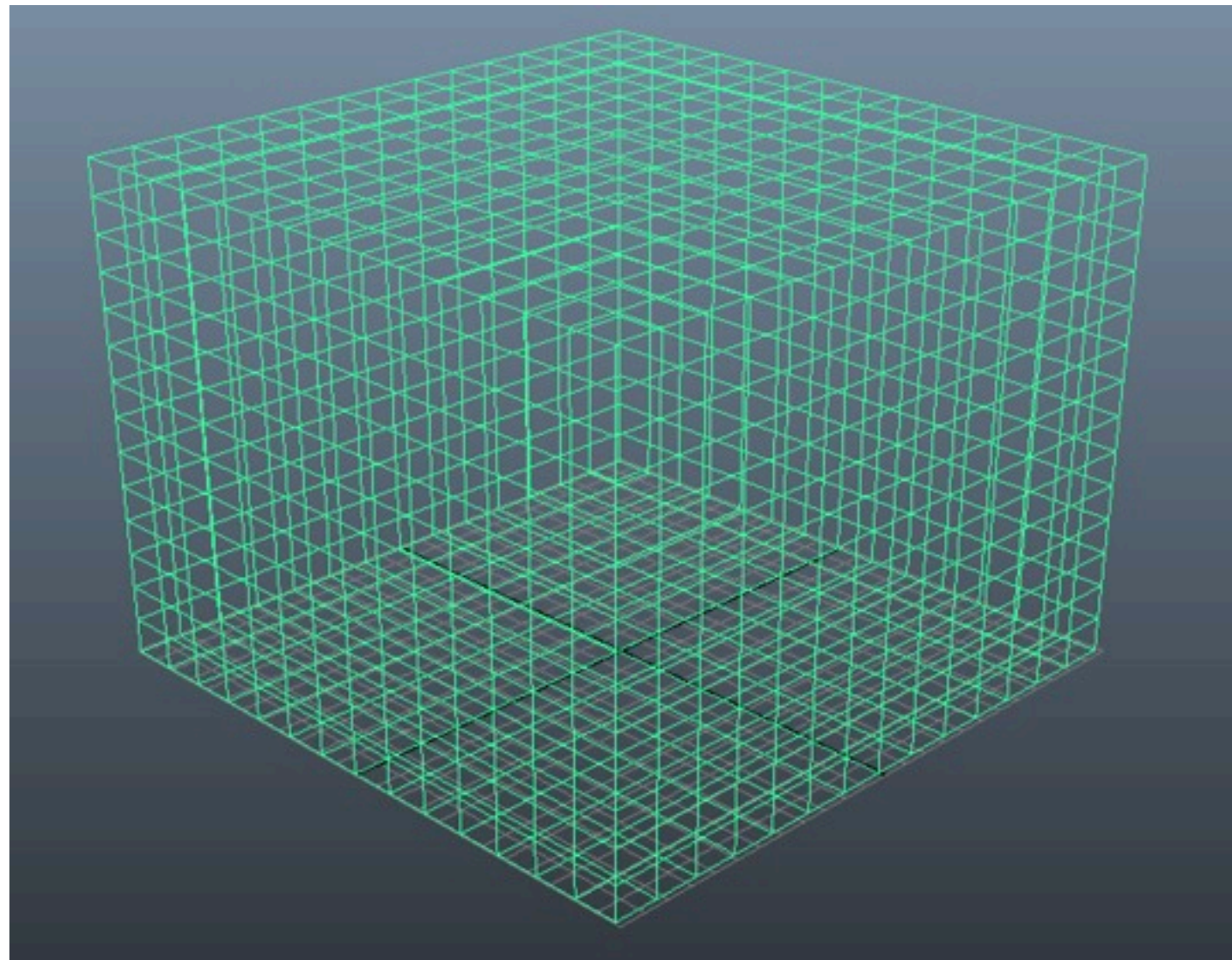


Newcombe, Izadi, Hilliges, Molyneaux, Kim,
Davison, Kohli, Shotton, Hodges, Fitzgibbon:

KinectFusion: Real-Time Dense Surface Mapping and Tracking,
ISMAR 2011

Rigid Scanning - Accumulation

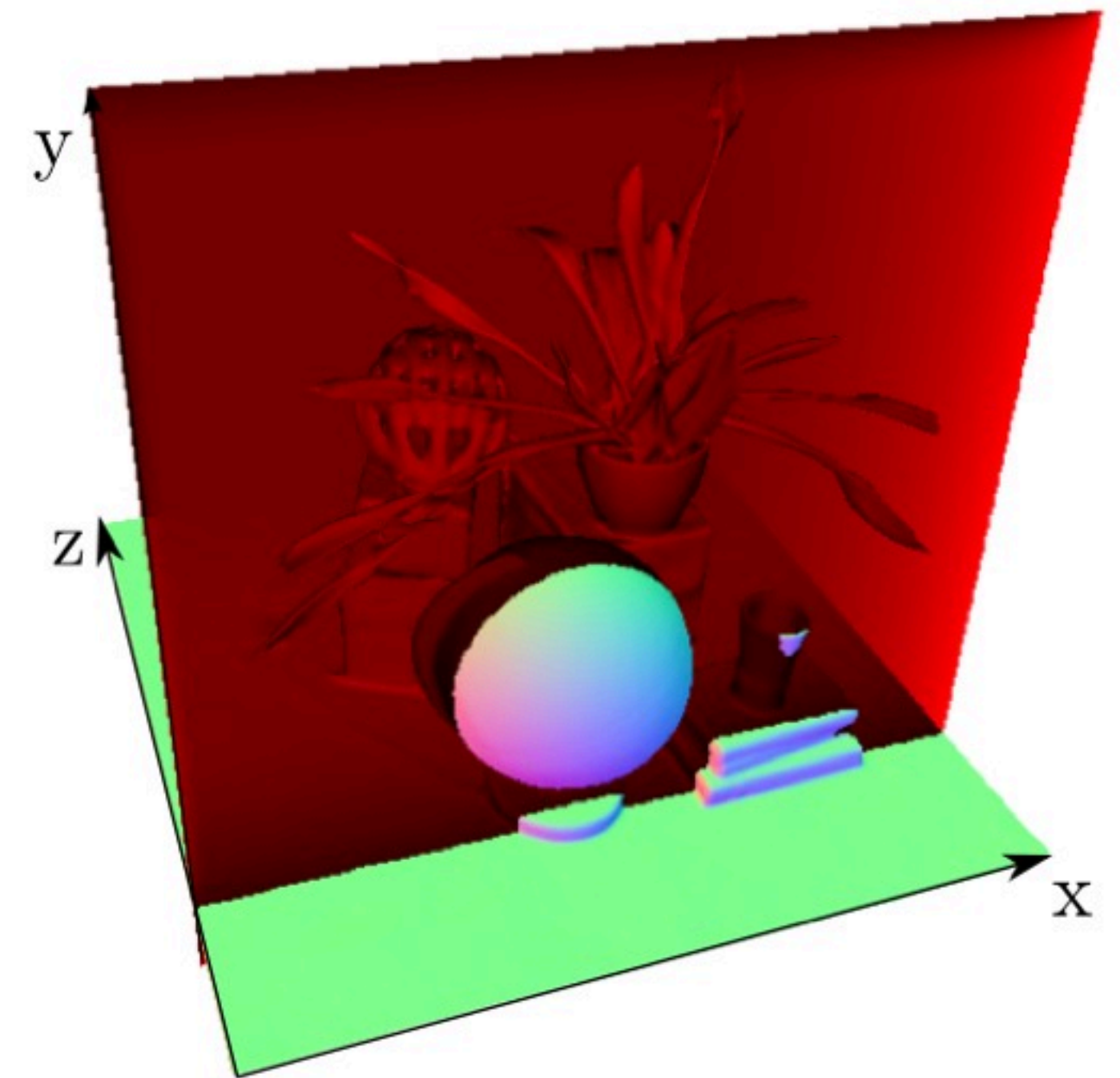
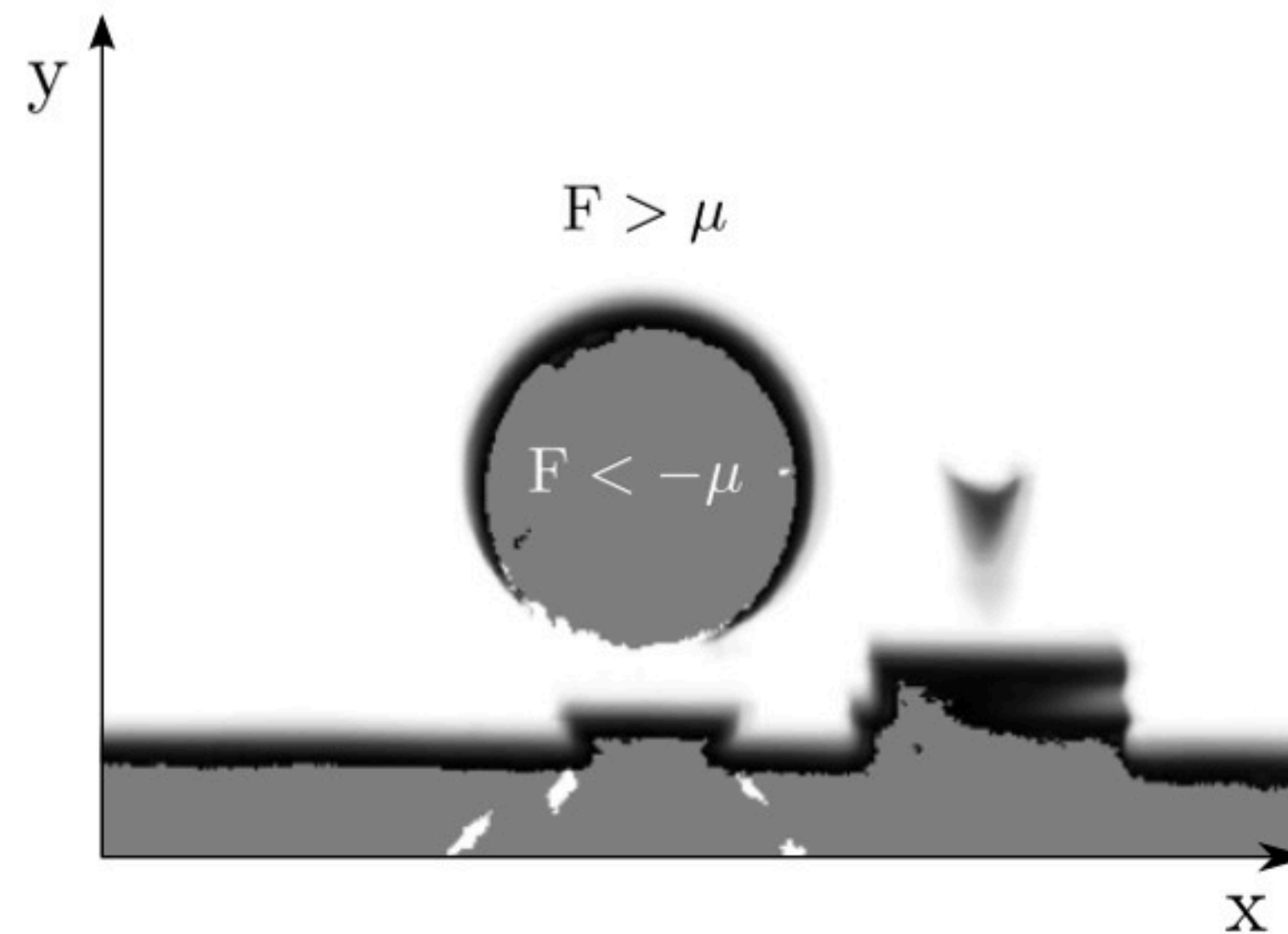
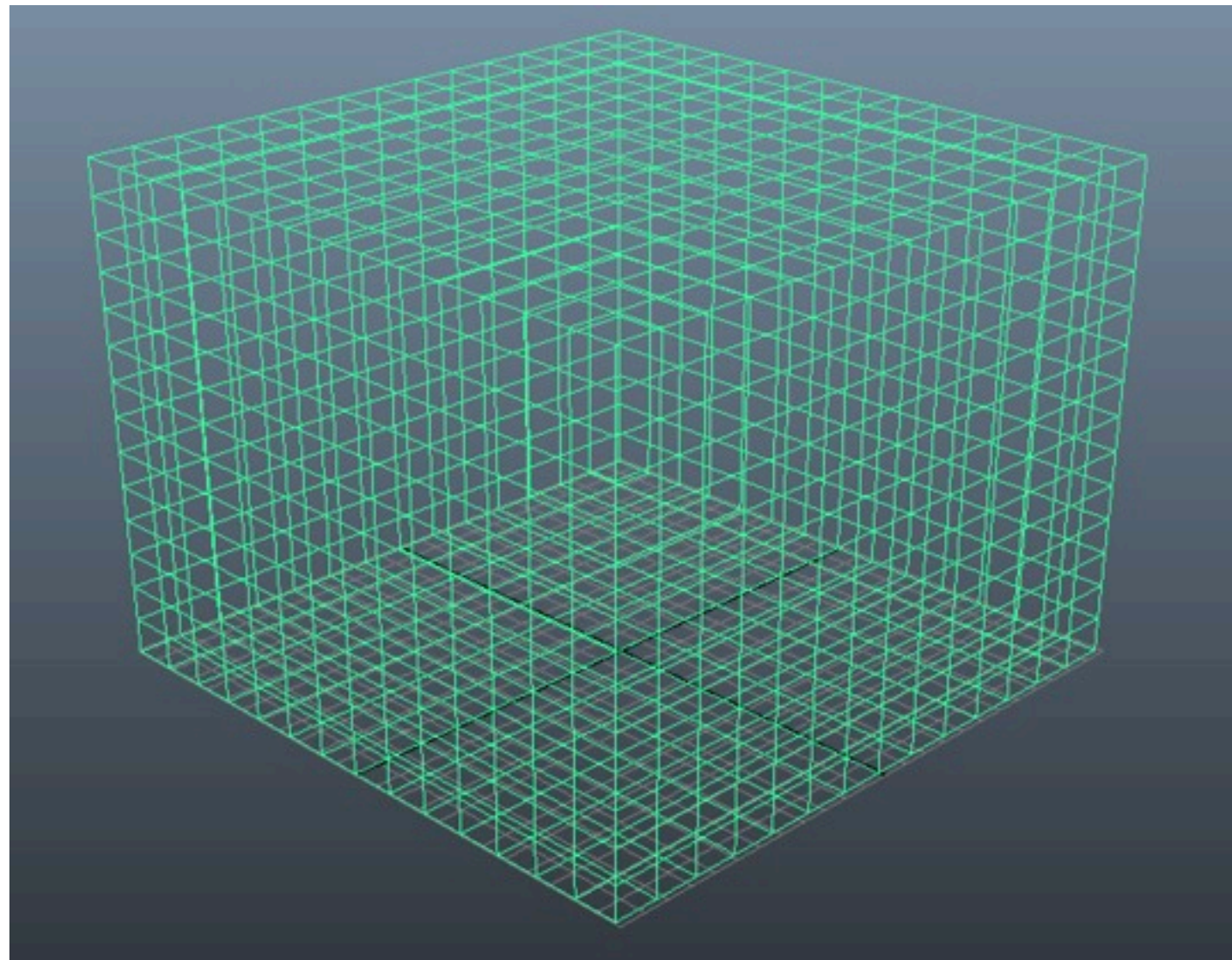
Volumetric accumulation



