Computational Design of Forced-Perspective Structures

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Abstract

This paper presents a computational method for designing forced-perspective structures. Our model focuses on empirical rules that the brain uses to perceive 3D scenes from 2D views and computes the width and height of each unit in man-made repeating objects (e.g., buildings and roads) that can intentionally emphasize or suppress our perspective from a specified viewpoint. Examples of generated structures and the results of a cognitive experiment are shown to demonstrate the flexibility and the effectiveness of our method.

CCS Concepts

• ***Computing methodologies*** → ***Shape modeling;***

1. Introduction

Forced perspective is an illusory technique that manipulates the sense of size and depth by adjusting object shapes and placements. By applying forced perspective, we can make targets appear larger and farther away (emphasis on perspective) or smaller and closer (suppression of perspective) than they actually are. Specifically, to emphasize perspective, the size of objects and the intervals between them should be designed to be smaller as the objects recede from the viewpoint. In contrast, the size and intervals of farther objects should be designed to be larger to suppress perspective. For example, as shown in Figure 1, Disney castles are usually designed using emphasized forced perspective, which makes them appear more imposing and immersive [Ste03]. In addition, using forced perspective in structural design can make a small space look larger and allow for more effective use of the space [RDM*12]. Forced perspective is also utilized to improve visibility [Fra93] and reduce slope illusions [YC02]. However, it is not trivial for users to adjust the shape and placement of man-made objects to achieve the desired forced perspective, as shown in Figure 2.

In this paper, inspired by how the brain perceives the shape of man-made objects, we introduce a first-step method for the computational design of forced-perspective structures. Although our method remains simple, it can be useful for various design purposes and targets, such as the emphasis or suppression of perspective on walls and floors. In addition, a user study is conducted to evaluate the effectiveness of the proposed method.

2. Related work

Various forced-perspective models have been proposed from early modern Europe [Pag16, SLB23]. For instance, Dürer [Dü25] suggested a spherical mapping system to suppress perspective, making text information on the upper part of a tower wall appear the same size as that on the lower part. Although their geometric rule is easy to understand, no formulas were defined. Thus, we must measure the lengths and proportions of blueprint buildings and empirically design them. In addition, the model does not support emphasized forced perspective. Later, Troili [Tro83] introduced a pyramidal model that shrinks toward the stage back to make the stage space larger than the actual space. This model enables users to han-

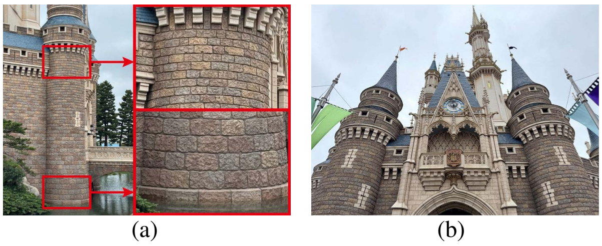
(a) (b)

Figure 1: *An example of forced perspective (Disney castle). (a) change in the texture density on the castle wall, and (b) forced-perspective results.*

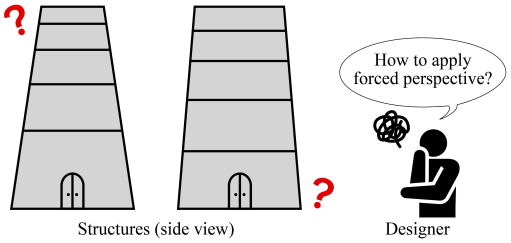

Figure 2: *A key issue in designing forced-perspective structures.*

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dle audiences' perspective by changing the shape of the pyramidal stage, but they did not discuss how to determine the strength of forced perspective. In addition, Jonsson [Jon22] proposed a forced-perspective system based on the fact that large objects at a distance and small objects nearby occupy the same view angle on the retina. However, their system is too simple to apply to man-made structures in which repeating units (e.g., floors of buildings or trees on streets) are arranged consecutively.

In summary, to our knowledge, existing methods remain difficult for efficiently designing forced-perspective structures. Therefore, we consider a computational method for designing forced perspective for man-made structures.

