

Visual Multiplexing

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Abstract

The majority of display devices used in visualization are 2D displays. Inevitably, it is often necessary to overlay one piece of visual information on top of another, especially in applications such as multi-field visualization and geo-spatial information visualization. In this paper, we present a conceptual framework for studying the mechanisms for overlaying multiple pieces of visual information while allowing users to recover occluded information. We adopt the term 'multiplexing' from tele- and data communication to encompass all such overlapping mechanisms. We establish 10 categories of visual multiplexing mechanisms. We draw support evidence from both perception literature and existing works in visualization to support this conceptual framework. We examine the relationships between multiplexing and information theoretic measures. This new conceptual categorization provides the much-needed theory of visualization with an integral component.

1. Introduction

Overlaying one piece of visual information on top of another is a common practice in visualization, ranging from having a few lines crossing each other in a line graph, to superimposing a vector field onto a surface in 3D flow visualization, and to integrating color-mapped scalar values at different depths in volume visualization. In abstraction, this suggests that visualization is often capable of delivering more than one piece of information at a screen location. More importantly, the *Human Visual System* (HVS) seems capable of decoding the combined visual signals into separate pieces of information. It is thereby highly desirable to harvest such a capability systematically in visualization.

The phenomenon of recognizing objects in various imperfect conditions has been studied in psychology, and in particular gestalt psychology (e.g., [Kof55, KS91]). In cartography, the concept of *map overlay*, which was formalized by McHarg [McH69], makes use of this phenomenon to facilitate data fusion in geo-spatial visualization. In visualization, a number of researchers have studied methods for multi-layer visualization (e.g., [BH07, War09, BCS11]), whilst allowing the presence of overlapping, consciously or unconsciously, is ubiquitous in many applications (e.g., [Tay02, KW06, CPC09, CPL*11]). In many cases, viewers can work out original information at ease, but in other cases, they may make incorrect observations because of various perceptual difficulties, such as occlusion, cluttering or dis-

tortion of a visual channel due to interference from spatially co-located visual channels. It is a fascinating as well as useful phenomenon in visualization, and will no doubt require a huge research effort for us to gain a full understanding.

In physical and social sciences, one of the necessary steps towards understanding a phenomenon is categorization. In this paper, we present a study of the mechanisms for overlaying multiple pieces of visual information while allowing users to recover occluded information. We adopt the term 'multiplexing' from the discipline of tele- and data communication to encompass all such overlapping mechanisms (Section 3). We classify these mechanisms into ten categories, which form the basis of the conceptual framework of *visual multiplexing* (Section 4). We draw evidence from perception literature to confirm that the concept of visual multiplexing has a scientific foundation, as well as from existing works in visualization to confirm that the concept has worked in practice (Section 5). As the evidence suggests that visual multiplexing may be able to deliver more information than uniplexing, we discuss how this new concept can be explained by the informational theoretic framework for visualization [CJ10] in Section 6. Finally, in Section 7, we offer our concluding remarks and discuss future work.

2. Related Work

In this section, we focus our literature review on existing visualization techniques that are closely related to this work.

We will discuss briefly the major references on multiplexing in Section 3. We defer the discussion on the relevant works in perception and cognition to Section 5 after presenting the conceptual framework of visual multiplexing.

Theoretical Context. As part of theoretical development, many taxonomies were proposed for different aspects of visualization. In addition to some 17 different proposals listed in [CJ10], the recent additions included user characteristics [GZ09], comparative visualization [GAW*11], video visualization [BCD*12], evaluation [LBI*12], mixed reality visualization [KOJC12], and task taxonomy [SNHS13]. In comparison, the classification proposed in this work addresses the topic of visual encoding at a much finer-grained level. At that level, the most relevant prior work includes the books by Bertin [Ber83], and Ware [War13]. This work complements Bertin's comprehensive study on several visual channels and Ware's broad coverage of perception in visualization with a detailed treatment of the mechanisms for delivery multiple pieces of visual information. In addition to taxonomies, there have also been theoretical developments in models and metrics. The most relevant is the information-theoretic framework proposed by Chen and Jänicke [CJ10]. This work represents an extension from this framework towards perception by focusing on a set of phenomena of visual encoding that are yet systemically explained or understood.

Overlaying Methodologies. There is a large collection of works on *map overlay* in geo-spatial visualization (e.g., [Mac04, CS11]). The concept was formalized by [McH69]. While the concept is generically applicable to any data fusion in visualizing geo-spatial data, much of the research has been focused on the construction of a composite map in the vector space by combining geometric primitives from different maps (e.g., [BE97]), or in the image space by combining other greyscale or color pixels/grids (e.g., [Dav01]). Such overlay operations are typically performed in a geographic information system (GIS), which usually assumes static and stable information in the database. Furthermore, although the concept of map overlay is simple and attractive, it is non-trivial to maintain the perceptual quality of the resulting composite map (e.g., [Bai88]).

There have been a number of perceptual studies on multi-layered methodologies in the context of visualization. These include the works by Hagh-Shenas *et al.* [HKIH07], Bair and House [BH07], Ware [War09] and Javed *et al.* [Jav10], which will be detailed in Section 5.