KinectFusion: Real-Time Dense Surface Mapping and Tracking, *ISMAR 2011*

Rigid Scanning - Accumulation

Volumetric accumulation



DEMO

KinectFusion: Real-Time Dense Surface Mapping and Tracking, *ISMAR 2011*

Rigid Scanning - Accumulation

- Reduce the number of points
- Reduce noise

Volume



Newcombe, Izadi, Hilliges, Molyneaux, Kim,
Davison, Kohli, Shotton, Hodges, Fitzgibbon:

KinectFusion: Real-Time Dense Surface Mapping and Tracking,
ISMAR 2011

Rigid Scanning - Accumulation

- Reduce the number of points
- Reduce noise

Volume



Newcombe, Izadi, Hilliges, Molyneaux, Kim,
Davison, Kohli, Shotton, Hodges, Fitzgibbon:
KinectFusion: Real-Time Dense Surface Mapping and Tracking,
ISMAR 2011

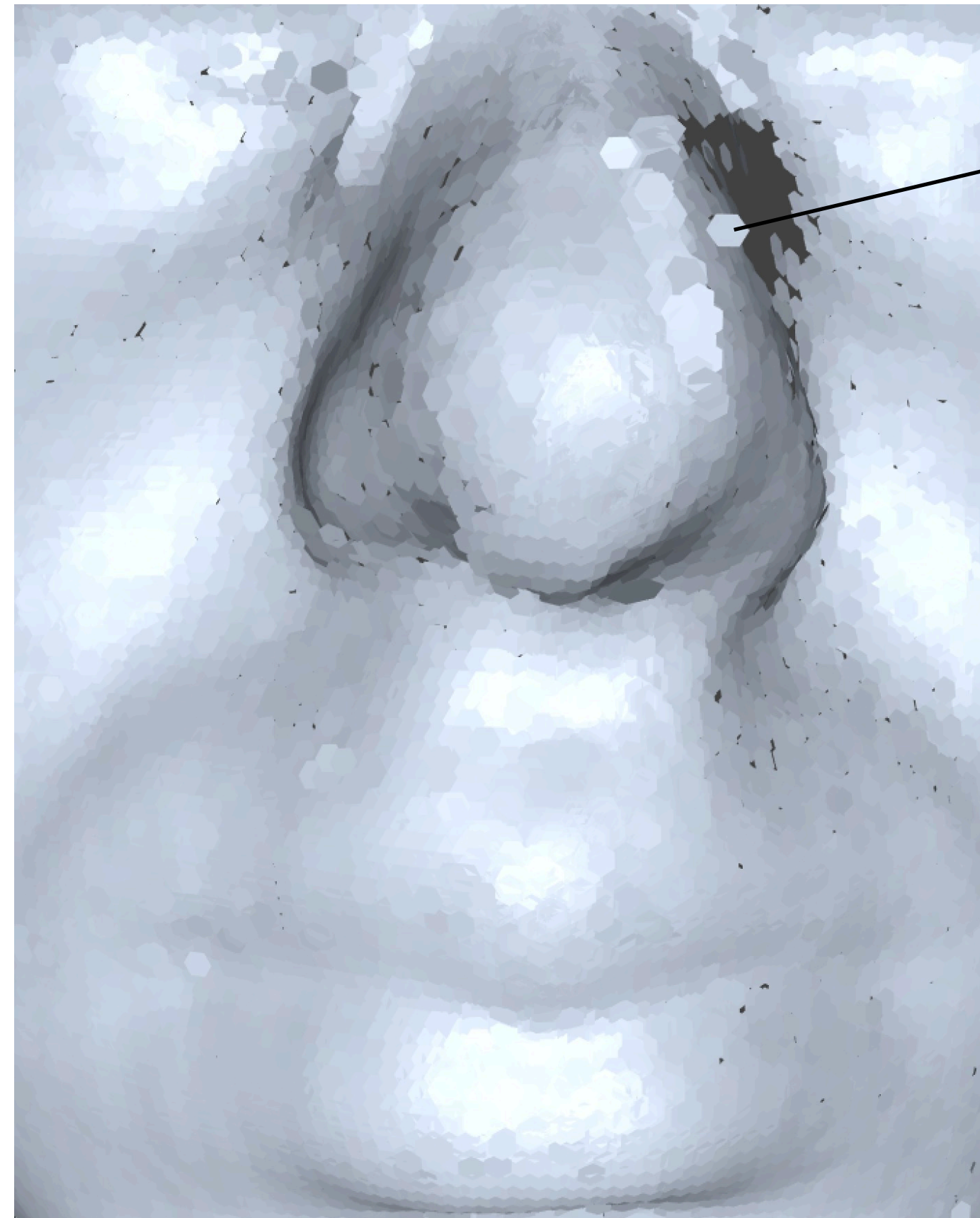
Surface



Weise, Wismer, Leibe, Van Gool:
In-hand Scanning with Online Loop Closure,
3DIM 2009

