

Topological Descriptor for CAD Models with Inner Cavities

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Abstract

The current work introduces an algorithm for constructing Reeb graph for CAD models and their inner cavities. The nodes of the graph represent areas of the manifold of the model where topological changes occur. Edges of the graph encode information about connections between such areas. The outline for the topological graph matching is given for detection of graph (sub)isomorphism. The proposed Reeb graph structure can be used as a descriptor of CAD models in the retrieval process.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling

1. Introduction

The need for shape description has risen significantly during the last years. Increasing amount of graphical objects in Internet and poor retrieval results using textual annotations [FMK*03] give a challenge to research in the area of shape description. In every day life for the definition of any object we use a set of its geometrical, topological and functional features. However, the domain of applications of a shape descriptor, i.e. Internet and large databases of models, restricts the number of ways for creating a descriptor. Being used for retrieval, recognition or alignment purposes [BMMP03], a shape descriptor should be compact enough to allow effective computer-human interaction. From the other side, a shape descriptor should be able to give a user the notion about the model it describes. Thus, if we just encode a model representation in order to obtain a unique identifier, e.g. applying some cryptographic function, a user will not have any clue about the models he is retrieving. In other words, a shape descriptor should be an optimum compromise between compactness and description of the model.

The existing solutions propose probabilistic approaches [OFCD01] [HK03] [ILSR02], based on sampling points of the shape, spatial mapping methods [KFR03] [VS01], which map data from the 3D domain into other domains like frequencies of Fourier transform. Some of the methods from these groups are designed to extract a shape descriptor invariant to affine transformations, whereas the others require preliminary model normalization. Usually the obtained shape descriptor has a structure of a vector or a matrix, which

is called feature vector. The similarity between models is measured as a distance value between the corresponding feature vectors.

In this article we present a graph-based approach for constructing a shape descriptor for CAD models. In the next section we present a brief survey of the graph-based approaches for building shape descriptors. In the second and the third sections we construct graph-identifiers for CAD models and we give an outline for performing their matching. Finally we discuss further improvements and provide directions for future research.

2. Related work

Skeleton-based descriptors are widely used in the fields of shape and image retrieval, recognition and alignment. The greatest advantage of such descriptors is that they are able to measure the similarity between two models, one of which is alike to the part of the other (subisomorphism detection), while those methods using shape distribution [OFCD01] and spatial mapping [VS01] give an estimation only of the model-to-model similarity. However, the matching of graph-based descriptors is a more complex problem than measuring the distance between feature vectors. Several approaches were suggested to detect (sub)isomorphism of graphs [HSKK01], [GR96], [HW02].

In this chapter we briefly describe Medial Axis Transform (MAT), shock and Reeb graphs, the three most used graph-based shape descriptors.

Medial Axis method was introduced nearly 30 years ago by H. Blum [KP03] and nowadays is widely used for image and 3D model recognition. The intuitive definition of medial axis given by the author is that of a curve where the frontiers of the fire coming from the boundary of the object and going towards the interior meet. In other words, medial axis is obtained by thinning the object. The original method proposed by Blum is an effective way for analyzing and describing the topology of the object. Still it has a strong weakness. More precisely, small perturbations in the boundary of the image/model lead to significant changes in the corresponding medial axis. Several solutions were proposed to overcome this lack, thus for example in [KP03] the authors introduce a substance measure for performing stable MAT calculation. In [SPB96] the authors perform the detailed analysis of the algorithm stating that its complexity for 3D polyhedral models is $O(n^2 \log n)$.

Shock graphs [SSDZ99] are based on MAT, where each vertex is labeled with the type of shock (protrusion, neck, bend and seed), and all edges are oriented with respect to the shock formation times. As a consequence, shock graphs carry more information about the shape of the object, but at the same time they inherit high complexity cost and sensibility to the small perturbations on the boundary from the MAT.