Comparative visualization often needs to convey different pieces of information at the same location (e.g., [VP04]). Gleicher *et al.* offered a categorization of three mechanisms [GAW*11]. This work, which is related to the mechanism of superposition, studies a broader range of low-level mechanisms relevant to many other forms of visualization (e.g., deformation, glyphs, maps and animation).

Multi-field Visualization. In scientific visualization, the need to depict multi-field data has inspired much effort

and creativity. Crawfis *et al.* [CA91] proposed to combine the visual representations of different data types in a texture space using a set of constructive operators. Laidlaw *et al.* [LAK*98] used a painting-inspired approach to visualize DTI images with three layers (image, textured image and glyph). Kirby *et al.* [KML99] used layered concept in oil painting to design multi-field flow visualization featuring overlapping glyphs and underpainting data fields. Healy and Enns [HE99] studied the use of different visual properties of color textures to represent multivariate data elements, and applied them to surface-based representations. Taylor [Tay02] used different visual channels in a layered approach. Bokinsky [Bok03] proposed to use 'data-driven spots' for layering sets of 2D single-valued functions.

3. Multiplexing in Communication

In the history of communication, *multiplexing* was introduced along with telegraph systems in the 19th century [Tuc71]. The concept enables efficient use of a transmission system, by allowing a number of signals at lower bit rate to share a single higher bit rate transmission medium [Sta04]. Multiplexing brings about a huge amount of benefits such as cost-effectiveness and convenience in sharing a common medium. In a multiplexing system, a *multiplexer* (mux) is an encoder that combines multiple input signals and forwards the combined signal onto a transmission medium. A *demultiplexer* (demux) is a decoder that takes a signal from a shared medium and separates it into multiple signals. The commonly used multiplexing schemes are:

- *Frequency-division multiplexing* (FDM) – It 'divides the transmission frequency range into narrow bands' and is widely used in radio, television, telephone, and satellite carrier systems [Bla89].
- *Time-division multiplexing* (TDM) – It 'provides a user with the full channel capacity but divides the channel usage into time slots' [Bla89]. It is commonly used in digital communication.
- *Space-division multiplexing* (SDM) – This is observable in wired communication systems, where a cable consists of a bundle of many wires carrying different signals.
- *Code-division multiplexing* (CDM) – It enables different signals encoded in different coding schemes to share a single transmission line at the same time over the same frequency band [SM03].

The extensive amount of research, development, deployment and benefit of multiplexing in communication inspired us to develop a similar concept in visualization.

4. Multiplexing in Visualization

Consider 2D media where visualizations are displayed. Such media may encompass many types of computer displays, projection screens and printed media. (Our consideration does not include any stereoscopic or holographic devices, which will make an interesting topic for future research.)

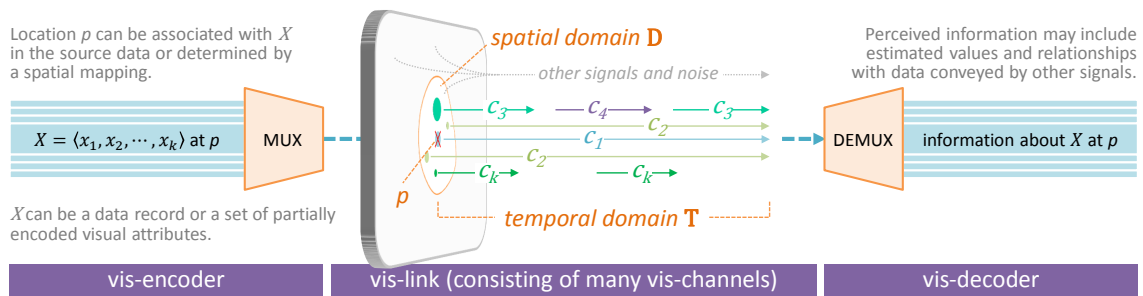


Figure 1: This illustration shows a multivariate data object X associated with a location p in a visualization. The visual encoder maps the data values of X to different visual signals that may be partitioned spatially and temporally and may use different amounts of spatial or temporal bandwidth. The visual decoder attempts to recover information about X , for example, by estimating its data values and determining its relationships with data conveyed by other signals. The mux and demux are part of the vis-encoder and vis-decoder respectively. Similar to the terminology in communication, the connection between the mux and demux is referred to as vis-link that consists of multiple visual channels (cf. [CJ10]).

Definition. Let p be a point on a medium, x be a valid value of a data variable, and c be a visual signal in a visual channel. Given a multivariate data object, $X = \langle x_1, x_2, \dots, x_k \rangle (k > 1)$, associated with a location p , the goal of a *visual multiplexer* is to map each data value x_i to a visual signal c_i , and deliver $\langle c_1, c_2, \dots, c_k \rangle$ through the corresponding visual channels. It is not essential for these visual signals be located at p and they can be distributed in a spatial domain \mathbf{D} surrounding p . Nor is it necessary for these visual signals be present at the same time, and they can be delivered over a short time interval \mathbf{T} . Different signals may be mixed in display. On the other hand, the goal of a *visual demultiplexer* is to obtain information about $\langle x_1, x_2, \dots, x_k \rangle$ at p by observing and interpreting visual signals depicted in \mathbf{D} during \mathbf{T} . Figure 1 illustrates the concept of visual multiplexing. In the following discussions, we will simply refer to the visual multiplexer and demultiplexer as *mux* and *demux* respectively as in communication. One major departure from a communication system is that a demux in a visualization pipeline is not a technical device, but a human-centered function that relies on the perception and cognition of a viewer.