Rigid Scanning - Accumulation

Surface accumulation

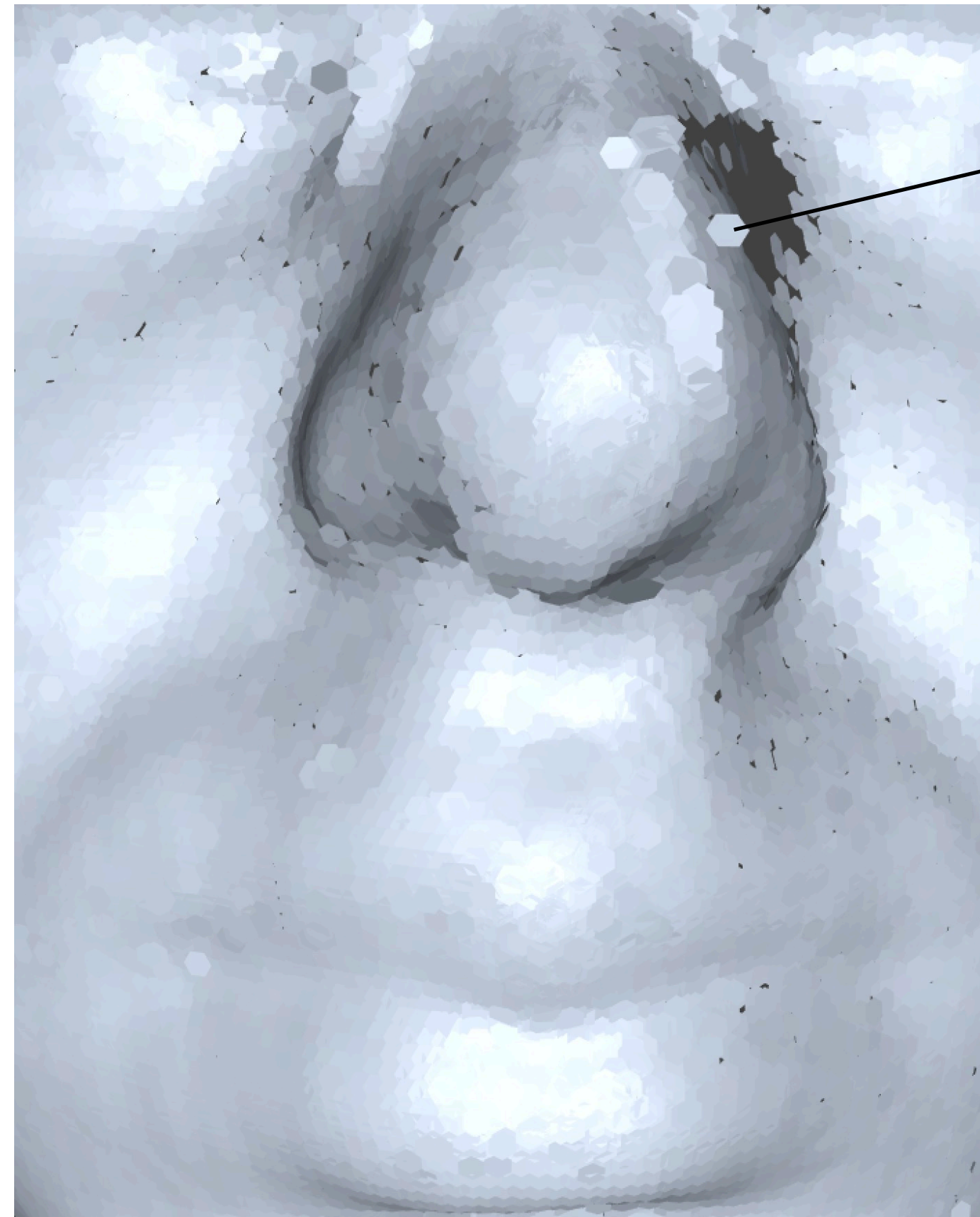


In-hand Scanning with Online Loop Closure, *3DIM 2009*

Rigid Scanning - Accumulation

Surface accumulation

Surfel update



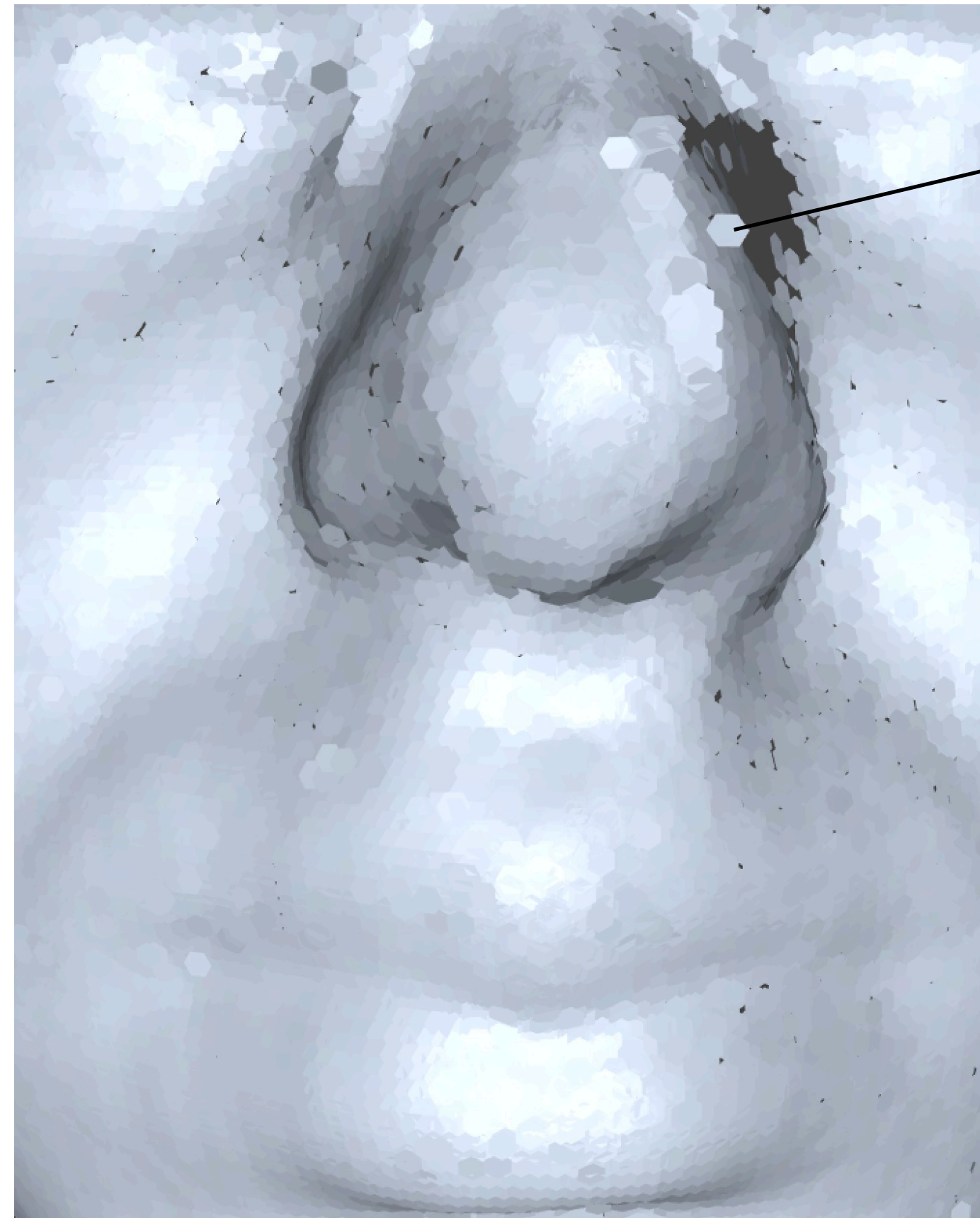
In-hand Scanning with Online Loop Closure, *3DIM 2009*

