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Smoke Surfaces of 4D Biological Dynamical Systems: Supplementary Material

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In this supplementary material, we showcase in detail the three dynamical systems that we discuss also in the paper. Two of these systems arise from biological phenomena (i.e., models related to bipolar disorder [Gol11] and immune response to infections [KJS06]) and the third one is a strange attractor with intricate dynamics [Bou15]. This strange attractor does not relate to any biological model and is included for demonstration purposes only, due to its interesting dynamics.

Implementation. We implemented a stand-alone application in C++ that builds upon OpenGL, making heavy use of compute shaders to calculate vertex normals, curvature, and projection. The computation of these values needs to be performed per vertex, and therefore benefits from parallelization. These values are computed for each frame in an individual pass, when the user changes the view or the unfolding progress. The approach uses one pass for the tesseract and trajectories, one for our transparent surfaces, and a final compositing screen pass. The implementation can be found in our repository[†].

1. The Goldbeter model of bipolar disorder

The Goldbeter model of bipolar disorder [Gol11] represents alternations between manic and depressive states with respect to medications, formalized in the parameters of the 4D model. For certain initial positions, the model exhibits oscillatory behavior between the two states. The system with our parameter choices can be seen in equation 1:

$$\begin{aligned} dx/dt &= \frac{0.16}{(0.16+y^2)}\frac{(2z)}{(1+2z)} - \frac{x}{(1+2x)} \\ dy/dt &= \frac{0.16}{(0.16+x^2)}\frac{(2w)}{(1+2w)} - \frac{y}{(1+2y)} \\ dz/dt &= 0.01(y-z) \\ dv/dt &= 0.01(x-w) \end{aligned} \tag{1}$$

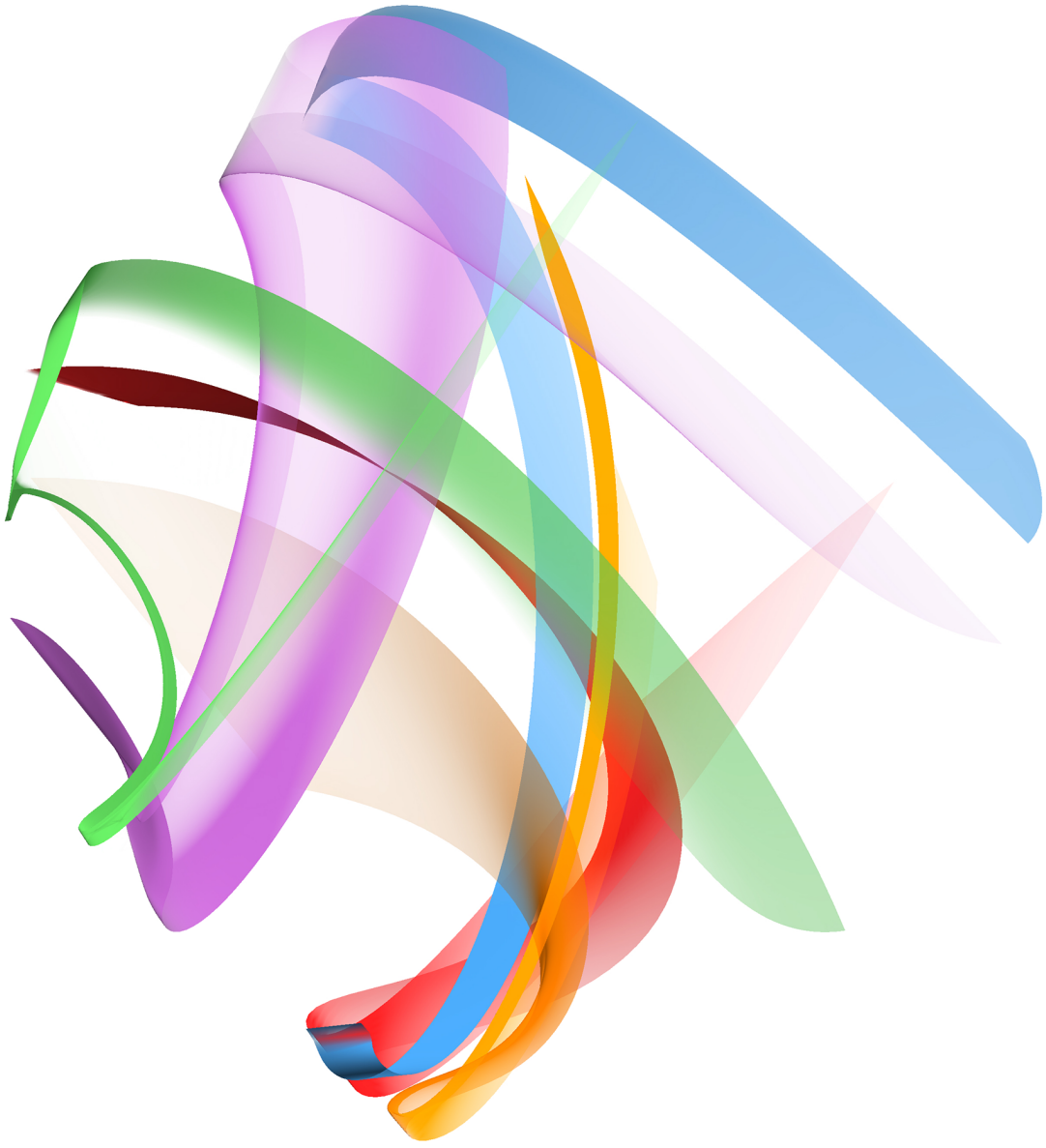
Initial conditions for the generation of the smoke surfaces
Five smoke surfaces (indicated with distinct colors) have been generated with five seeding lines that provide the following initial conditions:

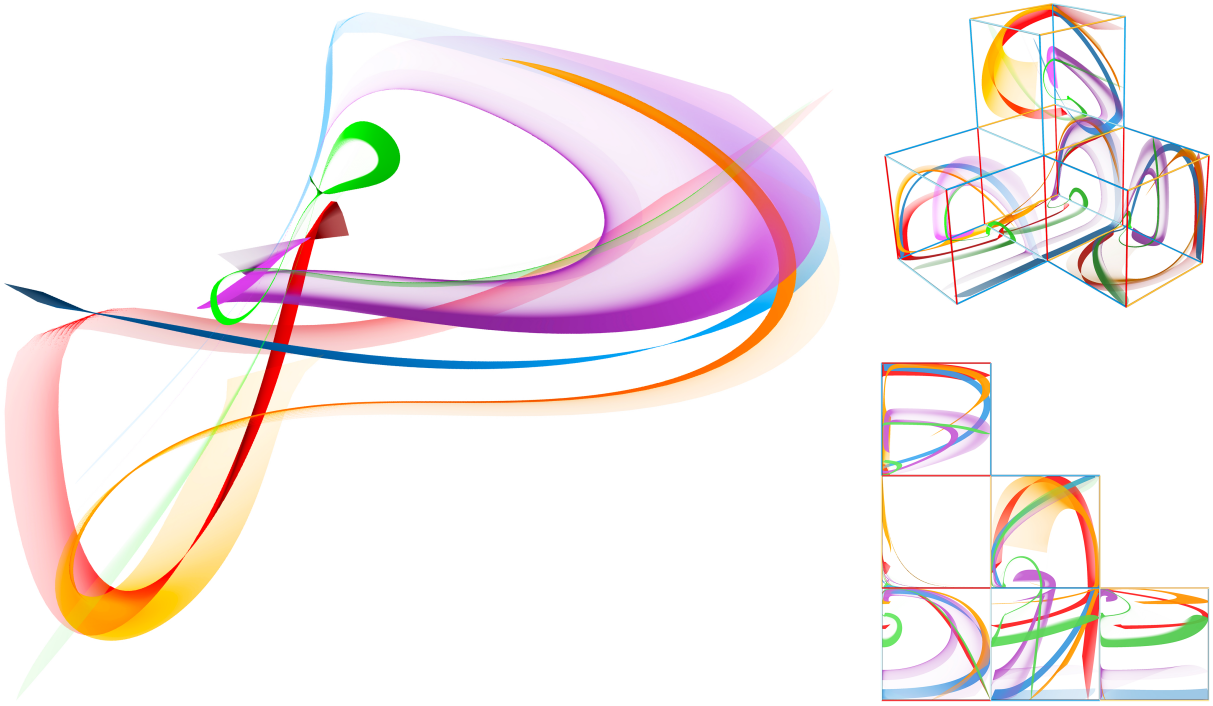
Red: From 0.939, 0.723, 0.512, 0.111 to 0.404, 0.868, 0.579, 0.268
Blue: From 0.116, 0.018, 0.184, 0.760 to 0.050, 0.161, 0.342, 0.835
Orange: From 0.211, 0.381, 0.272, 0.241 to 0.177, 0.487, 0.288, 0.025
Purple: From 0.246, 0.408, 0.31, 0.234 to 0.064, 0.396, 0.268, 0.434
Green: From 0.275, 0.061, 0.419, 0.414 to 0.164, 0.125, 0.469, 0.373

Integration duration: 100.0
Step size: 1.0

[†] https://github.com/MarwinSc/ManyLands-SmokeSurfaces

The following renderings result from various rotations in 4D space:





2. The NF-κB Pathway

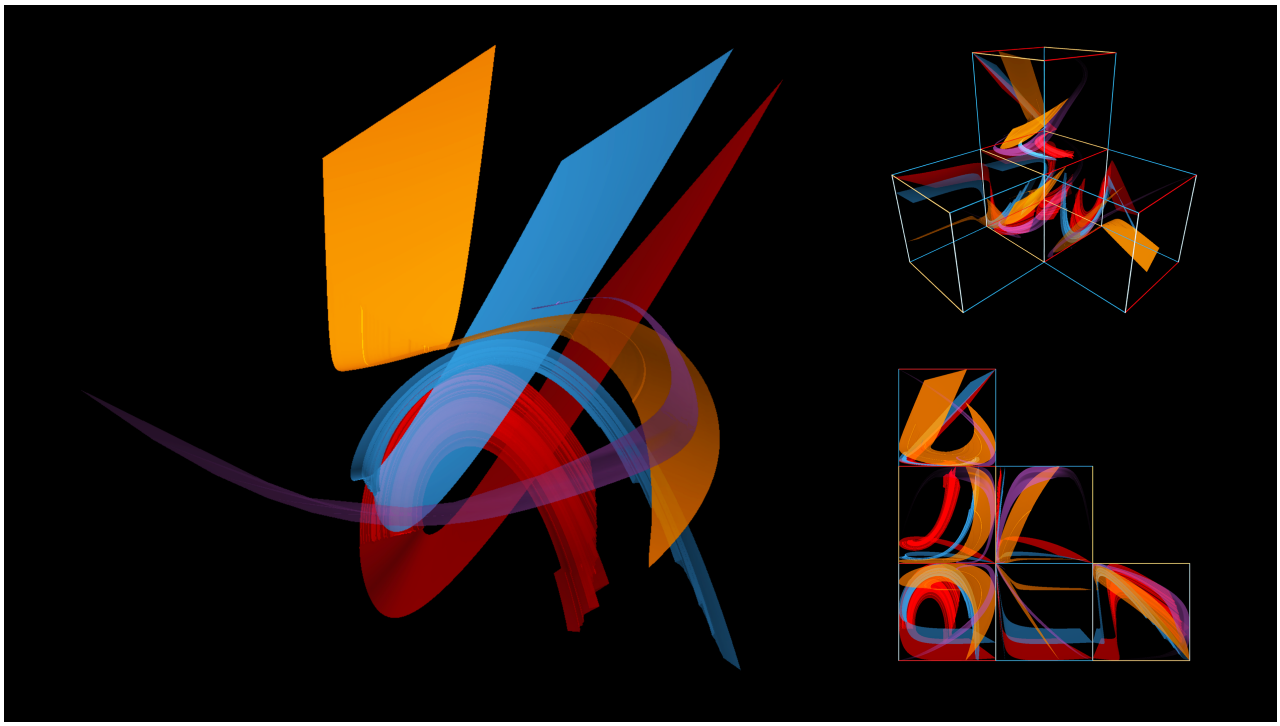
The NF-κB Pathway [KJS06] plays a key role in regulating our immune responses to infections. This system also exhibits oscillatory behavior for certain initial conditions. The system with our parameter choices can be seen in equation 2:

$$
\begin{aligned}
&kNin = 5.4\\
&kIin = 0.018\\
&kIout = 0.012\\
&kNIout = 0.83\\
&kt = 1.03\\
&kt1 = 0.24\\
&kf = 30\\
&kfn = 30\\
&kb = 0.03\\
&kbn = 0.03\\
&alpha = 1.05 * 0.5\\
&gamma = 0.017\\
&k3 = 0.00001\\
&k1 = 0.00001
\end{aligned}
\tag{2}
$$

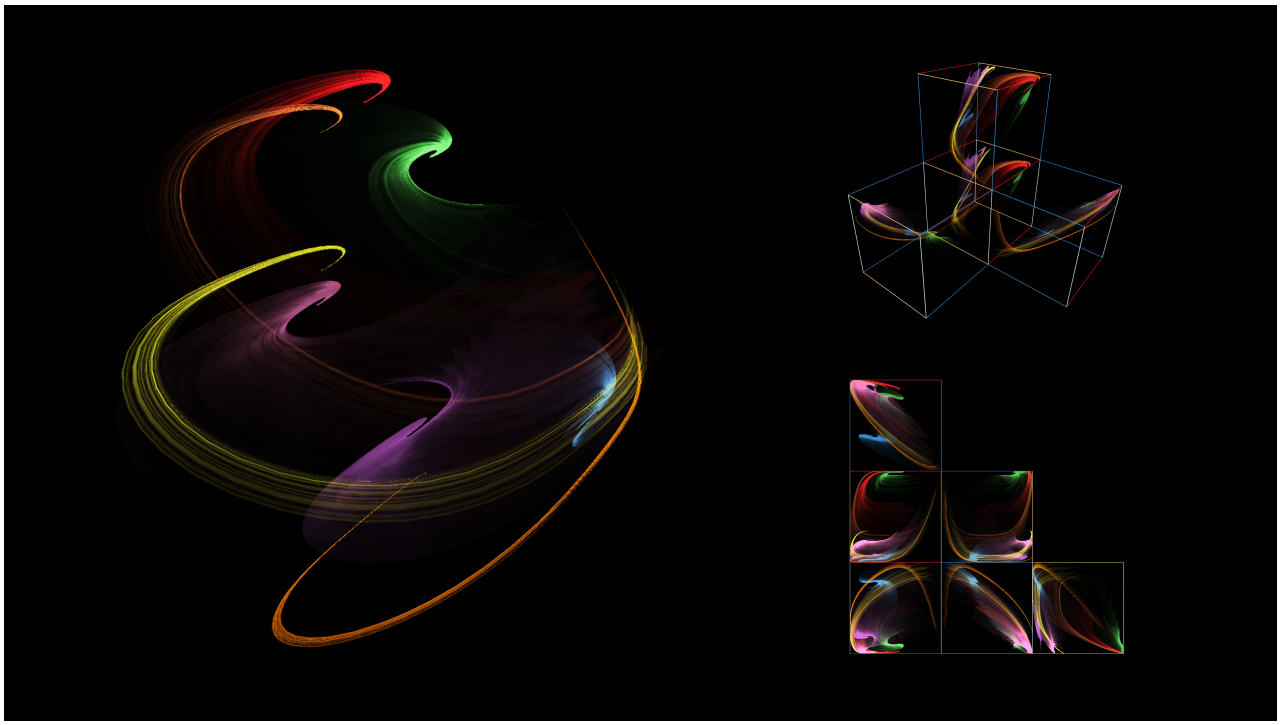
$$
\begin{aligned}
dx/dt &= \frac{kNin * k3 * (1-x)}{k3+z} - \frac{kNIout * w * x}{k1+x}\\
dy/dt &= kt * x * x * x - gamma * y\\
dz/dt &= kt1 * y - \frac{alpha * (1-x) * z}{k3+z} - kIin * z + \frac{kIout * w * k1}{k1+x}\\
dv/dt &= kIin * z - \frac{kIout * w * k1}{k1+x} - \frac{kNIout * w * x}{k1+x}
\end{aligned}
$$

The following images have been generated with different settings (different seeding lines and different parameters for the generation and rendering of smoke surfaces):