3. Our method

When predicting 3D scenes from the 2D views reflected on the retina, we often use empirical rules (or assumptions) in our brain. For example, (i) buildings are perpendicular to the ground, (ii) the ground plane is horizontal, (iii) man-made quadrangles (e.g., walls) are rectangles [TS18], and (iv) textures of man-made quadrangles are homogeneous [Gib50]. That is, even if buildings are slightly tilted, such as the Leaning Tower of Pisa, or the shape of each floor (e.g., width/height) is deformed, people will still perceive a distorted building as a normal building. We therefore consider a method to computationally make the effects of forced perspective by projecting trapezoidal buildings perpendicular to the ground onto a tilted plane.

Figure 3 illustrates our forced-perspective model. The model begins with a virtual surface (pink) that consists of stacked rectangles tilted from the ground by an angle of $\varphi \in \mathbb{R}$, like the Leaning Tower of Pisa. The width and height of each rectangle are w_0 and L, respectively. Next, given the location of the user's viewpoint (i.e., distance $\boldsymbol{d}$ and height $\boldsymbol{h}$), we project the virtual surface onto a physical plane tilted from the ground by an angle $\theta \in \mathbb{R}$ and divide the resulting trapezoid horizontally into several units (blue). Using the above variables as input, the height $l_n \in \mathbb{R}^+$ and top base $w_n \in \mathbb{R}^+$ of the n-th trapezoid from the bottom can be computed as follows.

$$l_n = f(n) - f(n-1) \tag{1}$$

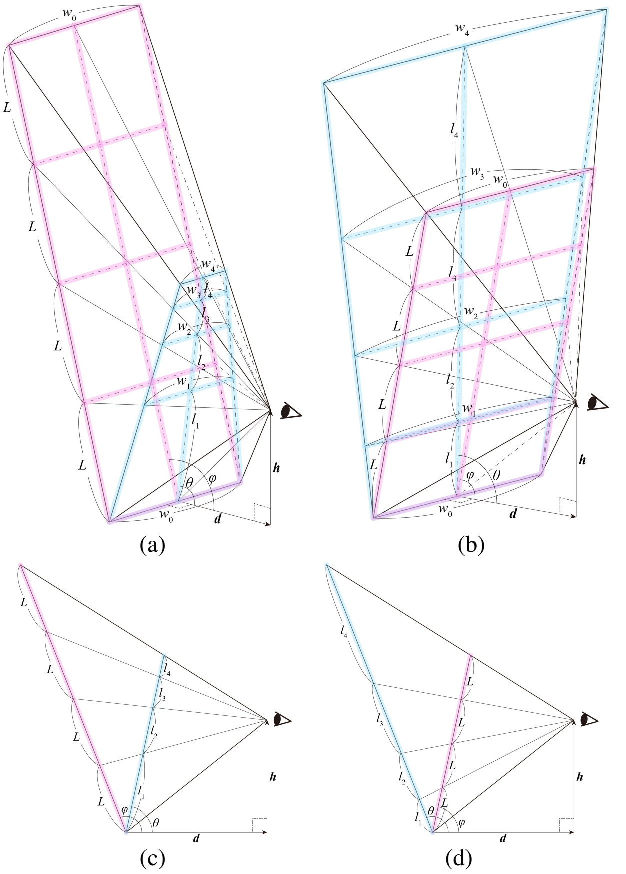
$$w_n = w_0 \frac{\boldsymbol{d}\sin\theta - \boldsymbol{h}\cos\theta}{nL\sin(\varphi-\theta) + \boldsymbol{d}\sin\theta - \boldsymbol{h}\cos\theta} \tag{2}$$

where

$$f(n) = \frac{nL(\boldsymbol{d}\sin\varphi - \boldsymbol{h}\cos\varphi)}{nL\sin(\varphi-\theta) + \boldsymbol{d}\sin\theta - \boldsymbol{h}\cos\theta} \tag{3}$$

$$(\boldsymbol{d}\sin\theta - \boldsymbol{h}\cos\theta)(\boldsymbol{d}\sin\varphi - \boldsymbol{h}\cos\varphi) > 0 \tag{4}$$

Here, Equation 4 represents a constraint that the projected and virtual surfaces are on the same side of a straight line connecting the viewpoint and the center of the surface base. In addition, we need to consider a constructive restriction on the total height of the projected trapezoid $C(=\sum_{n=1}^{N} l_n)(\in \mathbb{R}^+)$, where the number of trapezoidal units is $N(\in \mathbb{N})$. In this case, the following equations are added as constraints.