Rigid Scanning - Accumulation

Surface accumulation

Surfel update

Surfel addition



In-hand Scanning with Online Loop Closure, *3DIM 2009*

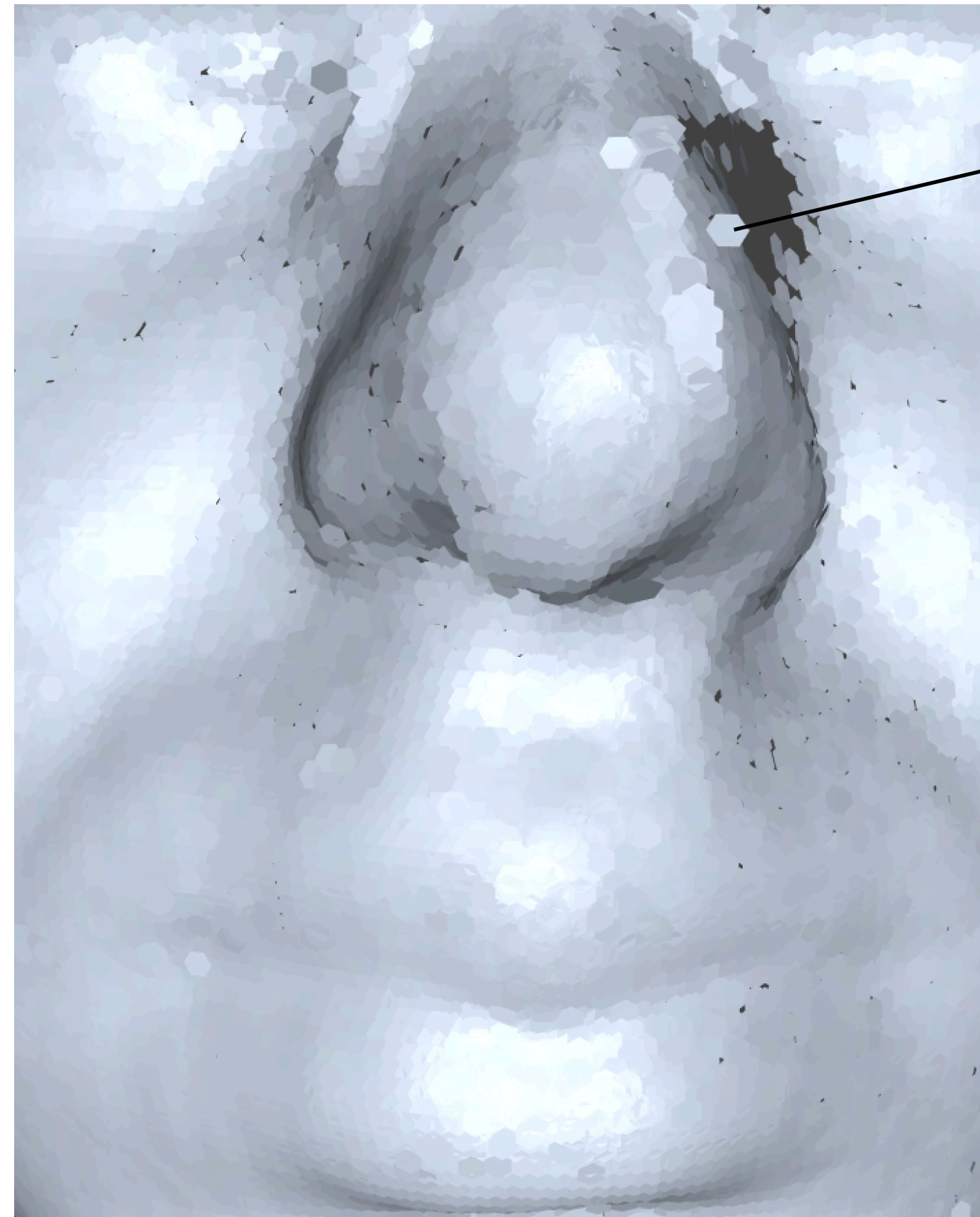
Rigid Scanning - Accumulation

Surface accumulation

Surfel update

Surfel addition

Surfel removal



In-hand Scanning with Online Loop Closure, *3DIM 2009*

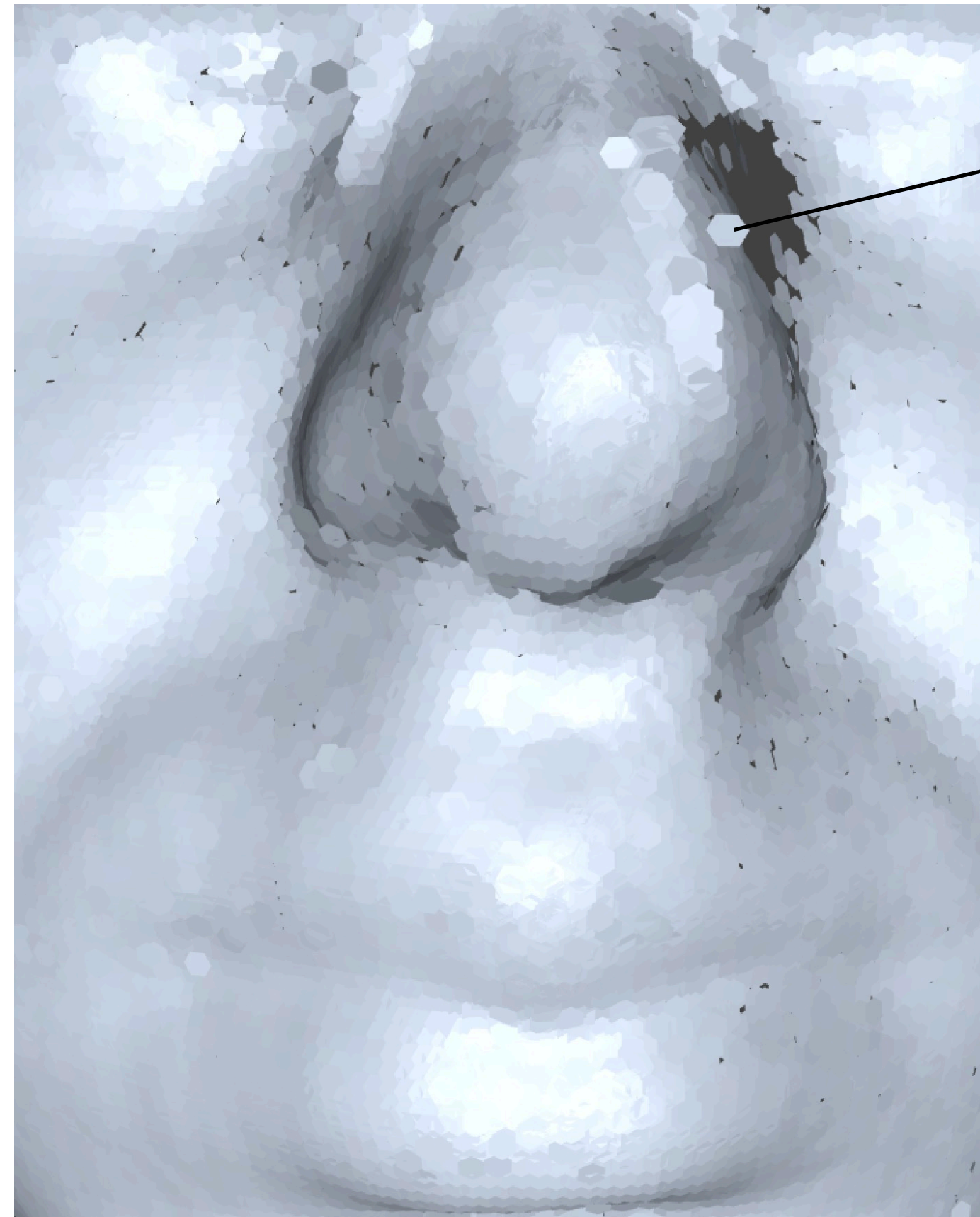
Rigid Scanning - Accumulation

Surface accumulation

Surfel update

Surfel addition

Surfel removal



DEMO

In-hand Scanning with Online Loop Closure, *3DIM 2009*

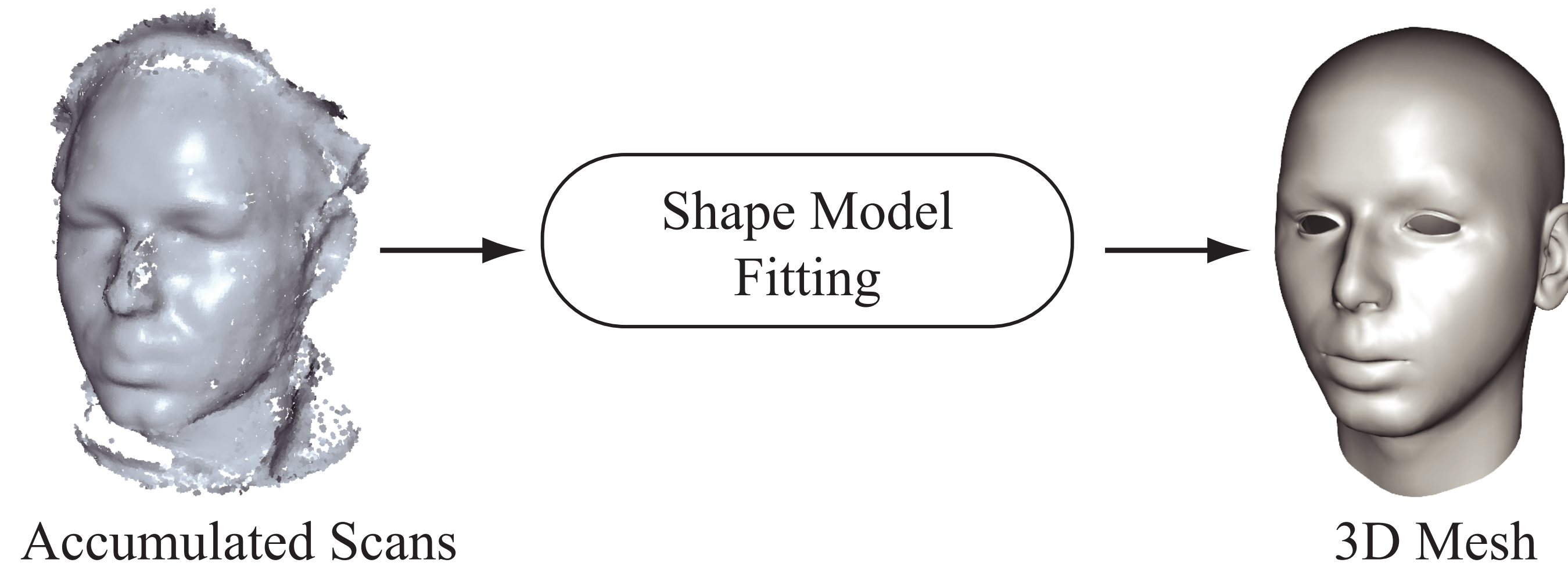
Overview

Introduction

- Registration
 - 3D Geometry
 - 2D Images
 - Combined 2D / 3D
- Applications
 - Rigid Scanning
 - **Non-rigid Modeling**
 - Realtime Face Tracking

Conclusions

Non-Rigid Modeling



Non-Rigid Modeling

$$\arg \min_{Z, \mathbf{R}_i, \mathbf{R}, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P\mathbf{y}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i\mathbf{d} + \mathbf{m}_i)\|_2^2$$

Non-Rigid Modeling

$$\arg \min_{Z, \mathbf{R}_i, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i \mathbf{d} + \mathbf{m}_i)\|_2^2$$

↑
matching term

Non-Rigid Modeling

$$\arg \min_{Z, \mathbf{R}_i, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i \mathbf{d} + \mathbf{m}_i)\|_2^2$$

↑
↑

matching term
rigid prior

Non-Rigid Modeling

$$\arg \min_{Z, \mathbf{R}_i, \mathbf{R}, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i \mathbf{d} + \mathbf{m}_i)\|_2^2$$

↑
↑
↑

matching term
rigid prior
non-rigid prior

Non-Rigid Modeling

$$\arg \min_{Z, \mathbf{R}_i, \mathbf{R}, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i \mathbf{d} + \mathbf{m}_i)\|_2^2$$

↑
↑
↑
↑

matching term
rigid prior
non-rigid prior
linear model

Non-Rigid Modeling

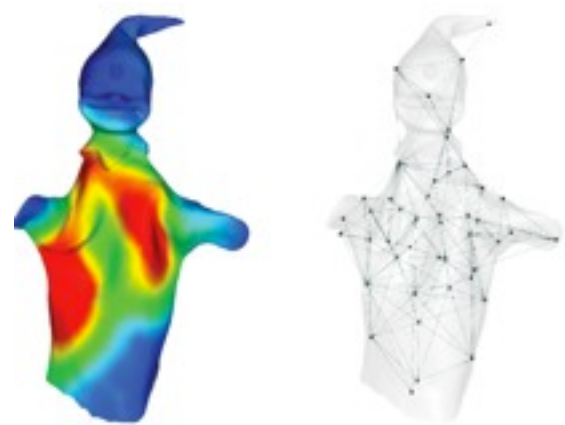
$$\arg \min_{Z, \mathbf{R}_i, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P\mathbf{y}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i\mathbf{d} + \mathbf{m}_i)\|_2^2$$

- How to set the weights for the different energies?

Non-Rigid Modeling

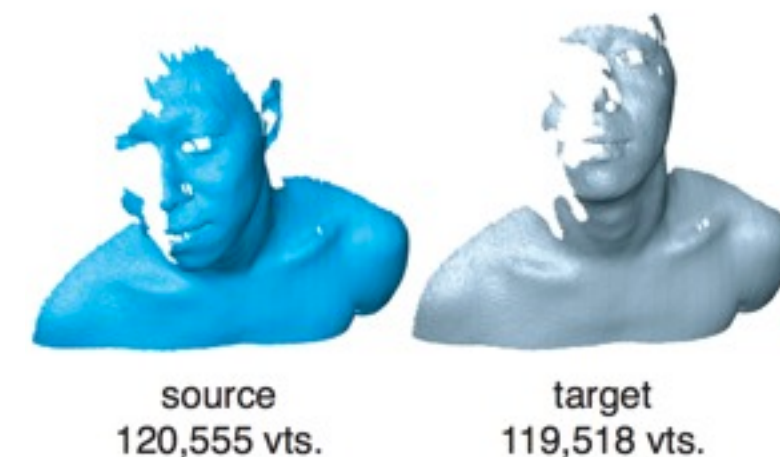
$$\arg \min_{Z, \mathbf{R}_i, \mathbf{R}, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i \mathbf{d} + \mathbf{m}_i)\|_2^2$$

- How to set the weights for the different energies?



Li, Adams, Guibas, Pauly:
**Robust Single-View Geometry
and Motion Reconstruction,**
ACM SIGGRAPH Asia 2009

Li, Sumner, Pauly:
**Global Correspondence Optimization for
Non-Rigid Registration of Depth Scans,**
SGP 2008

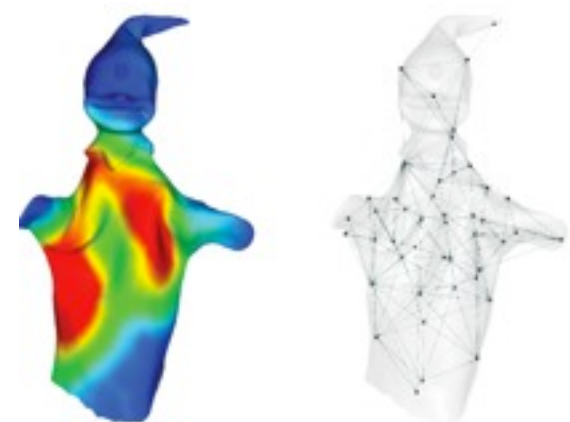


Non-Rigid Modeling

$$\arg \min_{Z, \mathbf{R}_i, \mathbf{t}, \mathbf{d}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_y(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}\mathbf{x}_i + \mathbf{t})\|_2^2 + w_3 \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \|(\mathbf{z}_j - \mathbf{z}_i) - \mathbf{R}_i(\mathbf{x}_j - \mathbf{x}_i)\|_2^2 + w_4 \sum_{i=1}^n \|\mathbf{z}_i - (P_i \mathbf{d} + \mathbf{m}_i)\|_2^2$$

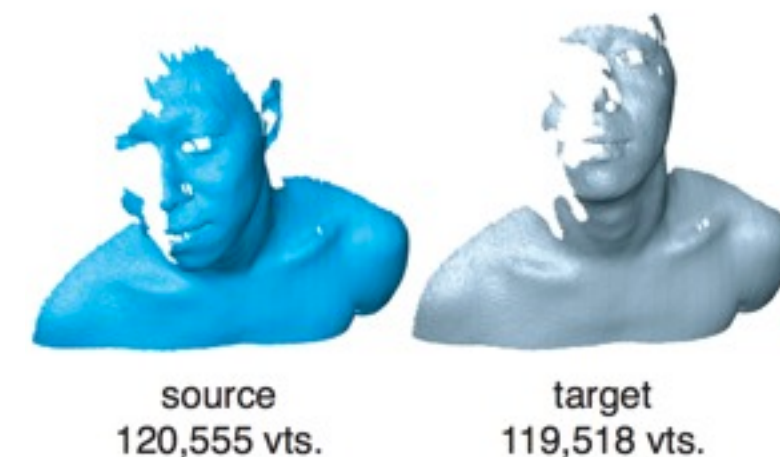
- How to set the weights for the different energies?

DEMO



Li, Adams, Guibas, Pauly:
**Robust Single-View Geometry
and Motion Reconstruction,**
ACM SIGGRAPH Asia 2009

Li, Sumner, Pauly:
**Global Correspondence Optimization for
Non-Rigid Registration of Depth Scans,**
SGP 2008



Overview

Introduction

- Registration
 - 3D Geometry
 - 2D Images
 - Combined 2D / 3D
- Applications
 - Rigid Scanning
 - Non-rigid Modeling
 - **Realtime Face Tracking**

