

# Preconditioned Single-step Transforms for Non-rigid ICP - Supplementary Document -

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In this document, we provide additional details and results. In Sec. 1, we explain additional details regarding the implementation of our algorithm. In Sec. 2, we provide additional analysis and results.

## 1. Additional Implementation Details

### 1.1. Implementation of Correspondence Search

Our ICP iterations update the matrices  $\mathbf{A}$  and  $\mathbf{b}$  at every iteration via a correspondence search between the source surface  $\mathcal{S}$  and the target surface  $\mathcal{T}$ . Here, we provide additional details for this procedure, which is denoted as  $\mathbf{A}, \mathbf{b} \leftarrow \text{correspondenceSearch}()$  in Alg. 2 in the main paper.

We perform two passes of nearest-neighbor search between the source mesh and the target mesh. The first pass establishes source-to-target correspondence  $\mathbf{c}_{st}$  by finding the closest target triangle centroid for each source triangle centroid. Then,  $\mathbf{c}_{st}(k)$  is the closest target triangle index for the  $k$ -th triangle in the source. The second pass establishes target-to-source correspondence  $\mathbf{c}_{ts}$  by finding the closest source mesh vertex for each target triangle centroid. Note that we use source mesh vertices instead of source triangle centroids in this search. Then,  $\mathbf{c}_{ts}(k)$  is the closest source vertex index for the  $k$ -th triangle in the target.

Using source vertices instead of source triangle centroids in the target-to-source search helps the registration reconstruct sharp details by reducing the influence region of each target triangle to a single source vertex; otherwise, each target triangle would influence three source vertices and may produce a more smoothed-out reconstruction if overlapping influences at source vertices are inconsistent. The source-to-target search suffers less from this phenomenon because each source triangle is influenced by exactly one target triangle.

Given the established correspondences  $\mathbf{c}_{st}$  and  $\mathbf{c}_{ts}$ , we compute a matrix  $\mathbf{A}$  and a vector  $\mathbf{b}$  for the linear equation  $\mathbf{A}\mathbf{x} = \mathbf{b}$  that defines the energy minimization problem. Our energy minimization is a linear least squares problem, whose optimum is defined as the

solution of a normal equation

$$\hat{\mathbf{A}}^T \hat{\mathbf{A}} = \hat{\mathbf{A}}^T \hat{\mathbf{b}}. \quad (1)$$

We define

$$\begin{aligned} \mathbf{A} &= \hat{\mathbf{A}}^T \hat{\mathbf{A}} \\ \mathbf{b} &= \hat{\mathbf{A}}^T \hat{\mathbf{b}}. \end{aligned} \quad (2)$$

In summary, the matrix  $\mathbf{A}$  and vector  $\mathbf{b}$  are computed using a rectangular matrix  $\hat{\mathbf{A}}$  and a vector  $\hat{\mathbf{b}}$ . The rectangular matrix  $\hat{\mathbf{A}}$  is constructed as a vertical concatenation of four smaller rectangular matrices:  $\mathbf{A}_{st}$ ,  $\mathbf{A}_{ts}$ ,  $\mathbf{A}_{lmk}$ , and  $\mathbf{A}_{reg}$ .  $\mathbf{A}_{st}$  is the matrix for the source-to-target point-to-plane error term,  $\mathbf{A}_{ts}$  is the matrix for the target-to-source point-to-plane error term,  $\mathbf{A}_{lmk}$  is the matrix for the landmark error term, and  $\mathbf{A}_{reg}$  is the matrix for the smoothness regularization term. Similarly, the vector  $\hat{\mathbf{b}}$  is constructed as the concatenation of four vectors:  $\mathbf{b}_{st}$ ,  $\mathbf{b}_{ts}$ ,  $\mathbf{b}_{lmk}$ , and  $\mathbf{b}_{reg}$ . The matrices and vectors are constructed so that each term in Eq. (4) in the main paper is computed as the squared  $l^2$ -norm of each residual, i.e.,  $E_{p2pl} = |\mathbf{A}_{st}\mathbf{x} - \mathbf{b}_{st}|^2 + |\mathbf{A}_{ts}\mathbf{x} - \mathbf{b}_{ts}|^2$ ,  $E_{lmk} = |\mathbf{A}_{lmk}\mathbf{x} - \mathbf{b}_{lmk}|^2$ , and  $E_{reg} = |\mathbf{A}_{reg}\mathbf{x} - \mathbf{b}_{reg}|^2$ .

### 1.2. Implementation of the Preconditioner Update

Our adaptive Sobolev preconditioner is iteratively updated at each iteration according to Eqs. 13-15 in the main paper. In Algorithm 1 of this document, we provide the pseudocode for the update of our adaptive preconditioner. This routine corresponds to `updatePreconditioner()` in Alg. 2 in the main paper.

## 2. Additional Results

### 2.1. Full Results for the Distortion Analysis

Tables 1 to 8 present a full exposition of the fitting error and distortion metrics for each instance in the 100 inputs from the 3DCaricShop dataset. Many original fittings included in the 3DCaricShop dataset [QXQ\*21] exhibit lower distortions than *Ours* (*Quality*) at the cost of higher fitting errors, while some instances show extreme distortions, e.g., *Arsene Wenger C00007*, *Arsene Wenger C00014*, and *Audrey Hepburn C00015*. The results of [LH22] show high fitting errors while consistently maintaining low distortions. In con-

<sup>†</sup> These authors contributed equally to this work.

