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Color reduction by using a new self-growing and self-organized neural network

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Abstract

A new method for the reduction of the number of colors in a digital image is proposed. The new method is based on the developed of a new neural network classifier that combines the advantages of the Growing Neural Gas (GNG) and the Kohonen Self-Organized Feature Map (SOFM) neural networks. We call the new neural network: Self-Growing and Self-Organized Neural Gas (SGONG). Its main advantage is that it defines the number of the created neurons and their topology in an automatic way. As a consecutive, isolated color classes, which may correspond to significant image details, can be obtained. The SGONG is fed by the color components and additional spatial features. To speed up the entire algorithm and to reduce memory requirements, a fractal scanning sub-sampling technique is used. The method is applicable to any type of color images and it can accommodate any type of color space.

Categories and Subject Descriptors (according to ACM CCS): I.4.1 [IMAGE PROCESSING AND COMPUTER VISION]: Digitization and Image Capture

1 Introduction

The reduction of the number of colors in digital image is an active research area. True type color images consist of more than 16 million of different colors. The image color reduction is an important task for presentation, transmission, segmentation and compression of color images. The proposed method can be considered as a Color Quantization (CQ) technique. The goal of the CQ techniques is to reduce the number of colors in an image in a way that minimizes the perceived difference between the original and the quantized image. Several techniques have been proposed for CQ which can be classified in the following three major categories. Firstly, there is a class of techniques that are based on splitting algorithms. According to these approaches, the color space is divided into disjointed regions by consecutive splitting up the color space. The methods of median-cut [HEC82] and variance-based algorithm [WAN90] belong to this category. The method of Octree [ASH94] is based on splitting the color space to smaller cubes. An optimized splitting technique is proposed by Wu [WU92] who utilizes the principal component analysis to split optimally the original color space. The second major class of CQ techniques is based on cluster analysis. Techniques in this category attempt to find the optimal palette by using vector classifiers. In this category belong methods like ACR [PAP02], FOSART [BAR99], Fuzzy ART [CAR91] and FCM [BEZ81]. As it is noticed by Buhmann et al. [BUH98], one of the basic disadvantages of the most classic CQ approaches is the fact that they neglect spatial, i.e. contextual, information. In order to overcome this disadvantage, the color reduction problem must be considered as a clustering problem with the input vectors describing not only the color information but also extra spatial features derived from the neighboring area of each pixel [PAP00] or [PAP99]. Artificial neural networks are very efficient approaches to create powerful vector classifiers and to solve clustering problems. A well-known unsupervised neural network classifier is the Kohonen SOFM [KOH90]. This network consists of two separate layers of fully connected neurons, i.e. the input and the output layer. Although, the Kohonen SOFM performs topology preserving mapping, there is a major drawback: the dimensions of the input space and the output lattice of neurons are not always identical and, consequently, the structure of the input data is not always preserved in the output layer.

Several implementations of the Kohonen SOFM have been proposed for color reduction. Dekker [DEK94]

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proposes a color reduction technique which is based on a Kohonen SOFM classifier. According to this approach, equal sized classes are produced. Papamarkos and Atsalakis propose a new technique according to which a Kohonen SOFM neural network is fed not only with the image gray-scale values but also with local spatial features extracted from the neighboring of the sampling pixels [PAP00] or [PAP99]. An extension to the methods mentioned above, is the Adaptive Color Reduction (ACR) technique [PAP02]. This technique, by applying a tree-driven splitting and merging strategy, decomposes the initial image into a number of color planes. A two-stage color segmentation methodology based on a Kohonen SOFM network is also proposed by Huang et al. [HUA02]. Ashikur et al. [RAH03] propose a CQ technique by combining the SOFM with a supervised counter propagation network. With the exception of the ACR algorithm, all the techniques mentioned above have the same drawback: the final number of colors should be predefined.