Conclusions

Face Tracking

- Blendshape model

Face Tracking

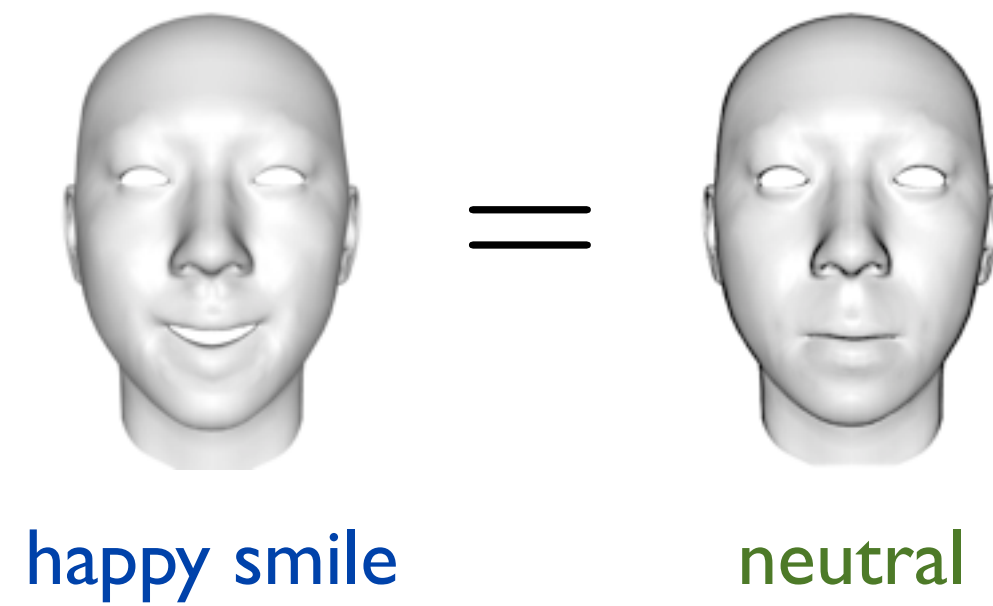
- Blendshape model



happy smile

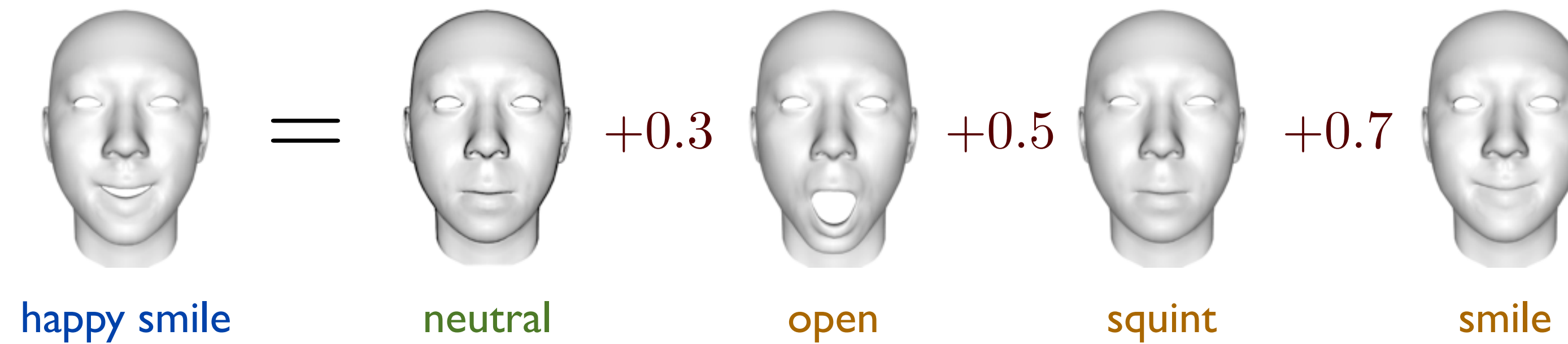
Face Tracking

- Blendshape model



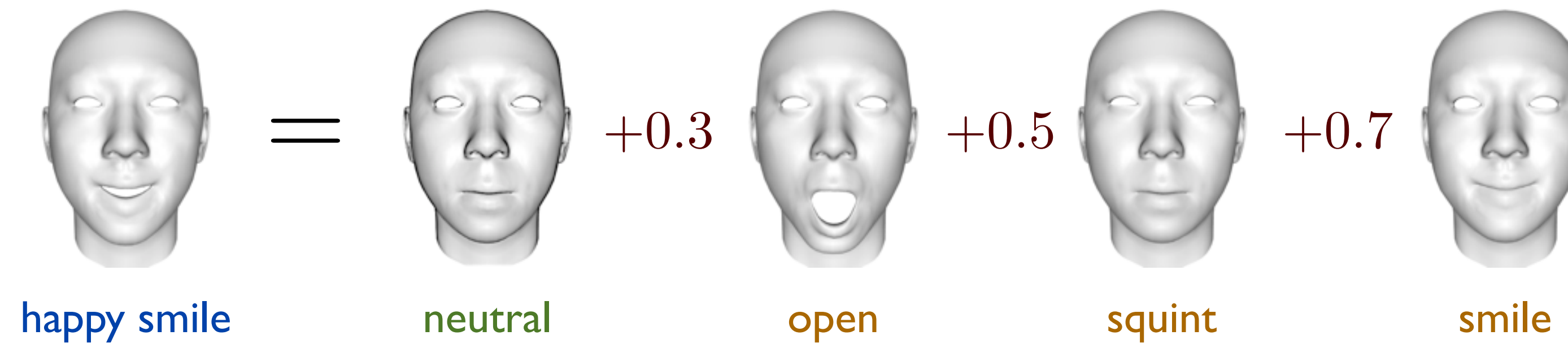
Face Tracking

- Blendshape model



Face Tracking

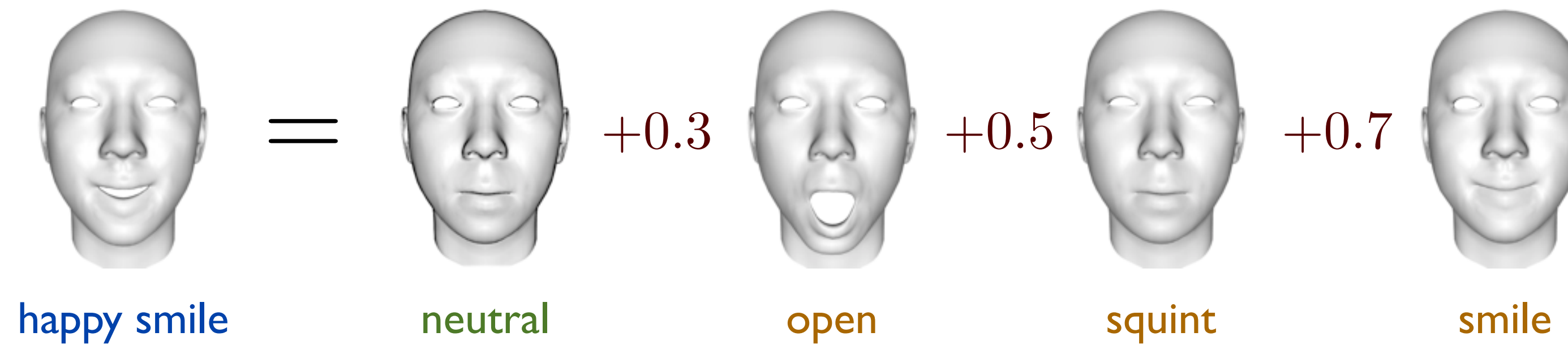
- Blendshape model



$$\mathbf{e} = \mathbf{Bd} + \mathbf{b}$$

Face Tracking

- Blendshape model

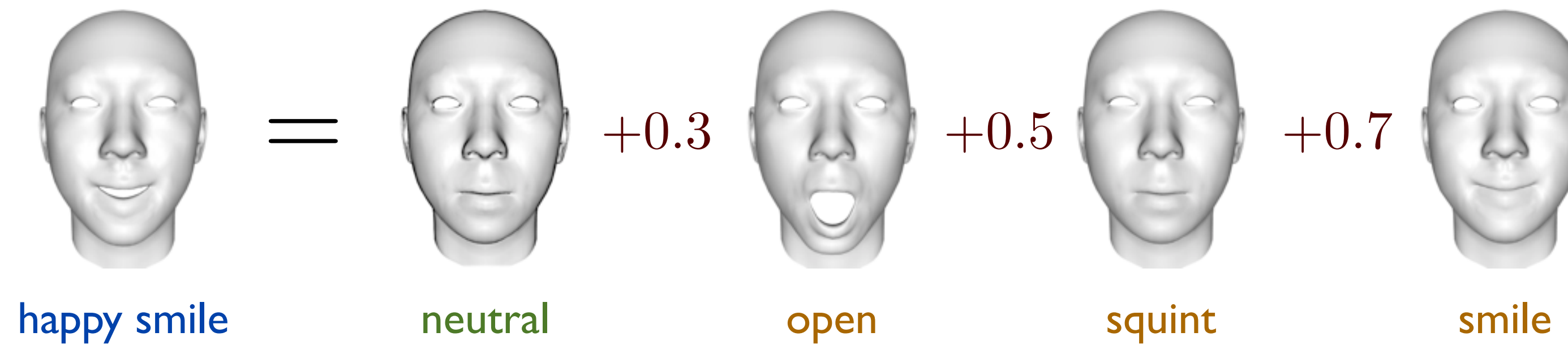


$$\mathbf{e} = \mathbf{Bd} + \mathbf{b}$$

novel expression

Face Tracking

- Blendshape model



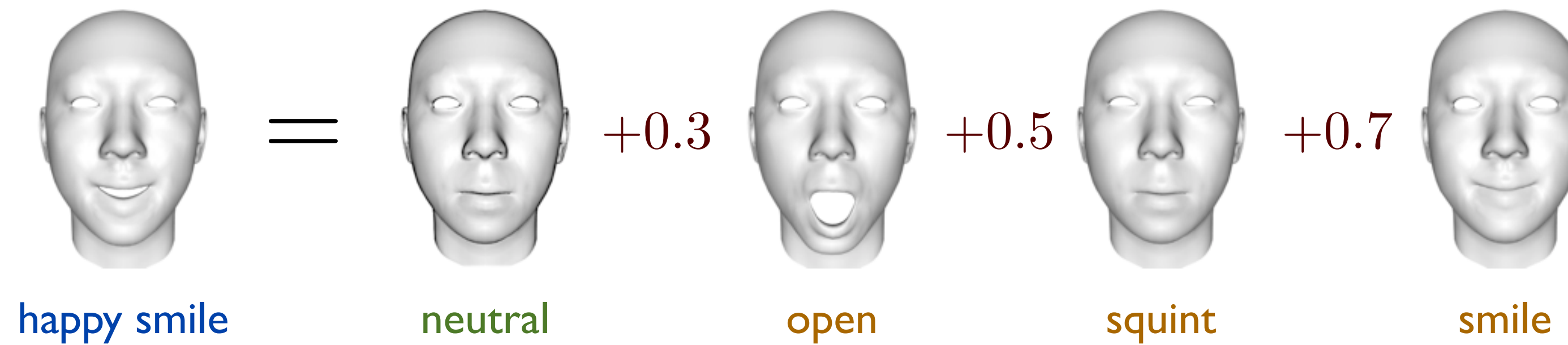
$$\mathbf{e} = \mathbf{Bd} + \mathbf{b}$$

novel expression

neutral

Face Tracking

- Blendshape model

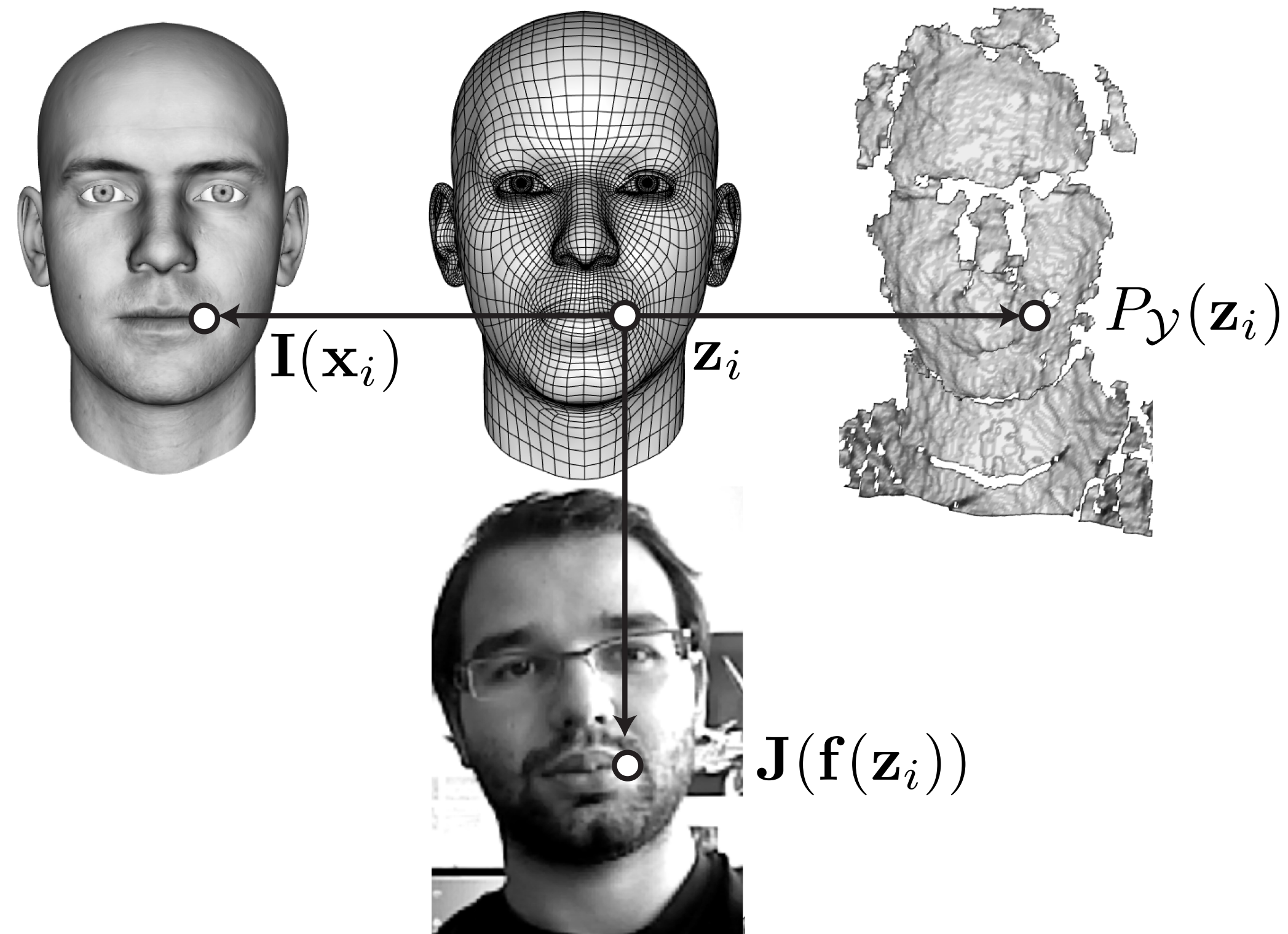


$$\mathbf{e} = \mathbf{B}\mathbf{d} + \mathbf{b}$$

basis → \mathbf{B}
 neutral → \mathbf{b}
 novel expression → \mathbf{e}
 coefficients → \mathbf{d}