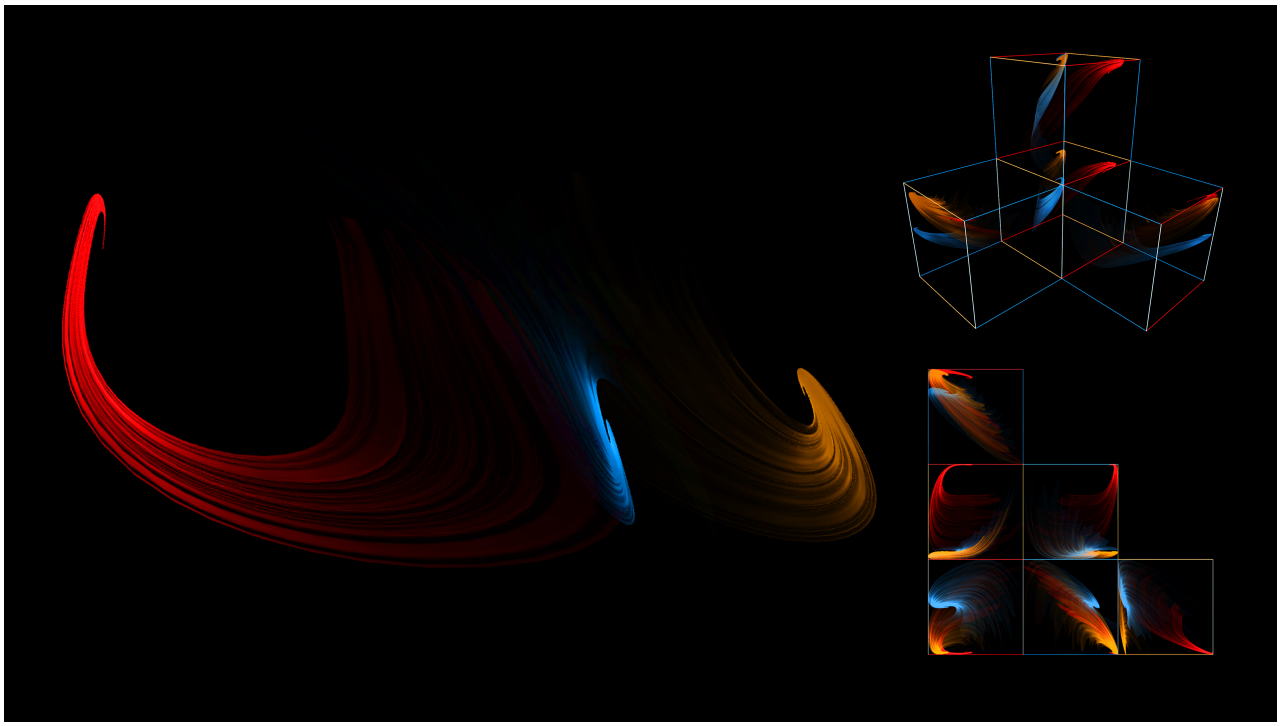
Tesseract (left), and its 3D and 2D unfoldings (right) for 4 initial conditions:



Tesseract (left), and its 3D and 2D unfoldings (right) for 7 initial conditions:



Tesseract (left), and its 3D and 2D unfoldings (right) for 3 initial conditions:



3. Bouali attractor

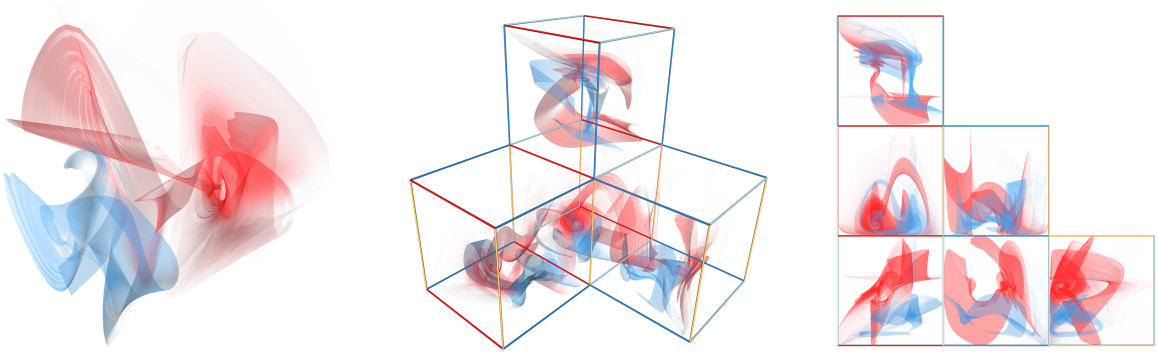
A Bouali attractor is a 4D strange attractor where the equation system is derived from the 2D Lotka-Volterra oscillator [Bou15]. The system with our parameter choices can be seen in equation 3:

$$\begin{aligned}
&\alpha = 1.0\\
&\beta = 0.7\\
&\gamma = -0.1\\
&\kappa = 1.0\\
&\eta = -0.2
\end{aligned} \tag{3}$$

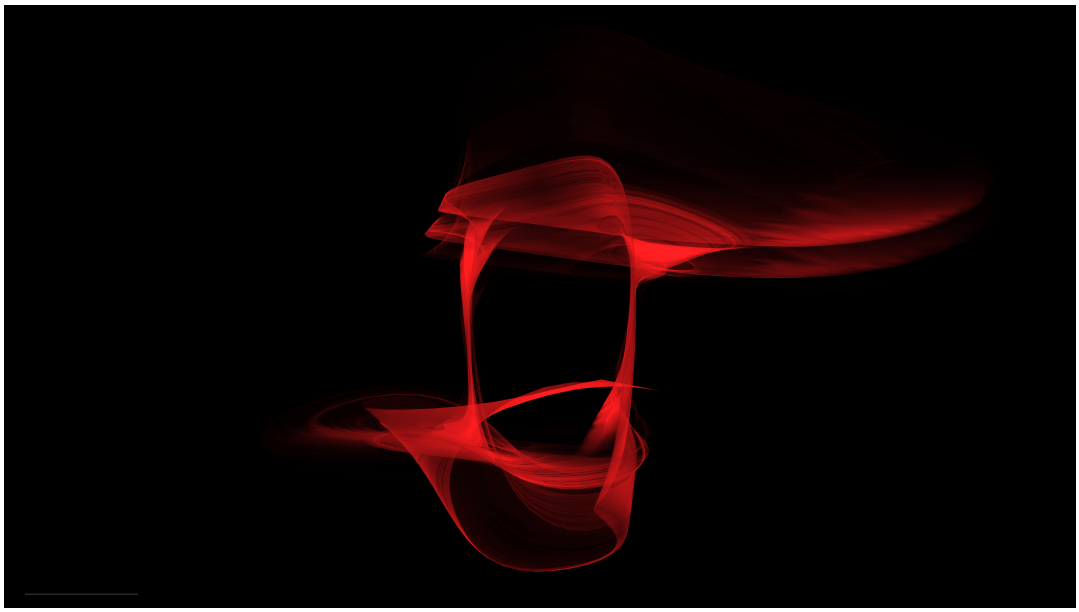
$$\begin{aligned}
dx/dt &= x(1-y) + \alpha * z\\
dy/dt &= \beta * (x^2 - 1)y\\
dz/dt &= \gamma * x + (1-y)z + \kappa * v\\
dv/dt &= \eta * xy
\end{aligned}$$