The proposed color reduction technique uses a new Self-Growing and Self-Organized Neural Gas (SGONG) network. We develop this neural classifier in order to combines the growing mechanism of the GNG algorithm [FRI95] and the learning scheme of the Kohonen SOFM. Specifically, the learning rate and the influence of neighboring neurons are monotonically decreased with the time. Furthermore, at the end of each epoch, three criteria are applied that improve the growing and the convergence of the network. These criteria define when a neuron must be removed or added to the output lattice. The proposed neural network is faster than the Kohonen SOFM. In contrast with the GNG classifier, a local counter is defined for each neuron that influences the learning rate of this neuron and the strength of its connections with the neighboring neurons. In addition, the local counters are also used to specify the convergence of the proposed neural network. The main advantage of the SGONG classifier is its ability to determine, in an adaptive way, the final number of neurons by using three suitable criteria. Thus, in the color reduction problem, the proper number of the image's dominant colors can be adaptively determined. Concluding, in this paper

A new self-growing and self-organized neural network is introduced. This SGONG neural network has the following characteristics:

- Is faster than the Kohonen SOFM.
- The dimensions of the input space and the output lattice of neurons are always identical. Thus, the structure of neurons in the output layer approaches the structure of the input data.
- Criteria are used to ensure fast converge of the neural network and detection of isolated classes.
- The proper number of the image dominant colors is automatically obtained and it is equal to the number of neurons in the output layer.
- Isolated color classes can be detected.
- Except for color components, the SGONG neural network can also be fed by additional local spatial features.
- The color reduction results obtained are better than previous reported techniques.

The proposed color reduction technique was tested by using a variety of images and the results are compared with other similar techniques. The implementation of the proposed technique can be found in http://ipml.ee.duth.gr/~papamark/.

2 The proposed color reduction technique

To simplify our approach, let us consider a digital image of n pixels and the corresponding data set X, consisting of n input vectors (feature vectors) X_k:

$$X_k = \begin{bmatrix} x_{k,1} & x_{k,2} & \dots & x_{k,D} \end{bmatrix}^T, \; k = 1,\dots,n \qquad (1)$$

where $X_k \in \Re^D$, with D the dimensionality of the input space. Each input vector X_k consists of the pixel's color components and additional spatial features extracted from a neighborhood region of this pixel. These feature vectors are the inputs to the SGONG network classifier. In color images, using for example the RGB color space ($n_c = 3$), the elements $x_{k,d} \in [0,255]$, $d = 1,\dots,n_c$ express the intensity values of red, green and blue color components of the k pixel. Apart from the RGB color space, which is not perceptually uniform, more advantageous and perceptually uniform color spaces like CIE-L*a*b* or CIE-L*u*v*, can be used.

The neighborhood region of each pixel can be defined using a mask of 3×3 or 5×5 dimensions, where the central mask's element expresses the pixel's position. Depending on the spatial features used, the color of each pixel is associated with the spatial color characteristics of the neighboring pixels. The usually used local features are derived from edge extraction, smoothing, noise reduction masks, min and max values, etc.

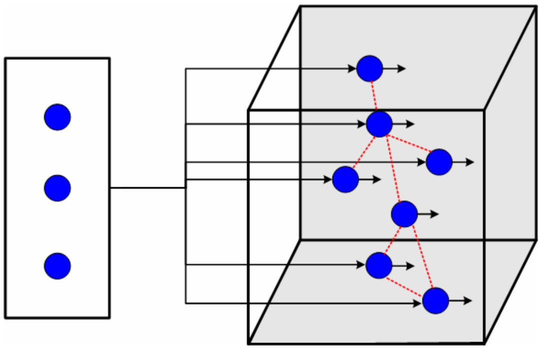

Figure 1: The form of the GNG and SGONG neural networks.