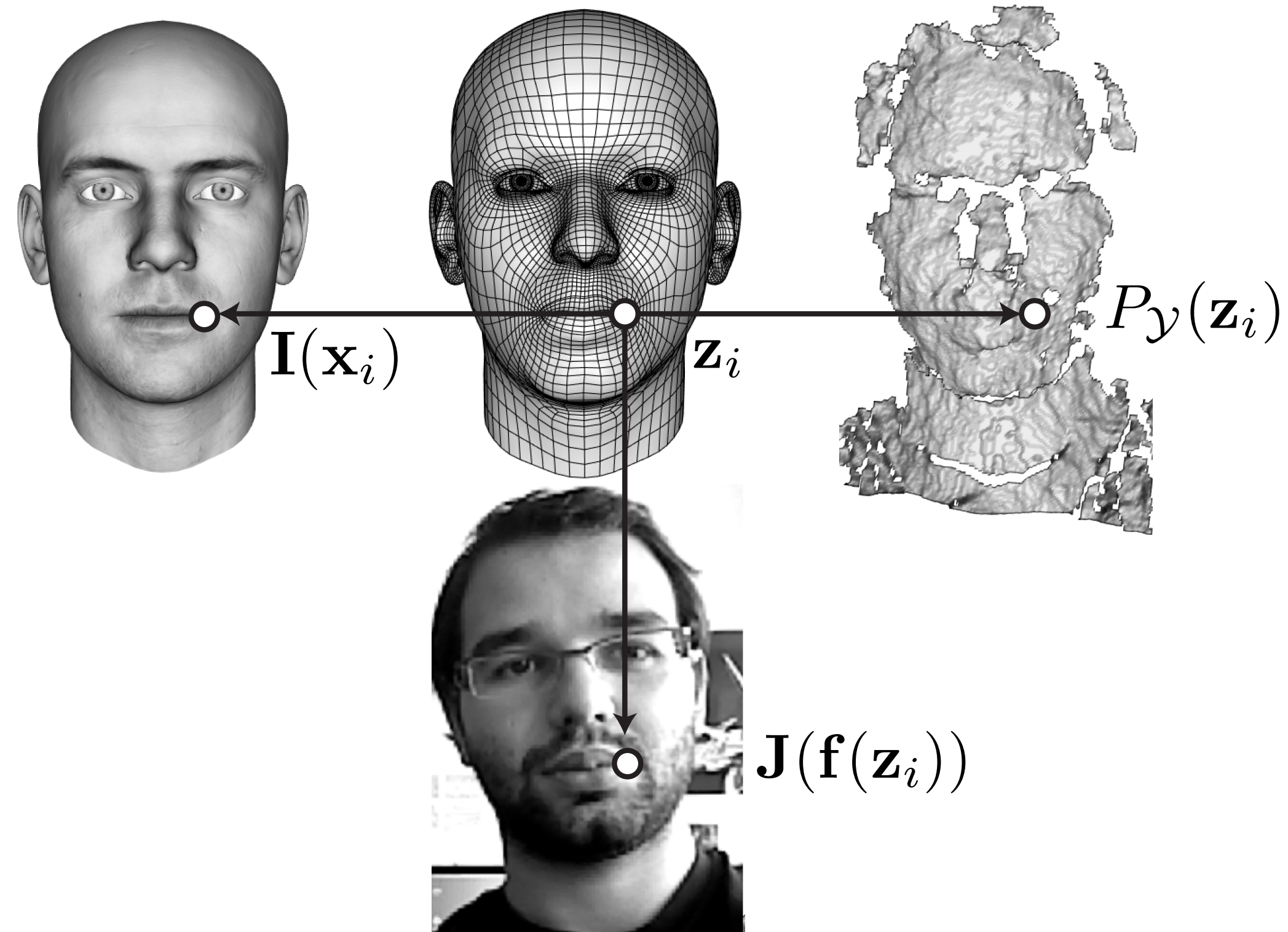
Face Tracking

$$\arg \min_{Z, \mathbf{d}, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i \mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$



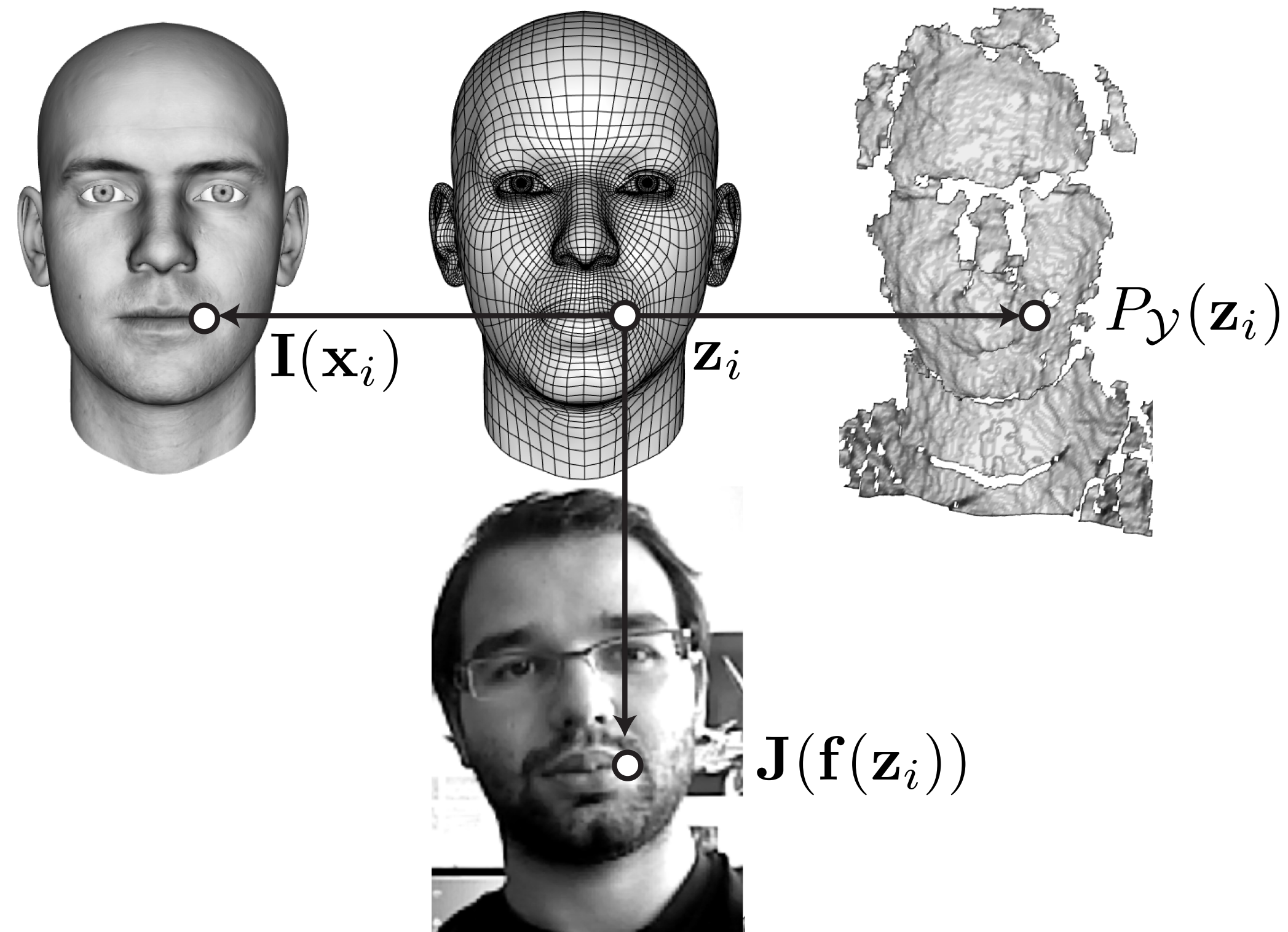
Face Tracking

$$\arg \min_{Z, \mathbf{d}, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i \mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$



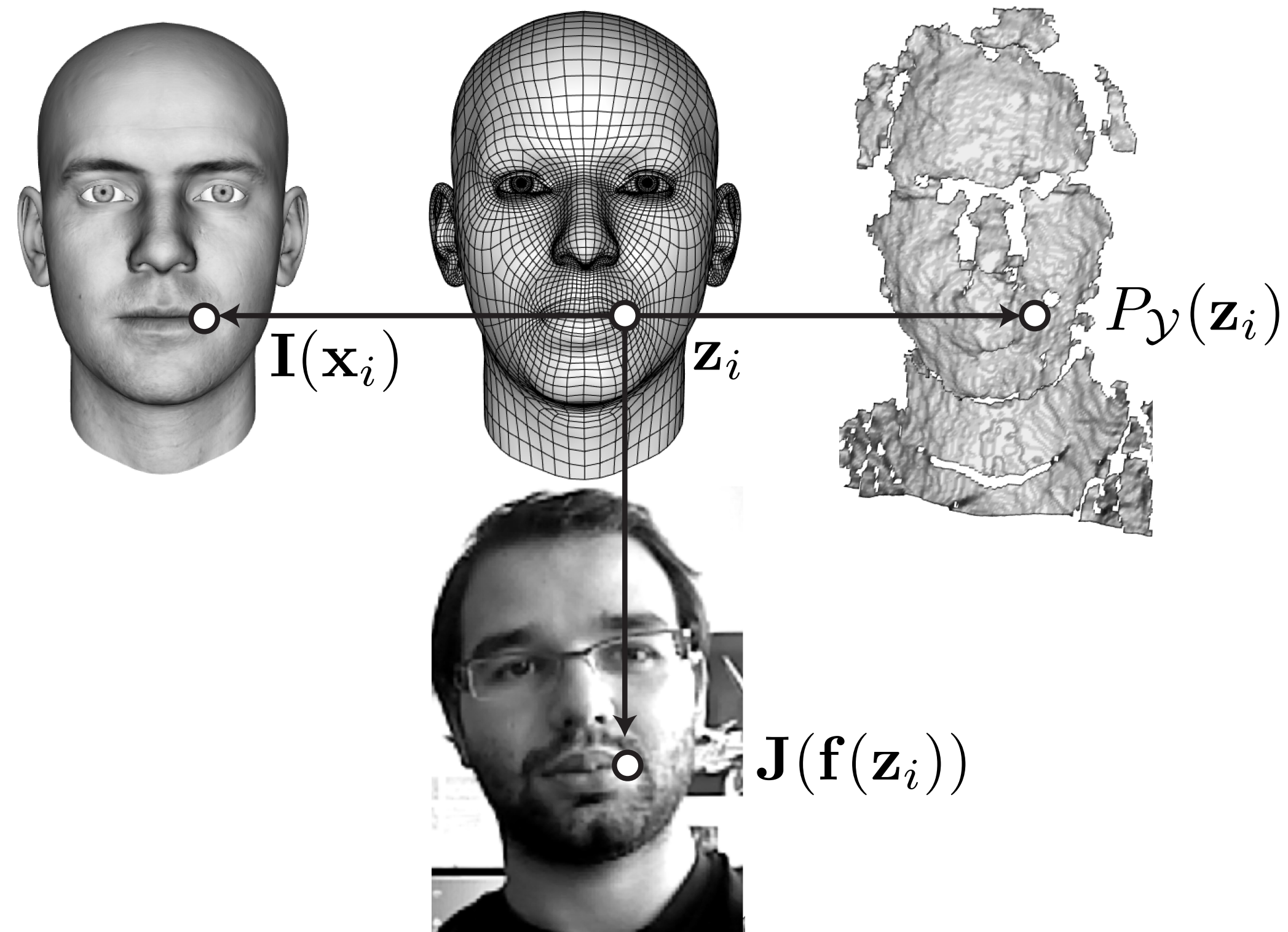
Face Tracking

$$\arg \min_{Z, \mathbf{d}, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i \mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$



