Fast Cloth Simulation with Implicit Contact and Exact Coulomb Friction

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Figure 1: *Our body-cloth contact solver captures exact Coulomb friction both efficiently and robustly. On this dress example featuring 6000 cloth vertices and 1000 contact points on average, our solver converges at each time step (dt = 1ms) in a few hundred milliseconds.*

Abstract

Cloth dynamics plays a major role in the visual appearance of moving characters. Properly accounting for frictional contact is of utmost importance to avoid cloth-body penetration and to capture folding behavior due to dry friction. We present here the first method able to account for contact with exact Coulomb friction between a cloth and the underlying character. Our key contribution is to formulate and solve the frictional contact problem merely on velocity variables, by leveraging some tools of convex analysis. Our method is both fast and robust, allowing us to simulate full-size garments with more realistic body-cloth interactions compared to former methods, while maintaining similar computational timings.

1 Related Work

Simulating cloth dynamics subject to contact and friction is commonly achieved through an iterative velocity filtering process at each time step [Bridson et al. 2002]. Within the main loop, the cloth dynamics $Mv + f = 0$ is first resolved as if no contact were applied, then each cloth vertex has its velocity sequentially modified so as to satisfy non-penetration and Coulomb frictional constraints locally. Although this method gives a contact-free configuration at the end of each time step, it fails to yield a consistent mechanical state w.r.t. both internal energy and Coulomb constraints. Typically, excessive bending energy may be localized between non-contacting and contacting vertices, and some vertices may be artificially constrained to stick whereas they should be sliding.

In contrast, treating frictional contact implicitly with constraints as

$$\left\{ \begin{array}{lll} Mv + f & = J^\top r & \\ Jv & = u & \\ (u,r) & \in \mathcal{C}(\mu) & \text{where } \mathcal{C}(\mu) \text{ is the Coulomb law} \end{array} \right. \quad (1)$$

alleviates such issues, since internal dynamics and frictional contacts are solved simultaneously. The classical way to solve such an implicit formulation is to first eliminate velocities v (primal variables) through the computation of the Delassus operator $W = J^\top M^{-1} J$, then solve the system in the contact forces r (dual variables), and finally compute v from r. In the case of curvature-based hair models, Daviet et al. [2011] introduced an efficient solver for the dual problem, able to take exact Coulomb friction into account. For cloth however, where primal variables are 3d velocities, such a method is computationally inefficient. Indeed, the matrix M is costly to invert for large systems and its inverse is dense.

2 A primal formulation with Coulomb friction

We assume body-cloth contacts occur at cloth vertices only, i.e., each contact involves only one vertex. A key observation is that each nonzero block of J related to a vertex i is simply a rotation matrix E_i, corresponding to the local contact basis for the vertex i. Our idea is then to build the square block-diagonal matrix G with $G_{i,i}$ equal to E_i if i is in contact, and to the $3x3$ identity matrix otherwise. G can thus be inverted trivially as $G^{-1} = G^\top$. Augmenting u and r so that their size matches that of v, we can write (1) as

$$\left\{ \begin{array}{llll} & GMG^\top u + Gf & = r & \\ & v & = G^\top u & \\ (u_i, r_i) \in \mathcal{C}(\mu_i) & \text{if } i \text{ in contact} & \text{and } r_i = 0 \text{ otherwise.} & \end{array} \right. \quad (2)$$

Denoting $\tilde{W} = GMG^\top$ and $b = Gf$, we obtain a system that is very close to the dual formulation of [Daviet et al. 2011], except that u and r have reversed roles. To retrieve symmetry, we apply De Saxcé's change of variable $u_i^* = u_i + s_i(u)$ so that u_i^* is orthogonal to r_i. For a fixed value of s, we identify (2) as the KKT conditions of the convex quadratic optimization problem [Acary et al. 2011]

$$\min_{u^*} \quad \tfrac{1}{2}(u^*)^\top \tilde{W} u^* + (b - \tilde{W} s^k)^\top u^*$$
$$u_i^* \in K_{\frac{1}{\mu_i}} \quad \text{if } i \text{ in contact} \quad (\text{with } K_\mu \text{ the Coulomb cone}),$$

which can be solved using [Daviet et al. 2011]'s solver, since this time u^* plays the exact same role as r (replacing K_{μ^i} with $K_{\frac{1}{\mu_i}}$). Following [Acary et al. 2011], we compute the solution of the full problem (1) by iteratively updating s using a fixed point algorithm. Unlike the Delassus operator W, our new operator $\tilde{W}$ is easy to assemble and sparse. Solving our primal problem (2) thus turns out to be orders of magnitude faster compared to the dual problem.

References

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