**Algorithm 1:** Update of Adaptive Sobolev-Jacobi Preconditioner

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**Input:** Cotangent Laplace-Beltrami Operator  $\mathbf{L}$ , Current iteration number  $k$ , Hyperparameters  $t_1 = 50, t_2 = 10, \gamma_1 = 0.9, \gamma_2 = 0.99, \omega = 0.5, \beta = 100$

**def** *updatePreconditioner*( $\mathbf{A}$ ):

```

 $\hat{\mathbf{M}}_{sob} \leftarrow (\mathbf{I} + \gamma_1^k t_1 \mathbf{L})(\mathbf{I} + \gamma_2^k t_2 \mathbf{L})$ 
if  $\gamma_1^k \geq \omega$  then
  |  $\hat{\mathbf{M}}_{jac} \leftarrow \mathbf{0}$ 
else
  |  $\hat{\mathbf{M}}_{jac} \leftarrow \text{diag}(\mathbf{A})$ 
end
 $\beta \leftarrow \beta / \text{median}(\text{diag}(\mathbf{A}))$ 
 $\mathbf{M}_{asj} \leftarrow \hat{\mathbf{M}}_{sob} + \beta \hat{\mathbf{M}}_{jac}$ 
return  $\mathbf{M}_{asj}$ 

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trast, our methods prioritize tight registrations, as evidenced by the low fitting errors.

*Ours* (*Cotan*) uses cotangent weights [PP93] in the regularization term  $E_{reg}$ , instead of uniform weights. Since the template consists of irregular triangles, *Ours* (*Cotan*) generally improves distortion, with some exceptions such as *Abraham Lincoln C00024* and *Cameron Diaz C00008*. While uniform weights provide stability across various templates, the cotangent-weighted Laplacian can be beneficial when preserving the shapes of template triangles is critical.

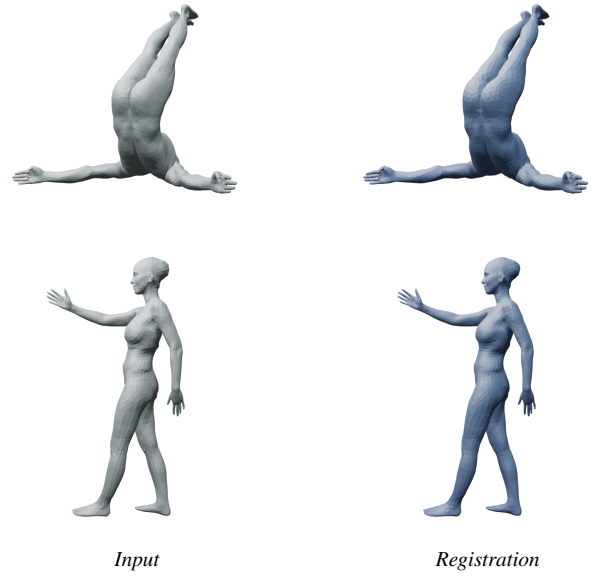
## 2.2. Registration of Human Bodies in SHREC

In this section, we showcase additional examples for human body fitting using the SHREC'19 dataset [MMRC20]. We first initialize the pose of the shape by fitting the SMPL model [LMR\*15] using rough correspondence maps [ELC20] for the rest-pose SMPL model. Then, we run our registration using the hyperparameters specified in Sec. 5.1 of the main paper. The results are presented in Fig. 1. Our registration method accurately reconstructs the input shape given the initial pose alignment.

Ideally, an end-to-end registration without pose initialization would be desirable in terms of computation time. Incorporating deformation graphs into our method to handle arbitrary pose alignment would be a promising future direction, as discussed in Sec. 6 of our main paper.

## References

- [ELC20] EISENBERGER, MARVIN, LAHNER, ZORAH, and CREMERS, DANIEL. "Smooth shells: Multi-scale shape registration with functional maps". *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2020, 12265–12274 2.
- [LH22] LI, YANG and HARADA, TATSUYA. "Non-rigid point cloud registration with neural deformation pyramid". *Advances in Neural Information Processing Systems* 35 (2022), 27757–27768 1, 3–6.
- [LMR\*15] LOPER, MATTHEW, MAHMOOD, NAUREEN, ROMERO, JAVIER, et al. "SMPL: A Skinned Multi-Person Linear Model". *ACM Trans. Graph.* 34.6 (2015). ISSN: 0730-0301 2.



**Figure 1:** Registration results of human body shapes in the SHREC dataset.

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- [QXQ\*21] QIU, YUDA, XU, XIAOJIE, QIU, LINGTENG, et al. "3DCaric-Shop: A Dataset and A Baseline Method for Single-view 3D Caricature Face Reconstruction". *Proc. CVPR*. 2021 1, 3–6.