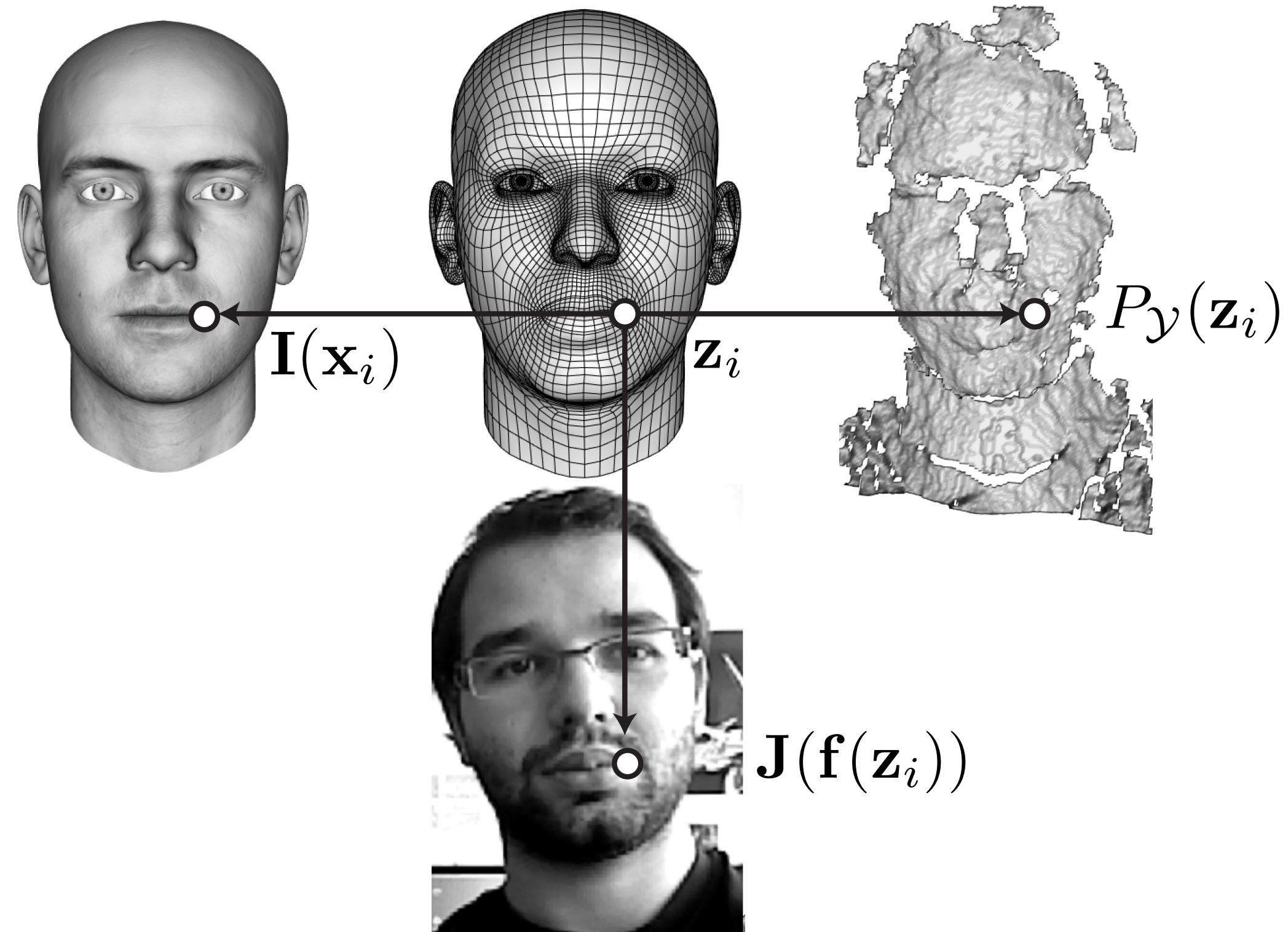
Face Tracking

$$\arg \min_{Z, \mathbf{d}, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i \mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$



Face Tracking

$$\arg \min_{Z, \mathbf{d}, \mathbf{R}, \mathbf{t}} w_1 \sum_{i=1}^n \|\mathbf{z}_i - P_{\mathcal{Y}}(\mathbf{z}_i)\|_2^2 + w_2 \sum_{i=1}^n \|\mathbf{I}(\mathbf{x}_i) - \mathbf{J}(\mathbf{f}(\mathbf{z}_i))\|_2^2 + w_3 \sum_{i=1}^n \|\mathbf{z}_i - (\mathbf{R}(\mathbf{B}_i \mathbf{d} + \mathbf{b}_i) + \mathbf{t})\|_2^2$$



DEMO

Overview

Introduction

- Registration
 - 3D Geometry
 - 2D Images
 - Combined 2D / 3D
- Applications
 - Rigid Scanning
 - Non-rigid Modeling
 - Realtime Face Tracking

Conclusions

Summary

- Registration as energy minimization

Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration

Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data

Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences

Summary

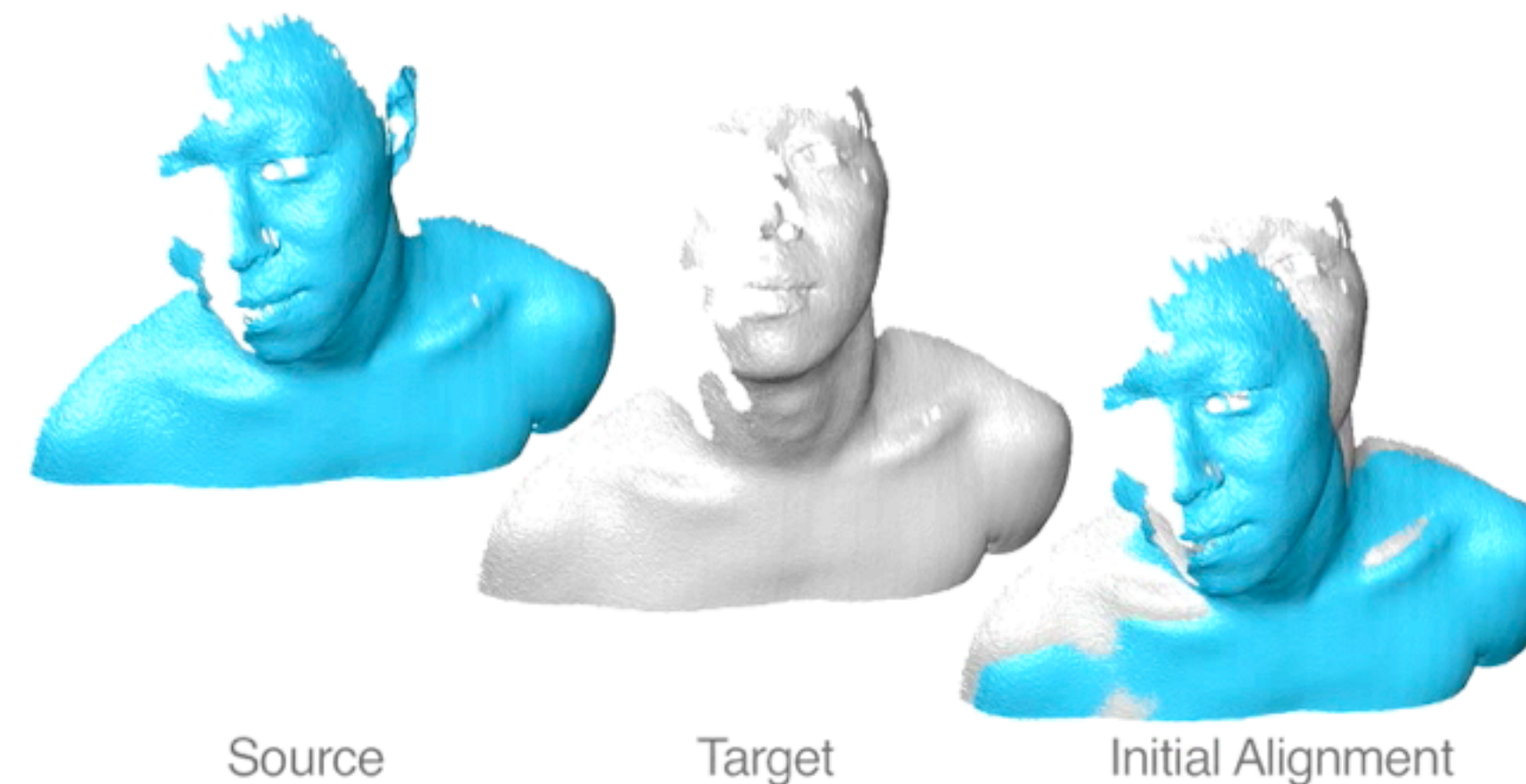
- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences

Li, Sumner, Pauly: **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences

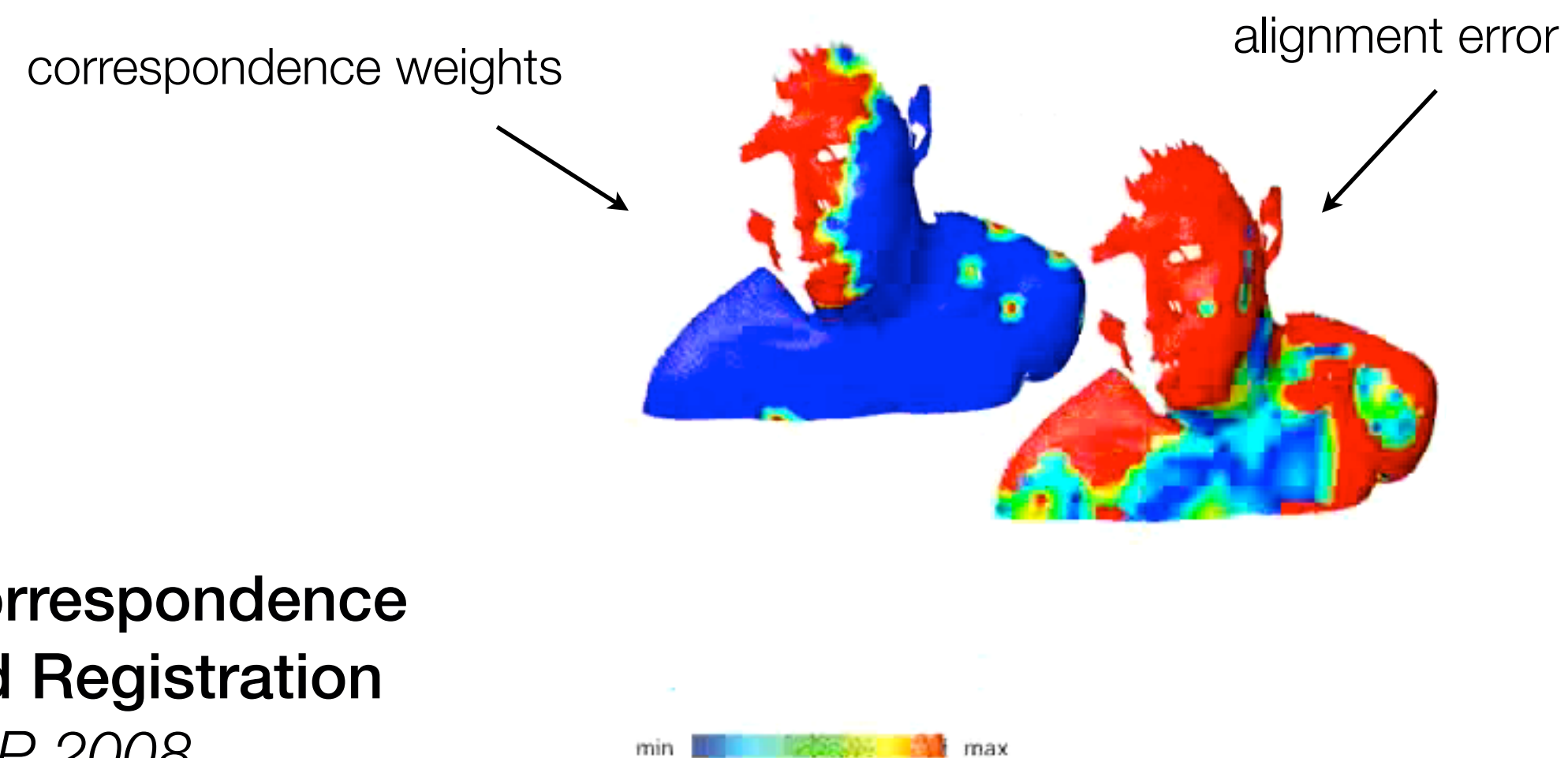
Upper Body (336 nodes, 120k vertices)



Li, Sumner, Pauly: **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

Summary

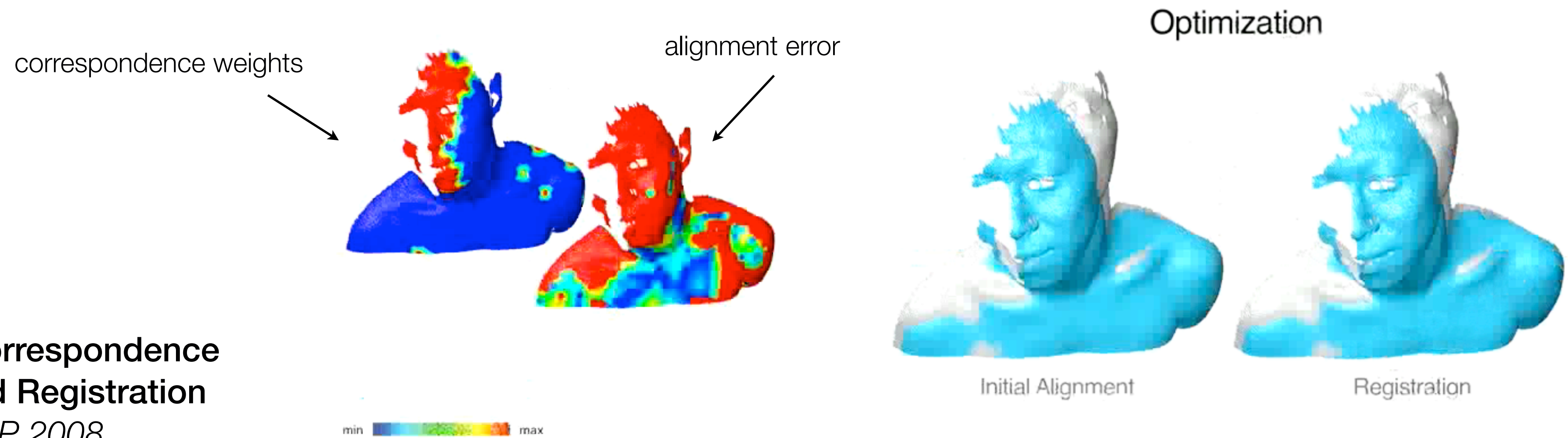
- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences



Li, Sumner, Pauly: **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences



Li, Sumner, Pauly: **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences
 - can the priors be learned?