(a) (b)

(c) (d)

Figure 3: *Our forced-perspective model. (a, c) emphasis on perspective, (b, d) suppression of perspective. Blue trapezoids and pink rectangles represent physical projected structures and virtual reference planes, respectively.*

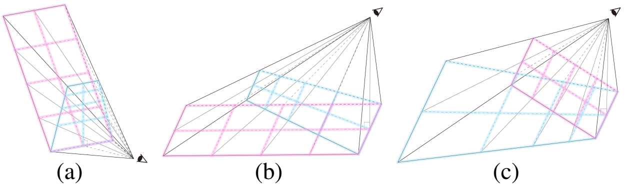

(a) (b) (c)

Figure 4: *Variations of our model. (a)* $\theta = \frac{\pi}{2}$, $\frac{\pi}{2} < \varphi < \pi$, $\boldsymbol{h} = 0$; *(b)* $\frac{\pi}{2} < \theta < \pi$, $\varphi = \pi$, $\boldsymbol{d} = 0$; *and (c)* $\theta = \pi$, $\frac{\pi}{2} < \varphi < \pi$, $\boldsymbol{d} = 0$.

$$C = f(N) \tag{5}$$

$$(\boldsymbol{d}\sin\theta - \boldsymbol{h}\cos\theta)[\boldsymbol{d}\sin\varphi - \boldsymbol{h}\cos\varphi - C\sin(\varphi-\theta)] > 0 \tag{6}$$

Equation 6 represents a condition that the projected surface fixed to height C fits within the square pyramid formed by the viewpoint and the virtual surface when perspective is emphasized.

The trapezoidal model has characteristics of forced perspective, which make it recognizable as the size of the virtual surface when