The proposed technique can be applied to color images without any sub-sampling. However, in the case of large-size images and in order to achieve reduction of the computational time and memory size requirements, it is preferable to have a sub-sampling version of the original image. We choose to use a fractal scanning process, based on the well-known Hilbert's space filling curve [PAP00] or [PAP99]. After training, the final color palette is obtained. Depending on the initial settings, the SGONG network converges to c neurons. In other words, c weight vectors, which express the position of the created neurons in the output space and the connections between neurons, are obtained. Finally, the quantized image is constructed by replacing the initial colors with the closer colors of the created palette.

3. The SGONG neural network

As depicted in Figure 1, the proposed SGONG network consists of two separated layers of fully connected neurons, the input layer and the output mapping layer. In contrast with the Kohonen SOFM, the space of the mapping layer has always the same dimensionality with the input space, and also, the created neurons take their position in order the structure of neurons to approximate the structure of the input training vectors. In other words, the topology of the created neurons always approximates the topology of the training vectors. All vectors of the training data set X' are circularly used for the training of the SGONG network. In contrast with the GNG network, where a new neuron is always inserted at the end of each epoch, in the proposed algorithm three new criteria are applied that control the growing of the output lattice of neurons. In summary, we introduce the following three criteria:

- A neuron is removed if, for a predefined number of consecutive epochs, none of the input vectors has been classified to the corresponding class.
- A new neuron is added, close to the neuron with the maximum contribution in the total quantization error if the average distance of the training vectors, classified to the corresponding class, from their neighboring classes is greater from a threshold value.
- All neurons are sorted according to their importance. In the less important class, the classified vectors have the smallest average distance from their neighboring classes. This class is removed if its importance is below a threshold value.

In accordance with the Kohonen SOFM, the proposed neural network uses the same equations for the adaptation of neuron's position and utilizes a cooling scheme for the learning rates used for weights readjustments. In contrast with the Kohonen SOFM, the learning rates are locally defined in such a way that the elimination of a neuron to be reversely proportional to the number of vectors that are classified in the corresponding class. This removes the problem of increased neuron's plasticity that networks with constant learning rates, like GNG, often suffer from. When the learning rates are constant the weights of even well-trained neurons continue to move slightly around the optimum solution and the network seems not to converge. The proposed strategy makes the convergence faster, as the weights of all neurons are stabilized and the network is forced to converge, when a predefined number of vectors have been classified to each neuron.

The adjacency relations between neurons are described by lateral connections between them. The Competitive Hebbian Rule (CHR) [KOH90] is used to dynamically create or remove the lateral connections during the training procedure. This approach improves the data clustering capabilities of the SGONG network, in comparison with the Kohonen SOFM, where the lateral connections are fixed and predefined. Taking into account that the new neurons are inserted in order to support these with the highest error, and that the neuron's lateral connections perfectly describe the topology of the input vectors, we conclude that the proposed technique, during the training procedure, always gives a good description of the data structure. In addition, as the new neurons are inserted near these with the maximum accumulated error across a single epoch, the proposed technique is robust to noisy data. The length of each epoch determines the robustness to noisy data. On the other hand, the convergence is not based on optimizing any model of the process or its data, as the proposed neural network share almost the same weight-update scheme with the Kohonen SOFM neural network.

3.1 The training steps of the SGONG network

The training procedure for the SGONG neural classifier starts by considering first two output neurons ($c = 2$). The local counters N_i , $i = 1,2$ of created neurons are set to zero. The initial positions of the created output neurons, that is, the initial values for the weight vectors W_i , $i = 1,2$ are initialized by randomly selecting two different vectors from the input space. All the vectors of the training data set X' are circularly used for the training of the SGONG network.

The training steps of the SGONG are as follows:

Step 1 At the beginning of each epoch the accumulated errors $AE_i^{(1)}$, $AE_i^{(2)}$, $\forall i \in [1,c]$ are set to zero. The variable $AE_i^{(1)}$ expresses, at the end of each epoch, the quantity of the total quantization error that corresponds to $Neuron_i$, while the variable $AE_i^{(2)}$, represents the increment of the total quantization error that we would have if the $Neuron_i$ was removed.