**Table 1:** Evaluation of symmetric Dirichlet energy.

**Table 2:** Evaluation of symmetric Dirichlet energy.

Input	Method	Chamfer	Normal	Dirichlet	Input	Method	Chamfer	Normal	Dirichlet
Abraham Lincoln C00024	[QXQ*21]	2.80e-04	6.26e-02	4.67E+00	Arnold Schwarzenegger C00015	[QXQ*21]	4.82e-04	4.20e-02	1.48E+00
	[LH22]	1.94e-04	8.88e-02	2.14E+00		[LH22]	1.36e-04	4.81e-02	1.23E+00
	Ours (Quality)	5.15e-05	1.34e-02	6.38E+00		Ours (Quality)	5.53e-05	9.27e-03	1.15E+01
	Ours (Cotan)	5.19e-05	1.38e-02	2.34E+01		Ours (Cotan)	5.56e-05	9.27e-03	4.53E+00
Albert Einstein C00014	[QXQ*21]	3.46e-04	6.35e-02	2.23E+00	Arsene Wenger C00001	[QXQ*21]	2.85e-04	5.37e-02	1.93E+00
	[LH22]	4.99e-04	1.38e-01	1.33E+00		[LH22]	2.75e-04	8.19e-02	1.22E+00
	Ours (Quality)	5.55e-05	1.28e-02	8.73E+00		Ours (Quality)	5.91e-05	1.62e-02	1.55E+02
	Ours (Cotan)	5.60e-05	1.27e-02	5.00E+00		Ours (Cotan)	5.92e-05	1.57e-02	7.70E+00
Angela Merkel C00039	[QXQ*21]	6.57e-04	5.50e-02	2.88E+00	Arsene Wenger C00002	[QXQ*21]	1.96e-04	4.03e-02	2.55E+00
	[LH22]	5.30e-04	9.32e-02	1.36E+00		[LH22]	2.25e-04	7.70e-02	1.45E+00
	Ours (Quality)	6.14e-05	1.33e-02	6.10E+00		Ours (Quality)	4.69e-05	1.09e-02	7.86E+01
	Ours (Cotan)	6.21e-05	1.38e-02	3.53E+00		Ours (Cotan)	4.73e-05	1.02e-02	4.82E+00
Angela Merkel C00054	[QXQ*21]	5.50e-04	5.08e-02	2.12E+00	Arsene Wenger C00007	[QXQ*21]	5.29e-01	6.16e-01	4.87E+02
	[LH22]	4.12e-04	8.97e-02	1.33E+00		[LH22]	1.62e-04	6.39e-02	1.48E+00
	Ours (Quality)	5.29e-05	1.48e-02	4.44E+01		Ours (Quality)	4.85e-05	9.64e-03	2.53E+01
	Ours (Cotan)	5.28e-05	1.31e-02	1.00E+01		Ours (Cotan)	4.88e-05	9.33e-03	6.43E+00
Angela Merkel C00065	[QXQ*21]	5.37e-04	5.08e-02	1.70E+00	Arsene Wenger C00014	[QXQ*21]	5.01e-01	6.92e-01	7.37E+02
	[LH22]	2.34e-04	7.58e-02	1.38E+00		[LH22]	4.32e-04	1.07e-01	1.32E+00
	Ours (Quality)	5.25e-05	1.45e-02	1.24E+01		Ours (Quality)	5.45e-05	1.22e-02	7.03E+00
	Ours (Cotan)	5.29e-05	1.42e-02	5.21E+00		Ours (Cotan)	5.49e-05	1.19e-02	2.90E+00
Angelina Jolie C00011	[QXQ*21]	3.89e-04	3.33e-02	1.98E+00	Ashley Olsen C00003	[QXQ*21]	4.17e-04	5.03e-02	1.68E+00
	[LH22]	2.98e-04	6.16e-02	1.14E+00		[LH22]	2.78e-04	7.28e-02	1.19E+00
	Ours (Quality)	5.72e-05	8.88e-03	2.79E+01		Ours (Quality)	5.27e-05	1.36e-02	7.73E+00
	Ours (Cotan)	5.70e-05	8.86e-03	2.80E+00		Ours (Cotan)	5.36e-05	1.35e-02	2.81E+00
Ann Hathaway C00007	[QXQ*21]	3.51e-04	3.91e-02	1.77E+00	Ashton Kutcher C00002	[QXQ*21]	3.40e-04	3.81e-02	1.55E+00
	[LH22]	1.90e-04	6.03e-02	1.20E+00		[LH22]	2.98e-04	6.20e-02	1.10E+00
	Ours (Quality)	5.52e-05	1.02e-02	9.40E+00		Ours (Quality)	5.39e-05	9.83e-03	5.63E+00
	Ours (Cotan)	5.53e-05	1.01e-02	2.19E+00		Ours (Cotan)	5.43e-05	9.94e-03	2.97E+00
Ann Hathaway C00012	[QXQ*21]	2.68e-04	4.49e-02	1.61E+00	Ashton Kutcher C00004	[QXQ*21]	3.75e-04	3.94e-02	1.41E+00
	[LH22]	2.10e-04	6.22e-02	1.18E+00		[LH22]	4.04e-04	7.08e-02	1.11E+00
	Ours (Quality)	4.87e-05	1.04e-02	1.41E+01		Ours (Quality)	6.13e-05	1.08e-02	5.65E+00