Visual multiplexing may take several different forms as illustrated in Figure 2. The process for deriving this categorization is given in the supplementary materials. There are two basic forms of multiplexing, conceptually similar to SDM and TDM in communication. Almost all visual designs in visualization involve partitioning a 2D medium into different regions, and delivering multiple visual signals simultaneously through these regions, which can be fine-grained at the level of pixels or ink dots. Figure 2(a) illustrates a coarse-grained space-division with a pixel-based visualization, and a fine-grained space-division with an elevation map. Animation is a basic form of time-division multiplexing. Theoretically, each pixel in every frame can act as a visual channel by transmitting a signal independently. In practice, it is well understood that a human viewer would not normally obtain all information at such a fine-grained level. Figure 2(b) shows a simple example, where signals for dif-

ferent data values (rain, temperature, wind speed and direction) can be transmitted and received in a short animation.

Let us now consider more complex forms of multiplexing by focusing on a single point p , marked by grey arrows on the bounding box of each illustration in Figure 2. One crude but often effective means of multiplexing is simply for the mux to plot all k visual channels at the same position p , allowing the last one being displayed to occlude all other visual channels at p . Figure 2(c) shows two illustrations, where the viewer can easily perceive three pieces of information at the center of the square due to some gestalt effects, e.g., in these two cases, *continuity*, *similarity* and *proximity*. Depending on the availability of a priori knowledge about the possible types of shapes in a visualization, the opaque occlusion may result in some uncertainty. For example, one may be uncertain about whether the orange rectangle does reach p , the center of the square in the right illustration.

Such uncertainty can be reduced by replacing filled shapes with outlines in Figure 2(c). Some visual channels do not necessarily demand the use of p in the medium in order to deliver information for p . As illustrated in Figure 2(d), visual channels such as *shape* and *size* can have a hollow interior, hence freeing the optical properties of p for delivering other signals. Some topological and relational channels, such as distance, closure and density may naturally leave a fair amount of space unblocked. In the right illustration of (d), the distance between arrows is a visual channel for depicting speed. At p , the center of the illustration, the demux delivers the speed information, while revealing the map information at p .

As illustrated in Figure 2(e), translucency allows some ‘non-hollow’ visual channels to benefit in a way similar to a ‘hollow’ visual channel. In addition to gestalt effects, the demux involves some cognitive reasoning, e.g., in these two cases, about the colors. For the left illustration, the demux has to determine if the purple square in the middle is an independent shape or the blending of the blue and red rectan-