Step 2 For a given input vector X_k , the first and the second winner neurons $Neuron_{w1}$, $Neuron_{w2}$ are obtained:

for $Neuron_{w1}$:

$$\|X_k - W_{w1}\| \le \|X_k - W_i\|, \ \forall i \in [1,c] \tag{2}$$

for $Neuron_{w2}$:

$$\|X_k - W_{w2}\| \le \|X_k - W_i\|, \ \forall i \in [1,c] \text{ and } i \ne w1 \tag{3}$$

Step 3 The local variables $AE_{w1}^{(1)}$ and $AE_{w1}^{(2)}$ change their values according to the relations:

$$AE_{w1}^{(1)} = AE_{w1}^{(1)} + \|X_k - W_{w1}\| \tag{4}$$

$$AE_{w1}^{(2)} = AE_{w1}^{(2)} + \|X_k - W_{w2}\| \tag{5}$$

$$N_{w1} = N_{w1} + 1 \tag{6}$$

Step 4 If $N_{w1} \le N_{idle}$ then the local learning rates $\varepsilon 1_{w1}$ and $\varepsilon 2_{w1}$ change their values according to equations (7), (8) and (9). Otherwise, the local learning rates have the constant values $\varepsilon 1_{w1} = \varepsilon 1_{\min}$ and $\varepsilon 2_{w1} = 0$.

$$\varepsilon 2_{w1} = \varepsilon 1_{w1} / r_{w1} \tag{7}$$

$$\varepsilon 1_{w1} = \varepsilon 1_{\max} + \varepsilon 1_{\min} - \varepsilon 1_{\min} \cdot \left(\frac{\varepsilon 1_{\max}}{\varepsilon 1_{\min}} \right)^{\frac{N_{w1}}{N_{idle}}} \quad (8)$$

$$r_{w1} = r_{\max} + 1 - r_{\max} \cdot \left(\frac{1}{r_{\max}} \right)^{\frac{N_{w1}}{N_{idle}}} \quad (9)$$

The learning rate $\varepsilon 1_i$ is applied to the weights of $Neuron_i$ if this is the winner neuron ($w1 = i$), while $\varepsilon 2_i$ is applied to the weights of $Neuron_i$ if this belongs to the neighborhood domain of the winner neuron ($i \in nei(w1)$). The learning rate $\varepsilon 2_i$ is used in order to have soft competitive effects between the output neurons. That is, for each output neuron, it is necessary that the influence from its neighboring neurons to be gradually reduced from a maximum to a minimum value. The values of the learning rates $\varepsilon 1_i$ and $\varepsilon 2_i$ are not constant but they are reduced according to the local counter N_i . Doing this, the potential ability of moving of neuron i toward an input vector (plasticity) is reduced with time. Both learning rates change their values from maximum to minimum in a period, which is defined by the N_{idle} parameter. The variable r_{wi} initially takes its minimum value $r_{\min} = 1$ and in a period, defined by the N_{idle} parameter, reaches its maximum value r_{max} .

Step 5 In accordance with the Kohonen SOFM, the weight vector of the winner neuron $Neuron_{w1}$ and the weight vectors of its neighboring neurons $Neuron_m$, $m \in nei(w1)$, are adapted according to the following relations:

$$W_{w1} = W_{w1} + \varepsilon 1_{w1} \cdot \left(X_k - W_{w1} \right) \quad (10)$$

$$W_m = W_m + \varepsilon 2_m \cdot \left(X_k - W_m \right), \ \forall m \in nei(w1) \quad (11)$$

Step 6 With regard to generation of lateral connections, SGONG employs the following strategy. The CHR is applied in order to create or remove connections between neurons. As soon as the neurons $Neuron_{w1}$ and $Neuron_{w2}$ are detected, the connection between them is created or is refreshed. That is

$$s_{w1,w2} = 0 \quad (12)$$

With the purpose of removing of superfluous lateral connections, the age of all connections emanating from $Neuron_{w1}$, except the connection with $Neuron_{w2}$, is increased by one:

$$s_{w1,m} = s_{w1,m} + 1, \ \forall m \in nei(w1), \text{ with } m \neq w2 \quad (13)$$