Reeb graphs are proved to give good representation of the topology of 3D models, [SKK91], [HSKK01] with lower complexity cost $O(n \log n)$ [ABS03]. However, the authors of [BRS03] claim that Reeb graph is not a suited solution for the description of the topology for CAD models. They use multiresolutional structure of the graph proposed in [SKK91].

Reeb graph matching algorithms are presented in [HSKK01], [BMMP03], [BM05]. In [HSKK01] matching of Multiresolutional Reeb graphs (MRG) proceeds in the coarse-to-fine direction. Similarity between two MRGs is defined as the sum of similarity measures for the pairs of the nodes which are chosen maintaining topological consistency of the graphs. In [BMMP03], [BM05] the authors construct the directed acyclic attributed Reeb graphs and perform their matching. The cornerstone of the matching algorithm is calculation of the maximum common subgraph. The bigger the common subgraph of two considered Reeb graphs the more similar they are. The detailed description and analysis of the matching algorithm is given in [MSF05].

In this work we propose a new shape descriptor for CAD models, which is composed from Reeb graphs of the model and its inner cavities. The algorithm for the construction of Reeb graph is similar to the one proposed in [ABS03]. The obtained descriptor carries more information about topology of a model and thus leads to finer retrieval results.

In the next section we provide the most important definitions and description of the process of Reeb graph construction. At the end we show several examples of the Reeb

graph obtained using the proposed algorithm and discuss the advantages it provides.

3. Topologically driven Reeb graph for CAD models

In the current section we give the most important definitions and explain the process of constructing a Reeb graph for CAD models and its inner cavities. For descriptive purpose we use height function as a mapping function for the construction of the Reeb graph. However in Section 6 we give an outline for the use of the function of the distance from barycenter.

3.1. Background

Before we get start with the detailed description of our algorithm we provide here the main definitions used in this work.

1. A smooth function f is Morse function if all its critical points are non-degenerate, or in other words, the Hessian matrix $\frac{\partial^2 f}{\partial x_i \partial x_j} \neq 0$. Consequently, if a Morse function f is an approximation of a smooth manifold M , then topological changes in the manifold (holes, joints and breaks of connected components) correspond to the critical points of the function f .
2. Reeb graph is defined as a quotient space of the compact manifold M , defined by the equivalence relation $(X, f(X)) \sim (Y, f(Y))$ iff $f(X) = f(Y)$, where $f(x)$ is a real-valued function defined on M , $f : M \rightarrow R$, and X and Y are from the same connected component $f^{-1}(X)$.

In the latter definition the smooth function f maps points from the manifold M to the real-valued manifold R . Then every node in the Reeb graph represents a critical point of the mapping function $f(M) \in R$. Moreover, if f is Morse function, then there are no topological changes of the manifold M along the edges of the Reeb graph.

Usually the height function or other distance-based functions are used for the construction of the Reeb graph [HSKK01] [BMM*03]. In general these functions do not guarantee non-degeneracy of their critical points. Figure 1 shows two possible situations.

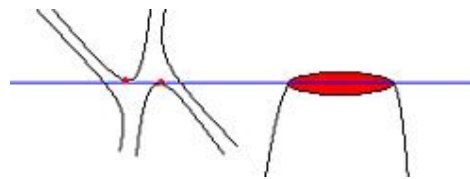


Figure 1: Convergence of several critical points of the height function.

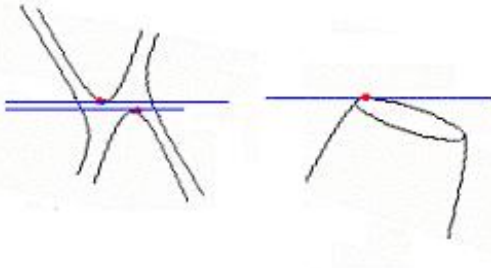


Figure 2: Handling convergent critical points.

In [FK97] the authors proved that almost all height and distance-based functions are Morse functions, where situations like the one showed in Figure 1 can be avoided by slight tilting or translating of the manifold, see Figure 2.