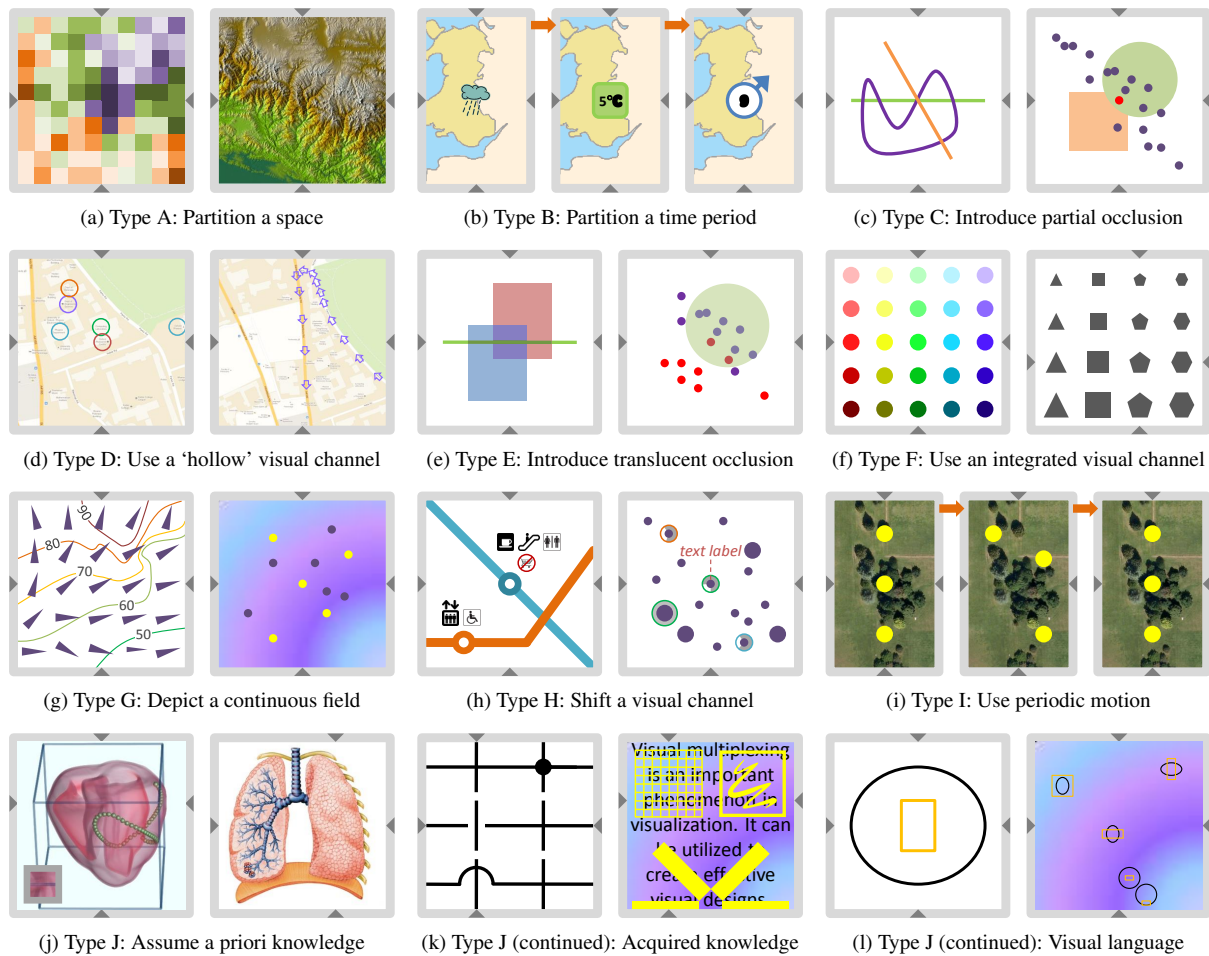


Figure 2: Different types of visual multiplexing. While multiplexing may occur in several places in each illustration, we focus on a specific point p indicated by arrows on the bounding box. For Type J (model-based), three sets of illustrations are shown.

gles. For the right illustration, the demux has to reason about whether there are 2, 3 or 4 types of dots, since there are 4 differently colored dots. The knowledge about color blending thus becomes an 'algorithmic' aspect of the demux.

At location p , translucency introduces an integration of different optical signals. In Figure 2(e), the demux attempts to differentiate one signal from another. In many other cases, the demux has to separate different signals at p more or less depending only on the integrated signal at p . Figure 2(f) illustrates two integrated visual channels, on the left, hue (x -axis) and luminance (y -axis); and on the right, shape or number of edges (x -axis) and size (y -axis). Although it is known that integrated channels have some shortcomings [She64], nevertheless, they provide a means of multiplexing.

In visualization, a 2D field, which assumes a continuous coverage of every point in a spatial domain, is usually depicted by using discretely placed objects. For example, the left illustration of Figure 2(g) shows a height field with a set

of contour lines and a vector field that is depicted using regularly spaced arrows. The demux can establish two pieces of information at the center p (and all other points) in a way that very much resembles the algorithmic function of bilinear interpolation. With the example on the right, similarly, the demux can infer the possible colors under the dots from the nearby colors that are not occluded.

In other cases, the demux may use the 'nearest neighbor' interpolation, as illustrated in Figure 2(h). The various signs shown in the left illustration are individual visual channels, which convey different information associated with the positions marked by hollow circles. At the center location, p , despite that the mux has deliberately displaced the 4 signs away from p , the demux has no difficulty to translate the information back to p .

In Figure 2(i), displacement is also used, but in a temporal manner rather than as a 'permanent' feature (cf. (h)). The three yellow dots move in different directions periodically,

thereby encoding three vectors. Because of the motion, none of the pixels in the aerial photograph are totally occluded within a time window. Conceptually, this may be considered similar to the use of ‘hollow’ visual channels in Figure 2(d), except the ‘hollowing out’ takes place in the temporal domain T . Note that this is not quite the same as the basic time-division multiplexing in Figure 2(b), where signals for different data values are delivered in an ordered manner.

While in Figure 2(a-i), the demux obtains some information at p from the surrounding domain D or a time interval T with little cognitive effort, often there are also cases where the demux relies on a known model about what is expected to be seen and how to see it. For example, Figure 2(j) shows a volume rendered object and a medical illustration. Once aware of the fact that the object on the left is a heart, the demux can perceive several surfaces along the ray passing through the center point p , though the actual quality of such signals is very poor (cf. the bottom-left inset). The cognitive reasoning relies on the a priori knowledge about the structure of a heart, such as the number of chambers. Similarly, with the right illustration, the demux can make use of the a priori knowledge of the relationship between two lungs to see multiple pieces of information displayed on the left and right lungs by applying a symmetry rule.

Specific knowledge for visual multiplexing can be acquired. Figure 2(k) shows two simple examples. In visualizing schematic logic diagrams, there is often uncertainty about connectivity between two crossing lines if one relies solely on gestalt effects (e.g., Type C). It is common to introduce a specialized depiction, such as a circle for a joint, and blank interval or distortion for unconnected lines. Similarly, a solid yellow box that would cause severe occlusion can be replaced by a wireframe mesh, a metaphoric illustration, or a piecewise deformed box. Such knowledge may not be intuitive at the beginning, but can be learned quickly. Note that the knowledge-based depictions in Figure 2(k) make use of other types of multiplexing, including Types C, D, and H.

