Visualization of Global Flow Structures Using Multiple Levels of Topology

Wim de Leeuw and Robert van Liere

Center for Mathematics and Computer Science CWI,
P.O. Box 94097, 1090 GB Amsterdam, Netherlands
{wimc,robertl}@cwi.nl

Abstract. The technique for visualizing topological information in fluid flows is well known. However, when the technique is used in complex and information rich data sets, the result will be a cluttered image which is difficult to interpet. This paper presents a technique for the visualization of multi-level topology in flow data sets. It provides the user with a mechanism to visualize the topology without excessive cluttering while maintaining the global structure of the flow.
Keywords: multi-level visualization techniques, flow visualization, direct numerical simulation.

1 Introduction

The importance of data visualization is clearly recognized in large scale scientific computing. However, the demands imposed by modern computational fluid dynamics (CFD) simulations severely test the limits of today's visualization techniques. This trend will continue as solutions to more complex problems are desired.

An important aspect of a flow field is its topology [1]. A technique for the visualization of vector field topology in fluid flows was introduced by Helman and Hesselink [2]. It is a technique that extracts and visualizes topological information, and combines simplicity of schematic depictions with the quantitative accuracy of curves computed directly from the data. Visualization of topology is impressive when applied to not too complex flow fields, but in high-resolution turbulent flows problems may arise. As fluid flow computations generate more complex and information rich data sets, the set of computed critical points will become very large. This results in a cluttered image which is difficult to interpet. For example, consider figure 1. The data is a 2D slice of a 3D data set turbulent flow around a square cylinder. A set of 322 critical points has been computed from the data. Small colored icons are used to display the set of critical points: a yellow spiral icon denotes a focus, a blue cross denotes a saddle point, and cyan/magenta disks denote repelling/attracting nodes. Spot noise is used to present the global nature of the flow. Note that most critical points are clustered in regions around the square cylinder. To prevent additional cluttering, streamlines linking critical points have been omitted.

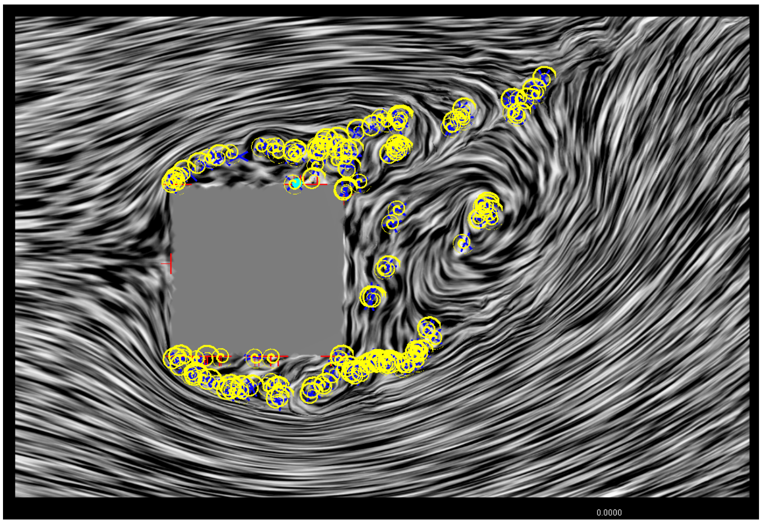

Fig. 1. Topological information of turbulent flow around a square cylinder. Spot noise is used to present the global nature of the flow.

In this paper we present a technique for the visualization of multi-level topology in flow data sets. The multi-level topology technique provides the user with a mechanism to select the set of critical points which define the global flow structure.

The format of this paper is: In the next section we review previous work on vector field topology. In section 3 discusses the multi-level topology technique. Finally, in section 4 we show how the technique has been used to explore a turbulent flow field.

2 Previous work

Vector field topology was introduced by Helman and Hesselink, [2]. It presents essential information by partitioning the flow field in regions using critical points which are linked by streamlines. Critical points are points in the flow where the velocity magnitude is equal to zero. Each critical point is classified based on the behavior of the flow field in the neighborhood of the point. For this classification the velocity gradient tensor is used. The velocity gradient tensor – or Jacobian – is defined as

$$\mathbf{J} = \nabla \boldsymbol{u} = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} \tag{1}$$

in which subscripts denote partial derivatives. The classification is based on the the two complex eigenvalues ($R1 + i\ I1,\ R2 + i\ I2$). Assuming that the the critical point is hyperbolic, i.e. the real part of the eigenvalues is non zero, five different cases are distinguished (see figure 2) :

1. *Saddle point*, the imaginary parts are zero and the real parts have opposite signs; i.e $R1 \ast R2 < 0$ and $I1 , I2 = 0$.
2. *Repelling node*, imaginary parts are zero and the real parts are both positive; i.e $R1 , R2 > 0$ and $I1, I2 = 0$.
3. *Attracting node*, imaginary parts are zero and the real parts are both negative; i.e $R1 , R2 < 0$ and $I1, I2 = 0$.
4. *Repelling focus*, imaginary parts are non zero and the real parts are positive; i.e $R1 , R2 > 0$ and $I1, I2 \neq 0$.
5. *Attracting focus*, imaginary parts are non zero and the real parts are negative; i.e $R1 , R2 > 0$ and $I1, I2 \neq 0$.