	Ours (Cotan)	4.89e-05	1.10e-02	4.21E+00		Ours (Cotan)	6.18e-05	1.13e-02	1.52E+00
Ann Hathaway C00020	[QXQ*21]	3.44e-04	4.36e-02	1.72E+00	Audrey Hepburn C00005	[QXQ*21]	3.82e-04	4.36e-02	1.77E+00
	[LH22]	2.02e-04	6.29e-02	1.13E+00		[LH22]	2.76e-04	7.75e-02	1.24E+00
	Ours (Quality)	5.51e-05	1.01e-02	9.93E+00		Ours (Quality)	5.90e-05	1.14e-02	3.51E+02
	Ours (Cotan)	5.55e-05	1.02e-02	6.12E+00		Ours (Cotan)	5.96e-05	1.14e-02	3.91E+00
Ann Hathaway C00022	[QXQ*21]	2.77e-04	3.66e-02	1.51E+00	Audrey Hepburn C00010	[QXQ*21]	3.74e-04	4.72e-02	1.70E+00
	[LH22]	2.72e-04	6.38e-02	1.16E+00		[LH22]	2.12e-04	7.06e-02	1.17E+00
	Ours (Quality)	5.13e-05	9.60e-03	2.29E+01		Ours (Quality)	5.69e-05	1.17e-02	1.61E+01
	Ours (Cotan)	5.15e-05	9.43e-03	4.48E+00		Ours (Cotan)	5.73e-05	1.13e-02	6.10E+00
Anthony Hopkins C00005	[QXQ*21]	4.49e-04	4.33e-02	2.25E+00	Audrey Hepburn C00012	[QXQ*21]	5.34e-04	4.03e-02	1.69E+00
	[LH22]	1.38e-04	5.90e-02	1.25E+00		[LH22]	4.54e-04	7.00e-02	1.16E+00
	Ours (Quality)	5.18e-05	1.10e-02	1.26E+01		Ours (Quality)	6.17e-05	1.10e-02	1.50E+01
	Ours (Cotan)	5.25e-05	1.15e-02	2.24E+00		Ours (Cotan)	6.16e-05	1.04e-02	4.43E+00
Arnold Schwarzenegger C00007	[QXQ*21]	4.94e-04	4.80e-02	1.50E+00	Audrey Hepburn C00013	[QXQ*21]	5.99e-04	5.19e-02	1.46E+00
	[LH22]	1.29e-04	4.37e-02	1.20E+00		[LH22]	3.29e-04	6.44e-02	1.17E+00
	Ours (Quality)	5.60e-05	9.17e-03	2.15E+01		Ours (Quality)	6.05e-05	9.42e-03	8.37E+00
	Ours (Cotan)	5.62e-05	8.83e-03	5.83E+00		Ours (Cotan)	6.08e-05	9.66e-03	3.01E+00
Arnold Schwarzenegger C00008	[QXQ*21]	5.23e-04	6.54e-02	2.42E+00	Audrey Hepburn C00015	[QXQ*21]	6.00e-01	6.24e-01	7.99E+02
	[LH22]	3.21e-04	9.12e-02	1.45E+00		[LH22]	1.35e-04	4.83e-02	1.21E+00
	Ours (Quality)	5.39e-05	1.39e-02	8.51E+00		Ours (Quality)	6.19e-05	1.01e-02	8.07E+00
	Ours (Cotan)	5.42e-05	1.34e-02	4.70E+00		Ours (Cotan)	6.27e-05	1.06e-02	1.84E+00

**Table 3:** Evaluation of symmetric Dirichlet energy.

Input	Method	Chamfer	Normal	Dirichlet
Ban Ki moon C00004	[QXQ*21]	3.46e-04	4.83e-02	1.74E+00
	[LH22]	3.62e-04	8.52e-02	1.25E+00
	Ours (Quality)	5.01e-05	1.70e-02	1.39E+01
	Ours (Cotan)	5.24e-05	1.64e-02	6.98E+00
Ban Ki moon C00008	[QXQ*21]	2.68e-04	5.02e-02	1.86E+00
	[LH22]	4.88e-04	9.51e-02	1.26E+00
	Ours (Quality)	6.12e-05	1.21e-02	4.16E+01
	Ours (Cotan)	6.25e-05	1.22e-02	4.02E+00
Barack Obama C00006	[QXQ*21]	5.36e-04	5.58e-02	1.43E+00
	[LH22]	2.31e-04	7.39e-02	1.19E+00
	Ours (Quality)	6.26e-05	1.20e-02	2.35E+01
	Ours (Cotan)	6.27e-05	1.24e-02	3.28E+00
Barack Obama C00010	[QXQ*21]	8.88e-04	1.55e-01	3.86E+00
	[LH22]	1.55e-03	2.50e-01	2.04E+00
	Ours (Quality)	4.37e-05	2.13e-02	4.51E+00
	Ours (Cotan)	1.05e-04	4.70e-02	3.01E+00
Barack Obama C00011	[QXQ*21]	3.18e-04	3.98e-02	1.96E+00
	[LH22]	2.66e-04	7.71e-02	1.19E+00
	Ours (Quality)	6.14e-05	1.06e-02	6.50E+00
	Ours (Cotan)	6.16e-05	1.00e-02	3.51E+00
Barack Obama C00015	[QXQ*21]	4.54e-04	6.09e-02	1.47E+00
	[LH22]	3.17e-04	9.46e-02	1.17E+00
	Ours (Quality)	5.81e-05	1.44e-02	5.92E+00
	Ours (Cotan)	6.09e-05	1.54e-02	2.85E+00
Barack Obama C00021	[QXQ*21]	3.81e-04	7.43e-02	1.70E+00