In some applications, the knowledge required for decoding complex multiplexing can be formalized into a *visual language*. To illustrate this, Figure 2(l) shows a simple glyph design with two graphical components. It consists of a number of visual channels, which can be used to encode different data variables. The relatively independent visual channels include shape, size, colors, aspect ratio of each outline, and relative location of the two outlines. Each visual channel is a hollow channel (i.e., Type D). In other words, they can encode multivariate data objects without causing much occlusion, for instance, to the continuous field below.

5. Evidence Supporting Visual Multiplexing

We considered possible correspondences between the 4 types of multiplexing in communication and the 10 types in visualization. It is not difficult to observe the correspondence between the SDM in communication with Types A

and H in visualization, and TDM with Types B and I. We can also consider that CDM corresponds to Types F and J in visualization and to some degree to the knowledge required for interpolation in Types G and D. There are also suggestions in the literature that the human visual system (HVS) has some demux capabilities for FDM signals (e.g., [CR68, MUP79]). To our knowledge, it will require a huge research breakthrough to confirm how such capabilities related to the gestalt effects that facilitate Types C, D, E and G multiplexing in visualization. We hence seek evidence mainly from perception and visualization literatures. Because of the space limit, it would not be feasible for this paper to provide a comprehensive list of all such evidences. In this section, we selectively discuss a number of previous works relevant to the different types of visual multiplexing.

5.1. Literary Evidence from Perceptual Studies

Following the same analogy of multiplexing in communication, *visual multiplexing* can bring about efficiency and effectiveness in conveying complex information in visualization. The efficiency is typically achieved by delivering more information under the space-time constraints of a visualization process. The effectiveness is typically achieved by delivering simultaneously multiple facets of the data required by a visualization task, hence reducing the cognitive load in hopping from one visualization to another. There is extensive evidence in the literature of cognitive sciences to suggest that the HVS provides intrinsic support to de-multiplexing (i.e., decoding multiplexed visual images).

It is well understood that the HVS supports spatial parallelism in low-level perception with some 130M photoreceptors for receiving visual signals and 1.2M axons for transmitting these signals. This confirms that the demux can receive spatially multiplexed signals. Several proposed theories, e.g., the *feature integration theory* [TG80], and the *guided search model* [WCF89], suggest parallelism in visual search. Although the serial-parallel distinction has been an unsolved problem, the presence of pseudo-parallelism (through rapid eye movement) is undeniable [Car88]. This form of pseudo-parallelism is similar to TDM in communication. Hence, the HVS may support Type A multiplexing with parallelism, pseudo-parallelism, or both.

Atkinson and Shiffrin proposed a multi-store model for human memory [AS68] that is commonly accepted in cognitive sciences. In this model, there are three levels of human memory, namely *sensory*, *working* (or short-term) and *long-term memory*. The sensory memory for vision (i.e., iconic memory), which retains visual information for less than 1 second [Bad66], enables temporal integration of visual signals, and assists in change detection and motion perception [UIY*10]. This is essential for Types B and I multiplexing.

The working memory retains information for 15-30 seconds [PP59]. Although its storage capacity is limited, 15-30 seconds are sufficient for the demux to infer informa-