3.2. Reeb graph for inner cavities

The decision to construct Reeb graph for the inner cavities is determined by the inability of the Reeb graph to detect through-holes if they are located in the direction of the mapping function (see Figure 3).

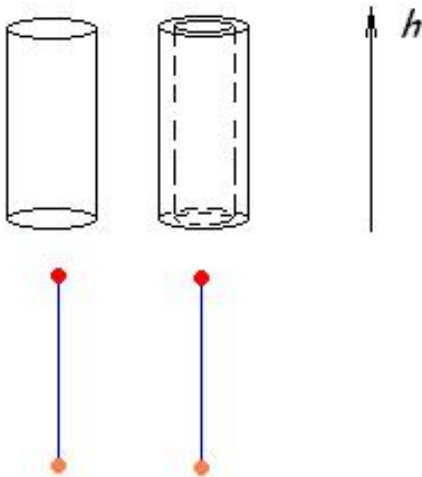


Figure 3: A solid model and a model with a through hole and their equal Reeb graphs.

Researchers from the area of graphical visualization use information about the complement of 3D models for the better shape encoding and repair [SL01] [CZ03]. We use data about inner cavities of the model in order to capture more information about its topology. The descriptor representing topology of a model and inner cavities characterize the model in more precise manner and as a consequence gives better classification and finer retrieval results. Here we

notice that graph representation of the inner cavities does not hold the requirement of the connectivity, according to which all nodes of the graph should be connected. The graph of the model is strictly connected as our method requires well-designed connected model, whereas the graph representation of inner cavities can be disjoint.

3.3. Reeb graph construction

For Reeb graph construction we were inspired by [ABS03] and [SKK91]. Below we provide the main steps for the construction of the Reeb graph based on the height mapping function.

Before starting the processing of the model we associate a scalar field with each vertex of the triangle mesh. The scalar field keeps the value of the mapping function for the corresponding vertex. As we pointed above, Reeb graph is a quotient space of the input manifold M according to the mapping function. Following this definition we collapse the manifold M , extracting the contours corresponding to the isolevels of the mapping function. We leave to the user the opportunity to decide the number of extracted contours. However as a future work we see implementation of automatic suggestion to extract more isocontours if it is needed for the better description of the topology. This can be achieved by comparing the genus value defined by the Euler formula with the number of loops in the Reeb graph. Here Euler formula for the connected model without holes in the faces of the mesh representation is $V - E + T = 2(2 - g)$, where V , E , T , and g are the number of vertices, edges, triangles and genus of the model correspondingly [FvDFH00]. Obviously, if genus is greater than the number of loops in the graph a user will receive an advice from the system to extract more contours for the better topological description. In this case a user has control over the accuracy of the model description.

In order to construct a contour for the particular isolevel of the mapping function, here called h_i , we search for the edges whose vertices are both equal to h_i , or located on the different sides from the current isolevel. In the former case the edge is a constraint of the contour, whereas in the latter case it is split into two and the splitting point belongs to the contour. Figure 4 shows the result of inserting contours into the triangulation, where the edges were split.

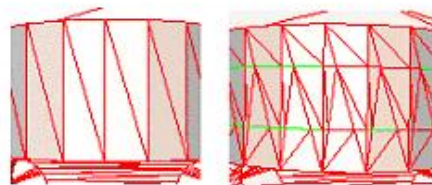


Figure 4: Isocontour insertion by splitting the edges.

The domain of CAD models guarantees that models are designed accurately, without holes and overlaps in the triangle mesh. In this case the model does not contain tangent or open contours as well as contours consisting only of one point, except the contours extracted at the critical points of the mapping function. In the latter case we can have degenerate critical points, which we handle by slight translation of the isolevel. Figure 5 depicts this situation.

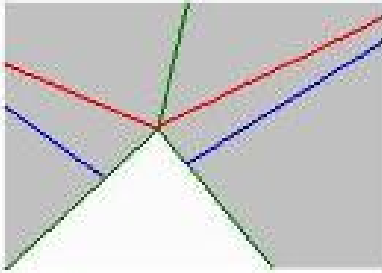


Figure 5: If the current isolevel coincides with the critical value of the mapping function, the isolevel is lightly translated. Here insertion of a contour at a saddle point (in red color) and subtle translation of the isolevel below the current position (in blue color).