	[LH22]	3.29e-04	9.98e-02	1.36E+00
	Ours (Quality)	5.71e-05	1.45e-02	9.03E+00
	Ours (Cotan)	5.79e-05	1.47e-02	2.00E+00
Barack Obama C00037	[QXQ*21]	4.31e-04	4.58e-02	1.52E+00
	[LH22]	3.42e-04	7.58e-02	1.13E+00
	Ours (Quality)	6.13e-05	1.11e-02	3.88E+00
	Ours (Cotan)	6.18e-05	9.65e-03	2.31E+00
Barack Obama C00040	[QXQ*21]	2.78e-04	5.52e-02	2.45E+00
	[LH22]	3.04e-04	9.79e-02	1.28E+00
	Ours (Quality)	5.08e-05	1.32e-02	4.48E+00
	Ours (Cotan)	5.18e-05	1.34e-02	2.69E+00
Ben Stiller C00002	[QXQ*21]	3.37e-04	5.25e-02	1.52E+00
	[LH22]	2.07e-04	7.56e-02	1.28E+00
	Ours (Quality)	5.66e-05	1.18e-02	2.12E+01
	Ours (Cotan)	5.68e-05	1.13e-02	1.35E+01
Ben Stiller C00006	[QXQ*21]	2.54e-04	5.31e-02	1.73E+00
	[LH22]	3.37e-04	8.86e-02	1.33E+00
	Ours (Quality)	4.69e-05	1.46e-02	5.91E+01
	Ours (Cotan)	4.70e-05	1.43e-02	1.41E+01
Ben Stiller C00009	[QXQ*21]	6.12e-04	6.00e-02	2.82E+00
	[LH22]	2.06e-04	7.84e-02	4.35E+00
	Ours (Quality)	5.65e-05	1.40e-02	6.52E+00
	Ours (Cotan)	5.72e-05	1.35e-02	4.55E+00
Ben Stiller C00012	[QXQ*21]	4.98e-04	5.70e-02	1.38E+00
	[LH22]	2.05e-04	6.83e-02	1.16E+00
	Ours (Quality)	5.86e-05	1.49e-02	3.88E+01
	Ours (Cotan)	5.86e-05	1.35e-02	9.59E+00

**Table 4:** Evaluation of symmetric Dirichlet energy.

Input	Method	Chamfer	Normal	Dirichlet
Ben Stiller C00015	[QXQ*21]	4.20e-04	5.11e-02	1.38E+00
	[LH22]	4.39e-04	8.43e-02	1.16E+00
	Ours (Quality)	5.72e-05	1.14e-02	1.33E+02
	Ours (Cotan)	5.73e-05	1.02e-02	6.80E+00
Benedict Cumberbatch C00001	[QXQ*21]	5.84e-01	6.84e-01	6.48E+02
	[LH22]	2.34e-04	7.59e-02	1.25E+00
	Ours (Quality)	5.79e-05	1.20e-02	2.69E+01
	Ours (Cotan)	5.83e-05	1.21e-02	3.68E+00
Benedict Cumberbatch C00002	[QXQ*21]	3.38e-04	4.91e-02	1.50E+00
	[LH22]	4.77e-04	8.71e-02	1.20E+00
	Ours (Quality)	4.72e-05	1.30e-02	2.58E+01
	Ours (Cotan)	4.76e-05	1.20e-02	4.46E+00
Benedict Cumberbatch C00008	[QXQ*21]	4.82e-04	5.11e-02	2.00E+00
	[LH22]	7.65e-04	1.16e-01	1.24E+00
	Ours (Quality)	5.36e-05	1.32e-02	5.94E+00
	Ours (Cotan)	5.48e-05	1.36e-02	3.78E+00
Benedict Cumberbatch C00009	[QXQ*21]	3.32e-04	4.17e-02	1.83E+00
	[LH22]	1.02e-04	4.31e-02	1.21E+00
	Ours (Quality)	4.69e-05	9.79e-03	7.32E+00
	Ours (Cotan)	4.68e-05	9.08e-03	4.07E+00
Benedict Cumberbatch C00012	[QXQ*21]	5.68e-04	5.40e-02	2.05E+00
	[LH22]	3.95e-04	8.72e-02	1.21E+00
	Ours (Quality)	5.54e-05	1.33e-02	1.12E+01
	Ours (Cotan)	5.60e-05	1.28e-02	4.22E+00
Benedict Cumberbatch C00013	[QXQ*21]	4.14e-04	4.87e-02	2.95E+00
	[LH22]	2.16e-04	7.29e-02	1.64E+00
	Ours (Quality)	4.13e-05	9.95e-03	5.50E+00
	Ours (Cotan)	4.16e-05	8.76e-03	4.80E+00
Benedict Cumberbatch C00016	[QXQ*21]	6.43e-04	5.00e-02	1.63E+00
	[LH22]	2.75e-04	7.75e-02	1.68E+00
	Ours (Quality)	5.46e-05	1.28e-02	8.84E+00
	Ours (Cotan)	5.49e-05	1.22e-02	2.49E+00
Benedict Cumberbatch C00020	[QXQ*21]	5.68e-04	5.61e-02	1.70E+00
	[LH22]	3.43e-04	8.41e-02	1.54E+00
	Ours (Quality)	5.64e-05	1.35e-02	1.06E+01
	Ours (Cotan)	5.76e-05	1.42e-02	4.98E+00
Benedict Cumberbatch C00024	[QXQ*21]	3.32e-04	4.74e-02	1.43E+00
	[LH22]	1.94e-04	6.12e-02	1.19E+00
	Ours (Quality)	4.92e-05	1.40e-02	4.18E+01
	Ours (Cotan)	4.95e-05	1.31e-02	1.17E+02