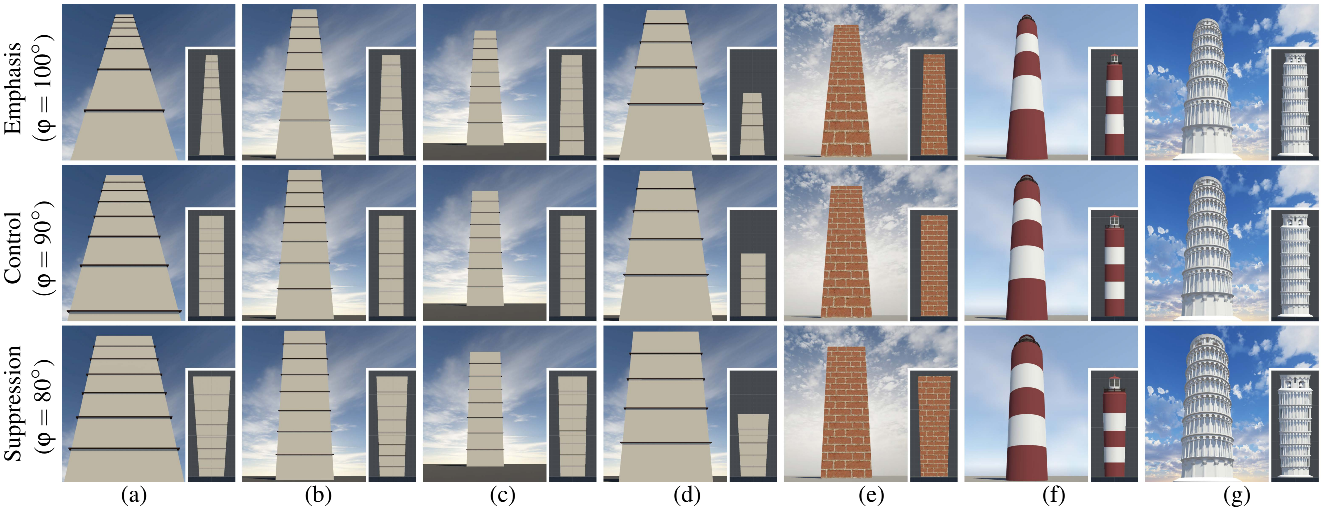


Figure 5: *Generated forced-perspective buildings from the user-specified viewpoint and their elevation views. (a)* $\frac{\boldsymbol{d}}{C} = \frac{1}{3}$*; (b)* $\frac{\boldsymbol{d}}{C} = \frac{2}{3}$*; (c)* $\frac{\boldsymbol{d}}{C} = 1$*, under* $N = 8$*; (d)* $\frac{\boldsymbol{d}}{C} = \frac{2}{3}$ *under* $N = 5$*; (e) brick texture; (f) lighthouse; and (g) the Leaning Tower of Pisa.*

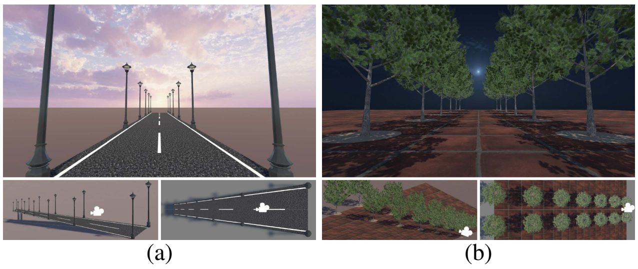


Figure 6: *Generated forced-perspective roads from the user-specified viewpoint. (a) emphasis, and (b) suppression.*

viewed from the user-specified viewpoint and as perpendicular to the ground under the above assumptions. In other words, we can determine the width or height of each unit (e.g., truncated cones and truncated pyramids) in building models with the desired strength of forced perspective (i.e., emphasis or suppression) by preparing a target surface. Note that in case of $\varphi = \frac{\pi}{2}$ or π, the apparent vanishing point is on the vertical or horizontal line, which may produce a natural-looking result, but the parameters tend to be difficult to explore. In addition, the strength of force perspective can be handled from the tiled angle parameter φ only while keeping the total height C and the angle θ of the input building fixed. The advantage of our model is that it can be applied to various scenes, such as walls and floors, as shown in Figure 4, so existing models [Dü25, Jon22, Tro83] can be mathematically treated. Figure 5 and Figure 6 show examples of forced-perspective buildings and roads generated using our method.

4. User study

To quantitatively evaluate the effectiveness of our forced-perspective model, we conducted a cognitive experiment. We invited 20 subjects (11 males and 9 females) aged from 20 to 24.

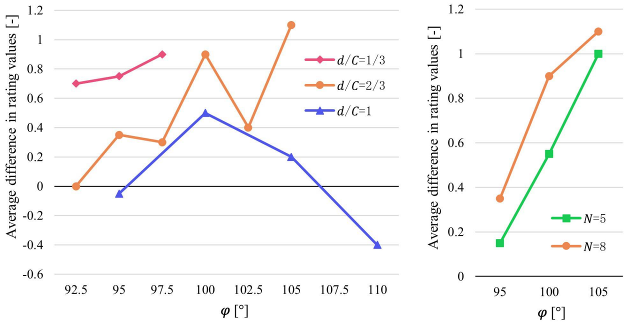
4.1. Experimental scenes

We implemented a system in Unity that enables users to make forced-perspective buildings using our method. Note that the building texture was a beige wall with black separators (see Figure 5(a)-(d)). Next, we rendered two types of images under $\theta = \frac{\pi}{2}$ and $\boldsymbol{h} = 0$: (A) a building with $\varphi = \frac{\pi}{2}$ without forced perspective, and (B) a building with $\frac{\pi}{2} < \varphi < \pi$ with emphasized forced perspective. The camera's elevation angle was set based on the $\frac{\boldsymbol{d}}{C}$ value to ensure that the entire building or its upper portion is clearly visible.