Step 7 At the end of each epoch it is examined if all neurons are in *idle state*, or equivalently, if all the local counters N_i , $\forall i \in [1,c]$ are greater than the predefined value N_{idle} and the neurons are well trained. In this case, the training procedure stops, and the convergence of SGONG network is assumed. The number of input vectors needed for a neuron to reach the "*idle state*" influences the convergence speed of the proposed technique. If the training procedure continues, the lateral connections between neurons with age greater than the maximum value α are removed. Due to dynamic generation or removal of lateral connections, the neighborhood domain of each neuron changes with time in order to include neurons that are topologically adjacent.

Step **8** At the end of each epoch, three criteria that modify the number of the output neurons and make the proposed neural network to become self-growing are applied. These criteria are applied in the following order:

- remove the inactive neurons,
- add new neurons,
- and finally, remove the non important neurons.

In order to make faster the network convergence it can be defied not to apply the above criteria when the total number of epochs is above a predefined value. This has as a result the rapid passing of all neurons to the "*idle state*" and therefore, the finalizing of the training procedure.

4. Experiments

The proposed method and the CQ methods that are based on the Kohonen SOFM, GNG, and FCM classifiers were implemented in software, called "ImageQuant", and can be downloaded and tested from the site http://ipml.ee.duth.gr/~papamark/ . In this, paper, due to the space limitation, we give only the following two experiments.

Experiment 1

This experiment demonstrates the ability of the proposed SGONG neural classifier to define, in an adaptive way, the number of created neurons, i.e. the number of final colors, according to the structure of the input data. The test image is the standard Lena image shown in Figure 2 which has 148279 unique colors. In order to find the number of the image dominant colors, we map the image colors, using the PCA neural network, to the first two principal components. Then, as it is depicted in Figure 3, 16 dominant colors are obtained. Thus the proposed SGONG is applied for only 16 colors and the image obtained is depicted in Figure 4.

Figure 2: The original image.

The proposed CQ technique is applied not only for 16 colors but also for 4, 8 and 32 colors in the RGB color space. For comparison reasons, except of the proposed technique, we have applied to the same problems the CQ techniques based on Kohonen SOFM, the GNG, and the Fuzzy C-Means.

Table 1 Comparative results

	SGONG	SOFM	GNG	FCM	MV	MC	Dekker	Wu
8 Colors								
MSE	400,75	400,75	426,82	401,08	507,01	710,55	503,18	522,08
ADC	20,36	20,36	20,73	20,37	22,16	26,10	21,92	22,46
SNR	50,05	50,05	49,42	50,04	47,70	44,32	47,77	47,40
PSNR	59,08	59,08	58,45	59,08	56,73	53,36	56,81	56,44
16 Colors								
MSE	209,77	218,07	215,68	211,99	272,90	402,32	291,02	282,42
ADC	14,94	15,07	15,10	14,99	16,55	19,57	17,04	16,81
SNR	56,52	56,13	56,24	56,42	53,89	50,01	53,25	53,55
PSNR	65,56	65,17	65,28	65,45	62,93	59,04	62,28	62,58
32 Colors								
MSE	118,36	121,37	123,07	119,14	158,80	252,67	153,60	172,12
ADC	11,27	11,38	11,43	11,33	12,74	15,37	12,57	13,18
SNR	62,24	61,99	61,85	62,18	59,31	54,66	59,64	58,50
PSNR	71,28	71,03	70,89	71,21	68,34	63,70	68,67	67,53

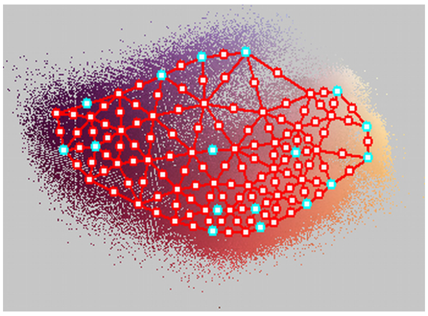

Figure 3: Estimation of the number of dominant colors.