Bette Midler C00011	[QXQ*21]	5.53e-04	4.80e-02	1.52E+00
	[LH22]	2.04e-04	6.43e-02	1.19E+00
	Ours (Quality)	6.17e-05	1.37e-02	4.73E+00
	Ours (Cotan)	6.26e-05	1.21e-02	3.28E+00
Bill Clinton C00011	[QXQ*21]	5.08e-04	5.59e-02	2.38E+00
	[LH22]	2.57e-04	8.31e-02	1.21E+00
	Ours (Quality)	5.58e-05	1.16e-02	8.03E+00
	Ours (Cotan)	5.58e-05	1.14e-02	3.35E+00
Bill Clinton C00013	[QXQ*21]	1.13e-03	7.19e-02	2.21E+00
	[LH22]	3.16e-04	8.90e-02	1.22E+00
	Ours (Quality)	5.96e-05	1.42e-02	9.76E+00
	Ours (Cotan)	5.98e-05	1.27e-02	3.41E+00

**Table 5:** Evaluation of symmetric Dirichlet energy.

**Table 6:** Evaluation of symmetric Dirichlet energy.

Input	Method	Chamfer	Normal	Dirichlet
Bill Murray C00003	[QXQ*21]	6.32e-04	3.83e-02	1.73E+00
	[LH22]	1.68e-04	5.95e-02	1.31E+00
	Ours (Quality)	5.41e-05	1.05e-02	1.85E+01
	Ours (Cotan)	5.45e-05	9.07e-03	6.81E+00
Bill Murray C00006	[QXQ*21]	6.11e-04	5.05e-02	1.88E+00
	[LH22]	4.49e-04	8.62e-02	1.18E+00
	Ours (Quality)	5.32e-05	1.21e-02	2.64E+01
	Ours (Cotan)	5.39e-05	1.16e-02	8.95E+00
Bill Murray C00020	[QXQ*21]	6.05e-04	4.97e-02	1.92E+00
	[LH22]	3.17e-04	8.61e-02	1.31E+00
	Ours (Quality)	4.60e-05	1.07e-02	1.69E+01
	Ours (Cotan)	4.64e-05	1.03e-02	7.81E+00
Bill Murray C00030	[QXQ*21]	5.19e-04	4.46e-02	2.39E+00
	[LH22]	1.96e-04	6.39e-02	1.26E+00
	Ours (Quality)	6.22e-05	1.27e-02	2.17E+01
	Ours (Cotan)	6.25e-05	1.20e-02	3.86E+00
Bill Murray C00032	[QXQ*21]	5.56e-01	8.41e-01	3.97E+02
	[LH22]	4.99e-04	1.26e-01	1.79E+00
	Ours (Quality)	4.84e-05	1.53e-02	2.68E+01
	Ours (Cotan)	4.86e-05	1.46e-02	2.15E+01
Bill Murray C00040	[QXQ*21]	7.42e-04	6.24e-02	1.71E+00
	[LH22]	2.52e-04	8.13e-02	2.06E+00
	Ours (Quality)	5.28e-05	1.17e-02	9.16E+00
	Ours (Cotan)	5.33e-05	1.15e-02	7.40E+00
Bill Murray C00041	[QXQ*21]	7.14e-04	3.75e-02	1.53E+00
	[LH22]	2.11e-04	5.89e-02	1.21E+00
	Ours (Quality)	5.80e-05	9.24e-03	7.72E+00
	Ours (Cotan)	5.80e-05	8.69e-03	6.59E+00
Billy Bob C00019	[QXQ*21]	6.68e-04	6.49e-02	2.13E+00
	[LH22]	2.47e-04	8.46e-02	4.26E+00
	Ours (Quality)	5.53e-05	1.38e-02	2.18E+01
	Ours (Cotan)	5.58e-05	1.38e-02	5.28E+00
Billy Idol C00007	[QXQ*21]	7.85e-04	6.59e-02	1.76E+00
	[LH22]	4.62e-04	8.88e-02	1.30E+00
	Ours (Quality)	5.45e-05	1.27e-02	3.68E+01
	Ours (Cotan)	5.54e-05	1.25e-02	3.55E+00
Bingbing Fan C00003	[QXQ*21]	4.54e-04	4.95e-02	1.50E+00
	[LH22]	1.80e-04	6.19e-02	1.20E+00
	Ours (Quality)	5.14e-05	1.14e-02	6.47E+00
	Ours (Cotan)	5.16e-05	1.05e-02	4.11E+00
Bingbing Fan C00004	[QXQ*21]	3.59e-04	4.23e-02	1.45E+00
	[LH22]	1.38e-04	5.35e-02	1.33E+00
	Ours (Quality)	5.88e-05	1.14e-02	1.29E+01
	Ours (Cotan)	5.93e-05	1.16e-02	8.64E+00
Bob Marley C00002	[QXQ*21]	5.72e-04	6.32e-02	2.19E+00
	[LH22]	4.55e-04	7.95e-02	1.57E+00
	Ours (Quality)	5.46e-05	1.82e-02	1.78E+01
	Ours (Cotan)	6.47e-05	1.77e-02	1.62E+01
Bob Marley C00038	[QXQ*21]	5.09e-04	5.53e-02	3.28E+00
	[LH22]	7.19e-04	9.46e-02	1.38E+00
	Ours (Quality)	4.86e-05	1.25e-02	1.15E+01
	Ours (Cotan)	4.94e-05	1.35e-02	5.77E+00

Input	Method	Chamfer	Normal	Dirichlet
Bono C00032	[QXQ*21]	7.11e-04	5.54e-02	1.89E+00
	[LH22]	5.05e-04	9.18e-02	1.23E+00
	Ours (Quality)	5.15e-05	1.25e-02	7.88E+00