If the real part of the eigen values is zero the type of the flow is determined by higher order terms of the approximation of the flow in the neighborhood of the critical point.

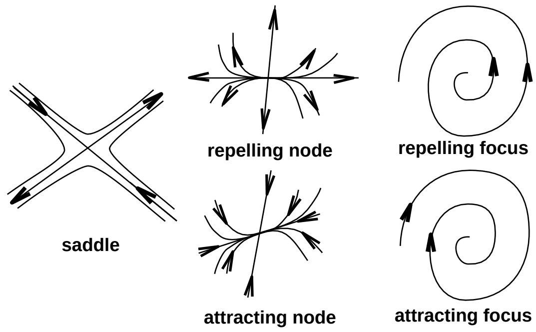

Fig. 2. Five different types of critical points.

Streamlines traced in the direction of the eigenvectors of the velocity gradient tensor will divide the flow field in distinct regions.

Implementation aspects of this technique can be found in [4].

3 Multi-level flow topology

Phenomena in turbulent flow fields are characterized by flow patterns of widely varying spatial scales. In terms of topological information, this means that a large set of critical points result from flow patterns at small spatial scales. However, the global structure of the flow can be described by a limited subset of all critical points. The governing idea of the multi-level flow topology method is that the displayed number of critical points should be limited to characterize only those flow patterns of a certain level of scale, while the other critical points are omitted.

In order to realize this idea, two distinct type of methods can be employed: implicit and explicit methods (see figure 3). Implicit methods are those that filter the input data set to obtain a derived data set to which the original topology algorithm applied. Explicit methods are those which first compute the set of critical points from the input data set and then use a filter to prune this set.

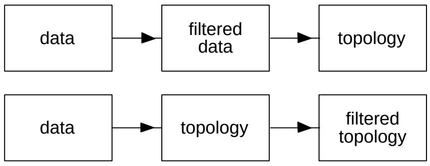

Fig. 3. Implicit (top) vs. explicit (bottom) methods of multi-level topology.

3.1 Implicit methods

In general, filters are used to enhance/suppress patterns in the data. For example, a low pass filter can be used to suppress high-frequency patterns; i.e. those flow patterns at small spatial scales. The idea is that by filtering the data and then doing the critical point analysis on the critical points caused by small disturbences in the flow will be filtered out. Nielson et al. obtained similar results with a method where wavelets where used to approximate the data [5].

For the images in this section, a simple box filter has been used: each data-point is replaced by an average over a small region in the orignal data. The motivation is that the box filter is a simple low-pass filter which will average out small scale patterns, while keeping large scale patterns intact.

Figure 4 illustrates the implicit method using a box filter. The left image shows the set of all 322 critical points of the original data set. The middle image shows the set of 179 critical points of the data set after being filtered with a 2x2 box filter. The right image shows the set of 40 critical points of the data set after being filtered with a 8x8 box filter. Although the implicit method is conceptually easy to understand and may achieve the desired effect in certain cases, a number of drawbacks can be mentioned. Due to filtering, there is no direct relation between critical points in the original and derived set. The position and type of a critical point can change after the filter is applied.

3.2 Explicit methods

Explicit methods use filters which prune the set of all critical points defining the topological structure. This is achieved by using specific knowledge about the characteristics of the flow topology.

For this purpose, a *pair distance filter* has been designed. The motivation of this filter is that an often occurring small disturbance of the flow is caused by

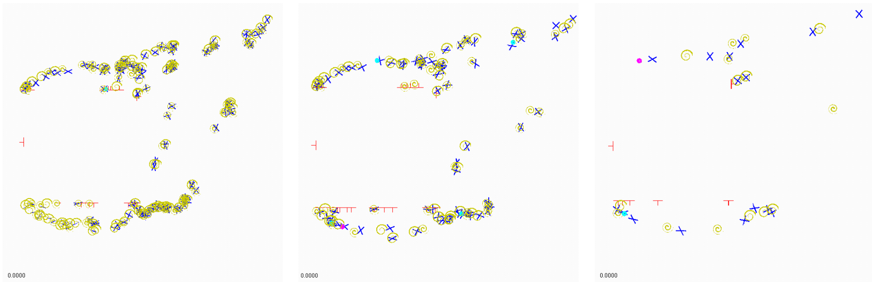

Fig. 4. Three views of the implicit method using a box filter. Left: original data set (322 critical points). Middle: 2x2 box filter (179 critical points). Right: 8x8 box filter (40 critical points).

