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Convergence Estimation of Markov-Chain Monte Carlo Rendering –Supplementary Document–

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In this supplementary document, we provide additional results, including an analysis of a 1D example (Sec. 1) to validate our theory, and additional performance evaluation on more test scenes (Sec. 2).

1. Analysis of a 1D example

We validate our theoretical analysis with a simplified example in a 1D case. As shown in Figure 1(d), $f(x)$ is a 2D discrete distribution defined on the integer points in $[0,n)\times[0,n)$. We discretize along one of the axes, resulting in n pixels. Another axis represents the sub-pixel path space. The state space contains n^2 states, and the reference value of pixel j can be written as:

$$I_j = \sum_{k=0}^{n-1} f(j,k)$$

We defined the Markov matrix and Markov process in this state space to render these n pixels. Subsequently, we calculated the eigenvalues and eigenvectors of the Markov matrix, thereby obtaining the theoretical prediction of the MSE for the corresponding Markov process.

As shown in Figure 1(a)(b), the orange curve is our predicted value, while the blue curve represents the actual values (with different shades of color indicating different numbers of chains). The result indicates that our prediction perfectly matches the real MSE and nMSE will converge to a constant.

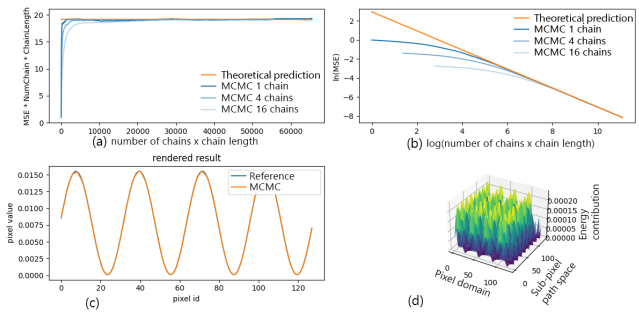

Figure 1: *1D example of* MSE *prediction. (a) shows that* nMSE *will converges to a constant. (b) indicates that our prediction perfectly matches* MSE *curve of MCMC process with different number of chains. (c) pixel value. (d) f(x) defined on 2D space.*

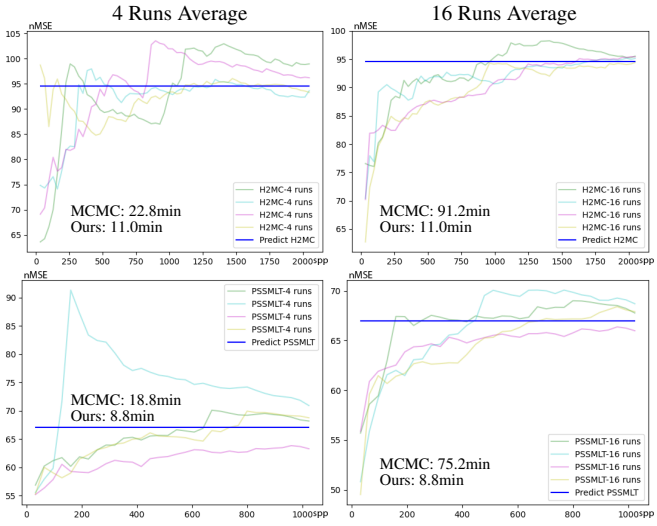

Figure 2: *Predicted* nMSE *on the Torus scene using our MC-estimator, alongside the measured* MSE *averaged from multiple MCMC runs. Tested with both H2MC [LLR*15] and PSSMLT [KSKAC02].*

2. More performance evaluations

Figure 2 shows the predicted nMSE on the *Torus* scene using our MC-estimator, alongside the measured MSE averaged from multiple MCMC runs. It also includes the computation time for each method.

References

[KSKAC02] KELEMEN C., SZIRMAY-KALOS L., ANTAL G., CSONKA F.: A simple and robust mutation strategy for the Metropolis light transport algorithm. In *Computer Graphics Forum* (2002), vol. 21, Wiley Online Library, pp. 531–540. 1

[LLR*15] LI T.-M., LEHTINEN J., RAMAMOORTHI R., JAKOB W., DURAND F.: Anisotropic Gaussian mutations for Metropolis light transport through Hessian-Hamiltonian dynamics. *ACM Trans. Graph. 34*, 6 (Nov. 2015). 1