	Ours (Cotan)	5.17e-05	1.20e-02	7.42E+00
Bono C00033	[QXQ*21]	3.67e-04	4.37e-02	1.66E+00
	[LH22]	2.48e-04	7.44e-02	1.22E+00
	Ours (Quality)	6.22e-05	1.74e-02	1.93E+01
	Ours (Cotan)	6.20e-05	1.51e-02	1.11E+01
Bono C00060	[QXQ*21]	5.10e-04	5.91e-02	2.29E+00
	[LH22]	5.60e-04	1.09e-01	1.28E+00
	Ours (Quality)	4.79e-05	1.34e-02	6.54E+00
	Ours (Cotan)	4.82e-05	1.21e-02	5.08E+00
Bono C00061	[QXQ*21]	6.31e-04	5.14e-02	2.16E+00
	[LH22]	3.56e-04	8.94e-02	1.23E+00
	Ours (Quality)	5.80e-05	1.11e-02	1.13E+01
	Ours (Cotan)	5.83e-05	1.06e-02	9.95E+00
Bono C00065	[QXQ*21]	5.35e-01	6.52e-01	7.40E+02
	[LH22]	4.36e-04	8.33e-02	1.17E+00
	Ours (Quality)	5.64e-05	1.02e-02	1.17E+01
	Ours (Cotan)	5.72e-05	9.67e-03	2.78E+01
Brad Pitt C00018	[QXQ*21]	6.93e-04	4.66e-02	1.57E+00
	[LH22]	4.86e-04	8.26e-02	1.13E+00
	Ours (Quality)	6.38e-05	1.19e-02	1.05E+01
	Ours (Cotan)	6.53e-05	1.29e-02	2.64E+00
Brad Pitt C00020	[QXQ*21]	4.98e-04	4.11e-02	2.08E+00
	[LH22]	2.28e-04	6.03e-02	1.22E+00
	Ours (Quality)	5.58e-05	9.40e-03	3.42E+01
	Ours (Cotan)	5.61e-05	9.16e-03	5.99E+00
Bruce Lee C00002	[QXQ*21]	4.19e-04	4.50e-02	1.57E+00
	[LH22]	1.67e-04	5.62e-02	1.19E+00
	Ours (Quality)	5.45e-05	1.23e-02	2.88E+01
	Ours (Cotan)	5.44e-05	1.12e-02	1.60E+01
Bruce Lee C00005	[QXQ*21]	3.72e-04	4.10e-02	3.78E+00
	[LH22]	3.62e-04	8.13e-02	1.74E+00
	Ours (Quality)	4.91e-05	1.32e-02	1.41E+01
	Ours (Cotan)	4.93e-05	1.29e-02	7.29E+00
Bruce Lee C00013	[QXQ*21]	4.93e-04	4.68e-02	2.86E+00
	[LH22]	3.88e-04	8.46e-02	1.60E+00
	Ours (Quality)	5.23e-05	1.25e-02	1.82E+01
	Ours (Cotan)	5.23e-05	1.13e-02	1.34E+01
Bruce Lee C00023	[QXQ*21]	3.58e-04	3.84e-02	2.23E+00
	[LH22]	2.21e-04	6.05e-02	1.22E+00
	Ours (Quality)	5.04e-05	1.09e-02	2.95E+01
	Ours (Cotan)	5.02e-05	9.57e-03	7.10E+00
Bruce Lee C00028	[QXQ*21]	5.42e-04	3.61e-02	1.67E+00
	[LH22]	1.75e-04	5.28e-02	1.29E+00
	Ours (Quality)	5.80e-05	9.19e-03	1.81E+01
	Ours (Cotan)	5.81e-05	8.60e-03	4.86E+00
Bruce Lee C00052	[QXQ*21]	4.32e-04	4.80e-02	1.77E+00
	[LH22]	3.19e-04	7.43e-02	1.16E+00
	Ours (Quality)	5.41e-05	1.16e-02	9.93E+00
	Ours (Cotan)	5.41e-05	1.12e-02	4.79E+00

**Table 7:** Evaluation of symmetric Dirichlet energy.

Input	Method	Chamfer	Normal	Dirichlet
Bruce Lee C00063	[QXQ*21]	5.49e-04	5.47e-02	3.18E+00
	[LH22]	2.08e-04	6.81e-02	1.47E+00
	Ours (Quality)	5.11e-05	1.23e-02	1.57E+01
	Ours (Cotan)	5.13e-05	1.11e-02	3.63E+00
Bruce Lee C00072	[QXQ*21]	4.63e-04	4.60e-02	3.05E+00
	[LH22]	3.03e-04	7.38e-02	1.21E+00
	Ours (Quality)	5.97e-05	1.77e-02	5.33E+01
	Ours (Cotan)	5.81e-05	1.36e-02	5.47E+00
Bruce Willis C00001	[QXQ*21]	3.50e-04	4.11e-02	1.73E+00
	[LH22]	2.35e-04	7.05e-02	1.18E+00
	Ours (Quality)	5.08e-05	1.06e-02	9.90E+00
	Ours (Cotan)	5.06e-05	9.86e-03	3.82E+00
Bruce Willis C00004	[QXQ*21]	4.25e-04	4.28e-02	2.12E+00
	[LH22]	2.02e-04	5.90e-02	1.24E+00
	Ours (Quality)	5.25e-05	1.17e-02	2.26E+01
	Ours (Cotan)	5.24e-05	1.01e-02	4.17E+00
Bruce Willis C00006	[QXQ*21]	4.69e-04	4.68e-02	2.63E+00
	[LH22]	2.87e-04	7.88e-02	1.62E+00
	Ours (Quality)	4.87e-05	1.19e-02	1.69E+02
	Ours (Cotan)	4.92e-05	1.12e-02	7.54E+01