pairs of critical points. For example, the topological structure of a two dimensional vortex consists of a focus (repelling or attracting) or a center combined with a saddle point (see figure 5). The size of the vortex is determined by the distance between the pair of critical points. Removing the pair from the topology does not influence the global structure of the flow.

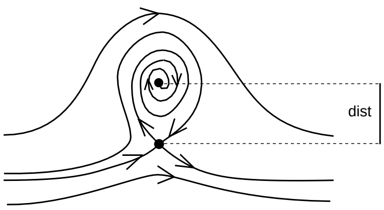

Fig. 5. An example of a critical point pair; a focus and saddle point forming a vortex. The distance defines the spatial scale of the point pair.

The pair distance filter can be implemented as follows: The set of all critical points are located and classified. Using the distance between saddles and attracting/repelling nodes, a pairwise distance matrix is constructed. The pair with the smallest distance is removed from the matrix. This is iteratively continued until a distance threshold is reached.

Figure 6 shows the pair distance filter in practice. The left image shows the set of all 322 critical points of the original data set. The middle image shows the critical points of the data set after being filtered with a distance threshold of $0.0001 * H$, in which H is the height of the set, resulting in a set of 114 critical points. The right image shows the critical points of the data set after being filtered with a distance threshold of $0.001 * H$, resulting in a set of 34 critical points.

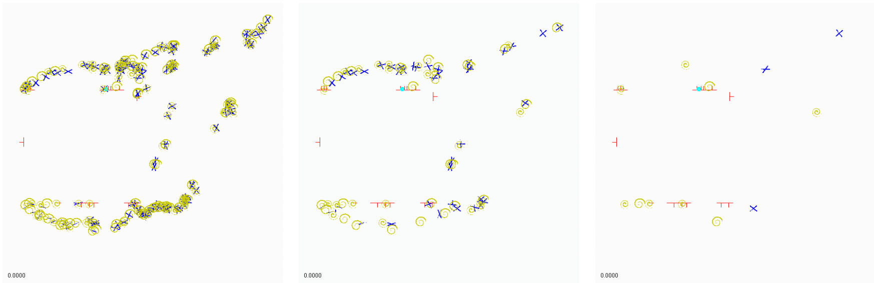

Fig. 6. Three views of the explicit method using the pair filter. Left: original data set (322 critical points). Middle: $0.0001 * H$ distance (114 critical points). Right: $0.001 * H$ filter (34 critical points).

4 Direct Numerical Simulation of Turbulent Flow

4.1 Problem and Data Set

We applied our method to the output direct numerical simulation (DNS) of a turbulent flow generated by A. Veldman and R. Verstappen [6]. DNS is an accurate technique for computing turbulent flow. Flow experts use the resulting visualizations to test hypotheses about flow phenomena and – after a detailed inspection of the animation – as a means to pose new hypotheses. Of particular interest is the detailed visualization of vortex formation and the transition from laminar to turbulent flow.

In this particular problem, a DNS of a turbulent flow around a square cylinder at Re = 22,000 (at zero angle of attack) has been performed. The size of the resulting data set is impressive: the resolution of the rectilinear grid is 314x538x64; the grid was finest near the cylinder. The number of time steps saved on disk are 7500.

4.2 Results

Figure 7 shows a view of the flow. This is the same 2D slice as in figure 1. The pair distance filter is used with a distance threshold of $0.001 * H$, in which H is the height of the data set. Now, streamlines can be drawn without excessive cluttering of the image while, simultaneously, maintaining the the global structure of the flow. Note, for example, the large vortex (consisting of a saddle and attracting node) behind the square cylinder.

Figure 8 shows a zoomed in view of the previous image. The distance threshold for the pair distance filter is adjusted to reflect structures at a smaller scale.