The following images have been generated with different settings (different seeding lines and different parameters for the generation and rendering of smoke surfaces):

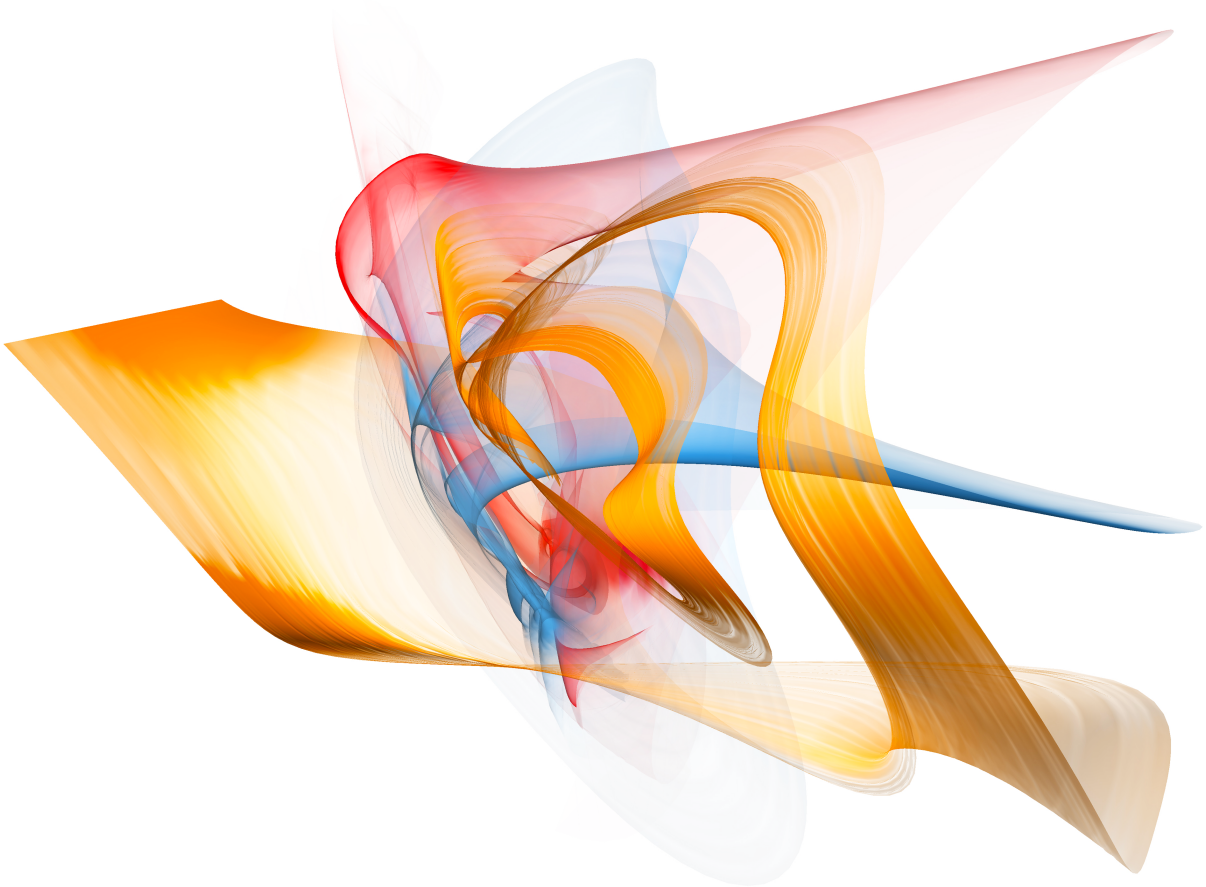
Tesseract (left), and its 3D (middle) and 2D unfoldings (right) for 2 initial conditions:



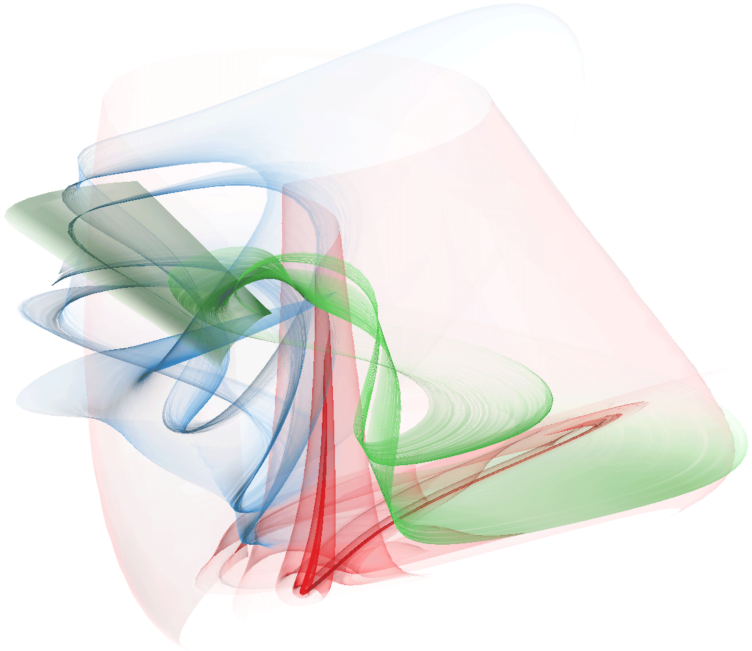
Tesseract for one initial condition depicting the chaotic behavior of the attractor:



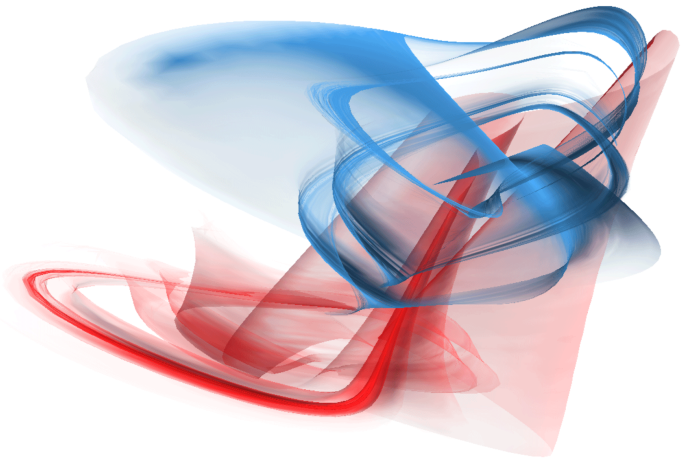
Tesseract for three initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:



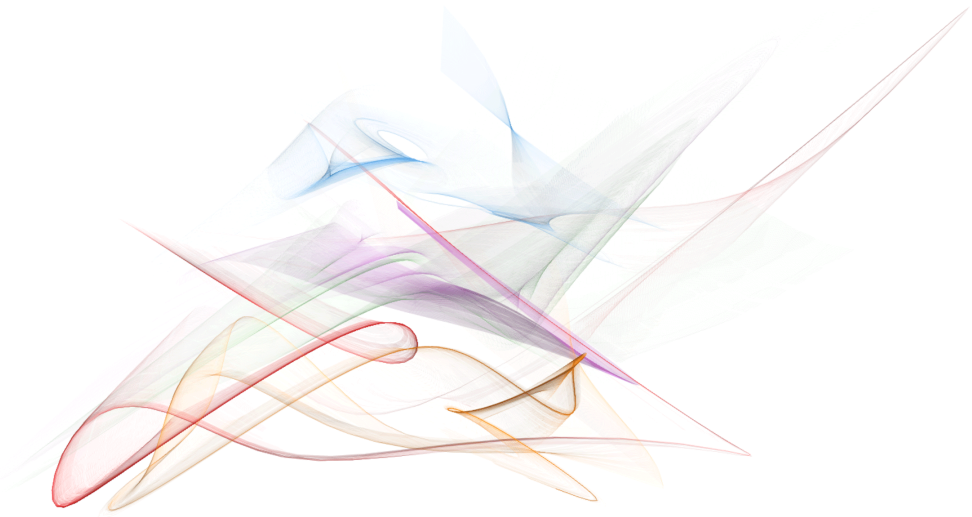
Tesseract for three initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:



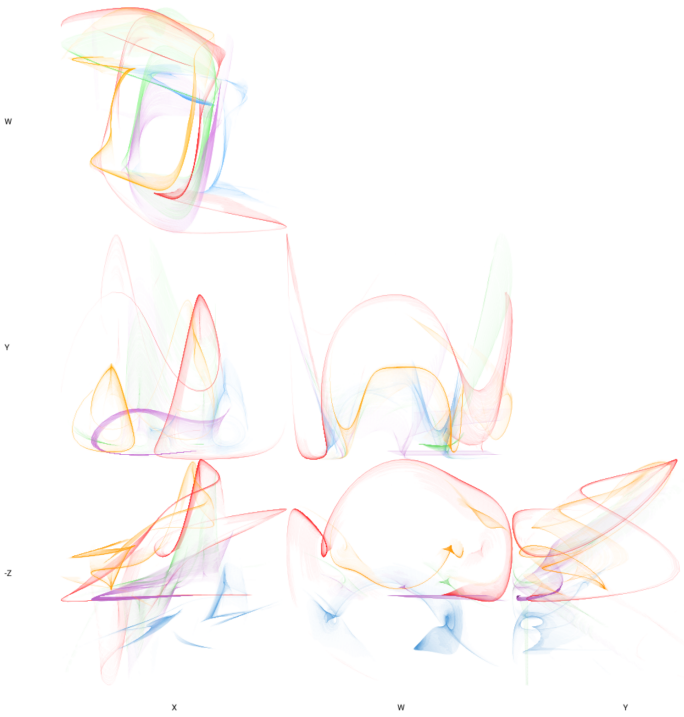
Tesseract for two initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:



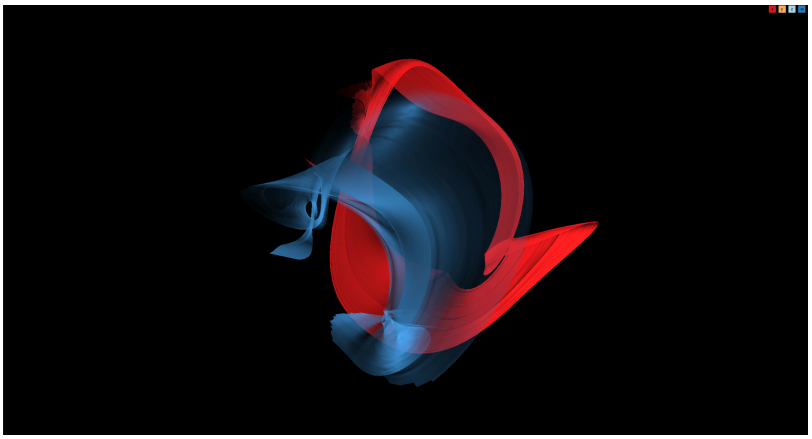
Tesseract for five initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:



2D unfolding of the tesseract of the previous figure with the same five initial conditions:

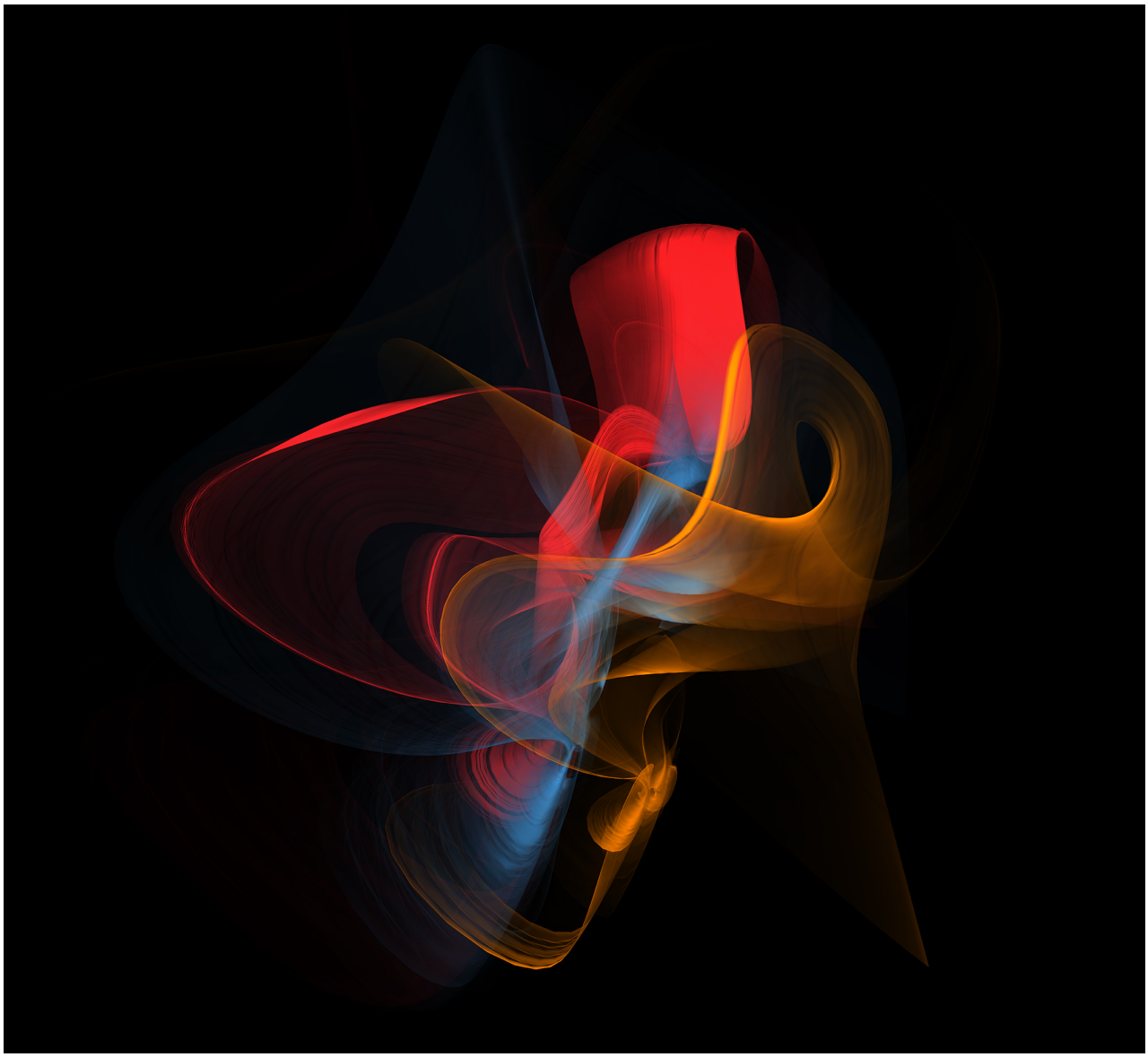


Tesseract for two initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:

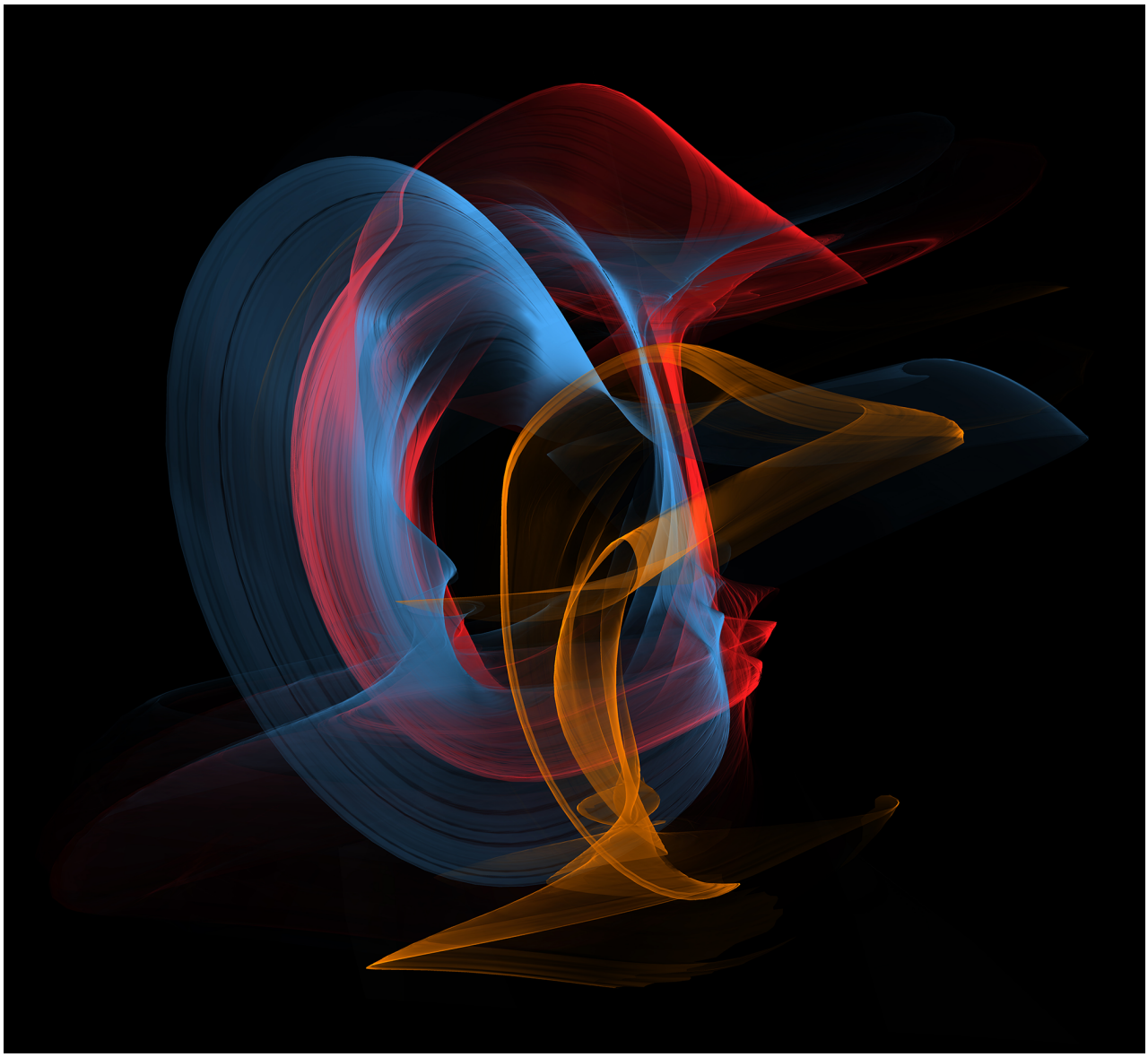


Tesseract for three initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:

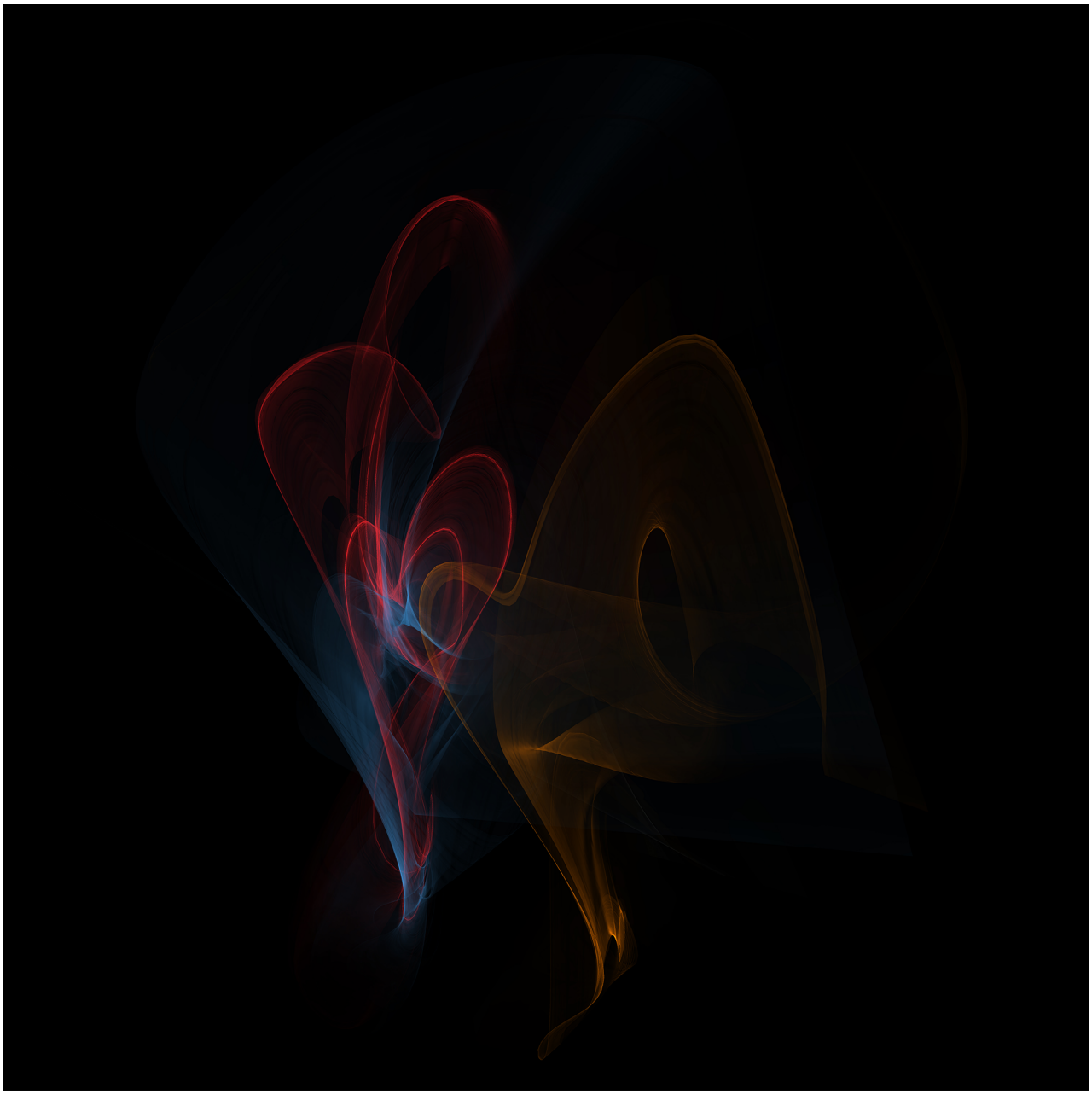
Tesseract for three initial conditions depicting the chaotic behavior and sensitivity of the attractor to initializations:



Tesseract for three initial conditions (same system as previous, but after 4D rotation):



Tesseract for three initial conditions (same system as previous, but with different parameterizations of the smoke surface):



References

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[Gol11] GOLDBETER A.: A model for the dynamics of bipolar disorders. *Progress in Biophysics and Molecular Biology 105*, 1 (2011), 119–127. BrainModes: The role of neuronal oscillations in health and disease. URL: https://www.sciencedirect.com/science/article/pii/S0079610710001070, doi:https://doi.org/10.1016/j.pbiomolbio.2010.11.007. 1

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