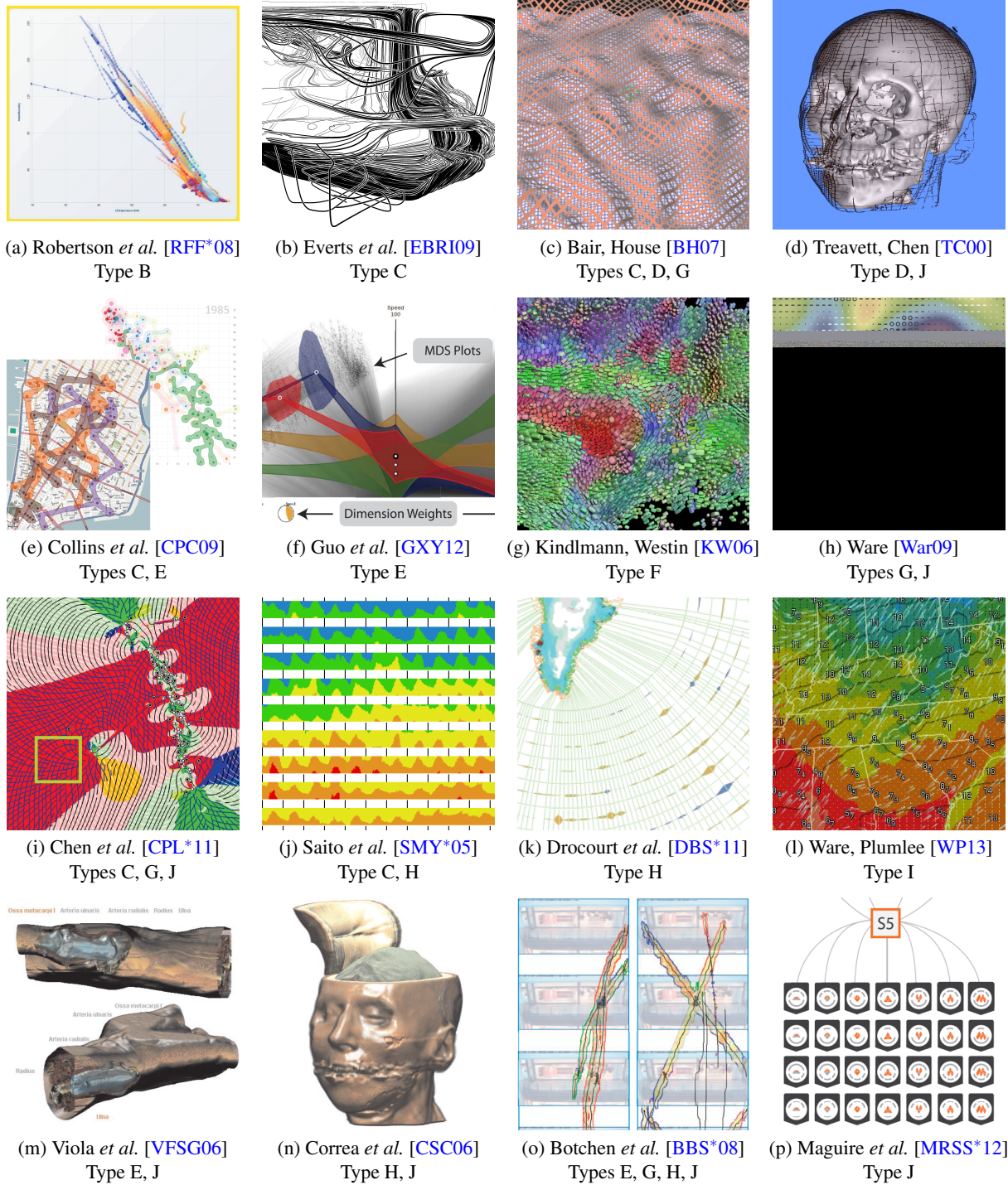


Figure 3: Examples of multiplexing in visualization. IEEE, Wiley-Blackwell and SAGE granted the permission to reprint the images in this figure, and the corresponding authors were informed. The image in (b) was provided by Dr. T. Isenberg, INRIA.

tion from a spatial domain surrounding p (Types C, D, G, H, J), or to cross-check a legend when decoding less intu-

itive visual signals, e.g., color mapping, glyphs, and integrated channels (Types E, F, J). Miller proposed that multiple

pieces of related information are grouped together in working memory [Mil56]. This suggests that multivariate glyphs may offer cost-effective visual encoding in order to hold more information in working memory. Franks and Bransford studied transformation of prototypes and suggested that humans can learn to recognize glyphs by rules consciously as well as unconsciously [FB71], confirming the learnability of multivariate glyphs as a visual language (Type J).

The gestalt principles of organization (e.g., proximity, similarity, continuity, closure, symmetry, and good form) have been shown to influence humans' perception of visual signals. It has been extensively studied for nearly a century (e.g., [Hum24, Kof55, KP81]). Although gestalt effects lack in an explanatory model, they have been confirmed by numerous experimental findings, e.g., about spatiotemporal relatability [PKS06], occluded objects and illusory figures [KS91], surface interpolation 3D relatability [FHGK08], and contour extrapolation and interpolation [HMS08]. Broadly, the existence of gestalt effects substantiates humans' ability to decode Types C, D, E, G and H multiplexing.

5.2. Literary Evidence from Use Cases in Visualization

There are also extensive evidences in the visualization literature to suggest that different types of multiplexing are commonly and successfully used as practical solutions in many applications. Although the term 'visual multiplexing' has not been used in visualization, it has been featured in almost every visualization image in the literature. The most common use of visual multiplexing is a space partition, as it is rare for a visualization to devote its entire space to a single data value. Figure 3 shows 16 example visualizations that feature different types of visual multiplexing. Most examples in Figure 3 deal with reasonably large/complex datasets and have a dense visual appearance, suggesting that the 'simplex' spatial resource may have been 'exhausted'.

Robertson *et al.* [RFF*08] categorized the uses of animation in visualization (i.e., Type B multiplexing) into four groups, namely depicting state transition, view transition, process steps, and trend. Heer and Robertson [HR07] summarized the potential benefits of animation as directing viewers' attention, facilitating learning and decision making, and increasing levels of engagement. For example, Robertson *et al.* [RCM91] showed that animation can help depth perception in visualizing hierarchical data representations. Meanwhile, several empirical studies (e.g., Figure 3(a) from [RFF*08]) showed that animation does not necessarily improve users' performance and may result in more errors.

A few visualization techniques such as wire-frame rendering and pen-and-ink style rendering (e.g., [EBRI09]) feature an extensive amount of Type C multiplexing. Figure 3(b) features the introduction of halos by Everts *et al.* [EBRI09] to aid line perception (cf. Figure 2(k)). Bair and House [BH07] conducted two empirical studies to compare layered surface visualizations with different textures. Figure 3(c),

from [BH07], shows two surfaces with different textures, confirming that textured surface rendering led to significant improvements over non-textured surfaces. As shown in Figure 3(c,d), wire-frame rendering and pen-and-ink style rendering can also facilitate the use of a 'hollow' visual channel (Type D). In the case of pen-and-ink volume rendering in [TC00], demux relies on a priori knowledge to guess the spatial coverage of the hollow channel.