Figure 4: Final image with only 16 dominant colors.

All the above techniques utilize a representative data set for their training procedure. Other applied techniques are the Minimum Variance (MV), the Octree, the Median Cut, the Dekker, and finally, the Wu. The representative training data set is constructed using the fractal filling curve of Hilbert. In order to have comparative results, the color reduction techniques that are based on the Kohonen SOFM, the GNG, the FCM classifiers and the proposed technique uses exactly the same samples in the same order.

Tables 1 gives the comparative results obtained using as quality measures the mean squared error (MSE), the average distortion per color (ADC), SNR and PSNR. As it can be observed in all cases the proposed technique gives better quantization results.

Experiment 2

In many cases, it is important to proceed in color reduction by taking into account the color human perception. This experiment demonstrates the improvement of color discrimination, simply using the perceptually uniform color spaces CIE-L*a*b* and CIE-L*u*v. In this experiment, the initial image, which is depicted in Figure 5(a), is quantized using the SGONG network with the same settings in all cases. Even if this image has 33784 unique colors, the human vision system perceives the colors: white, black, gray, red, green and blue as the image dominant colors. Therefore, in the specific experiment, the maximum number of created neurons is c = 6. Figs. 5(b), (c) and (d) depict the resultant quantized images (with only 6 colors) after using the RGB, CIE-L*a*b* and CIE-L*u*v* color spaces, respectively. As it can be observed, in contrast with the CIE-L*a*b* and CIE-L*u*v*, the RGB color space fails to detect the image's principal colors. For comparison reasons, Figs. 5(e) and (f) depict the quantization results obtained, for 6 colors, by the methods of median-cut [HEC82] and Wu's [WU92], respectively. As it can be observed, these methods fail to obtain the proper dominant colors.

Experiment 3

In this experiment, in order to have perceptually comparative results, the image of Fig. 6(a) is quantized to only 8 colors using the SGONG, the GNG, the Kohonen SOFM, and finally, the FCM classifier. The proposed neural network is applied for $m_2 = 0.1$ and $m_2 = 0.3$. The quantization results obtained are depicted in Fig. 6. Despite the fact that the FCM algorithm, for this image, gives the best comparative statistical results, it fails to detect the obvious image dominant colors. On the other hand, the proposed method, with $m_2 = 0.3$, gives perceptually the best color quantization results. Giving the greater value to parameter m_2, increases the possibility to remove the neurons with the smallest distance with its neighboring neurons, while the isolated classes are retained. Doing this, the final colors, are perceptually best match to the image dominant colors, even if the RGB color space is used.

5. Conclusions

This paper proposes a new CQ technique which is based on a new neural network classifier (SGONG). The SGONG network classifier is suitable for CQ applications. Each pixel is considered as a multidimensional vector which contains the color components and additional spatial characteristics derived from the neighborhood domain of each pixel. The main advantage of the SGONG network is that it controls the number of created neurons and their topology in an automatic way. This advanced feature makes feasible the automatic extraction of the image's dominant colors. The convergence speed of SGONG classifier is comparable to the convergence speed of the GNG classifier, while the stability of SGONG classifier is comparable to the stability of Kohonen SOFM classifier. In order to speed up the entire algorithm, a fractal sub-sampling procedure based on the Hilbert's space filling curve is applied to initial image, taking samples only from the fractal peaks and their neighboring pixels. The proposed CQ technique has been extensively tested and compared to other similar techniques.

Figure 5: (a) Original image.