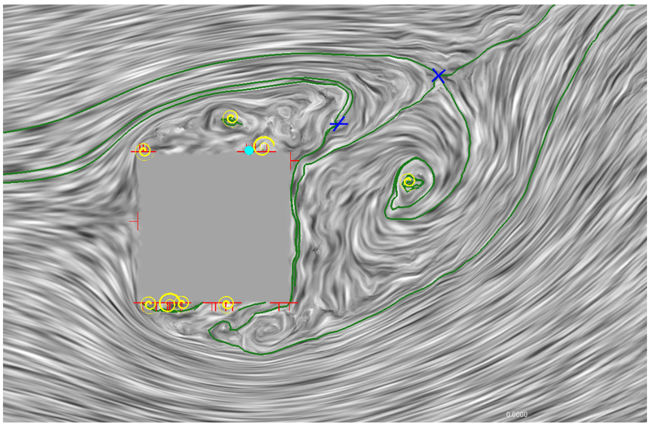

Fig. 7. A view of global flow structure around a square cylinder. The pair distance filter is used with a distance of $0.001 * H$.

4.3 Evaluation

This application clearly benefits from the added value of the multi-level flow topology technique, [7]. The data set contains an abundance of detailed information. Using traditional topology visualization methods on these data sets, excessive cluttering can not be avoided. With the multi-level approach, simplified views of the topology can be obtained without cluttering.

The multi-level topology method has been parallelized, so that interactive rates can be obtained. The interactive multi-level topology method can be used combination with interactive spot noise [8] to realize real-time animation of the time-dependent field and interactive zooming into details one time step.

In the near future DNS can be applied to flows with a Reynolds number in the order of 10^5. The increased size and detailed information in the resulting data sets will require multi-level flow visualization techniques.

5 Conclusion

In this paper, a technique for the visualization of multi-level topology in flow data sets was presented. It provides the user with a mechanism to visualize the topology without excessive cluttering while maintaining the global structure of the flow. Two methods have been introduced. Implicit methods can be used to filter data in order to suppress flow patterns at small spatial scales. Explicit methods can be used to prune a set of critical points, making use of specific knowledge about characteristics of the flow topology. The technique has been succesfully applied to a complex data set resulting from a direct numerical simulation.

In the future, two enhancements to the explicit method are envisioned. First, filtering will take not only distance information between critical points into ac-

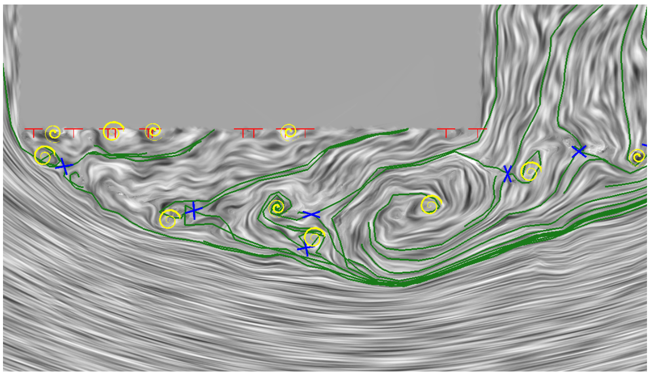

Fig. 8. A zoomed in view of flow topology around a square cylinder. The pair distance filter is used with a distance of $0.0002 * H$.

count, but also topological information about the computed links (i.e. streamlines) between the critical point pairs. Second, temporal information about critical points can be used. Due to flow patterns at small temporal scales, the existance of a critical point may vary over time. By tracking the critical points over time, the lifetime of a point can be determined. The threshold metric can be the minimum lifetime of a critical point.

References

1. V.I. Arnold. *Ordinary Differential Equations.* MIT Press, 1973.
2. J.L. Helman and L. Hesselink. Visualizing vector field topology in fluid flows. *IEEE Computer Graphics and Applications*, 11(3):36–46, May 1991.
3. D. Asimov. Notes on th topology of vector fields and flows. Technical Report Technical Report RNR-93-003, NASA Ames Research Center, 1993.
4. A. Globus, C. Levit, and T. Lasinski. A tool for visualizing the topology of three-dimensional vector fields. In G.M. Nielson and L.J. Rosenblum, editors, *Proceedings Visualization '91*, pages 33–40. IEEE Computer Society Press, Los Alamitos (CA), 1991.
5. Gregory M. Nielson, Il-Hong Jung, and Junwon Sung. Wavelets over curvilinear grids. In David Ebert, Hans Hagen, and Holly Rushmeier, editors, *Proceedings Visualization '98*, pages 313–315. IEEE Computer Society Press, 1998.
6. R.W.C.P. Verstappen and A.E.P. Veldman. Spectro-consistent discretization of navier-stokes: a challenge to RANS and LES. *Journal of Engineering Mathematics*, 34:163–179, 1997.
7. A.E.P. Veldman. Private communication.
8. W.C. de Leeuw and R. van Liere. Divide and conquer spot noise. In *Proceedings Super Computing '97 (http:// scxy.tc.cornell.edu / sc97 / program / TECH / DELEEUW / INDEX.HTM)*, 1997.