Figure 3(e) shows a visual design for depicting complex set relationships by Collins *et al.* [CPC09]. This is an excellent example of using translucency in visualization (cf. Type E). Figure 3(f) shows a visual design, by Guo *et al.* [GXY12], featuring dot, line and translucent shapes that exhibit different spatial frequency. Integrated visual channels have often been used in glyph-based visualization. As shown in Figure 3(g), for example, Kindlmann and Westin used this approach to encode tensors [KW06] (cf. Type F).

Hagh-Shenas *et al.* color weaving with blending for multi-field visualization [HKIH07]. Ware studied glyph-based texture in [War09], and oscillatory motion of glyphs in [WB06]. Figure 3(h) shows an example of a texture pattern made of a collection of simple glyphs, called textons [War09] (cf. (g) and (l) in Figure 2). As shown in Figure 3(i), Chen *et al.* combined a number of visual channels such as patches, lines, textures and glyphs to visualize a tensor field [CPL*11].

Javed *et al.* compared several multiplexing methods for time series visualization [JME10], namely line graph (cf. Type C), stacked graph (cf. C), multiples (cf. A), braided graph (cf. C, G), and horizon graph (cf. H, C). In a horizon graph [SMY*05], as shown in Figure 3(j), higher-value sections are shifted downwards (Type H) and placed upon the lower-value sections (Type C). Drocourt *et al.* transformed 2D Cartesian coordinates of marine terminating glaciers to 1D angular coordinates. As shown in Figure 3(k), by shifting the actual locations of these glaciers, they were able to free one dimension for depicting time [DBS*11]. Using motion to encode variables is common in vector field visualization. Recently, Ware and Plumlee showed that encoding wind direction (Figure 3(l)) using animated traces results in a lower perceptual error rate than three static methods [WP13].

Figures 3(m,n,o,p) show four examples of visual multiplexing by utilizing a priori knowledge. Viola *et al.* proposed to make important features in volume data more visible [VFSG06]. As this alters the perception of objects that occlude important features, it relies on the viewers' knowledge about the corresponding anatomic model (Type J) and/or the non-uniform mapping of transparency by the algorithm. Correa *et al.* enabled users to peel away part of a volumetric object to reveal the previously occluded interior [CSC06]. It relies on a user's knowledge to decode the fact that the peeled-away parts are multiplexed with the revealed interior. Botchen *et al.* [BBS*08] proposed *VideoPerpetuoGram* (VPG) for video visualization. It discards most of the frames and distorts the shape of remaining key frames to create

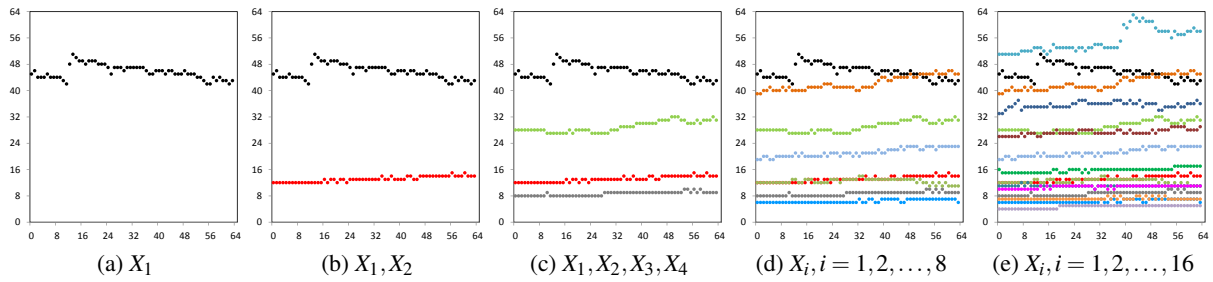


Figure 4: Uniplexed and multiplexed dot plots for time series visualization. Plots (a), (b), (c), (d), and (e) show 1, 2, 4, 8, 16 time series respectively, each of which is an instance of an independent data space $X_i, i = 1, 2, \dots, 16$. Although the original data was obtained from DataMarket.com, it is not intended here to visualize that dataset. In order to illustrate the calculation of information theoretic measures, the data has been mapped to the integer value range $[0, 63]$.

‘hollow’ intervals between them, making the tracking information easier to perceive. This design utilizes several types of visual multiplexing. It makes use of viewers’ common knowledge about frames in a video to facilitate omission and deformation (Types H and J). It creates a frame-level continuous field by using discrete depiction (Type G). In addition, it utilizes translucent tracking lines and highlighting shapes (Type E). Maguire *et al.* proposed an algorithmic approach for creating a large collection of glyphs, facilitating knowledge-assisted multiplexing [MRSS^{*}12].