4.2. Procedure and results

For each pair of images, we consecutively showed the subjects images (A) and (B) in random order for 4 seconds each, separated by a blackout for 2 seconds. The presentation time was adjusted based on feedback from a pilot study. The subjects were then asked to rate the perceived height of each building on a 5-point scale, from 1 (very short) to 5 (very tall). Note that we recorded the relative value, obtained by subtracting the rating value for (A) from that for (B), as an indicator of forced-perspective effects. The above step was repeated for all image pairs generated based on conditions outlined below. The evaluation period lasted approximately 60 minutes.

CONDITION 1. The first condition investigated the effect of changing φ at each viewpoint distance $\boldsymbol{d} = 8, 16, 24$. Note that all other parameters were the same in each image pair (i.e., $C = 24$ and $N = 8$), but we set parameter ranges of φ and L based on the constraints described in Equation 5 and Equation 6. Figure 7(a) shows the results. The horizontal axis represents the value of angle φ, and the vertical axis represents the average difference in rating values (i.e., the strength of the emphasized forced-perspective effect). From the results, our model generally produced emphasized forced-perspective effects, with most difference values being positive. In addition, the effects became greater when viewed from a shorter distance, probably due to the difficulty in distinguishing



(a) (b)

Figure 7: *Transition of subjective forced-perspective effects for φ angles. (a) Condition 1, and (b) Condition 2.*

trapezoids from rectangles from a closer viewpoint. On the other hand, when viewing buildings with larger values of φ from a grater distance, it can be easier to see through the trick of our model, which may have led to the less effects. In fact, a questionnaire survey following the evaluation task showed that several subjects felt a sense of strangeness from images rendered based on this condition.

CONDITION 2. The second condition investigated the effects of changing φ for the number of building floors $N = 5, 8$. Note that in the case of $N = 5$, we set $\boldsymbol{d} = 10$ and $C = 15$, and in the case of $N = 8$, we set $\boldsymbol{d} = 16$ and $C = 24$, aiming to match the value of $\frac{\boldsymbol{d}}{C} = \frac{2}{3}$ in all rendered images. In addition, the value of L was derived from Equation 5. The results are shown in Figure 7(b). From the results, we confirmed that the forced-perspective effect was stronger when buildings had a larger number of floors. It is thought that as the repetition of building floors increased, the subjects became more likely to perceive the texture as having uniform intervals, making them more susceptible to the illusion.

From this experiment, we confirmed that our method provides sufficient forced-perspective effects by setting appropriate parameters. That means our method has been demonstrated to be useful from a cognitive perspective. Furthermore, the subjects recognized the shape of the buildings by correcting the distorted trapezoids into homogeneous rectangles in their brains, confirming the validity of our approach. Note that our analysis focused on general trends derived from mean values, due to the substantial variance observed across all conditions (SD $\approx$ 1.0).

5. Limitations and future work

In the current implementation, we must set one fixed position from which viewers observe the target object, but each viewer's location is slightly different in the real world. It might be better to explore parameter ranges robust to viewpoint changes or to extend our model considering various viewpoints. In addition, in our cognitive experiment, we prepared 2D images of forced-perspective buildings, but the experimental results may have been affected by the way the generated buildings were presented to the subjects. In the future, we plan to use VR techniques or 3D printers to present the generated buildings in 3D.

Designs involving forced perspective have often relied on intuition rooted in experience or repeated trial and error. Thus, we plan to integrate our system into standard CAD tools.

6. Conclusion

This paper details an initial computational method for designing forced-perspective structures, such as castle walls and floors. Inspired by the characteristics of the Leaning Tower of Pisa, we revisited how people understand 3D scenes through their eyes and constructed a parametric model of forced perspective. Our systematic method is grounded in clear evidence and robust against various design purposes, such as creating the illusion of grandness or tininess. In addition, a cognitive experiment demonstrated that our model enables subjects to experience forced-perspective effects. Our efficient, convincing, and comprehensive work, as demonstrated, will broaden the application range of forced perspective by making it easier to be incorporated into practical design.

Acknowledgments

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