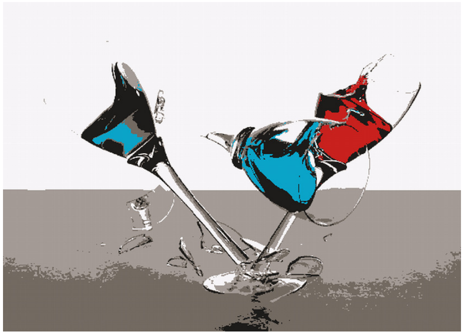

Figure 5: (b) CQ results in the RGB color space.

Figure 5: (c) CQ results in the L*a*b* color space.

Figure 5: (d) CQ results in the L*u*v* color space.

Figure 5: (e) Median-cut result

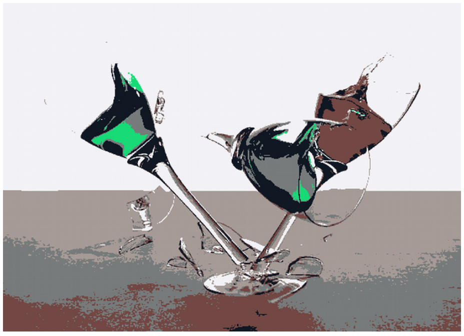

Figure 5: (f) Application of the Wu's method

Figure 5: Quantization results for Experiment 2.

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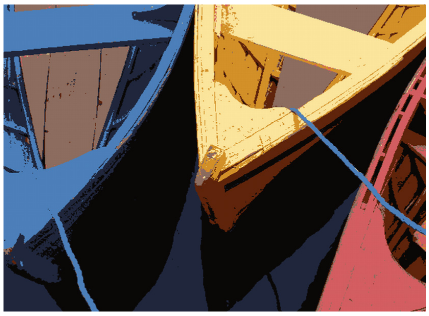
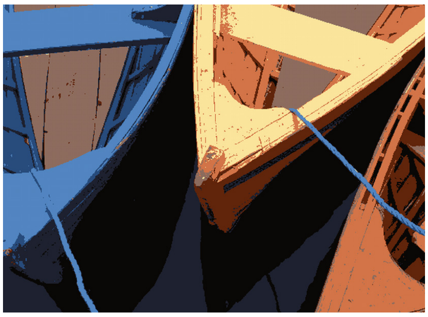
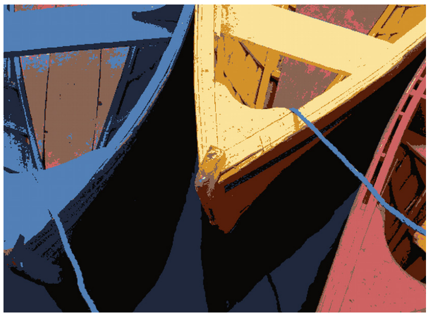
(a) 101098 unique colors



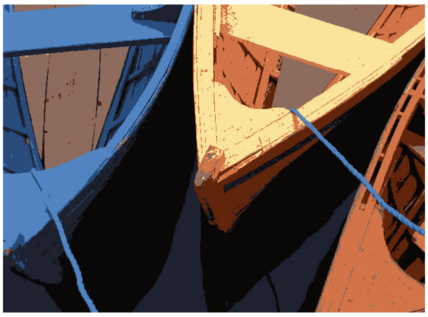
(b) SGONG with $m_2 = 0.3$



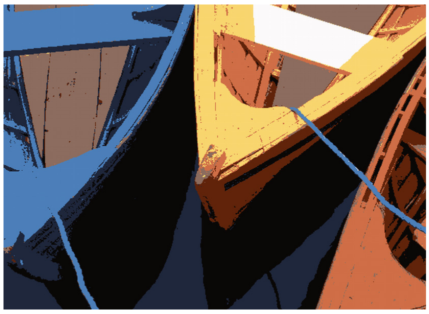
(c) SGONG with $m_2 = 0.1$



(d) GNG



(e) Kohonen SOFM



(f) FCM

Figure 6: The original image of Fig. 6(a) is quantized to only 8 colors using the SGONG, the GNG, the Kohonen SOFM, and finally, the FCM classifier.

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