6. Multiplexing and Information Theoretic Measures

Multiplexing is an integral component of modern communication technologies. This suggests that visual multiplexing may also be an integral part of visualization. In their work on an information theoretic framework [CJ10], Chen and Jänicke suggested that multiplexing may be relevant to comparative visualization, volume rendering and multi-field visualization. In particular, they proposed several entropy-based measures, one of which is *Display Space Utilization* (DSU), which is defined as:

$$\text{Display Space Utilization (DSU)} = \frac{\mathcal{V}(G)}{\mathcal{D}}. \quad (1)$$

where \mathcal{D} is the *Display Space Capacity*, and $\mathcal{V}(G)$ is the *Visualization Capacity* of a graphical mapping G . Using a time series plot as an example, they showed that the DSU in that particular condition is 0.03125. In other words, from an information theoretic perspective, only 3.1% of \mathcal{D} is utilized by that time series plot in [CJ10]. Hence visual multiplexing merely provides a means for making more efficient use of the apparently wasteful display space capacity. Also using time series plots as an example, we demonstrate how this is achieved in terms of DSU.

Figure 4 shows five simple time-series plots. Assume that the graph plotting area (i.e., excluding axis regions) in each plot is given as 64×64 pixels. The medium (or device) used is in 24 bit color. Hence $\mathcal{D} = 24 \times 2^{12} = 3 \times 2^{15}$ bits.

In Figure 4(a), a dot plot shows one time series with 64 independent samples, each of which has an integer value range

between 0 and 63. We choose the dot plot as it is more intuitive than a line graph for illustrating the calculation of various probabilistic measures. Samples are taken at a regular temporal step, so the dot plot displays 64 pixels corresponding to the data samples. The probability mass function of each sampling value is independent and identically-distributed, i.e., $p = 1/64$. Let X_1 denote the data space encompassing all time series with 64 samples in the integer range $[0, 63]$. The time series in Figure 4(a) is just an instance of X_1 . It is not difficult to calculate the entropy of X_1 , yielding $\mathcal{H}(X_1) = 384$ bits. Using the same reasoning as [CJ10], we obtain that $\mathcal{V}(G) = \mathcal{H}(X_1) = 384$ bits. Therefore $\text{DSU} = 384 / (3 \times 2^{15}) = 0.0039$. \mathcal{D} thus is very much under-utilized and has more than 99% spare capacity.

Let X_2 be the data space for a second time series with the same numerical and probabilistic constraints. The entropy of X_2 is also 384 bits. As it has been commonly done in practice, we can multiplex two time series in the same plot area. As shown in Figure 4(b), the two instances of X_1 and X_2 happen to occupy different parts of the plot area without any overlapping. This can be considered as spatial multiplexing (i.e., Type A). If we use this visual mapping G for any arbitrary pair of time series from X_1 and X_2 , we will nearly double the visualization capacity $\mathcal{V}(G)$, such that $\mathcal{V}(G) = \mathcal{H}(X_1) + \mathcal{H}(X_2) - \varepsilon_{1,2}$, where $\varepsilon_{1,2}$ is the average information loss due to overlapping. As discussed in the previous section, gestalt effects (i.e., Type C) can alleviate a fair amount of the perceptual difficulties caused by overlapping unless that one series is largely occluded by another. In fact, the probability of such a situation is rather low. In our example, at each of the 64 sampling positions, the probability of two series coinciding is $p = 1/64$. Considering all 64 positions, the probability of k coincident positions is:

$$Q(k) = \binom{64}{k} p^k (1-p)^{64-k} = \frac{64!}{k!(64-k)!} p^k (1-p)^{64-k}$$

Hence, the probability for two time series coincide at 4 or fewer positions is $\sum_{k=0}^4 Q(k) = 0.997$. We are reasonably certain that gestalt effects can comfortably help separate two time series coinciding at 4 or fewer positions (out of 64).

From Figures 4(d,e), we can see that the situation deteriorates when the amount of multiplexing increases. Most people have found, or would find, that visualizing 10 or more time series in the same plot is not easy. Even if we ignore the information loss ε , multiplexing 10 time series would result in $DSU = 0.039$. The utilization of \mathcal{D} is more efficient, but it is nowhere near the capacity limit.

Some may wonder where the spare capacity of \mathcal{D} comes from for visualization that makes full use of every pixels, e.g., in Figures 3(c,h). Let us consider that Y is the data space encompassing all background heatmaps in Figure 3(h). In comparison with the data space Ω for all possible images that can be displayed on the same display, Y is usually tiny because most images (e.g., portraits) are not probable in that specific context. Hence the entropy $\mathcal{V}(G) = \mathcal{H}(Y)$ is much smaller than $\mathcal{H}(\Omega) = \mathcal{D}$, therefore $DSU \ll 1$.

7. Conclusions

In this paper we proposed a new concept of visual multiplexing. From this work, we can make the following observations. (a) Visual multiplexing is a common phenomenon in visualization. It provides an intrinsic way to access underutilized display bandwidth, while supporting various task needs such as comparative visualization and multivariate visualization. (b) Humans can decode many types of visual multiplexing effortlessly, and this categorization can assist visualization designers in exploring different visual encoding schemes systematically. (c) A priori knowledge can help deploy visual multiplexing more effectively. It is important for visualization designers to understand the users and recognize and utilize such knowledge. Designers may also explore opportunities to create new knowledge, such as a visual language. The theory of visualization remains to be an inspiring and challenging topic. We wish to continue this study by obtaining quantitative measures about multiplexing through controlled user studies.

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