Having extracted the isocontours we perform the parity check method in order to understand which contour belongs to the model and which to the cavities. For the detailed description of the parity check algorithm we refer to [NT03]. We mark all edge-constraints of the extracted isocontours with labels indicating if a contour belongs to a model or its cavities. Having broken one set of contours into two subsets of those belonging to the model and those of the inner cavities we construct the Reeb graph separately for each of these subsets as we would have two different models.

During the process of extracting isocontours, we also calculate geometrical information which we associate with the node representing a contour. Such information includes: perimeter and area of the contour, average curvature and compactness [TS04] [LWL04]. Each extracted contour is represented by a node in the Reeb graph. The next step in the graph construction process is to decide which nodes should be connected by an edge in the Reeb graph. As Reeb graph is a topological structure of the graphical object, the nodes of the graph should be incident if the contours they represent are connected through the mesh. This compels us to trace the connectivity between triangles representing the model. Tracing the connectivity of the mesh has complexity $O(N * E)$, where N is the number of the inserted contours, and E is the number of the edges in the triangulation. As a result we receive a graph structure (V, E) of the vertices and edges. However the obtained graph representation is not unique for the 3D model, because the number of nodes and edges depends on the number of extracted isocontours. To

avoid this we determine which nodes represent contours of critical values of the mapping function. Then we leave "critical" nodes in the graph structure, and we connect them with edges, eliminating other "regular" nodes. Finally we obtain a unique graph-based representation of the model, where each node represents the critical point of the mapping function. Figure 6 shows the result.

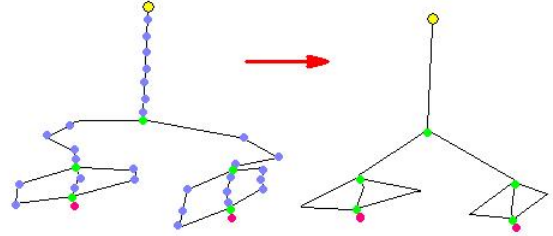


Figure 6: Smoothing Reeb graph.

Using a mapping function invariant to rotation, translation and scaling we will obtain a unique graph representation of a 3D model. The graph structure reflects the topology of the model; moreover it is enriched with several geometrical parameters, which helps to distinguish models with equal topology but different geometry, like size and average curvature.

Figure 7 provides some examples of our Reeb graph.

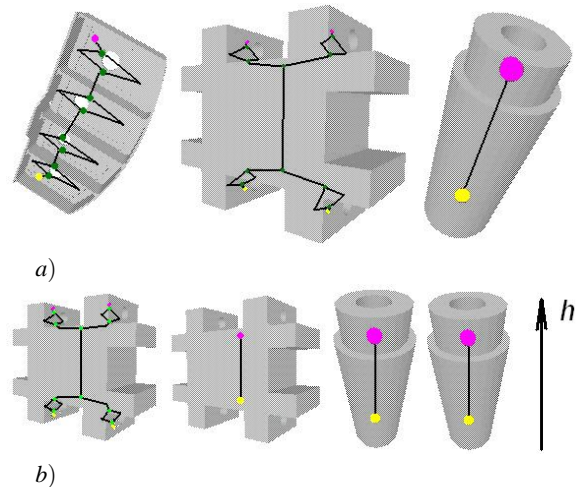


Figure 7: a) Examples of our Reeb graphs. b) Reeb graphs of models and their through-holes (for the model on the right the graphs coincide).