Bruce Willis C00011	[QXQ*21]	5.26e-04	3.74e-02	2.35E+00
	[LH22]	2.62e-04	6.25e-02	1.21E+00
	Ours (Quality)	5.65e-05	8.83e-03	5.59E+01
	Ours (Cotan)	5.63e-05	8.44e-03	2.96E+00
Bruce Willis C00015	[QXQ*21]	6.13e-04	5.79e-02	2.81E+00
	[LH22]	2.63e-04	7.48e-02	1.45E+00
	Ours (Quality)	5.20e-05	1.13e-02	1.41E+01
	Ours (Cotan)	5.19e-05	1.07e-02	7.06E+00
Bruce Willis C00019	[QXQ*21]	2.96e-04	5.11e-02	1.95E+00
	[LH22]	3.12e-04	9.22e-02	1.33E+00
	Ours (Quality)	4.74e-05	1.30e-02	1.03E+01
	Ours (Cotan)	4.73e-05	1.24e-02	5.16E+00
Bruce Willis C00024	[QXQ*21]	4.82e-04	4.12e-02	2.21E+00
	[LH22]	3.98e-04	8.21e-02	1.23E+00
	Ours (Quality)	5.00e-05	1.08e-02	2.81E+01
	Ours (Cotan)	5.02e-05	1.01e-02	8.14E+00
Cameron Diaz C00004	[QXQ*21]	7.37e-04	4.04e-02	2.97E+00
	[LH22]	1.68e-04	5.19e-02	1.20E+00
	Ours (Quality)	6.20e-05	1.02e-02	1.63E+01
	Ours (Cotan)	6.28e-05	1.00e-02	9.04E+00
Cameron Diaz C00008	[QXQ*21]	7.15e-04	4.83e-02	2.19E+00
	[LH22]	1.55e-04	6.17e-02	1.52E+00
	Ours (Quality)	6.09e-05	1.47e-02	8.76E+00
	Ours (Cotan)	6.85e-05	1.30e-02	2.02E+01
Cameron Diaz C00011	[QXQ*21]	5.34e-04	4.55e-02	2.52E+00
	[LH22]	1.66e-04	6.19e-02	1.23E+00
	Ours (Quality)	6.02e-05	1.41e-02	5.88E+00
	Ours (Cotan)	6.21e-05	1.30e-02	4.82E+00
Cate Blanchett C00001	[QXQ*21]	2.64e-04	4.95e-02	1.44E+00
	[LH22]	2.12e-04	7.31e-02	1.21E+00
	Ours (Quality)	4.87e-05	1.29e-02	6.49E+00
	Ours (Cotan)	4.92e-05	1.27e-02	2.39E+00

**Table 8:** Evaluation of symmetric Dirichlet energy.

Input	Method	Chamfer	Normal	Dirichlet
Cate Blanchett C00002	[QXQ*21]	5.25e-04	5.10e-02	1.80E+00
	[LH22]	2.34e-04	7.28e-02	1.73E+00
	Ours (Quality)	6.18e-05	1.45e-02	5.01E+00
	Ours (Cotan)	6.25e-05	1.37e-02	2.98E+00
Cate Blanchett C00003	[QXQ*21]	4.45e-04	3.88e-02	1.63E+00
	[LH22]	2.14e-04	5.75e-02	1.22E+00
	Ours (Quality)	6.11e-05	1.24e-02	1.54E+01
	Ours (Cotan)	6.09e-05	1.10e-02	2.74E+00
Cate Blanchett C00008	[QXQ*21]	2.39e-04	4.49e-02	1.42E+00
	[LH22]	2.10e-04	7.42e-02	1.21E+00
	Ours (Quality)	4.85e-05	1.13e-02	4.52E+01
	Ours (Cotan)	4.92e-05	1.19e-02	4.47E+00
Cate Blanchett C00009	[QXQ*21]	3.04e-04	4.52e-02	1.93E+00
	[LH22]	3.19e-04	7.67e-02	1.18E+00
	Ours (Quality)	5.02e-05	1.16e-02	1.32E+01
	Ours (Cotan)	5.08e-05	1.08e-02	3.30E+00
Cate Blanchett C00012	[QXQ*21]	4.39e-04	4.99e-02	2.14E+00
	[LH22]	5.40e-04	9.07e-02	1.20E+00
	Ours (Quality)	5.60e-05	1.32e-02	1.08E+01
	Ours (Cotan)	5.60e-05	1.24e-02	3.34E+00
Cate Blanchett C00013	[QXQ*21]	4.88e-04	3.54e-02	2.70E+00
	[LH22]	2.13e-04	5.20e-02	4.16E+00
	Ours (Quality)	5.36e-05	1.05e-02	3.10E+01
	Ours (Cotan)	5.37e-05	1.02e-02	7.24E+00
Cate Blanchett C00014	[QXQ*21]	2.78e-04	5.78e-02	1.80E+00
	[LH22]	2.46e-04	8.89e-02	1.26E+00
	Ours (Quality)	5.31e-05	1.37e-02	6.47E+00
	Ours (Cotan)	5.34e-05	1.34e-02	4.44E+00
Cate Blanchett C00017	[QXQ*21]	6.93e-04	6.70e-02	3.05E+01
	[LH22]	2.71e-04	7.14e-02	1.35E+00
	Ours (Quality)	6.18e-05	1.26e-02	8.65E+00
	Ours (Cotan)	6.20e-05	1.17e-02	3.17E+00
Charles Bronson C00014	[QXQ*21]	4.96e-04	5.10e-02	1.59E+00
	[LH22]	1.93e-04	6.69e-02	1.21E+00
	Ours (Quality)	6.21e-05	1.59e-02	3.19E+01
	Ours (Cotan)	6.15e-05	1.37e-02	3.50E+00