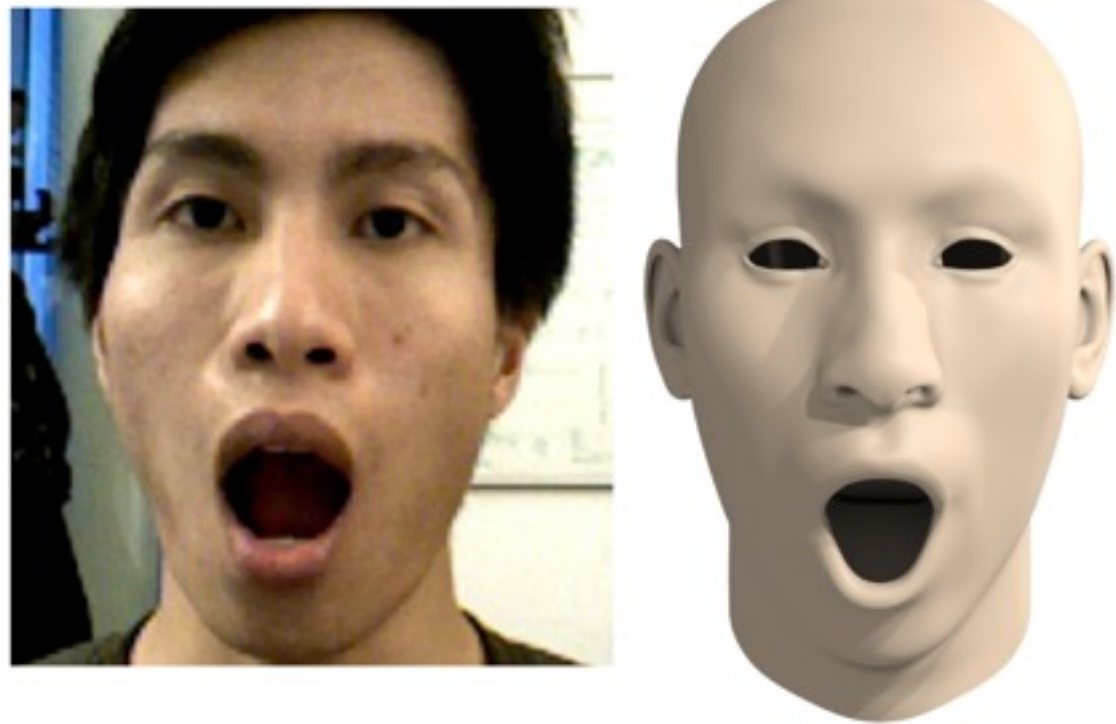
Summary

- Registration as energy minimization
 - general framework for integrating image and geometry registration
 - matching cost and priors need to be adapted to input data
 - fundamental challenge: correspondences
 - can the priors be learned?



Bouaziz, Wang, Pauly: **Online Modeling For Realtime Facial Animation**, *SIGGRAPH 2013*

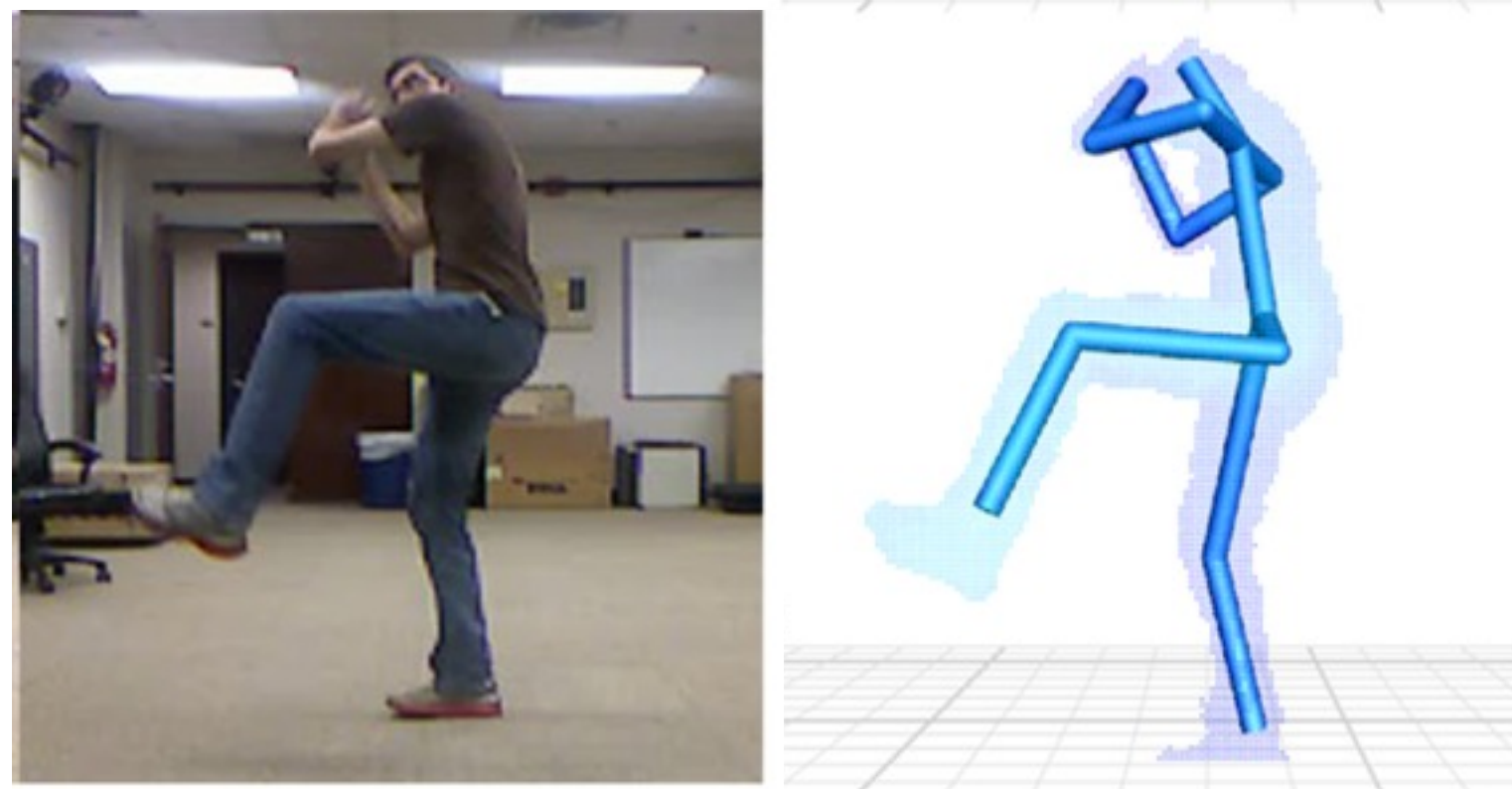
Conclusion



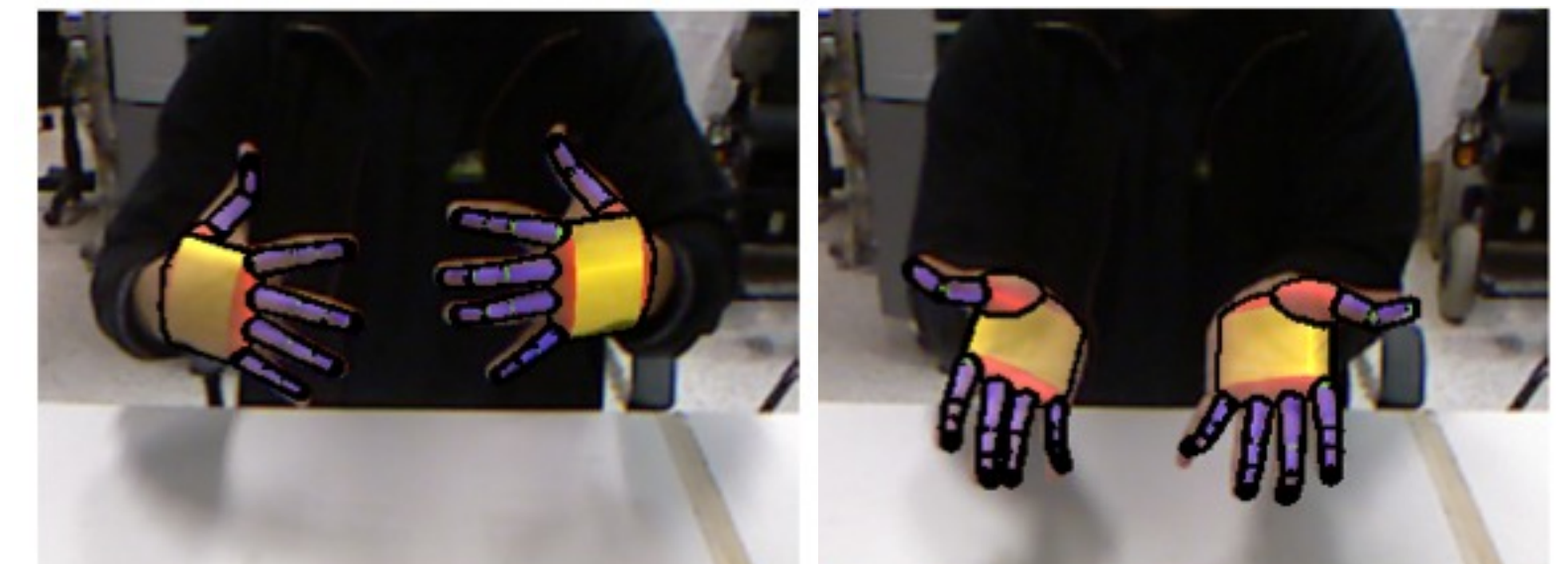
[Bouaziz et al, 2013]



[Newcombe et al, 2011]



[Wei et al, 2012]



[Oikonomidis et al, 2012]

Thanks!

<http://lgg.epfl.ch/2d3dRegistration>

Sumner, Schmid, Pauly: **Embedded Deformation for Shape Manipulation**, *SIGGRAPH 2007*

Li, Sumner, Pauly: **Global Correspondence Optimization for Non-Rigid Registration of Depth Scans**, *SGP 2008*

Weise, Li, Pauly: **Face/Off: Live Facial Puppetry**, *SCA 2009*

Li, Adams, Guibas, Pauly: **Robust Single-View Geometry and Motion Reconstruction**, *SIGGRAPH Asia 2009*

Weise, Bouaziz, Li, Pauly: **Realtime Performance-based Facial Animation**, *SIGGRAPH 2011*

Bouaziz, Deuss, Schwartzburg, Weise, Pauly: **Shape-Up: Shaping Discrete Geometry With Projection**, *SGP 2012*

Bouaziz, Tagliasacchi, Pauly: **Sparse Iterative Closest Points**, *SGP 2013*

Bouaziz, Wang, Pauly: **Online Modeling For Realtime Facial Animation**, *SIGGRAPH 2013*