4. Graph matching

Usually in the retrieval process graph-based shape descriptors carry more information than only topology. The topological structure of such descriptors can be enriched with several geometrical characteristics of the model. Such parameters allow performing shape comparison using the sum of the similarity measure between pairs of nodes and edges of the graphs as a degree of matching between different models. Following this approach we associate to each node the values of the perimeter, area, compactness, average curvature and topological index of the corresponding isocontour. Our graph matching algorithm is similar to the one proposed in [HW02], which gives good results even for matching large graphs and it is applied to content-based image retrieval.

Moreover, according to the different goals of shape retrieval (search for the exact shape or search for the topologically equivalent object), we decide to preserve the possibility to perform graph matching using only topological data of the Reeb graph, which is the adjacency matrix.

We think of the elements of the adjacency matrix as about an estimation of the possibility to perform a walk between nodes in the graph [RKH01]. In such way we consider the adjacency matrix as a transition matrix. According to the Markov's chain processes, the elements of the matrix of the eigenvectors of the transition matrix represent the probability to transfer from one node to another in a steady state. The important role here has the principal eigenvector (the vector corresponding to the largest positive eigenvalue of the transition matrix). Its elements represent the probability of being at a correspondent node in the steady state after performing certain number of transfers in the graph. Consequently the similarity of two transition matrices can be measured as minimal deviation between the vectors representing their steady-state probabilities considering the connectivity constraints.

Precisely, we calculate the principal eigenvector of the transition matrix, and find the minimal deviation error between elements of their eigenvectors, checking the constraints of adjacency of matching nodes.

As a result we propose to use balanced matching $Match(u, v) = \lambda Geometry_Match(u, v) + (1 - \lambda) Topology_Match(u, v)$, where a user can decide the weighting factor λ .

5. Discussion

As we have mentioned in Section 2 the authors of [BRS03] analyzed the multiresolutional structure of the Reeb graph applying it to CAD models. As a result they conclude that Reeb graph built using geodesic distance function is more sensitive to the geometrical changes of the model representation than to the topological ones. Moreover, the authors discovered that Reeb graph is very sensitive to the resolution of the model. The main reason for this are changes of

the mapping function which appear together with different model resolution. In our opinion, a high sensibility of the topological structure Reeb graph to the geometrical changes is determined by the fact that all nodes of the graph carry information about geometrical features of the corresponding levelset. Then the matching algorithm is based on founding the smallest error between weights (which are geometrical characteristics) of the nodes and edges of the graph. As a consequence, small changes in the geometry of the model are immediately reflected in the weights of the nodes and thus lead to the different results of the matching process.

In our work we introduced a Reeb graph which nodes represent only critical areas of the mapping function and thus topological changes in the model. The number of nodes is also less than in the multiresolutional Reeb graph, where each node corresponds to a certain levelset of the mapping function. As a result, we expect to have our Reeb graph to be more sensitive to the topological changes in the model representation than to the geometrical ones. Moreover, due to the reduced number of nodes in the graph, we foresee that the graph matching algorithm will take less time than the one introduced in [BRS03].

6. Conclusions and future work

In this work we have provided the description of the process of building Reeb graphs using the height mapping function for a model and its inner cavities. The decision to construct Reeb graph for the inner cavities was determined by the need to capture information about through holes of the model which are located in the direction of a mapping function. As a result, the descriptor composed from Reeb graphs of a model and its inner cavities characterizes the model more precisely and will give better retrieval results. The constructed Reeb graph corresponds to the intuitive guess of the skeleton of the model, however it is not rotation invariant. Now we outline the Reeb graph construction based on the mapping function of the distance from the barycenter. The approach is very similar. We preprocess the model by translating its barycenter to the point of origin, and then transform original cartesian coordinates into the spherical coordinates.

After the indicated model processing the extraction of Reeb graph is performed in the same way as for the height mapping function.

The other improvement we are going to introduce is a control of the representation of all holes of a model in its Reeb graph using Euler formula (See Section 3.3)

As a perspective we are going to investigate and to improve the geometrical characteristics with which the topological structure of the Reeb graph can be enriched.

Moreover, we will implement the matching algorithm presented in this work which uses the adjacency matrix of the Reeb graph. We will make experiments and perform analysis of their results.

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